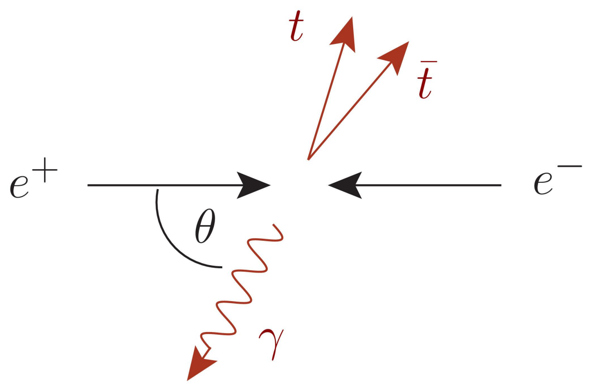
Top Quark Mass Determination at Future Lepton Colliders using Radiative Events

arXiv:1912.01275



M. Boronat, E. Fullana, J. Fuster, P. Gomis, A.H. Hoang, V. Mateu,
M. Vos, Angelika Widl (University of Vienna)

13th International Workshop on Top-Quark Physics
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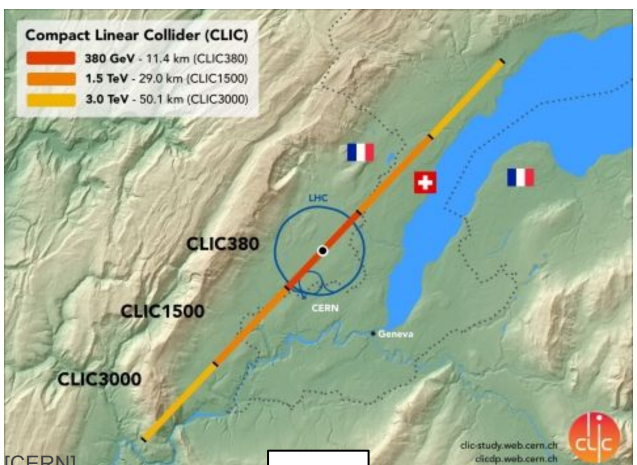
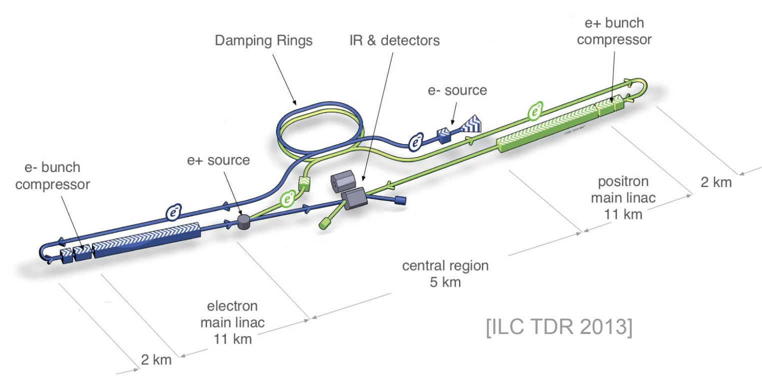
Top Quark Mass Measurements at Lepton Colliders
(main methods)

- threshold scan $\sqrt{s} \sim 350\ \mathrm{GeV}$ $\sigma(e^+e^- \to t\bar{t})$ < 75 MeV precision [Simon 2019]
- radiative events $\sqrt{s} = 380, 500\ \mathrm{GeV}$ $\sigma(e^+e^- \to t\bar{t}\gamma)$ ~ 110 - 150 MeV precision [Boronat et al. 2019]
- direct reconstruction $\sqrt{s} = 380, 500\ \mathrm{GeV}$ invariant mass ~ 50 - 100 MeV precision

[Abramowicz et al. 2019], [Seidel et al. 2013]



CLIC



[ILC TDR 2013]

