Top quark mass and pair production near threshold

Li Lin Yang
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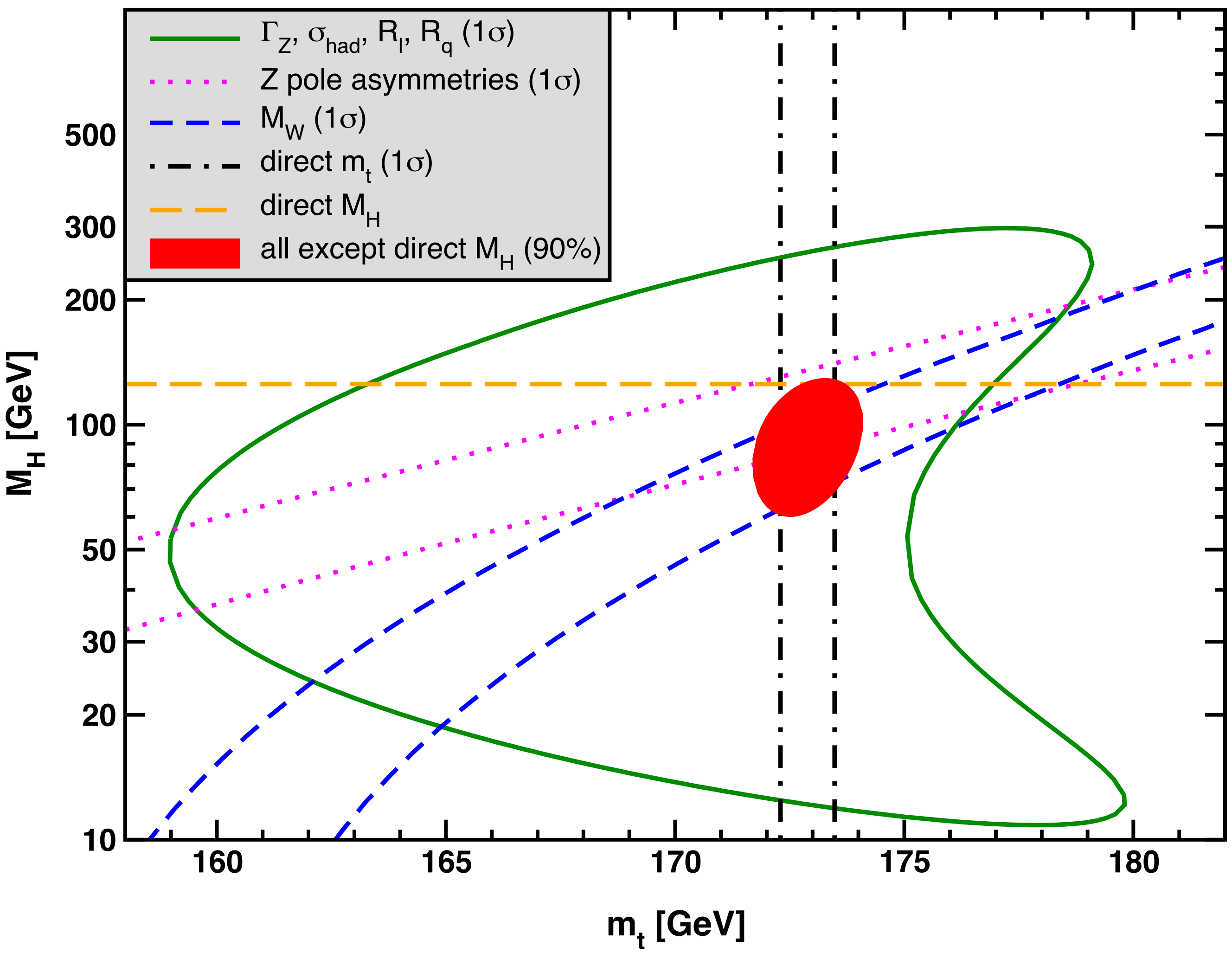
Based on Ju, Wang, Wang, Xu, Xu, **LLY**: 1908.02179, 2004.03088

13th International Workshop on Top-Quark Physics (TOP2020)

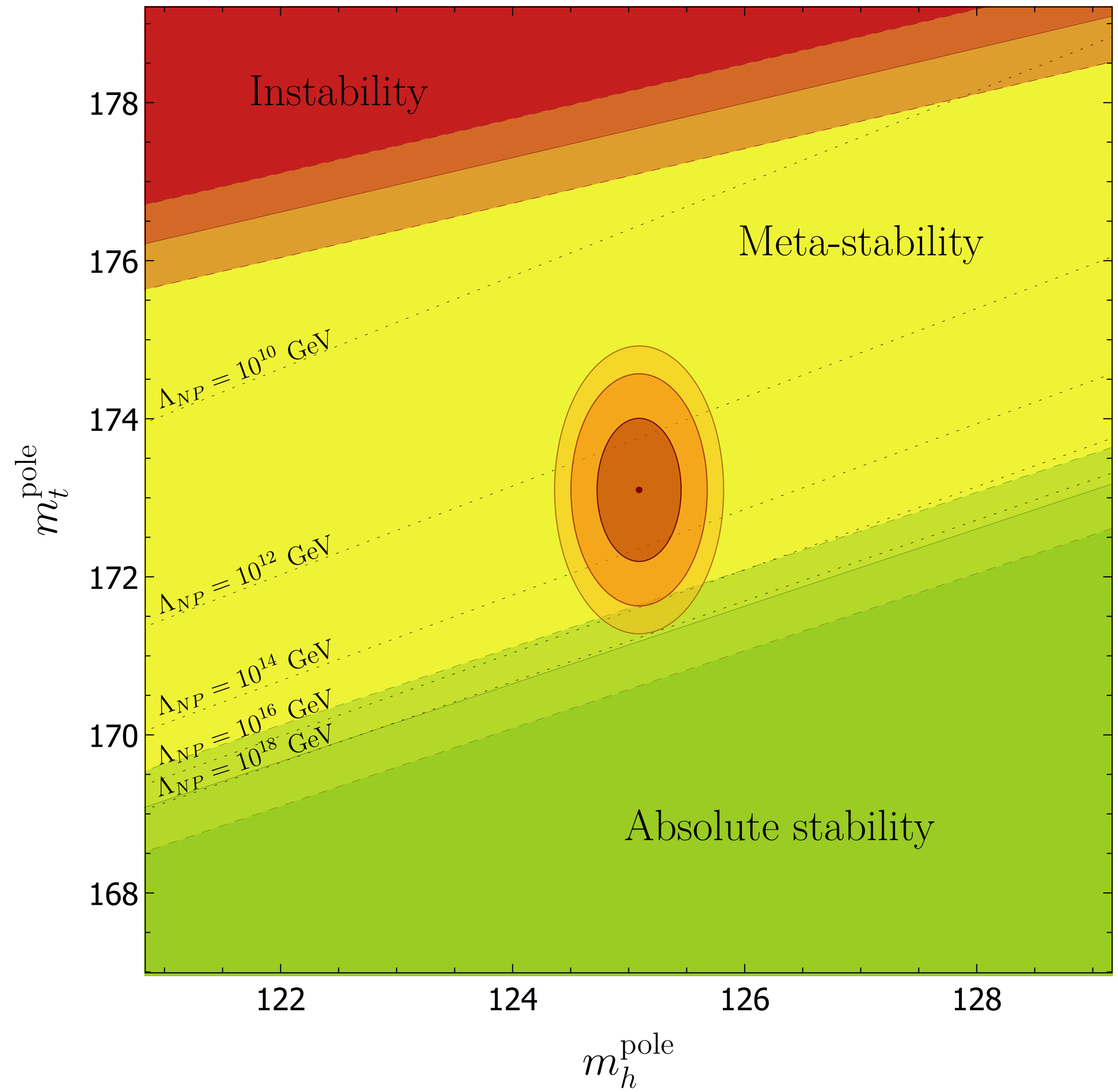
The top quark mass *a highly important parameter of the SM*

Constraints on new physics



2020 Review of Particle Physics

The fate of our universe



A. Andreassen, W. Frost, M. D. Schwartz: 1707.08124

What is the top quark mass?

- There are different definitions for a mass
- Kinematic: reconstructed object fitted to Monte Carlo event generators (so-called MC mass) Direct measurements
- Field theoretic: a (renormalized) parameter in the Lagrangian density (scheme-dependent) Indirect measurements
  - Pole (on-shell) mass
  - $\overline{\text{MS}}$ mass
  - …

t-QUARK MASS

t-Quark Mass (Direct Measurements)	172.76 ± 0.30 GeV (S = 1.2)
t-Quark Mass from Cross-Section Measurements	$162.5^{+2.1}_{-1.5}$ GeV
t-Quark Pole Mass from Cross-Section Measurements	172.4 ± 0.7 GeV

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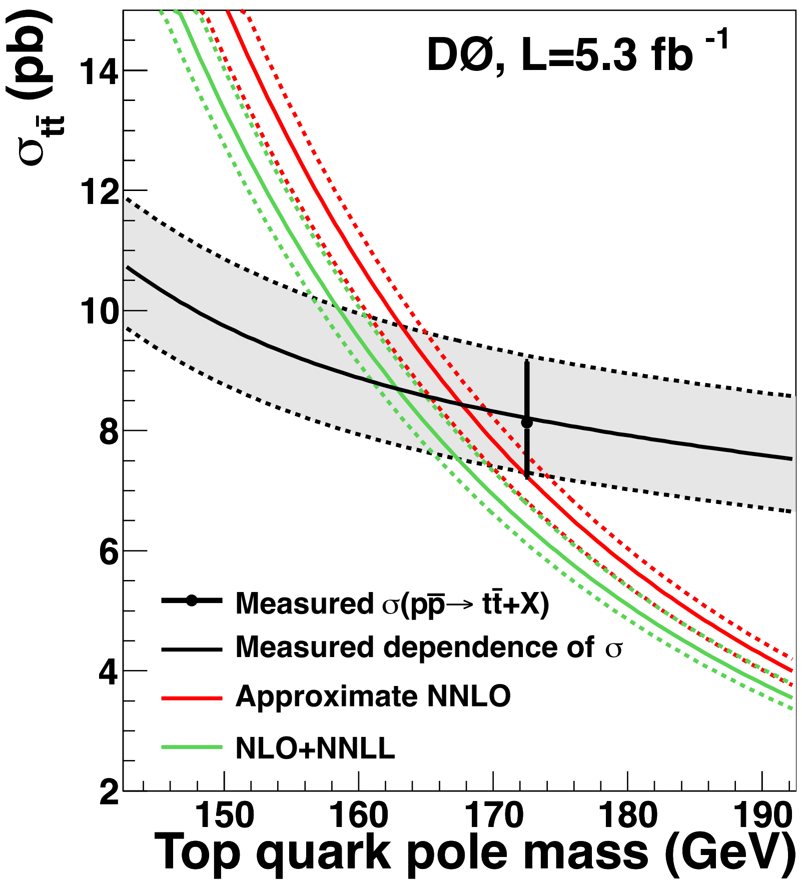
Indirect measurements of the pole mass

Extract m_t^{pole} from cross sections



D0 Collaboration: 1104.2887

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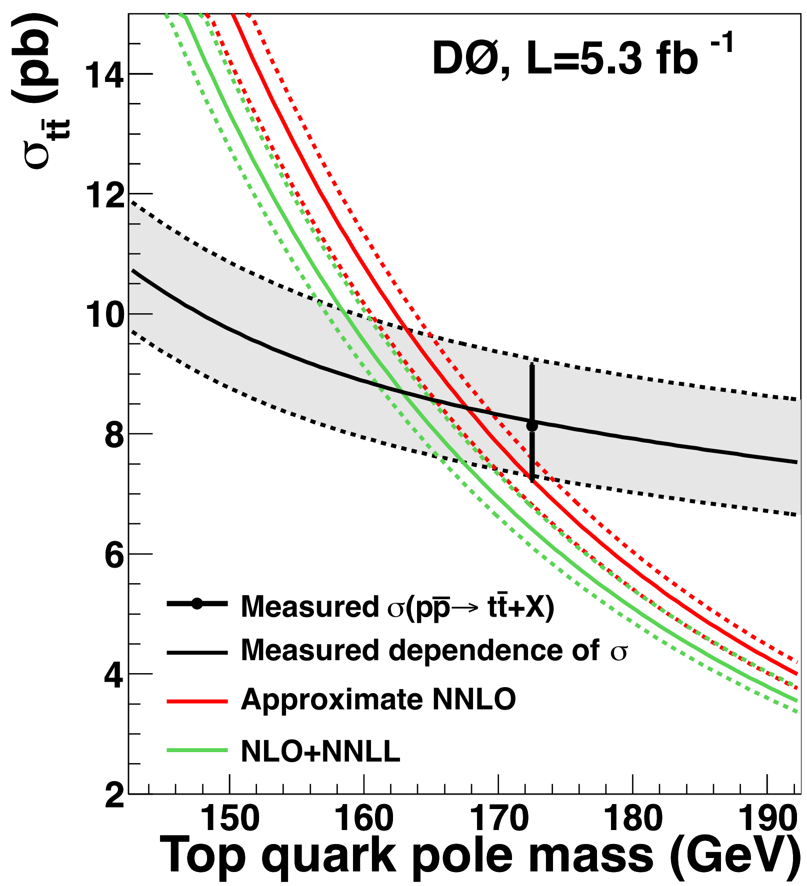

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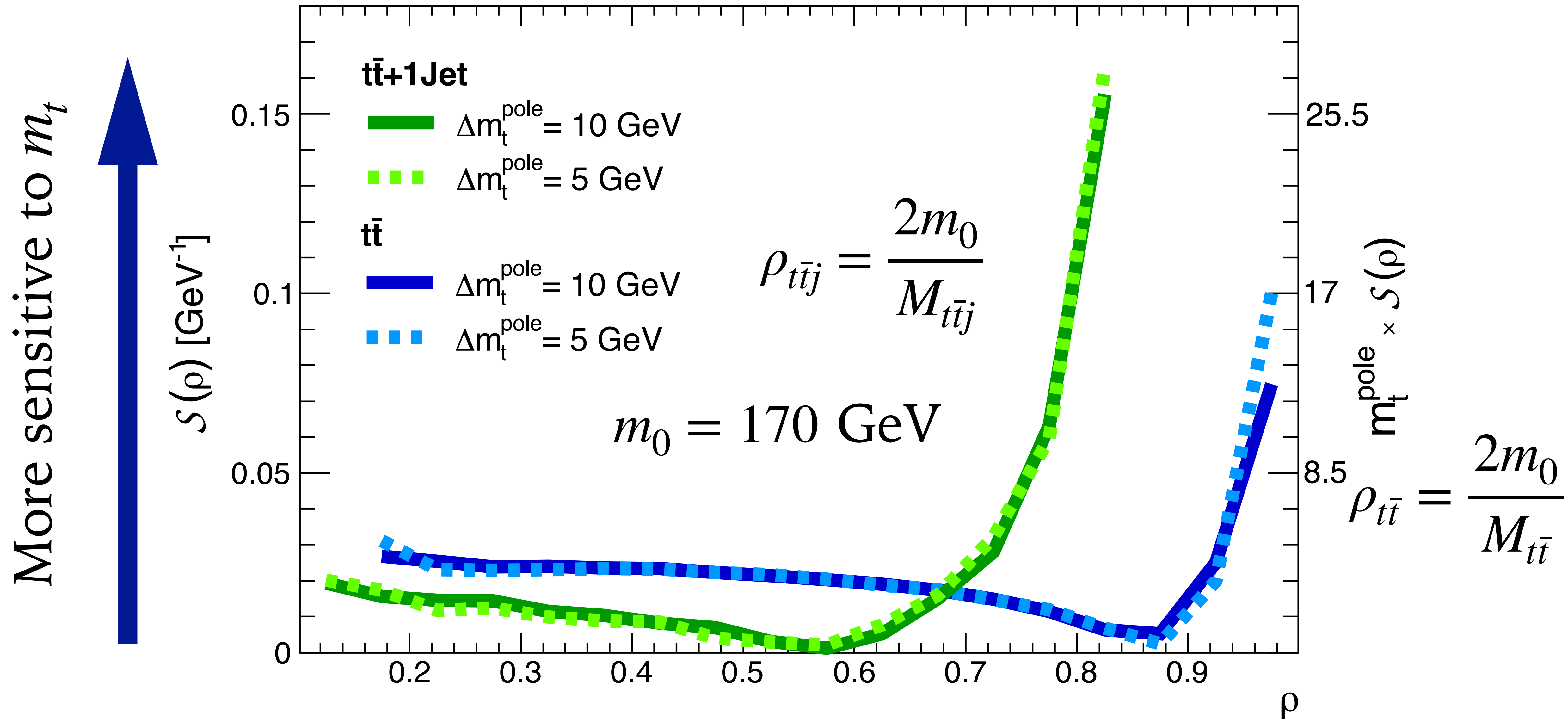
The next natural step is to use more differential observables…

…should choose observables most sensitive to m_t

Threshold region and top quark mass

Alioli et al.: 1303.6415



Threshold region and top quark mass

$M_{t\bar{t}} \to 2m_t$ in $t\bar{t}$+jets production

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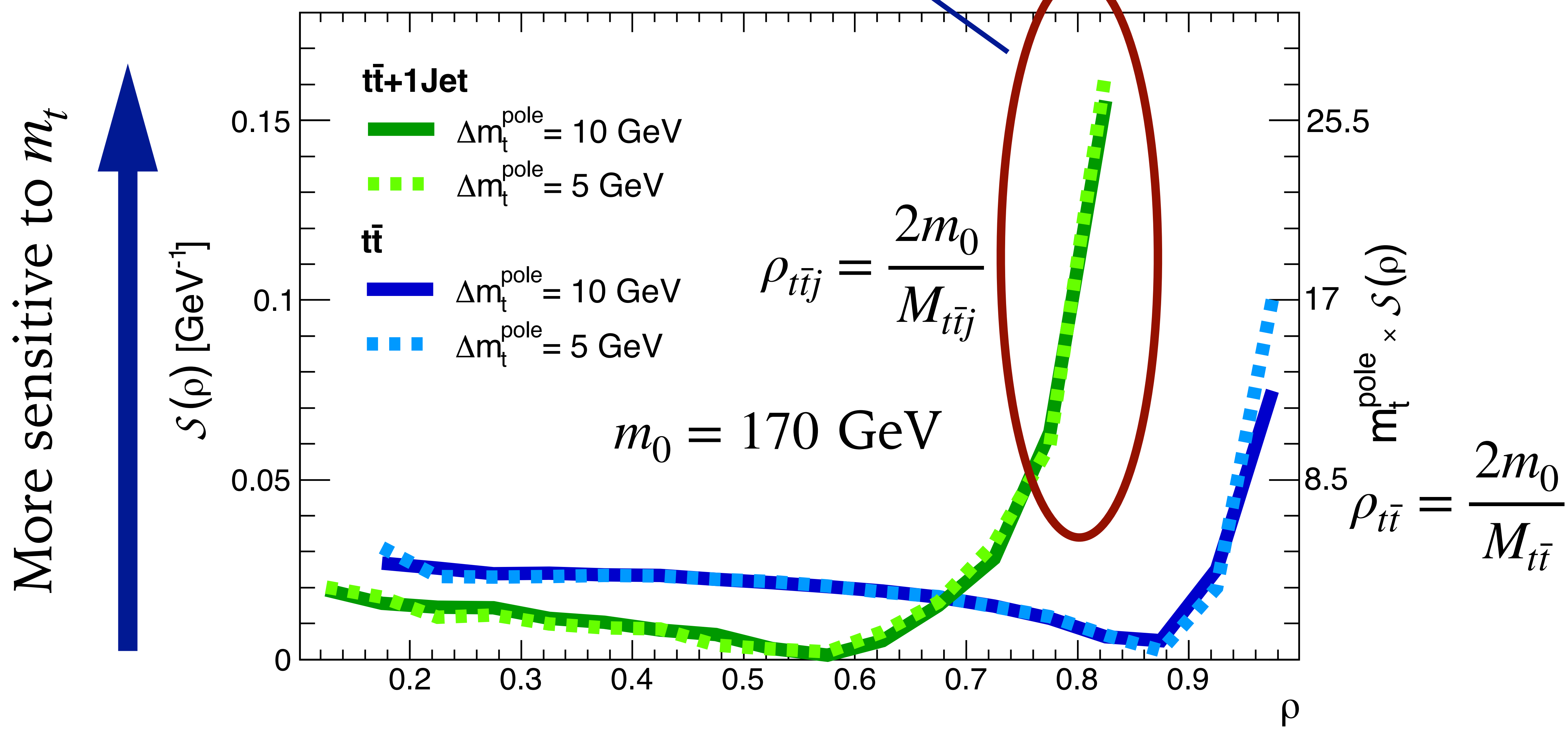

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$M_{t\bar{t}} \to 2m_t$ in $t\bar{t}$ production

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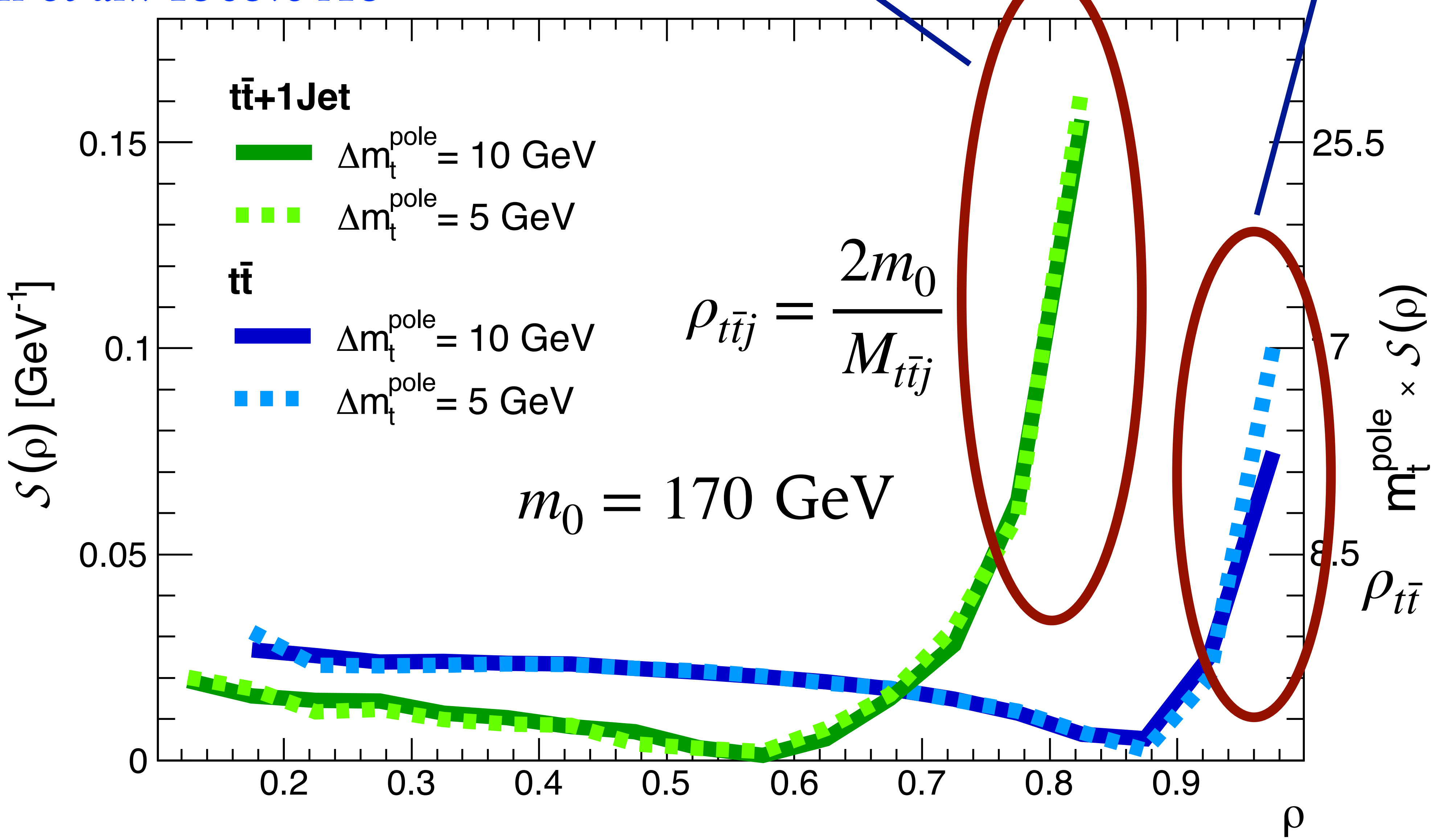
More sensitive to m_t



$$\rho_{t\bar{t}} = \frac{2m_0}{M_{t\bar{t}}}$$

Threshold region and top quark mass

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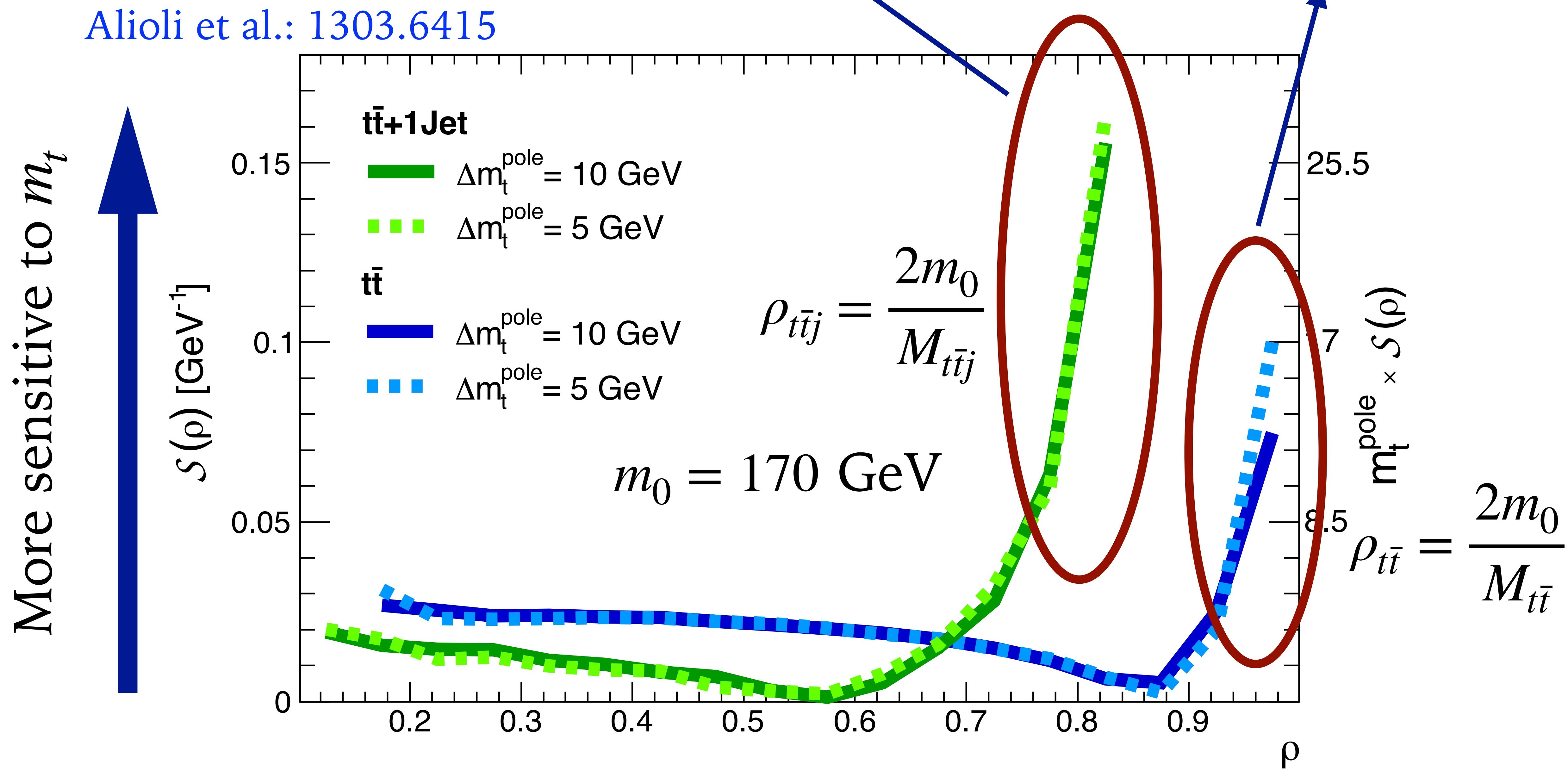
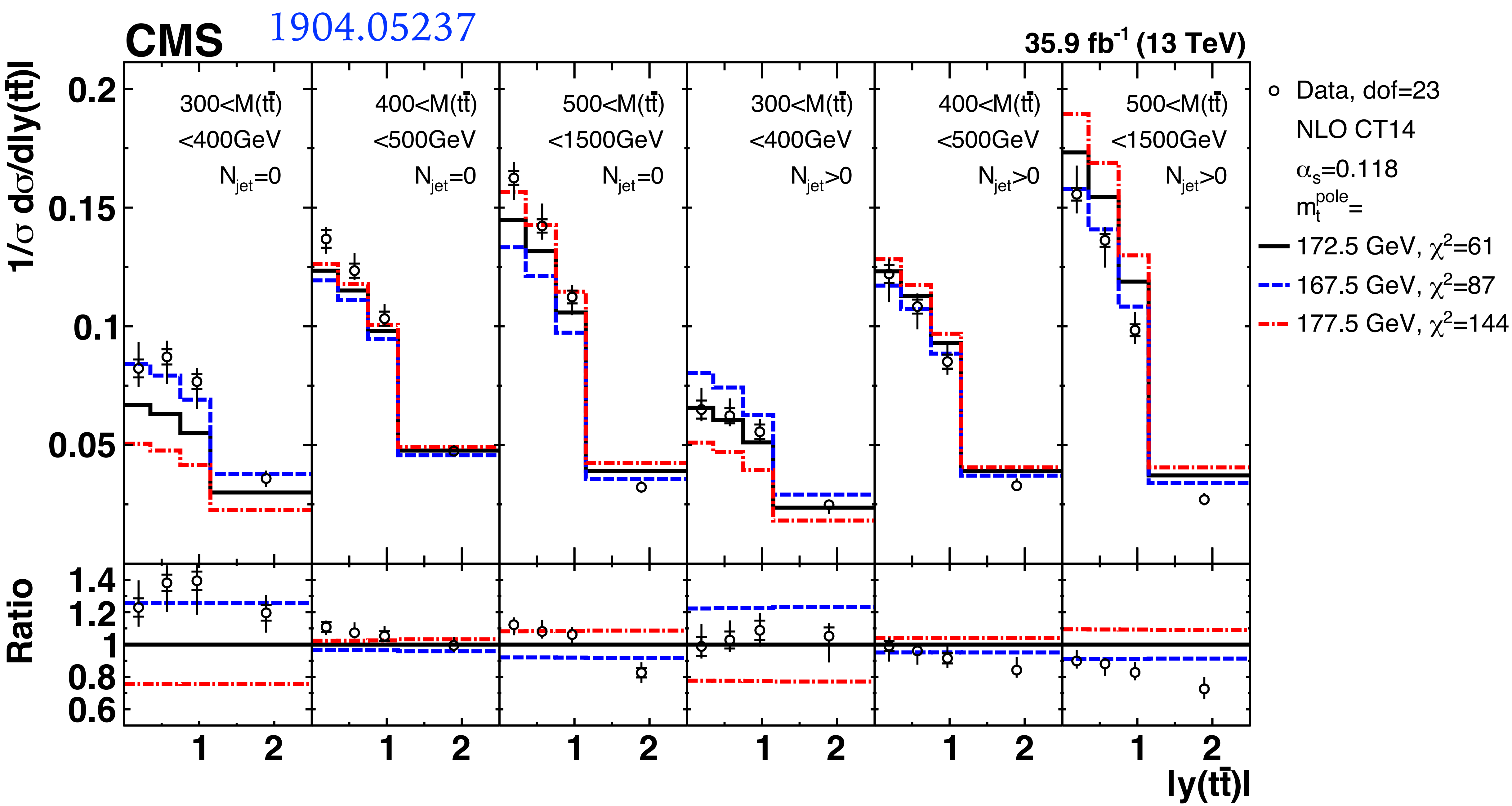