ILC

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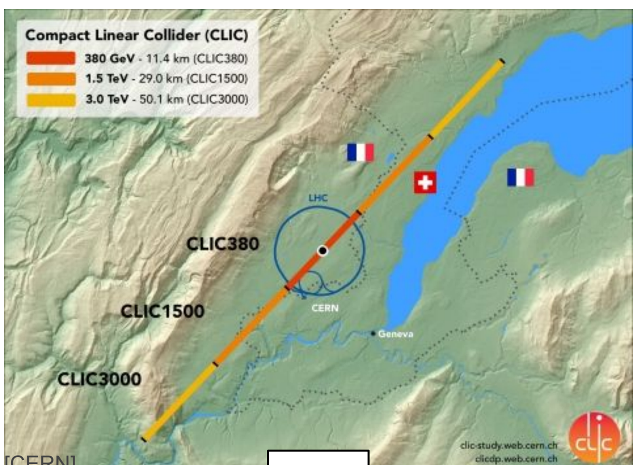
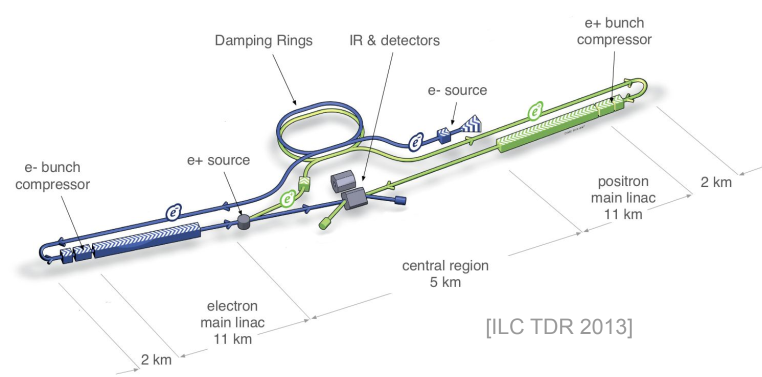
this talk:
1) mass measurement
2) test of top quark mass running



[CERN]

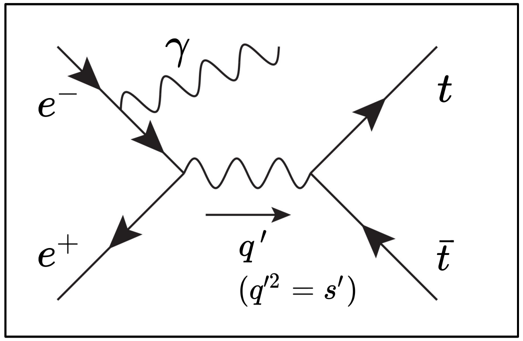
CLIC



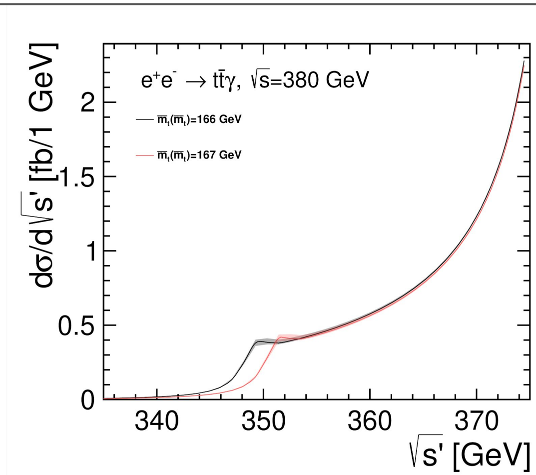
[ILC TDR 2013]

ILC

Mass Measurement



$$(q')^2 = s' = s\left(1 - \frac{2E_\gamma}{\sqrt{s}}\right)$$



[Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW 2019]

mass sensitivity of $\sigma_{t\bar{t}\gamma}$

→ initial state radiation (ISR) photon reduces the invariant mass of the top quark pair

→ high mass sensitivity for $s' \sim 4m_t^2$ (radiative return to the threshold)

→ makes high precision mass measurements above the top pair production threshold ($s > 4m_t^2$) possible

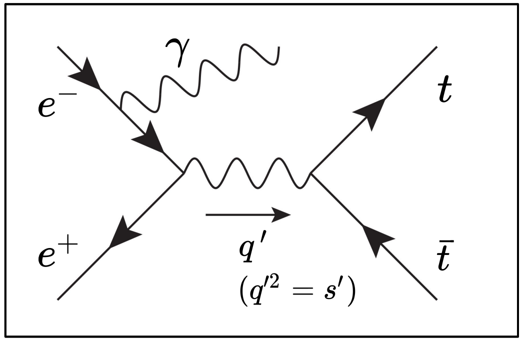
→ cross section factorizes (in the ISR approximation):

$$\frac{d\sigma_{t\bar{t}\gamma}}{d\sqrt{s'}} = f(E_\gamma)\,\sigma_{t\bar{t}}(s')$$

→ method only needs an identified top quark pair and an exact measurement of the photon energy E_γ

→ the theory cross section $\sigma_{t\bar{t}}(s')$ uses a matched cross section including QCD threshold effects up to NNLL and continuum effects up to NNNLO [Dehnadi, Hoang, Mateu, Stahlhofen, AW - in preparation]

Mass Measurement



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cms energy	CLIC, $\sqrt{s} = 380\,\mathrm{GeV}$		ILC, $\sqrt{s} = 500\,\mathrm{GeV}$	
luminosity $[\mathrm{fb}^{-1}]$	500	1000	500	4000
statistical	140 MeV	90 MeV	350 MeV	110 MeV
theory	46 MeV		55 MeV	
lum. spectrum	20 MeV		20 MeV	
photon response	16 MeV		85 MeV	
total	150 MeV	110 MeV	360 MeV	150 MeV

expected uncertainties of the $\overline{\mathrm{MS}}$ top quark mass

- 50 % selection efficiency
- $E_\gamma > 5$ GeV
- photon polar angle between 8° and 172°

- theoretical error from variation of renormalization scales

Top Quark Mass Running

- for precision measurements, the scale μ of the running coupling constant $\alpha_s^{(nf)}(\mu)$ is set to the relevant dynamical scale of the process and the flavour scheme n_f is chosen according to this scale
- similarly, the correct renormalization scale and scheme have to be chosen for the top mass: we use a running top quark mass to avoid the renormalon of the pole mass
- the relevant mass scale for $\sigma_{t\bar{t}}(s')$ is the non-relativistic 3-momentum of the top quark in the CM frame of the top quark pair:

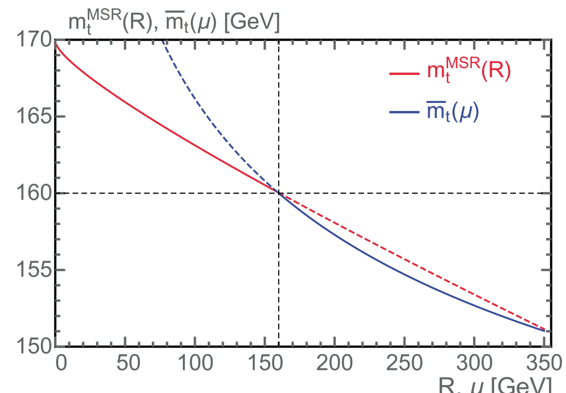
$$\boxed{\mathrm{p}_t = m_t v \leq m_t}$$

$$v = \sqrt{(\sqrt{s'} - 2m_t)/m_t}$$

- but: the $\overline{\text{MS}}$ mass only works well for mass scales $\mu \geq m_t$

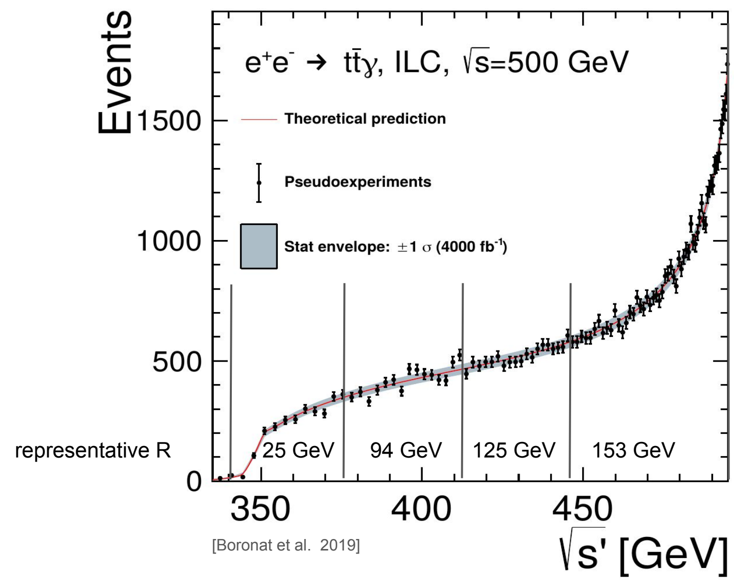
Top Quark Mass Running

- solution: we use the running MSR mass $m_t^{\mathrm{MSR}}(R)$ as a natural extension of the $\overline{\mathrm{MS}}$ mass for scales below the top mass:



Hoang et. al 2018 [arXiv:1704.01580]