Most of the mass-sensitivity comes from the threshold region

Threshold region and top quark mass



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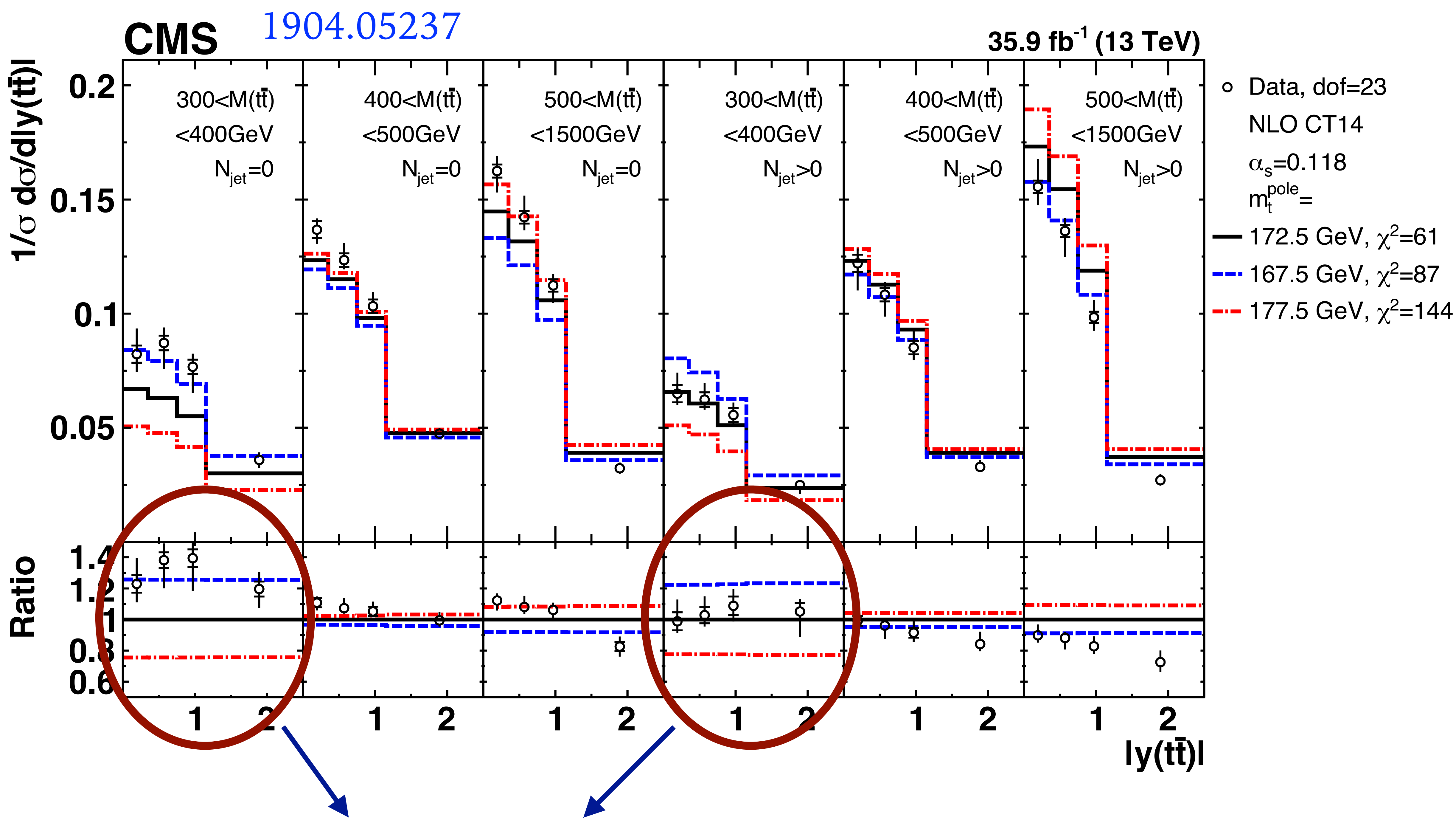
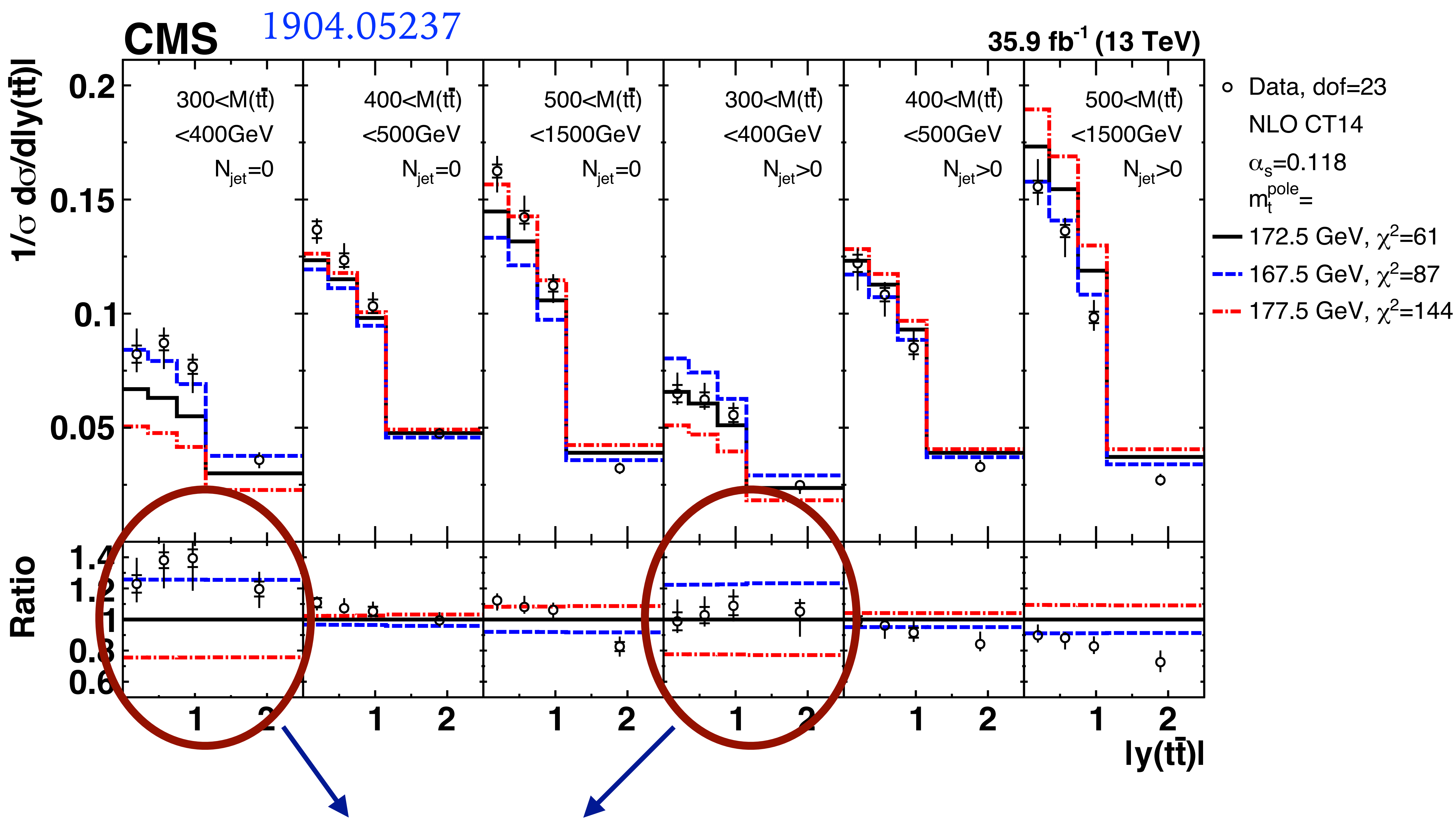


Most of the mass-sensitivity comes from the low $M_{t\bar{t}}$ region

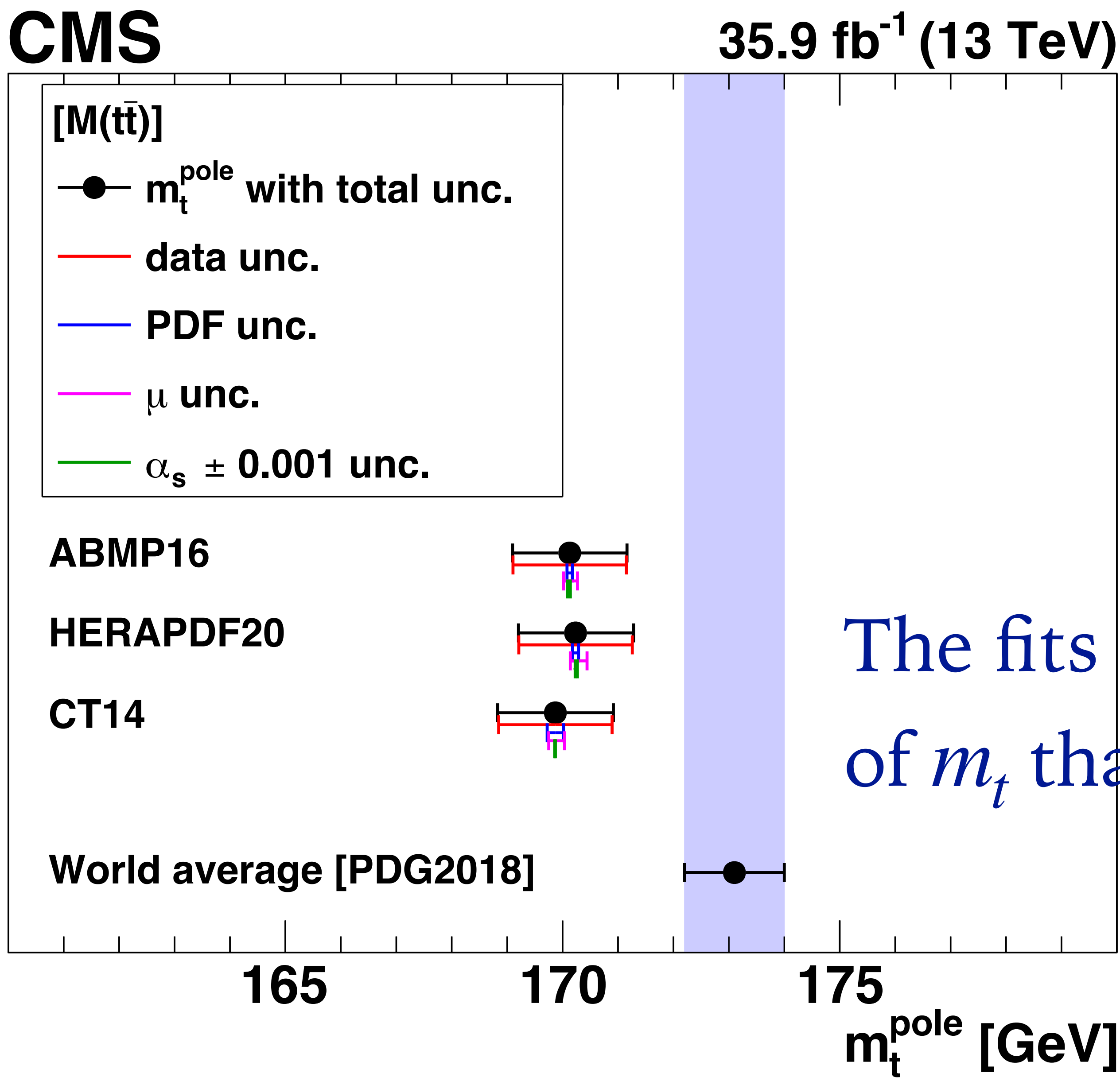
Threshold region and top quark mass



Most of the mass-sensitivity comes from the low $M_{t\bar{t}}$ region

My focus in the rest of the talk

Threshold region and top quark mass

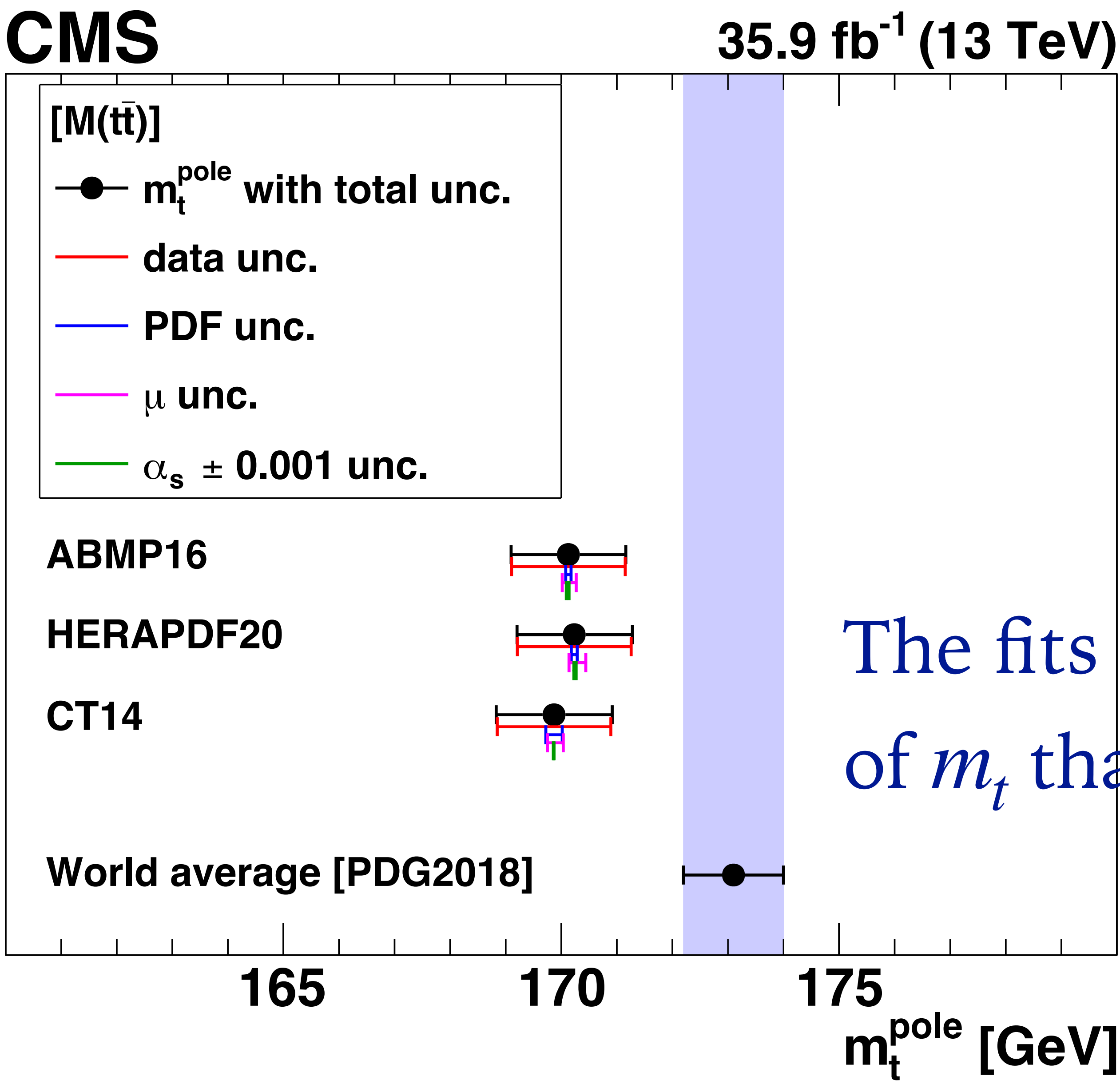


ATLAS collaboration: 1905.02302

CMS collaboration: 1904.05237

The fits favor much lower values of m_t than the world average!

Threshold region and top quark mass



ATLAS collaboration: 1905.02302

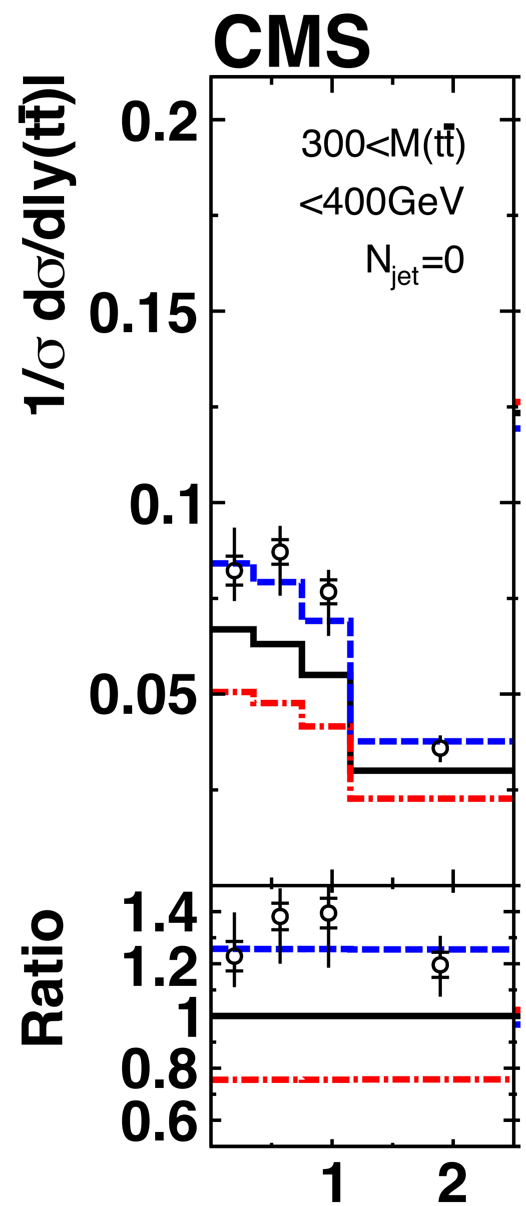
CMS collaboration: 1904.05237

The fits favor much lower values of m_t than the world average!

The difference is much larger than the estimated uncertainties…

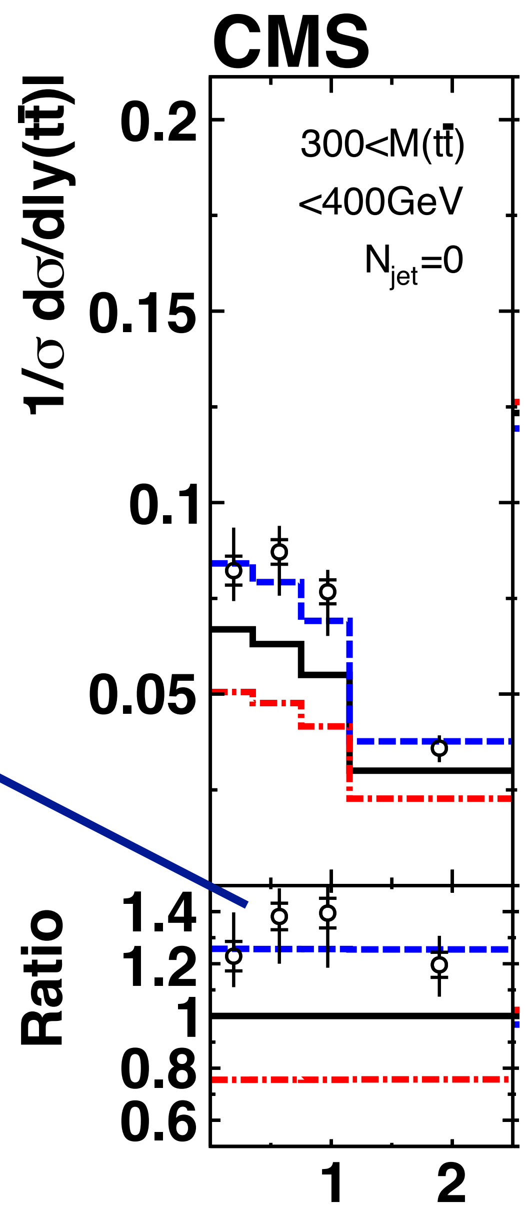
What could possibly be the reason?

The method of indirect mass measurements relies heavily on **both** the experimental side **and** the theoretical side



The method of indirect mass measurements relies heavily on **both** the experimental side **and** the theoretical side

The experimentalists need to measure the mass-sensitive observables to high precisions



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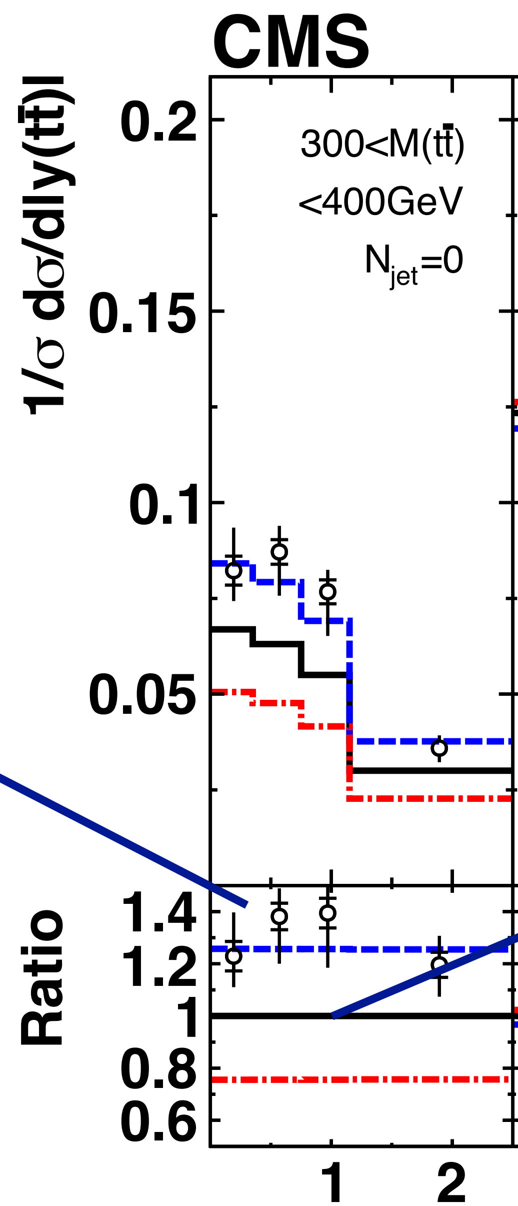

The theorists need to provide high-precision predictions for these observables

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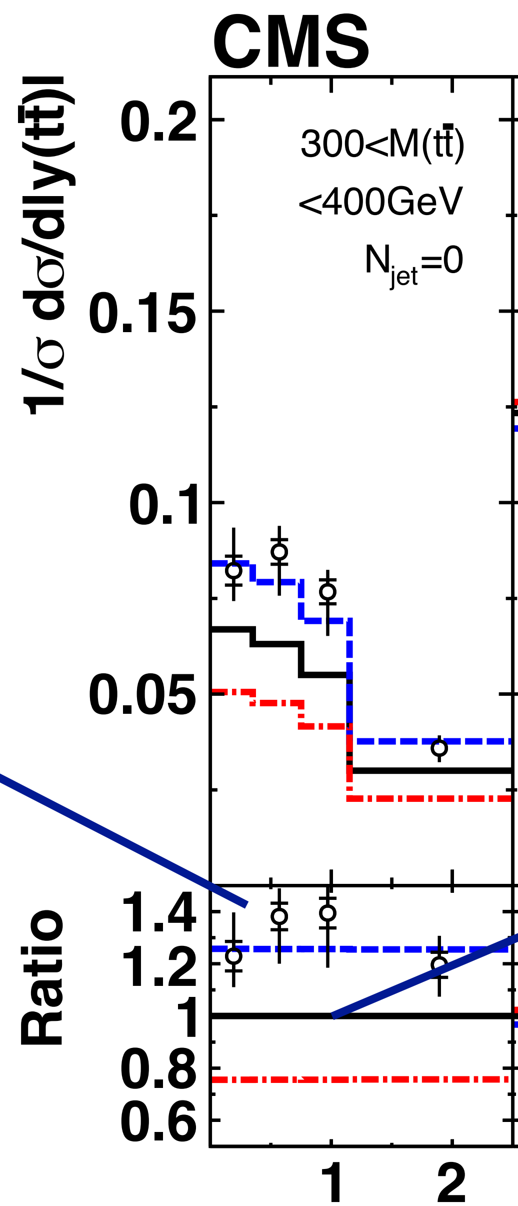


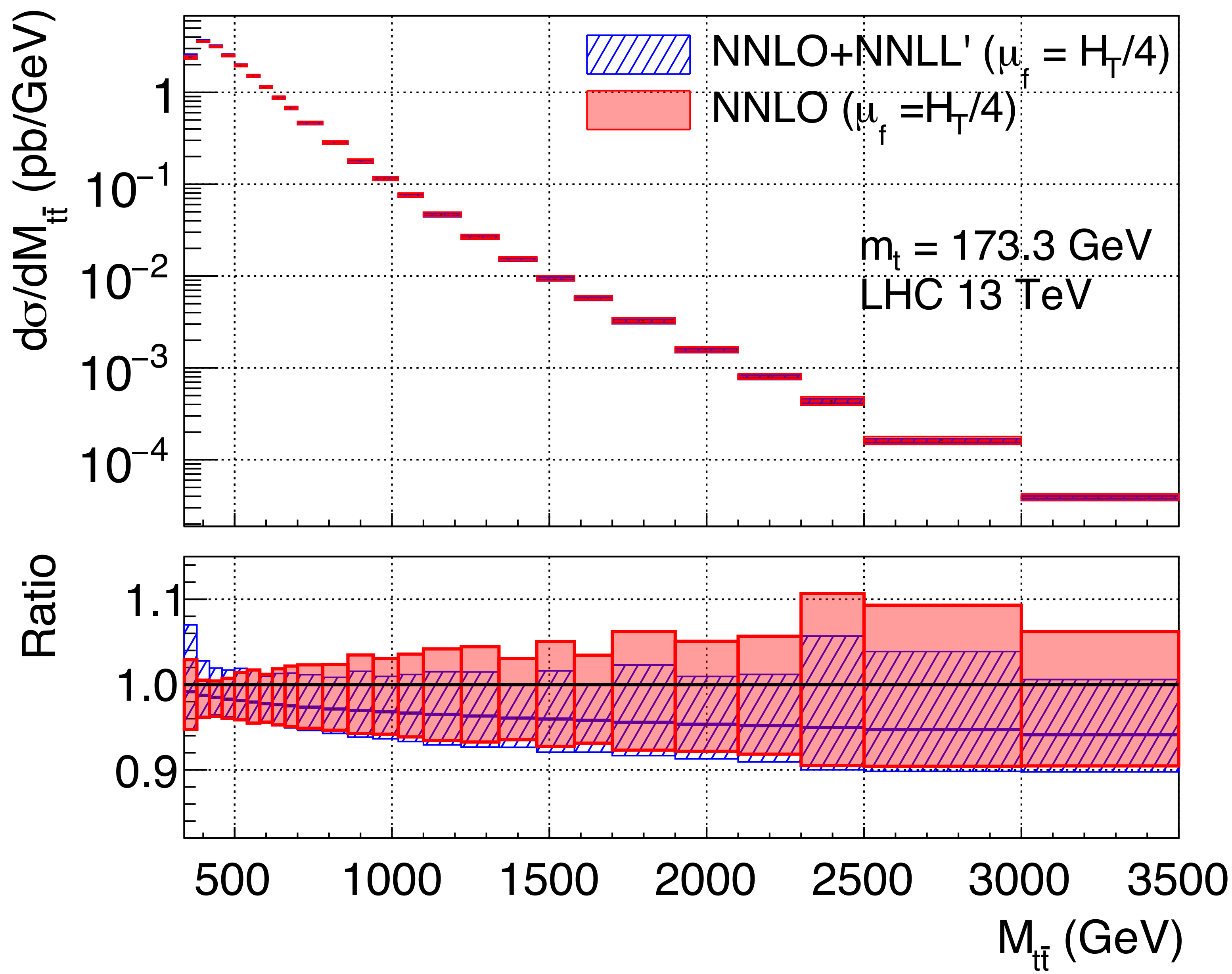
The theorists need to provide high-precision predictions for these observables

There are various possible reasons for this discrepancy, e.g., definition of $M_{t\bar{t}}$, unfolding, higher order corrections, …

Up-to-date perturbative predictions

NNLO+NNLL' in QCD

Czakon, Ferroglia, Heymes, Mitov, Pecjak, Scott, Wang, **LLY**: 1803.07623

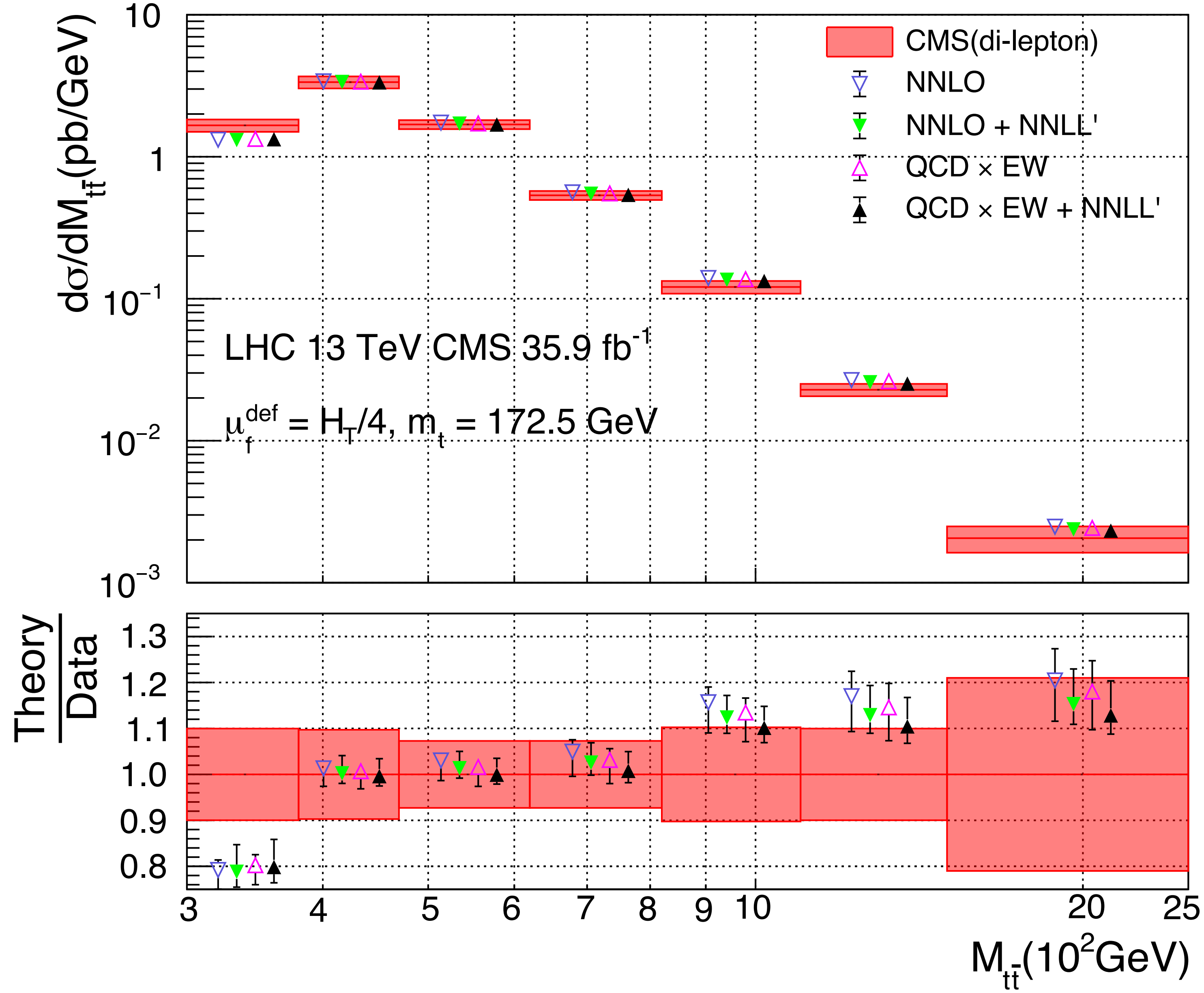


See also: Ahrens, Ferroglia, Neubert, Pecjak, **LLY** 2009; Pecjak, Scott, Wang, **LLY** 2016; Czakon, Heymes, Mitov 2016; Pecjak, Scott, Wang, **LLY** 2018; Catani, Devoto, Grazzini, Kallweit, Mazzitelli 2019

Up-to-date perturbative predictions

Further combined with the full NLO (QCD+electroweak) results

Czakon, Ferroglia, Mitov, Pagani, Papanastasiou, Pecjak, Scott, Tsinikos, Wang, **LLY**, Zaro: 1901.08281