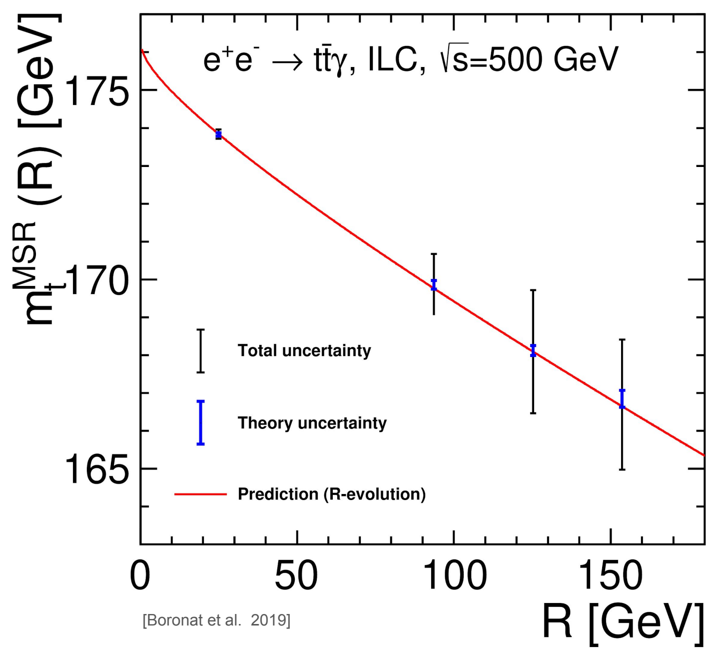
- to examine the running of the top quark mass, we extracted the MSR mass from 4 bins for the ILC run at 500 GeV, and determined the MSR mass in each bin:



[Boronat et al. 2019]

Top Quark Mass Running

- radiative events make the extraction of the MSR mass $m_t^{\mathrm{MSR}}(R)$ at different values of the invariant mass of the top pair, and therefore at different scales *R*, possible
- this provides a consistency check of QCD with a running MSR mass
- ILC at 500 GeV can test the running of the MSR mass with over 5σ significance



[Boronat et al. 2019]

Conclusions

- Radiative events allow for high precision measurements of the top quark mass above the top pair production threshold with a precision of:

  $\Delta m_t = 110\ \mathrm{MeV}$ for CLIC at 380 GeV
  $\Delta m_t = 150\ \mathrm{MeV}$ for ILC at 500 GeV

- The running of the MSR mass can be checked with over 5σ significance using radiative events.

Conclusions

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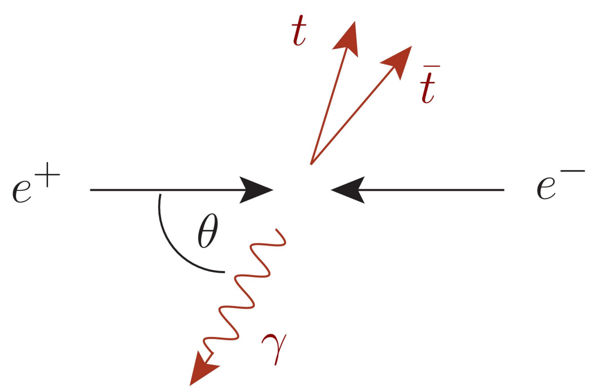


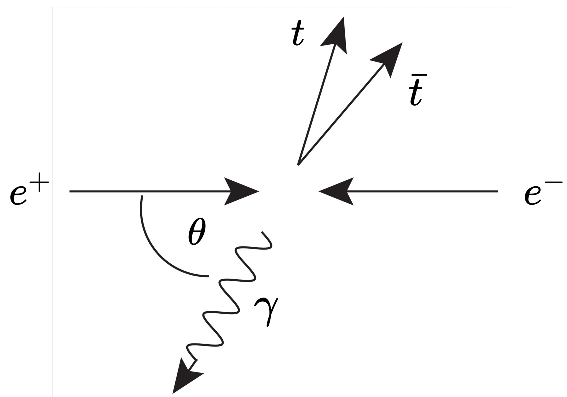
Thank you for your attention!

Backup

Backup: Factorization Formula

$$\frac{d\sigma_{t\bar{t}\gamma}}{d\sqrt{s'}} = f(E_\gamma)\,\sigma_{t\bar{t}}(s')$$

$$\frac{\mathrm{d}\sigma_{t\bar{t}\gamma}}{\mathrm{d}\cos\theta\,\mathrm{d}\sqrt{s'}} = \frac{\alpha_{\mathrm{em}}}{\pi\sqrt{s}}\,g(x,\theta)\,\sigma_{t\bar{t}}(s') + \mathcal{O}(\alpha_{\mathrm{em}}^2)\,, \quad g(x,\theta) = \frac{2\sqrt{(1-2x)}}{x\sin^2\theta}\left[1-2x+(1+\cos^2\theta)x^2\right], \quad x = \frac{E_\gamma}{\sqrt{s}}$$

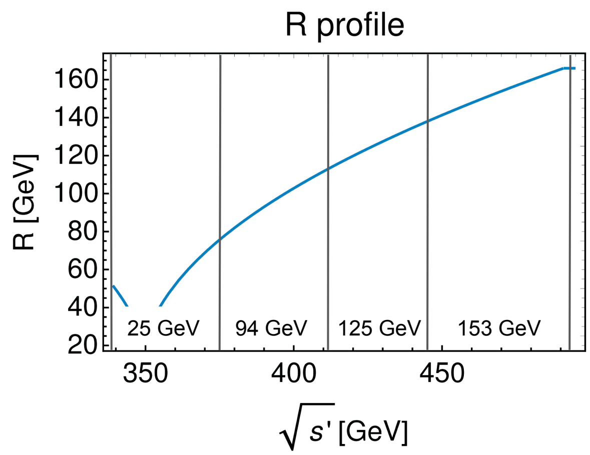


- large photon energy $E_\gamma > 5$ GeV
- θ integrated from 8° to 172°
- highest mass sensitivity for collinear top quarks
  - $s' \sim 4\,m_t^2$
  - radiative return to threshold

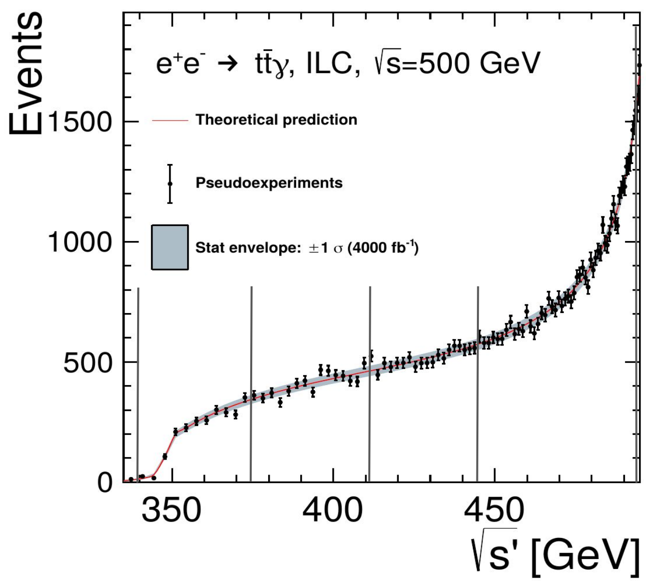
Backup: Mass Scale R

- the relevant mass scale for the cross section is the 3-momentum of the top quark in the CM frame of the top quark pair:

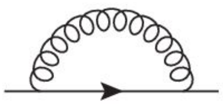
$$\boxed{R \sim p_t}$$



representative R



Backup: Mass Schemes - $\overline{\text{MS}}$ Mass

Full propagator: $$S_F = \frac{i}{\not{p} - \overline{m}(\mu) + \Sigma_{\text{finite}}(\not{p}, \overline{m}(\mu))}\,, \qquad \Sigma(\not{p}, m_0) =$$  $+ \ldots$

Conversion:

$$m_{pole} = \overline{m} + \overline{m}\sum_{n=1}^{\infty} a_n(n_l, n_h)\,\alpha_s(\overline{m})^n$$

$$= \overline{m} + \underbrace{\overline{m}\,\alpha_s\,a_1}_{\sim\, mv} + \ldots \qquad \left(\,\overline{m} = \overline{m}^{(n_l+1)}(\overline{m}^{(n_l+1)})\,\right)$$

➞ works only in the continuum

Breaking of non-relativistic power counting in the $\overline{\text{MS}}$ scheme:

$$v_{\text{pole}} = \sqrt{\frac{\sqrt{s} - 2\,m_{\text{pole}}}{m_{\text{pole}}}}$$

$$= \sqrt{\frac{\sqrt{s} - 2\,(\overline{m} + \overline{m}\,a_1\,\alpha_s)}{\overline{m} + \overline{m}\,a_1\,\alpha_s}}$$

$$= v_{\overline{\text{MS}}} - a_1\left(\frac{\alpha_s}{v_{\overline{\text{MS}}}}\right)\left(1 + \frac{1}{2}v_{\overline{\text{MS}}}^2\right) + a_1^2\left(\frac{\alpha_s^2}{v_{\overline{\text{MS}}}^3}\right)\left(-\frac{1}{2} + \frac{1}{2}v_{\overline{\text{MS}}}^2 + \frac{3}{8}\,v_{\overline{\text{MS}}}^4\right) + \mathcal{O}(\alpha_s^3)$$