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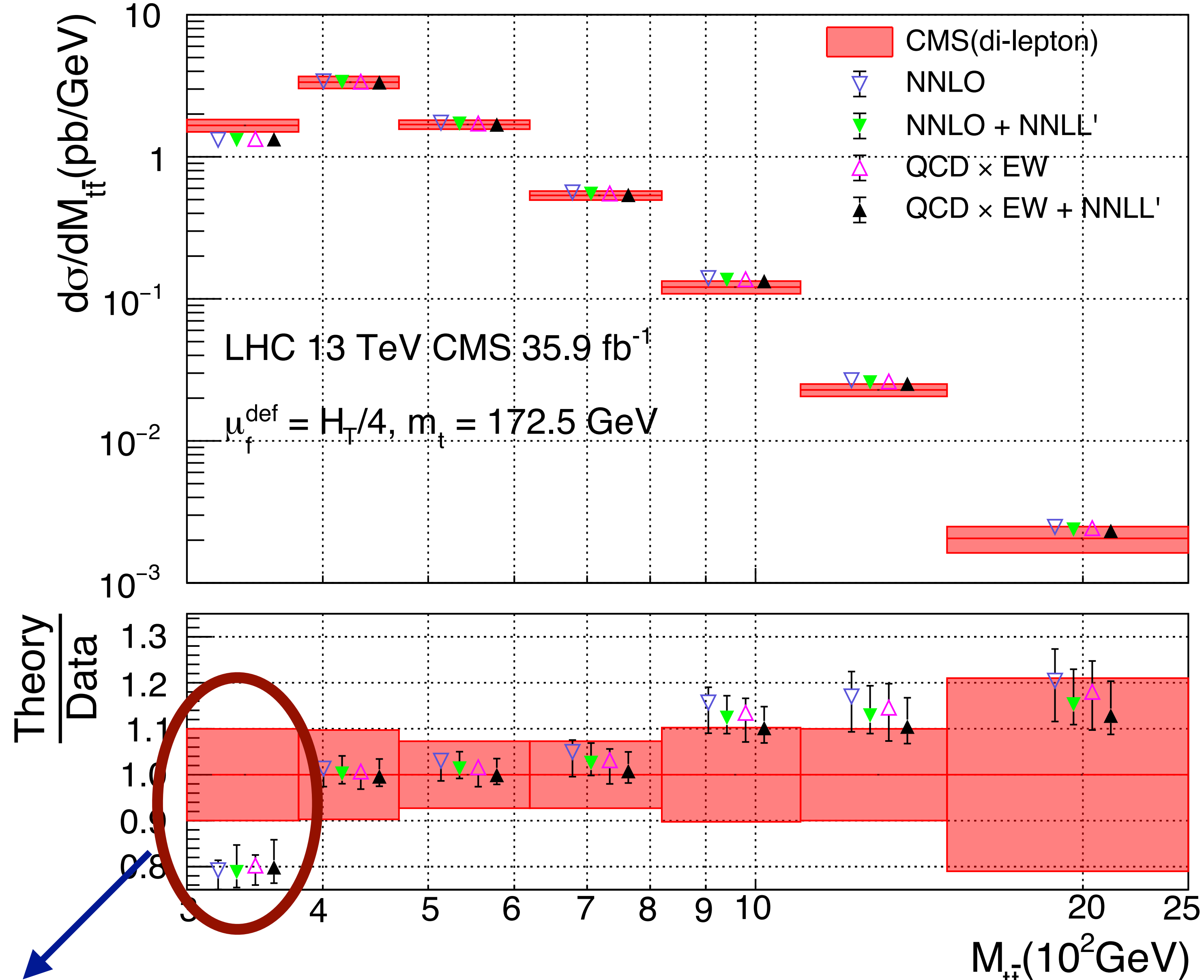

Still a discrepancy precisely in the m_t-sensitive region

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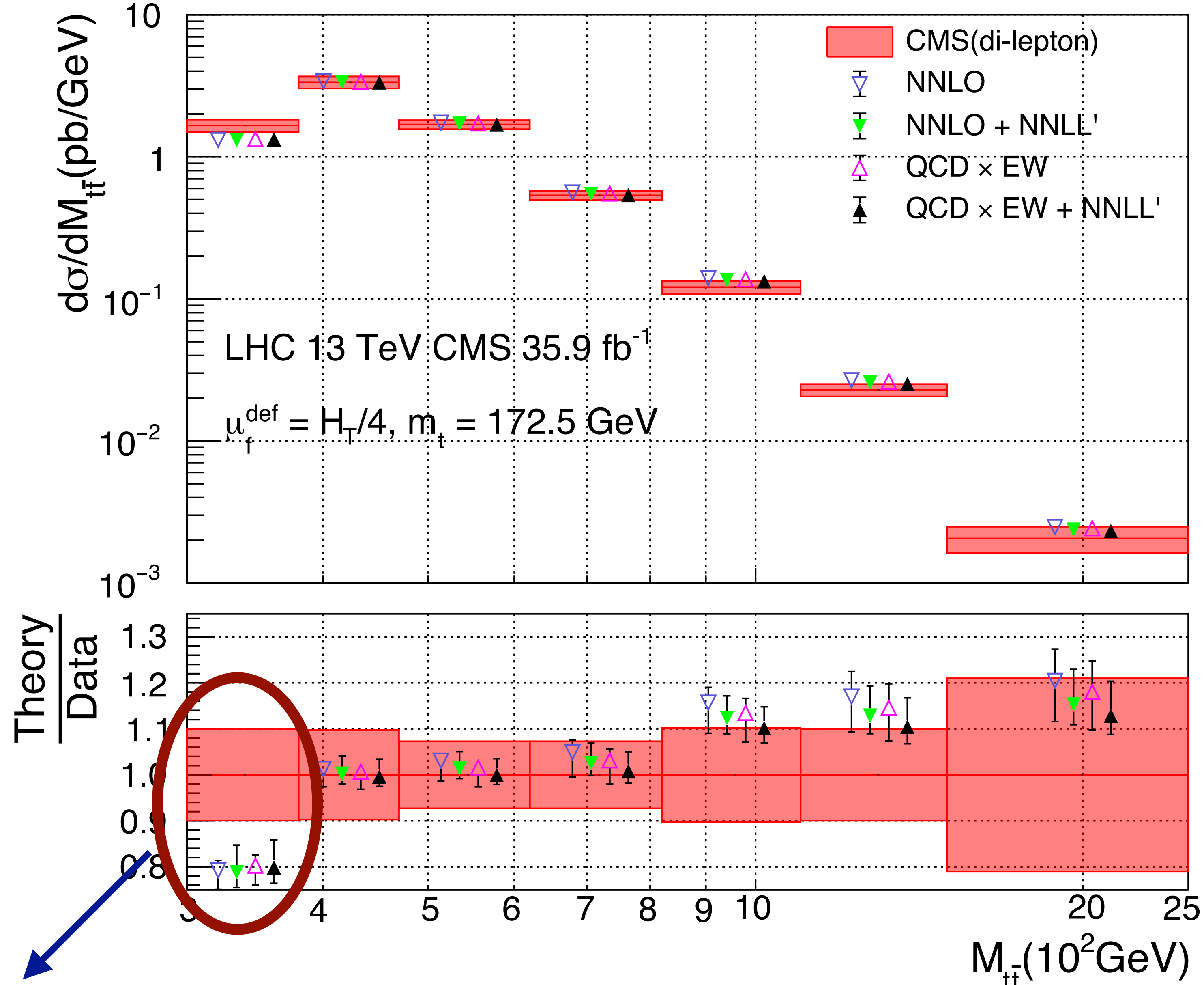
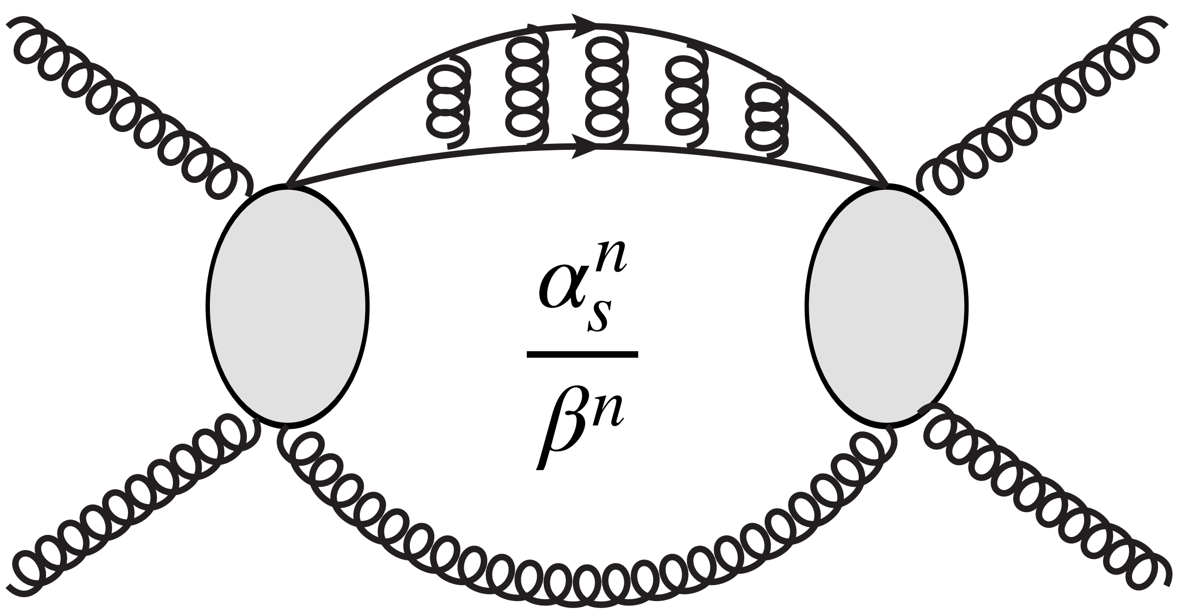


Still a discrepancy precisely in the m_t-sensitive region

What else have we missed?

Non-relativistic Coulomb corrections

When the top and anti-top quarks move slowly with respect to each other, exchanges of gluons in between lead to "Coulomb corrections" or "Sommerfeld enhancement"



$$\beta = \sqrt{1 - \frac{4m_t^2}{M_{t\bar{t}}^2}} \to 0$$

Kind of "non-perturbative" bound-state effects, but still calculable for top quarks

Resummation to all orders in α_s

A note on technical details

The basic EFT framework to resum these Coulomb corrections has been laid out in, e.g., Fadin et al. 1990; Bodwin et al. 1994; Petrelli et al. 1997; Hagiwara et al. 2008; Kiyo et al. 2008; Beneke et al. 2010

- We have derived a next-to-leading power (NLP) resummation formula with full kinematic dependence (and have calculated a new hard function for that) which allows us to:
  - Use dynamic renormalization and factorization scales, and consequently combine our resummed result with existing NNLO calculations
  - Study double differential distributions

Momentum regions and EFTs

Using EFTs to describe physics at different scales

$$
\begin{aligned}
\text{hard:} \quad & k^\mu \sim M_{t\bar{t}}\,, \\
\text{potential:} \quad & k^0 \sim M_{t\bar{t}}\beta^2\,,\ \vec{k} \sim M_{t\bar{t}}\beta\,, \\
\text{soft:} \quad & k^\mu \sim M_{t\bar{t}}\beta\,, \\
\text{ultrasoft:} \quad & k^\mu \sim M_{t\bar{t}}\beta^2\,, \\
\text{collinear:} \quad & k^\mu = (\bar{n}_i \cdot k, n_i \cdot k, k_\perp) \sim M_{t\bar{t}}(1, \beta^2, \beta)\,.
\end{aligned}
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Wilson coefficients

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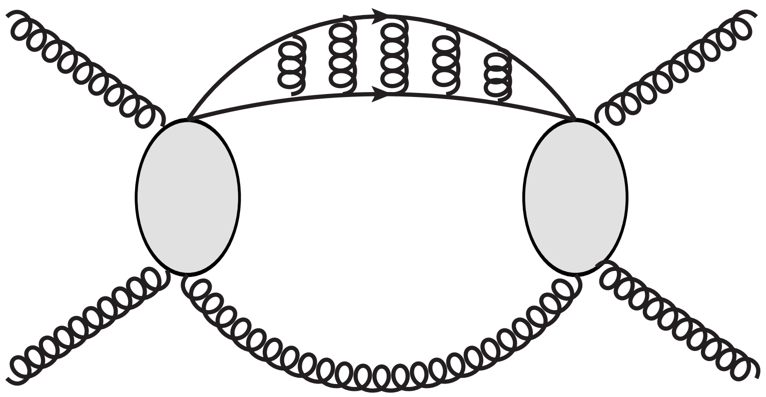
\end{aligned}
$$

pNRQCD

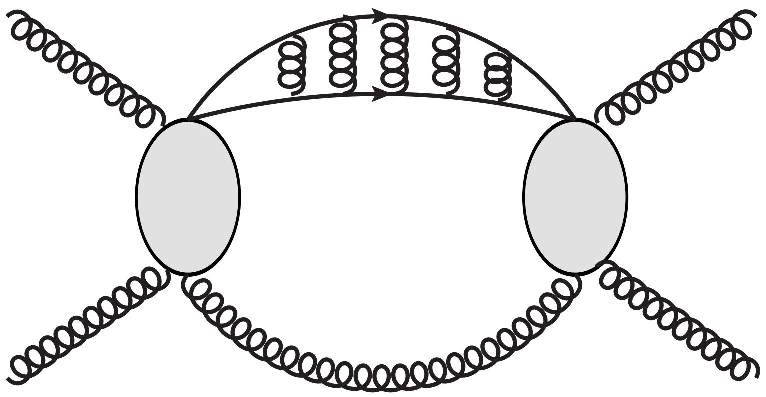
SCET, irrelevant at next-to-leading power,
possibly relevant at higher powers

Factorization formula



$$\frac{d\sigma}{dM_{t\bar{t}}d\Theta} \sim \int H \times J \times f \times f$$

Factorization formula



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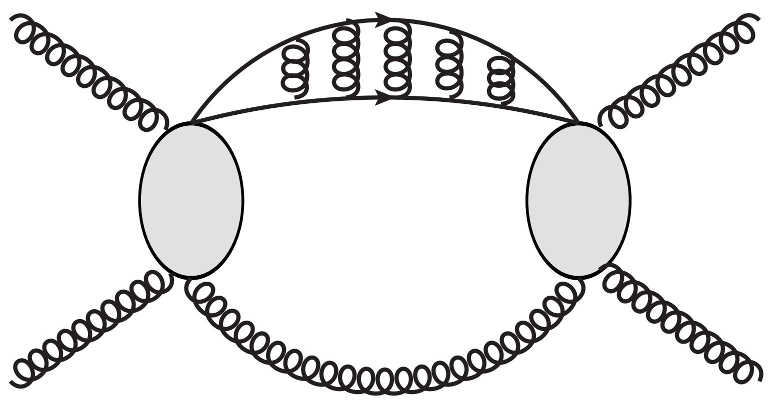
Other kinematic variables

Factorization formula



Potential function

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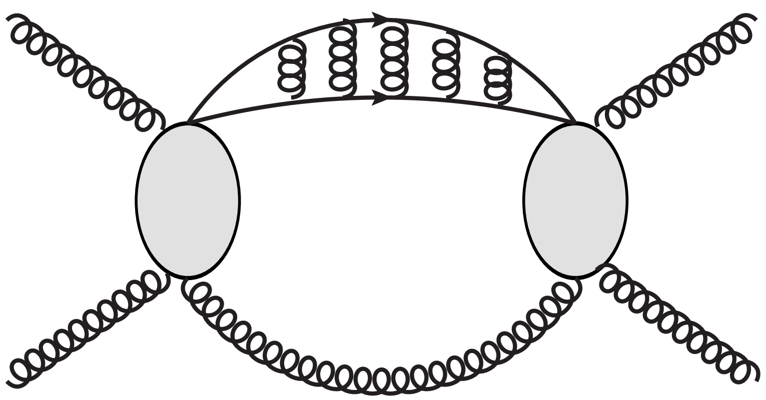
Other kinematic variables

Factorization formula



Potential function

PDFs

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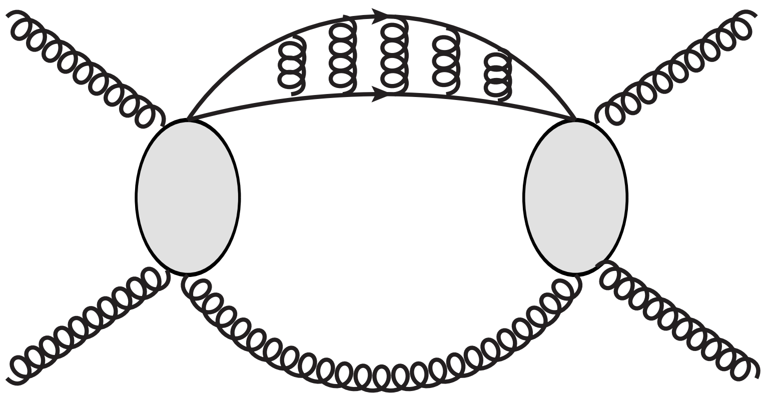
Other kinematic variables

Factorization formula



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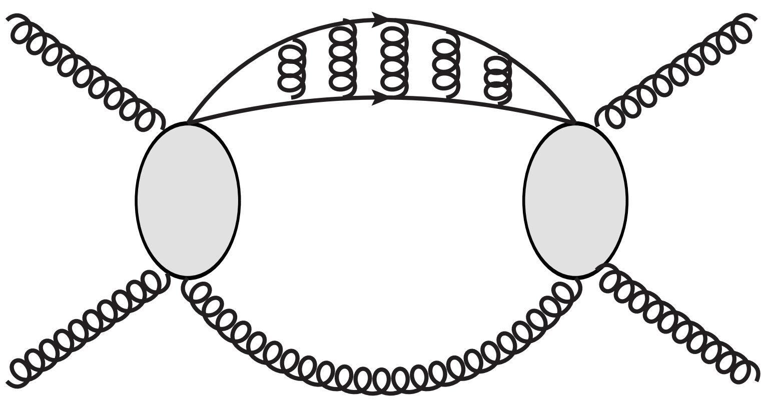
Other kinematic variables

Kinematics-dependent hard function (different from known ones in literature)

We calculate it analytically to next-to-leading order

Factorization formula



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We calculate it analytically to next-to-leading order

Note: no soft function at NLP! What about higher powers?

Factorization and resummation

Pre-factors to account for the exact leading order

$$\frac{d\hat{\sigma}_{ij}}{dM_{t\bar{t}}\,d\Theta} = \frac{16\pi^2\alpha_s^2(\mu_r)}{M_{t\bar{t}}^5}\sqrt{\frac{M_{t\bar{t}}+2m_t}{2M_{t\bar{t}}}}\sum_\alpha c_{ij,\alpha}(\cos\theta_t)$$
$$\times H_{ij,\alpha}(z, M_{t\bar{t}}, Q_T, Y, \mu_r, \mu_f)\, J^\alpha(E) + \mathcal{O}(\beta^3)$$

$$c_{q\bar{q},8}(\cos\theta_t) = \frac{1}{4}\left[2-\beta^2(1-\cos^2\theta_t)\right]$$
$$c_{gg,1}(\cos\theta_t) = \frac{1}{2(1-\beta^2\cos^2\theta_t)^2}\left[4-2(1-\beta^2)^2-2\beta^2(1-\beta^2\cos^2\theta_t)-(1+\beta^2\cos^2\theta_t)^2\right],$$
$$c_{gg,8}(\cos\theta_t) = 2c_{gg,1}(\cos\theta_t)\left[\frac{16}{5}-\frac{9}{10}(3-\beta^2\cos^2\theta_t)\right],$$

Benefit of the pre-factor: re-expansion the resummation formula to the first order reproduces the exact LO differential cross section

NLP resummation and matching

$$\frac{d\sigma^{\rm NLP}}{dM_{t\bar{t}}} = \int_\tau^1 \frac{dz}{z} \int_{-1}^1 d\cos\theta_t \int_0^{2\pi} \frac{d\phi_t}{2\pi} \int_0^{Q^2_{T,\max}} dQ_T^2 \int_{-Y_{\max}}^{Y_{\max}} dY \frac{16\pi^2\alpha_s^2(\mu_r)}{s\, M_{t\bar{t}}^3} \sqrt{\frac{M_{t\bar{t}}+2m_t}{2M_{t\bar{t}}}}$$
$$\times \sum_{ij,\alpha} c_{ij,\alpha}(\cos\theta_t)\, f\!f_{ij}(\tau/z,\mu_f)\, \frac{1}{z}\, K^{\rm NLP}_{ij,\alpha}(z, M_{t\bar{t}}, m_t, Q_T, Y, \mu_r, \mu_f) + \mathcal{O}(\beta^3)\,,$$

$$H^{(0)}_{ij,\alpha}\big(J_0^\alpha(E) + J_1^\alpha(E)\big) + \frac{\alpha_s(\mu_r)}{4\pi} H^{(1)}_{ij,\alpha}\, J_0^\alpha(E) = \frac{M_{t\bar{t}}^2}{8\pi}\sqrt{\frac{2E}{M_{t\bar{t}}}} \sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu_r)}{4\pi}\right)^n K^{(n)}_{ij,\alpha}$$

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Fixed-order expansion of the resummation formula:

$$\frac{d\sigma^{{\rm n}^k{\rm LO}}}{dM_{t\bar{t}}} = \int_\tau^1 \frac{dz}{z} \int_{-1}^1 d\cos\theta_t \int_0^{2\pi} \frac{d\phi_t}{2\pi} \int_0^{Q^2_{T,\max}} dQ_T^2 \int_{-Y_{\max}}^{Y_{\max}} dY \, \frac{16\pi^2\alpha_s^2(\mu_r)}{s\, M_{t\bar{t}}^3} \sqrt{\frac{M_{t\bar{t}}+2m_t}{2M_{t\bar{t}}}}$$
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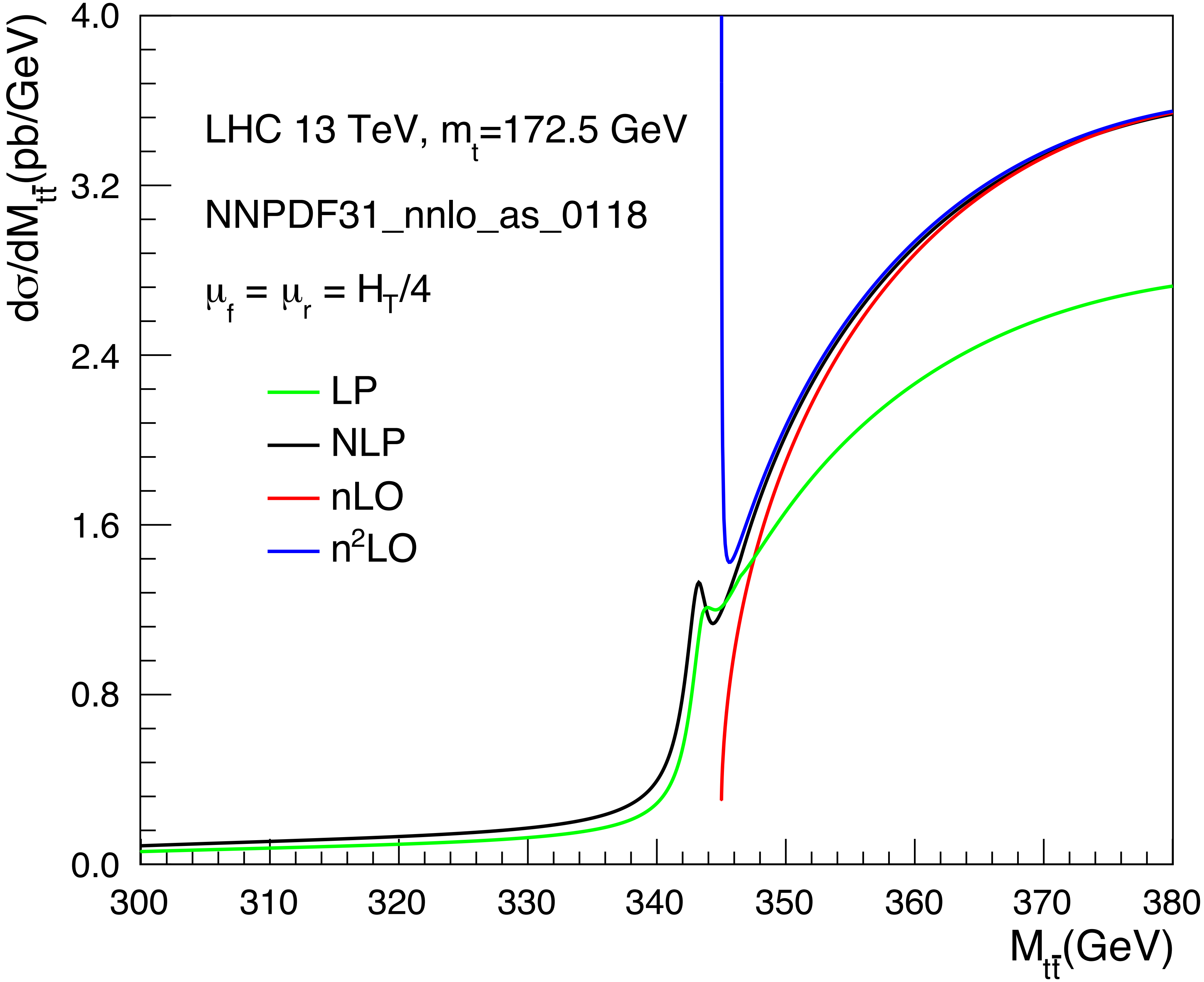
$$\times \sum_{ij,\alpha} c_{ij,\alpha}(\cos\theta_t)\, f\!f_{ij}(\tau/z,\mu_f)\, \frac{1}{z}\, \frac{M_{t\bar t}^2}{8\pi}\sqrt{\frac{2E}{M_{t\bar t}}} \sum_{n=0}^k \left(\frac{\alpha_s(\mu_r)}{4\pi}\right)^n K^{(n)}_{ij,\alpha}\,.$$

Match to exact NLO and NNLO calculations:

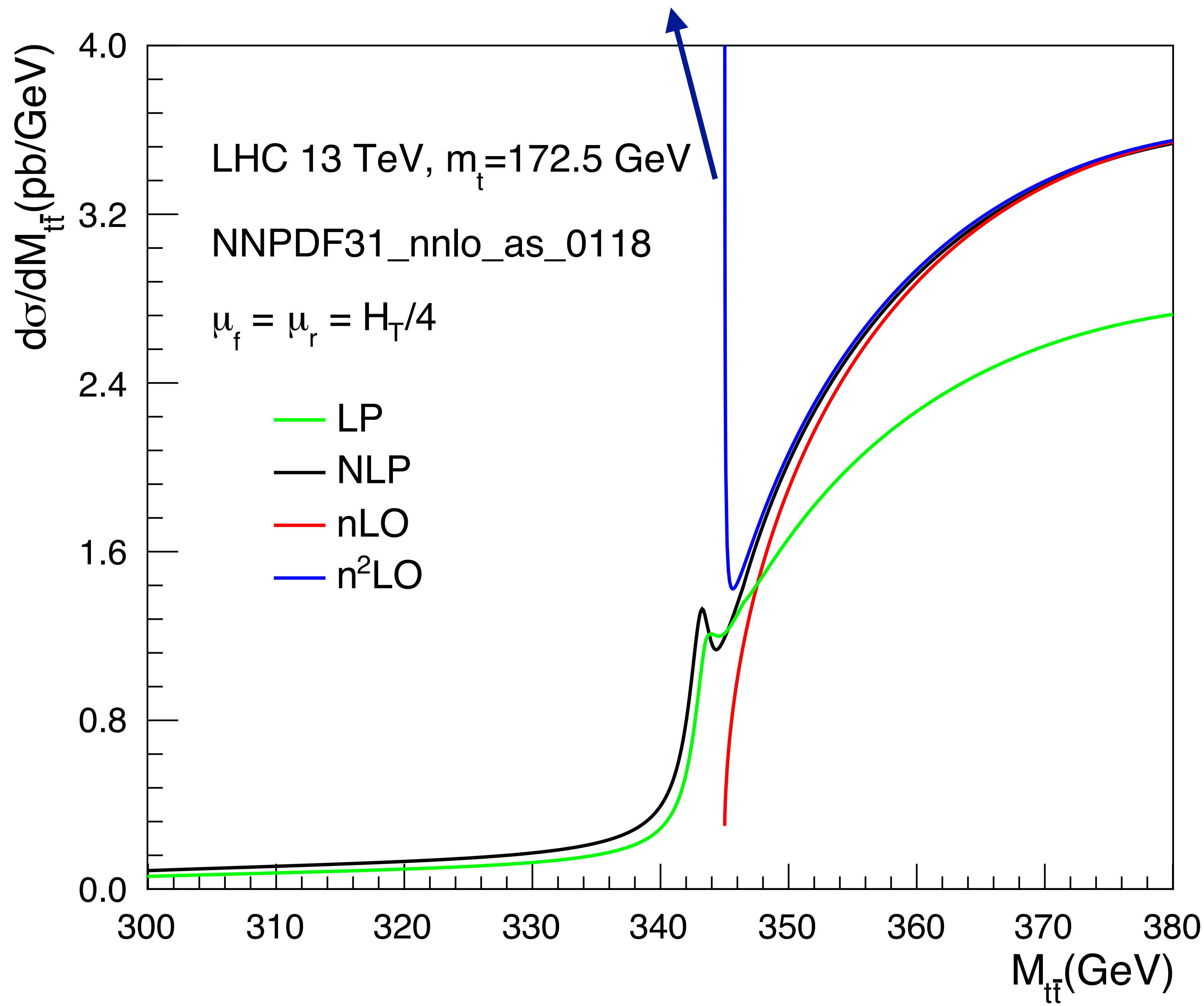
$$\frac{d\sigma^{\rm (N)NLO+NLP}}{dM_{t\bar t}} = \frac{d\sigma^{\rm NLP}}{dM_{t\bar t}} - \frac{d\sigma^{\rm (n)nLO}}{dM_{t\bar t}} + \frac{d\sigma^{\rm (N)NLO}}{dM_{t\bar t}}$$

Perturbative behavior



Perturbative behavior

Fixed-order expansion divergent in the threshold limit



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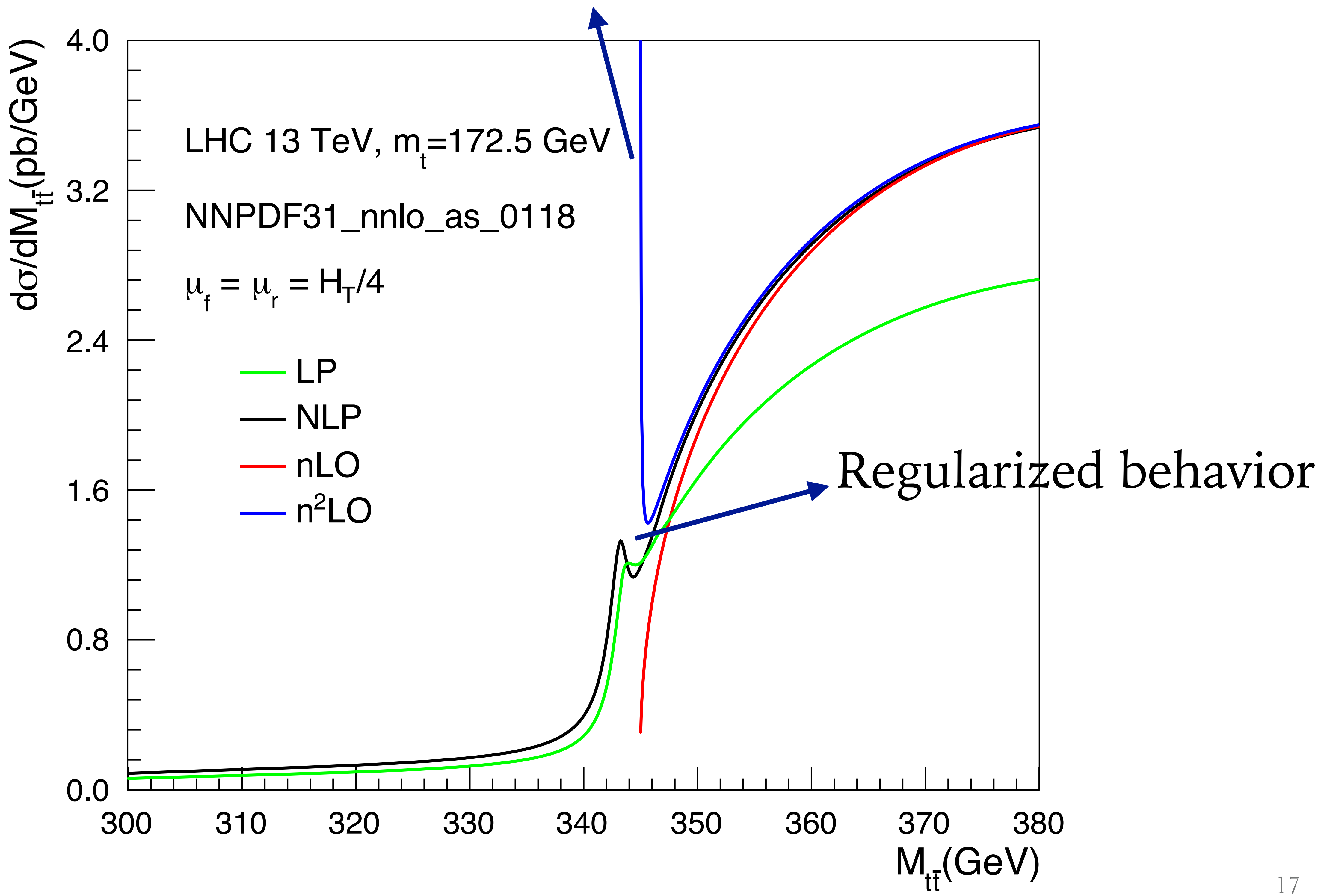