At threshold: $\sim \alpha_s$ $\quad\sim \alpha_s^0$ $\quad\sim \alpha_s^{-1}$

Backup: Mass Schemes - MSR Mass

[Hoang, Jain, Scimemi, Stewart 2008], [Hoang, Jain, Lepenik, Mateu, Preisser, Scimemi, Stewart 2017]

Conversion: $$m_{pole} = \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n \, \alpha_s(\overline{m})^n = \overline{m} + \overline{m} \, \alpha_s \, a_1 + \ldots$$

$$m_{pole} = m_{MSR}(R) + R \sum_{n=1}^{\infty} a_n \, \alpha_s(R)^n = m_{MSR}(R) + R \, \alpha_s \, a_1 + \ldots$$

→ no breaking of the non-relativistic power counting at threshold

$$\begin{aligned} v_{\text{pole}} &= \sqrt{\frac{\sqrt{s} - 2\, m_{\text{pole}}}{m_{\text{pole}}}} \\ &= \sqrt{\frac{\sqrt{s} - 2\,(m_{\text{MSR}} + R\, a_1\, \alpha_s)}{m_{\text{MSR}} + R\, a_1\, \alpha_s}} \\ &= \underbrace{v_{\text{MSR}}}_{\sim \alpha_s} - a_1 \underbrace{\alpha_s}_{\sim \alpha_s} \left(\frac{R}{m_{\text{MSR}} v_{\text{MSR}}}\right)\left(1 + \frac{v_{\text{MSR}}^2}{2}\right) + a_1^2 \underbrace{\frac{\alpha_s^2}{v_{\text{MSR}}}}_{\sim \alpha_s} \left(\frac{R}{m_{\text{MSR}} v_{\text{MSR}}}\right)^2 \left(-\frac{1}{2} + \frac{1}{2} v_{\text{MSR}}^2 + \frac{3}{8} v_{\text{MSR}}^4\right) + \mathcal{O}(\alpha_s^3) \end{aligned}$$

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$$v_{\rm pole} = \sqrt{\frac{\sqrt{s} - 2\, m_{\rm pole}}{m_{\rm pole}}}$$
$$= \sqrt{\frac{\sqrt{s} - 2\,(m_{\rm MSR} + R\, a_1\, \alpha_s)}{m_{\rm MSR} + R\, a_1\, \alpha_s}}$$
$$= v_{\rm MSR} - a_1\, \alpha_s \left(\frac{R}{m_{\rm MSR} v_{\rm MSR}}\right)\left(1 + \frac{v_{\rm MSR}^2}{2}\right) + a_1^2 \, \frac{\alpha_s^2}{v_{\rm MSR}} \left(\frac{R}{m_{\rm MSR} v_{\rm MSR}}\right)^2 \left(-\frac{1}{2} + \frac{1}{2} v_{\rm MSR}^2 + \frac{3}{8} v_{\rm MSR}^4\right) + \mathcal{O}(\alpha_s^3)$$

→ no power counting breaking for $R \sim mv$

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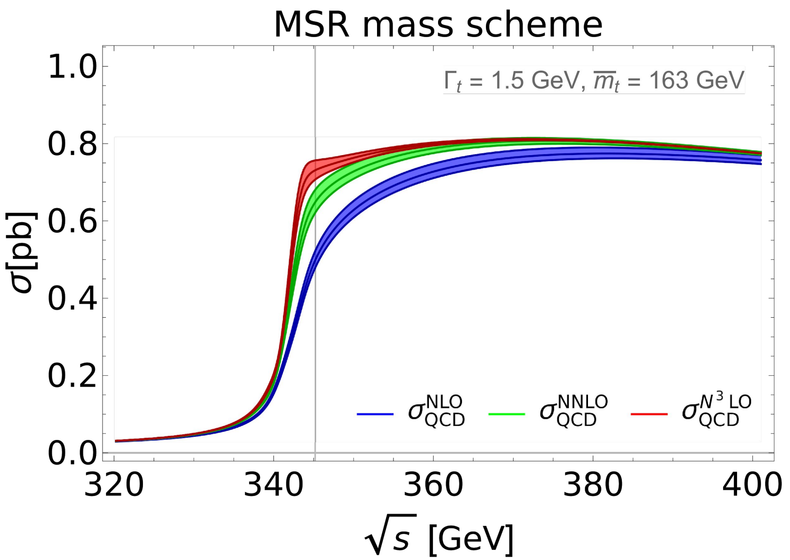
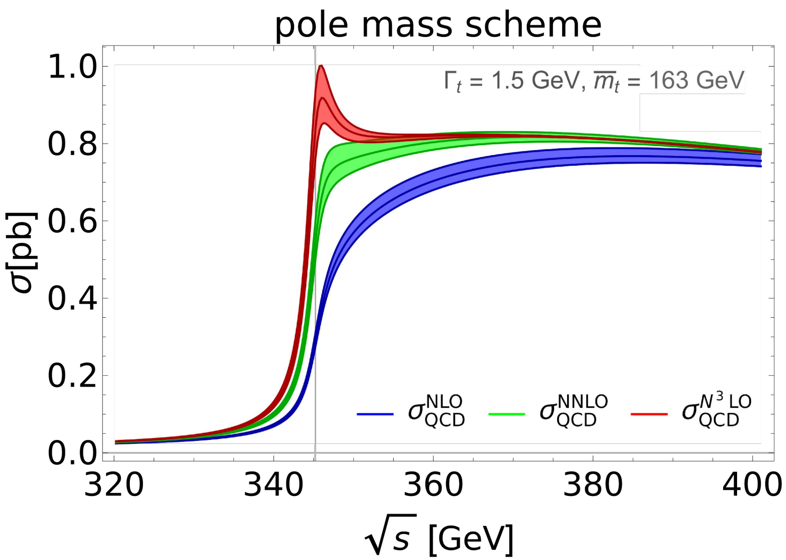
→ no breaking of the non-relativistic power counting at threshold
→ improves convergence of the continuum cross section in the intermediate region:





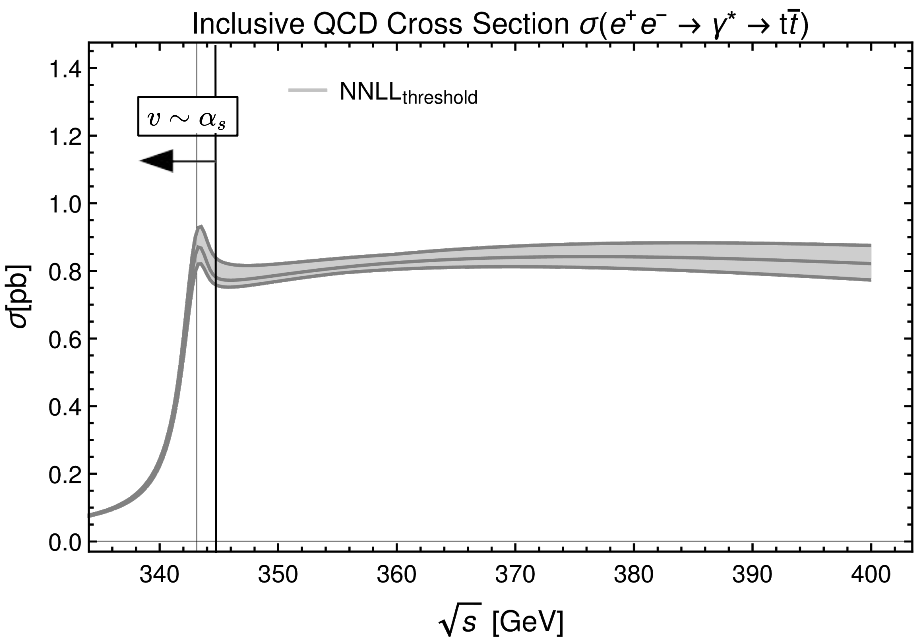
Backup: $\sigma_{t\bar{t}}$ - Theory Overview

<u>non-relativistic QCD</u>
(NRQCD)

NNLL$_{\text{threshold}}$
[Hoang, Stahlhofen 2013]
- QCD + LO electroweak (double-resonant)
- vNRQCD
- resummation of velocity-logarithms
- complete for all practical purposes
- 1S mass