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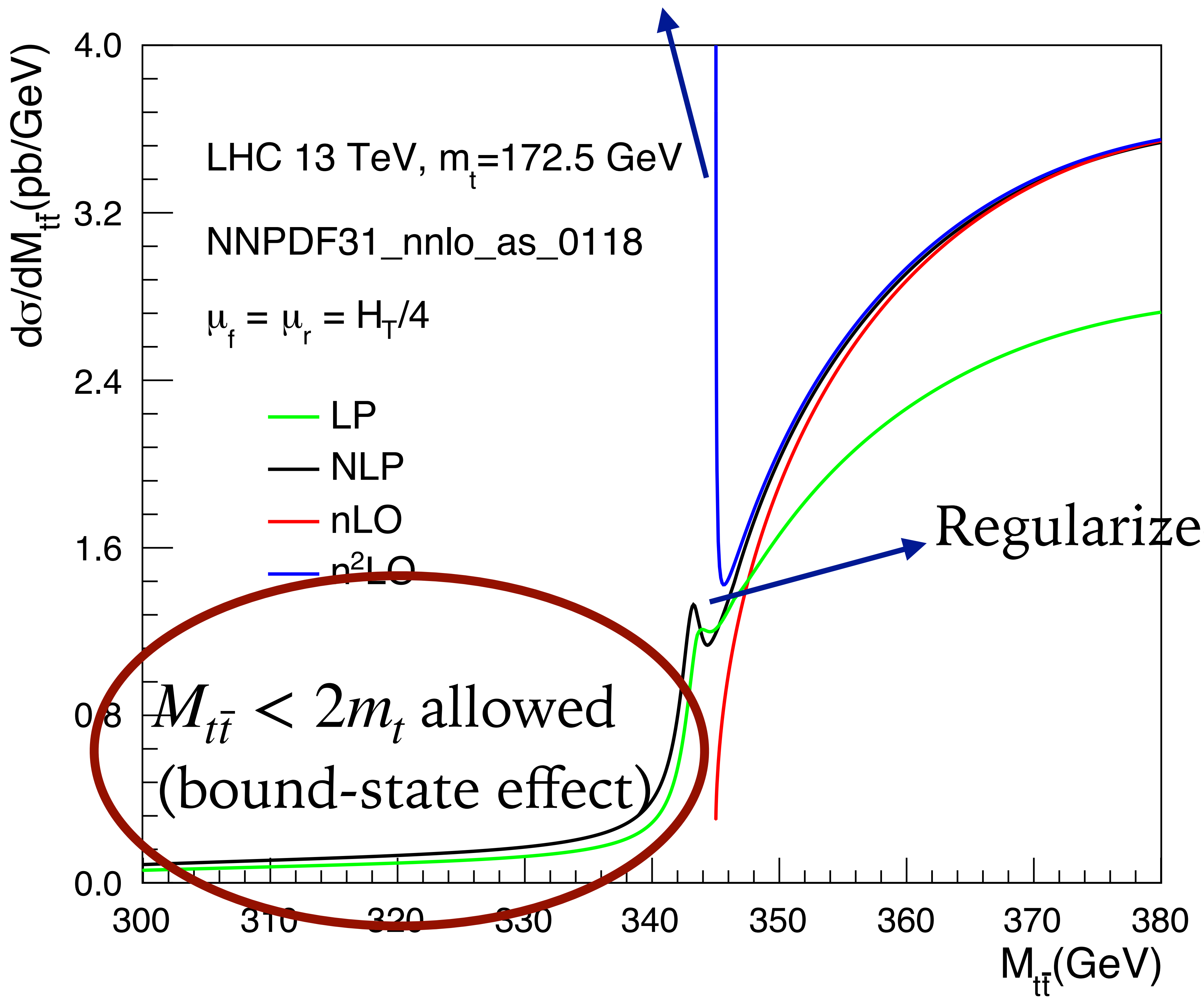
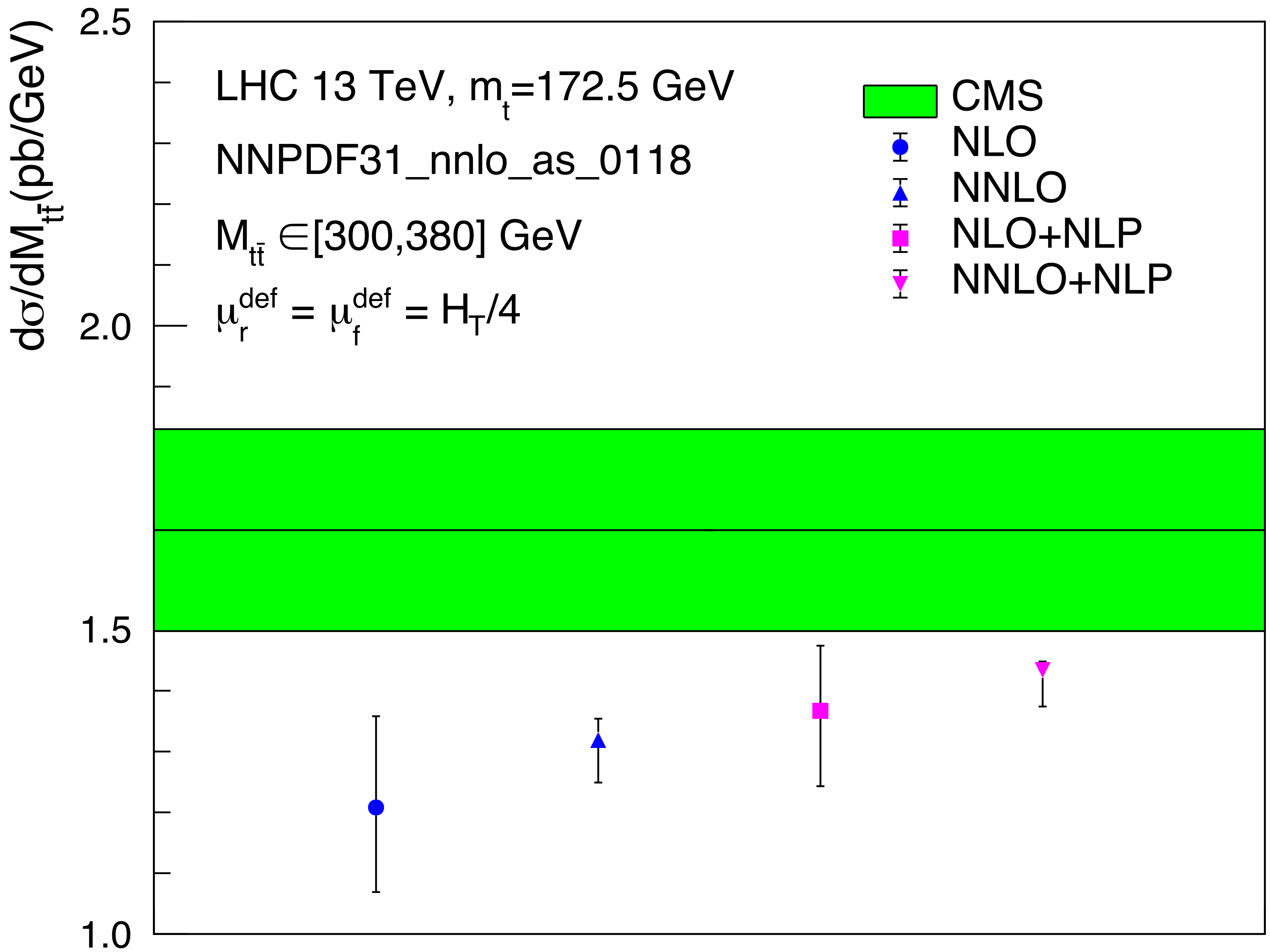


Regularized behavior

$M_{t\bar{t}} < 2m_t$ allowed (bound-state effect)

Compare to data: absolute distribution



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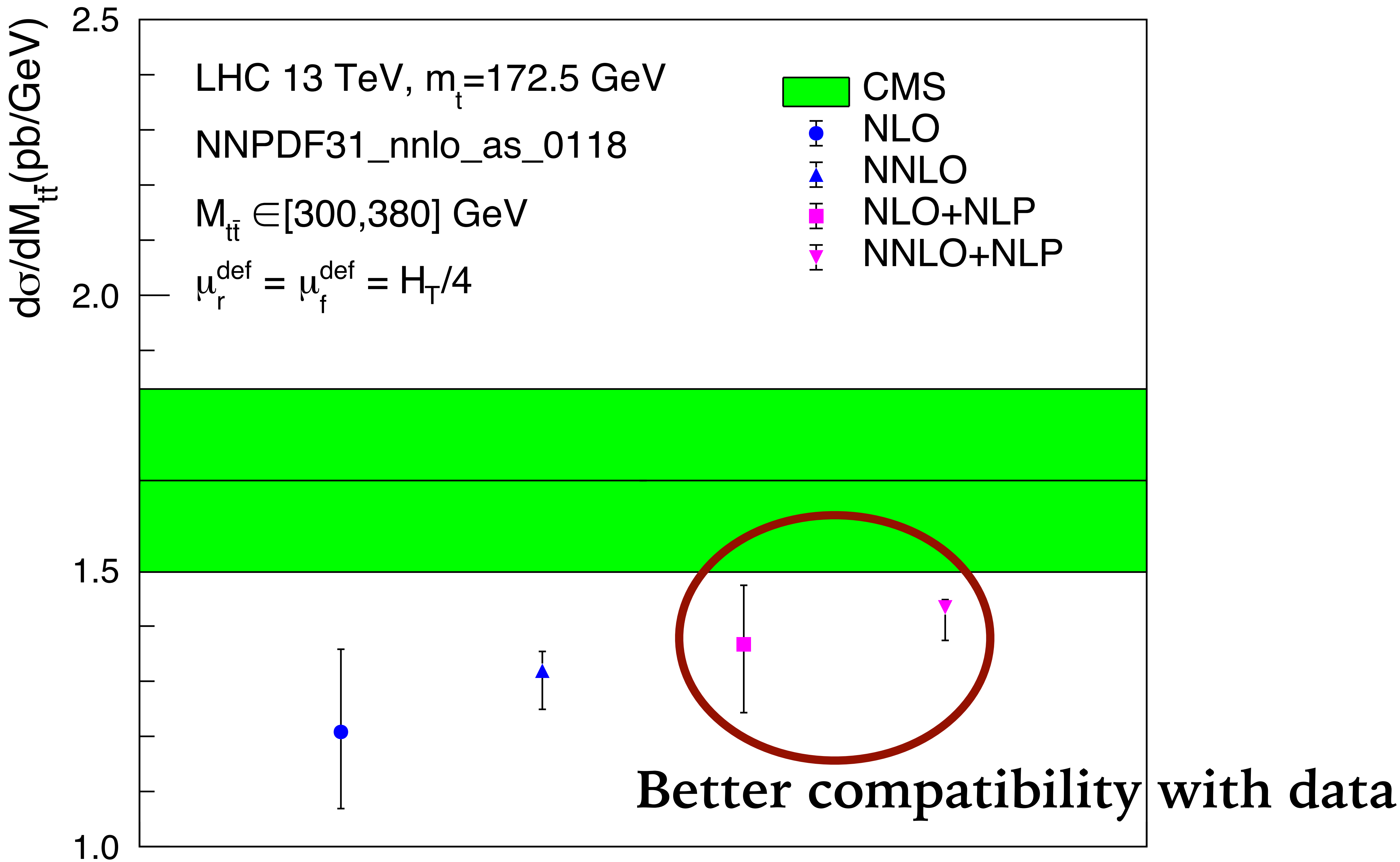

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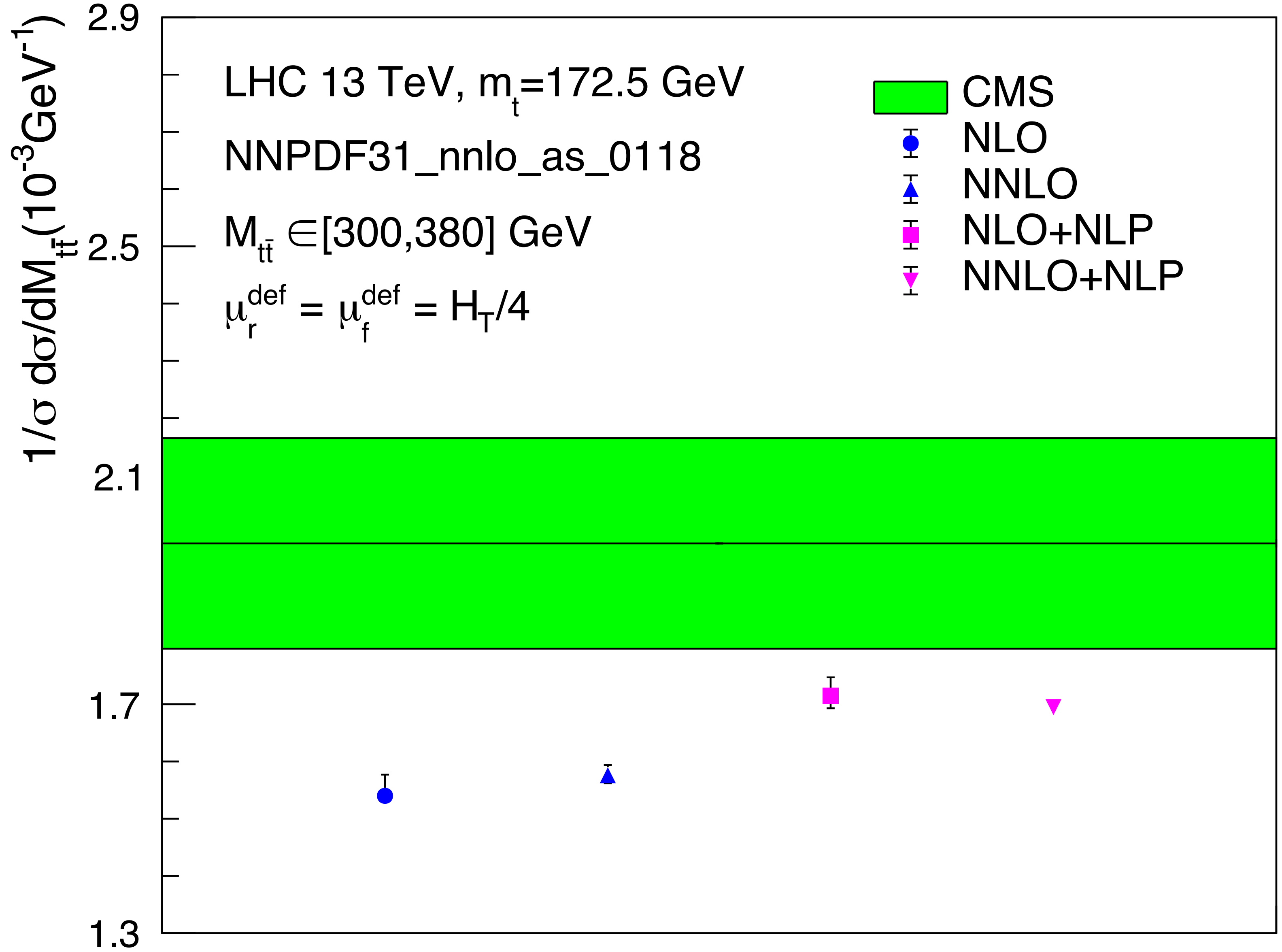
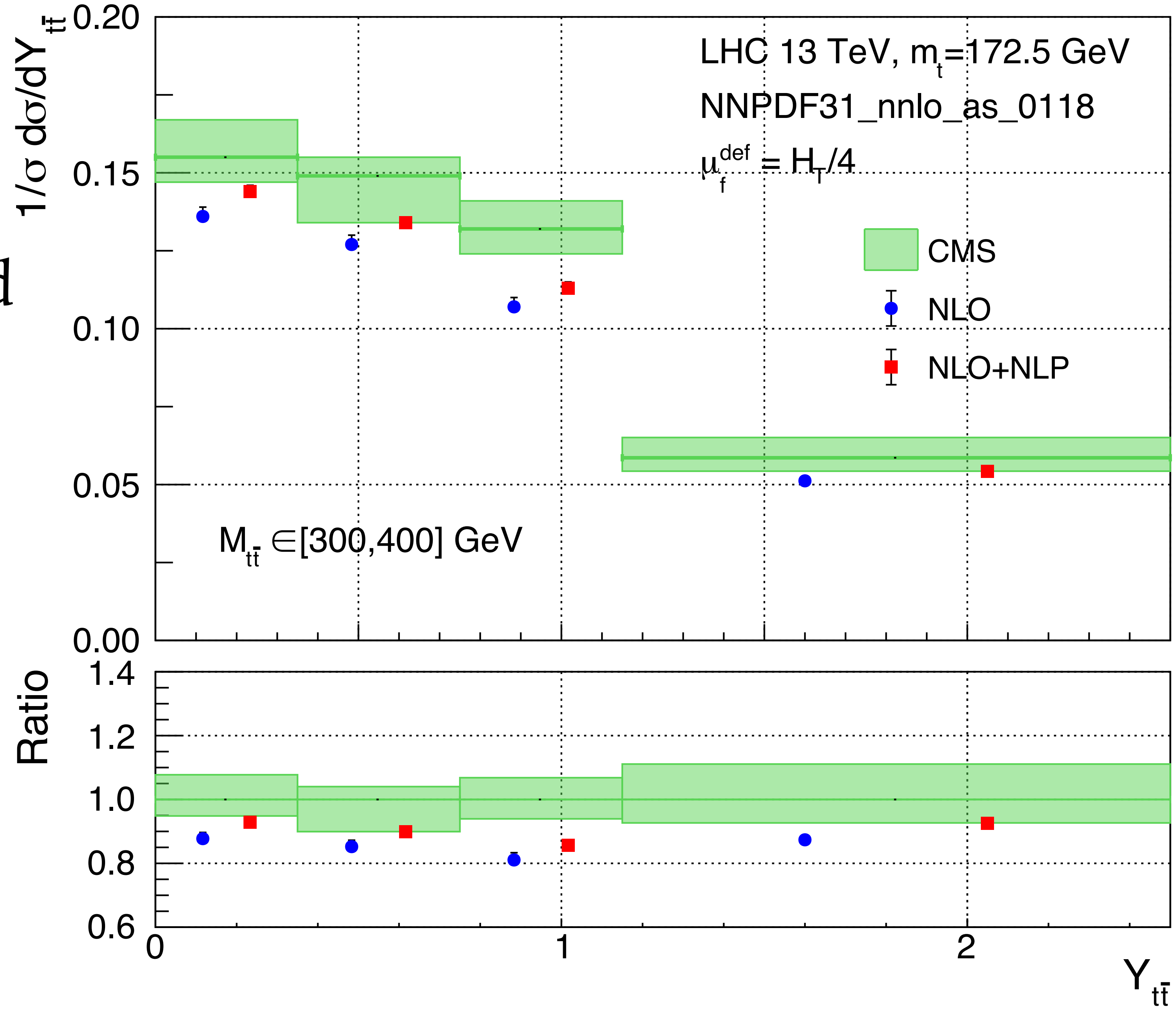