NNNLO$_{\text{threshold}}$
[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015]
[Beneke, Kiyo, Maier, Piclum 2016]
- QCD + EW + Higgs
- pNRQCD
- PS mass

threshold



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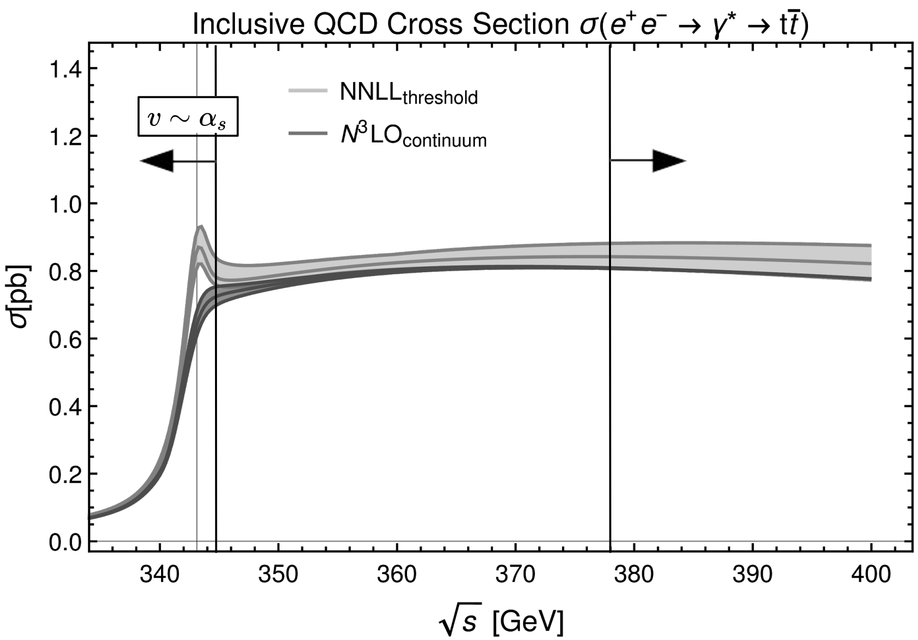
- PS mass

threshold

continuum



full QCD

NNNLO$_{\text{continuum}}$

[Hoang, Mateu, Zebarjad 2009]
[Kiyo, Maier, Maierhofer, Marquard 2009]

- QCD corrections

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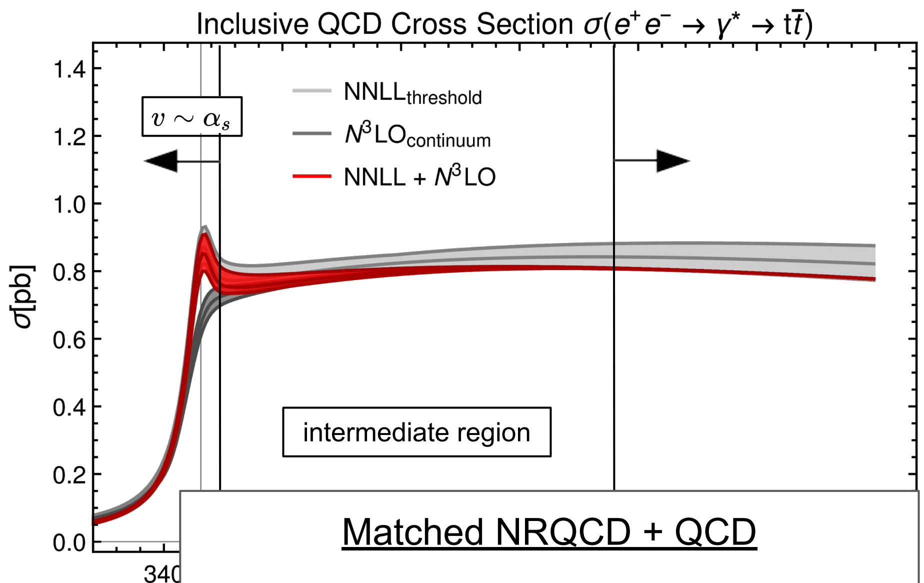

<u>Matched NRQCD + QCD</u>

LL$_{\text{threshold}}$ + NLO$_{\text{continuum}}$ (differential cross section)
[Bach et al. 2017]

NNLL$_{\text{threshold}}$ + NNNLO$_{\text{continuum}}$ this talk
- photon-induced cross section
- QCD + LO EW at threshold
- 1S + MSR mass scheme

<u>full QCD</u>

NNNLO$_{\text{continuum}}$
[Hoang, Mateu, Zebarjad 2009]
[Kiyo, Maier, Maierhofer, Marquard 2009]
- QCD corrections