For normalized distribution, the NNLO corrections are small in this region, but the Coulomb corrections are still significant

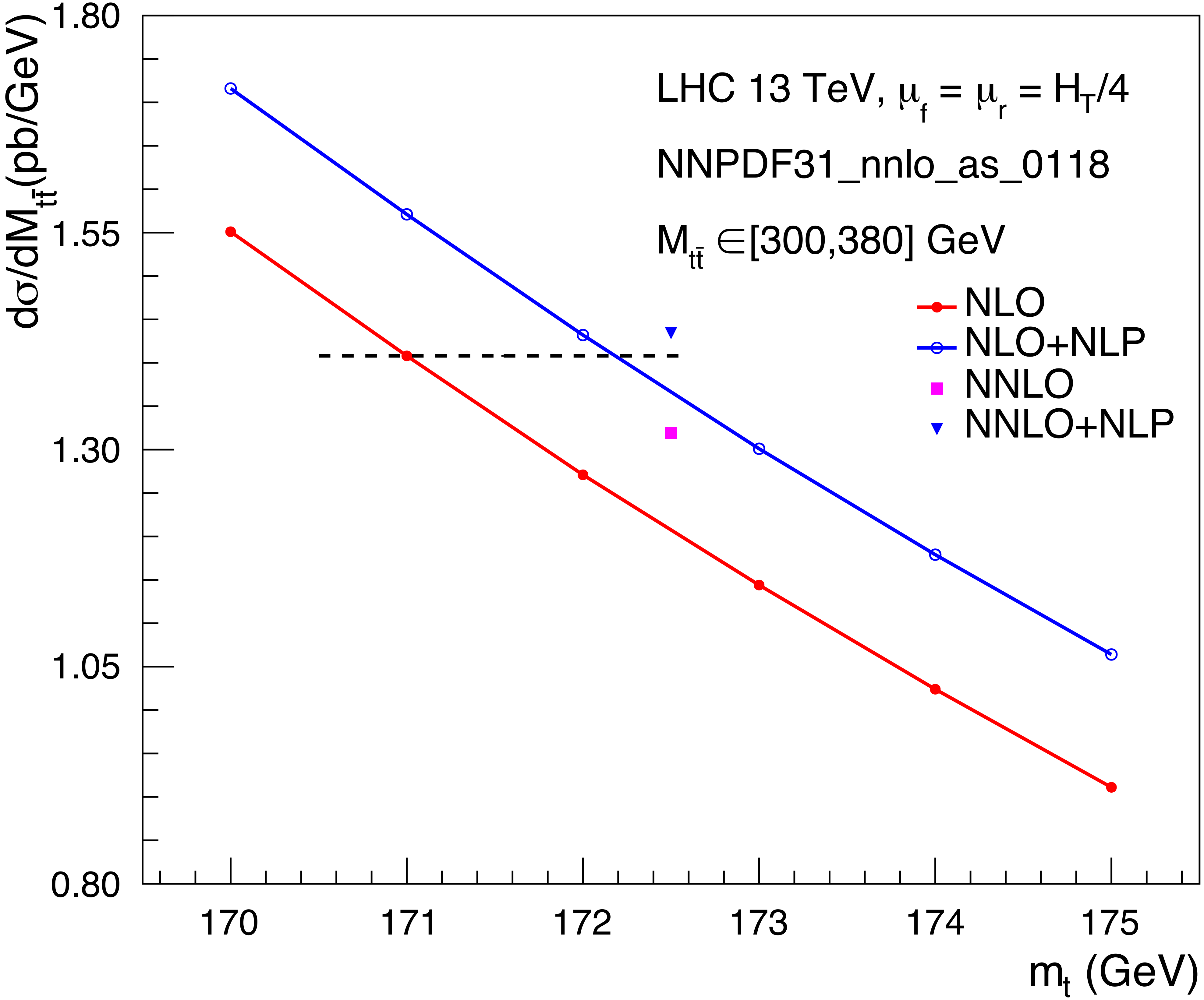
Compare to data: double distribution

Normalized



Observing similar effects as in the single distribution

Influence on mass extraction: absolute distr.



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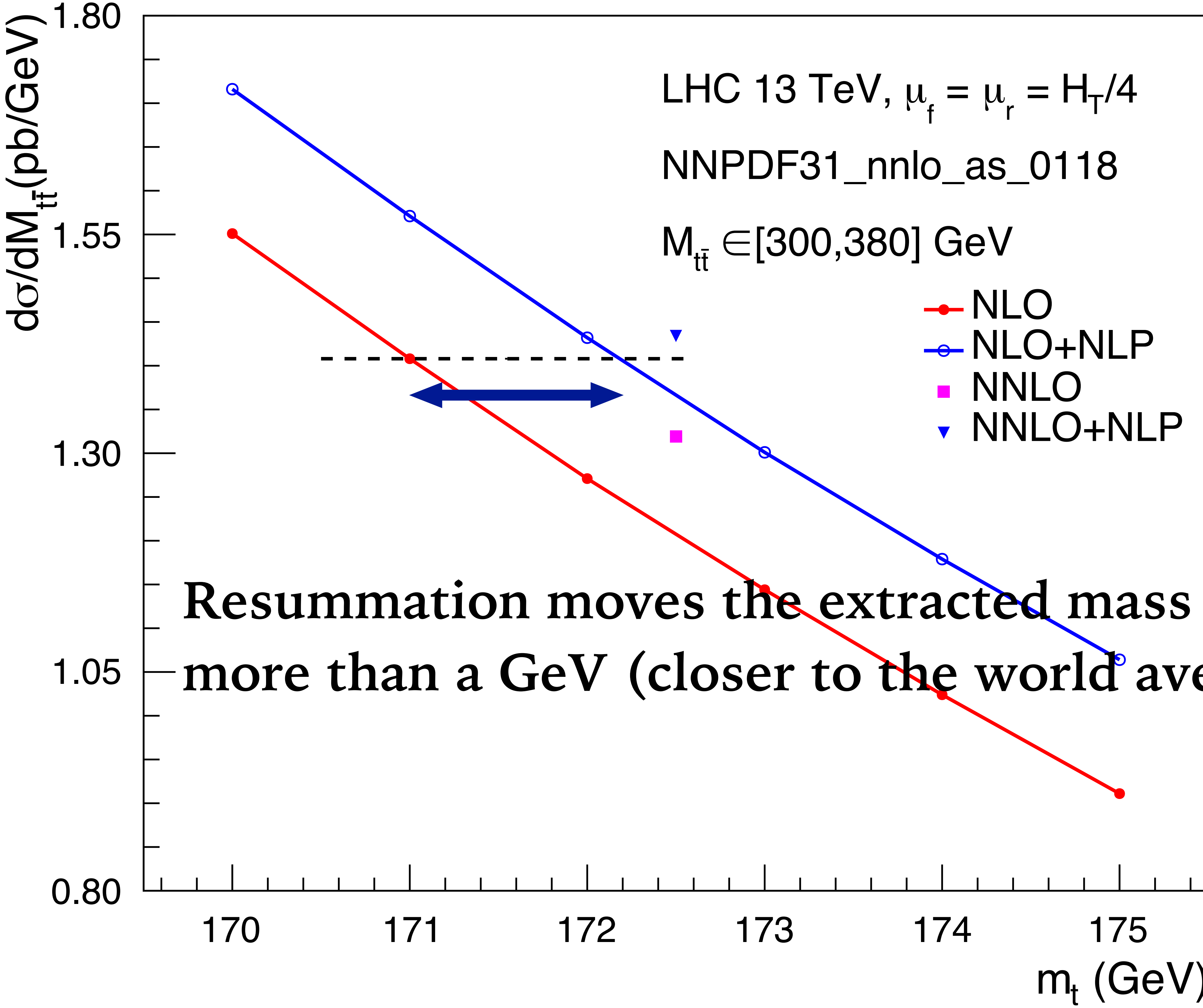
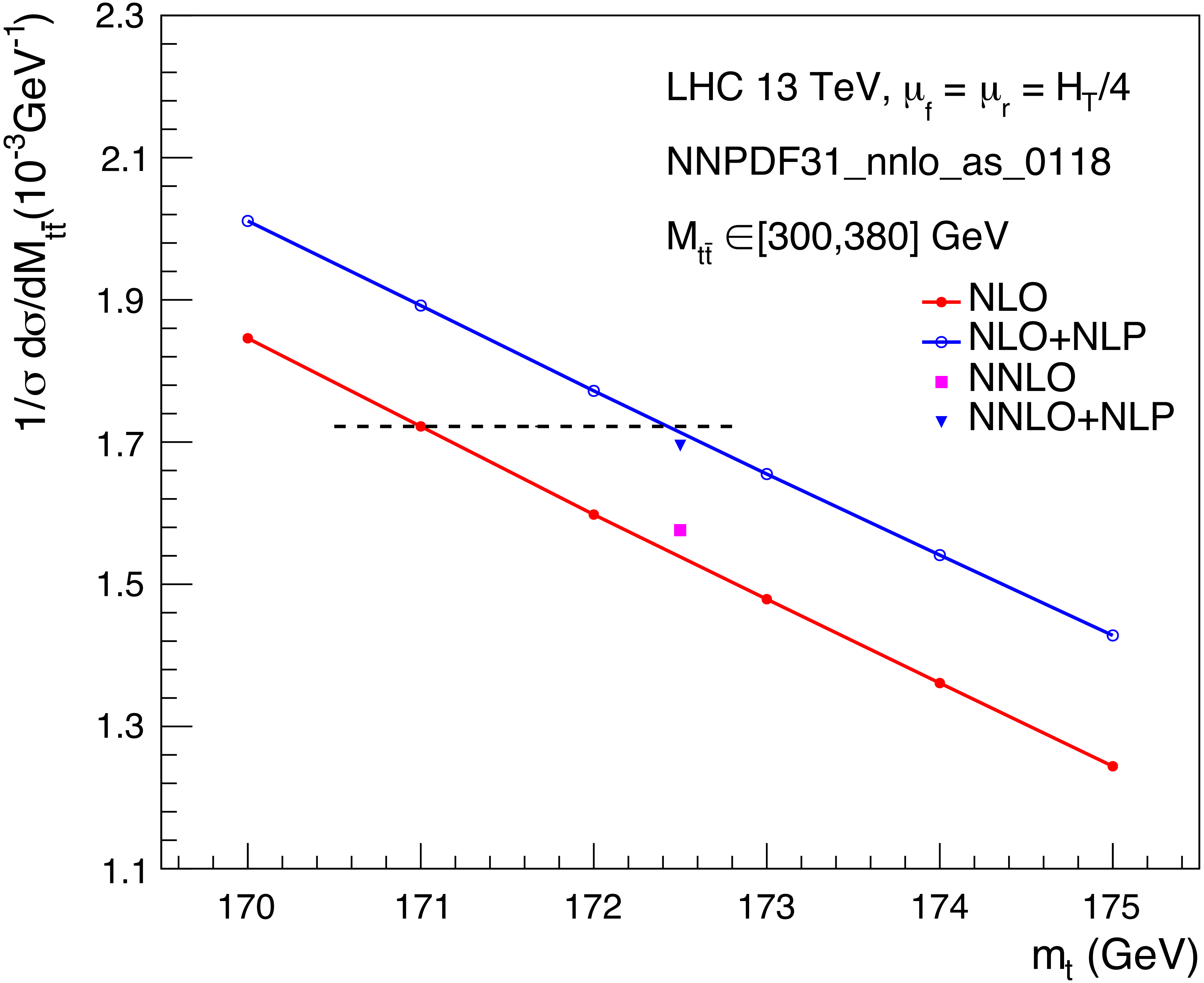


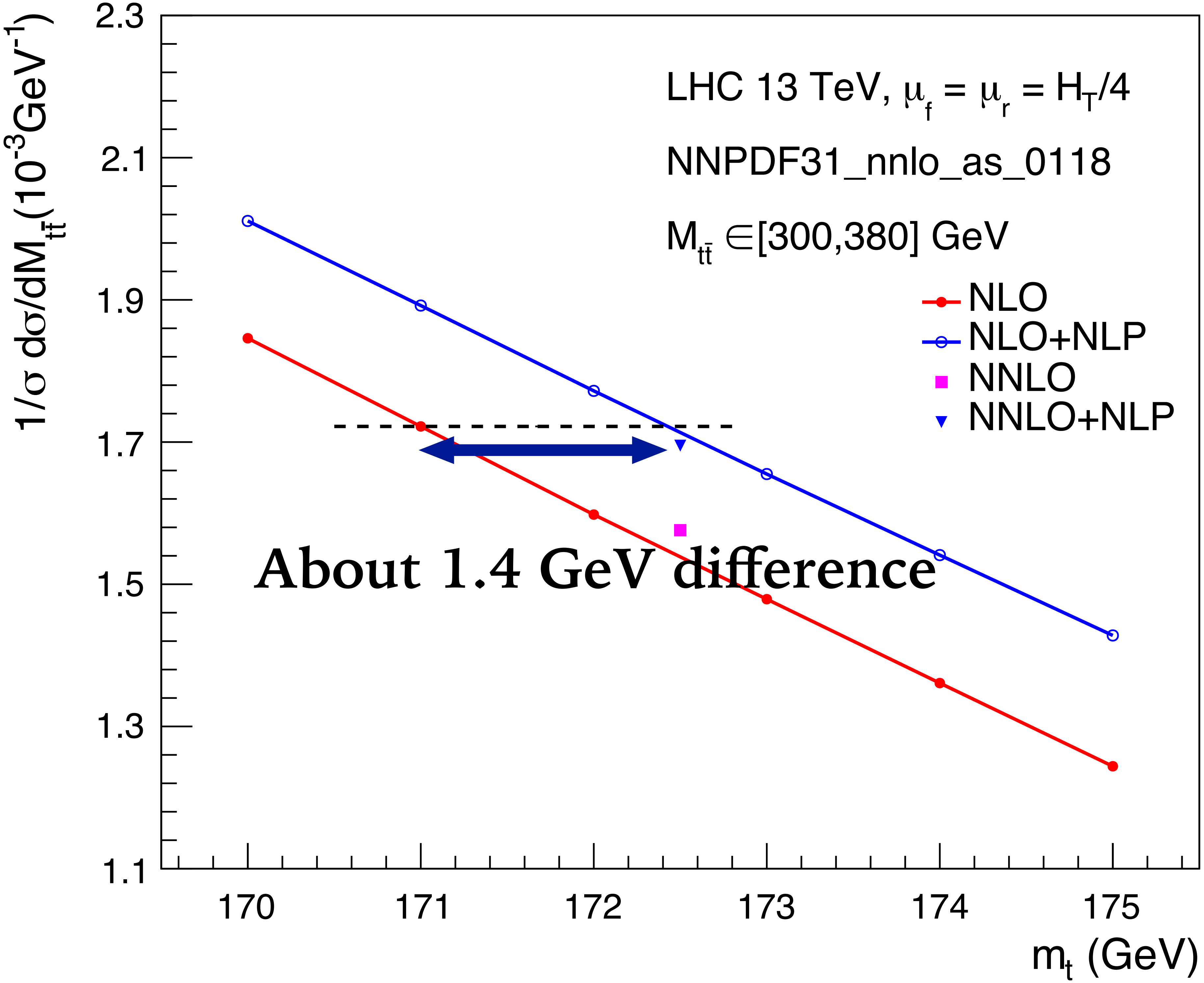
Resummation moves the extracted mass up by more than a GeV (closer to the world average)

Influence on mass extraction: normalized distr.



Similar behavior for [300,400] GeV range

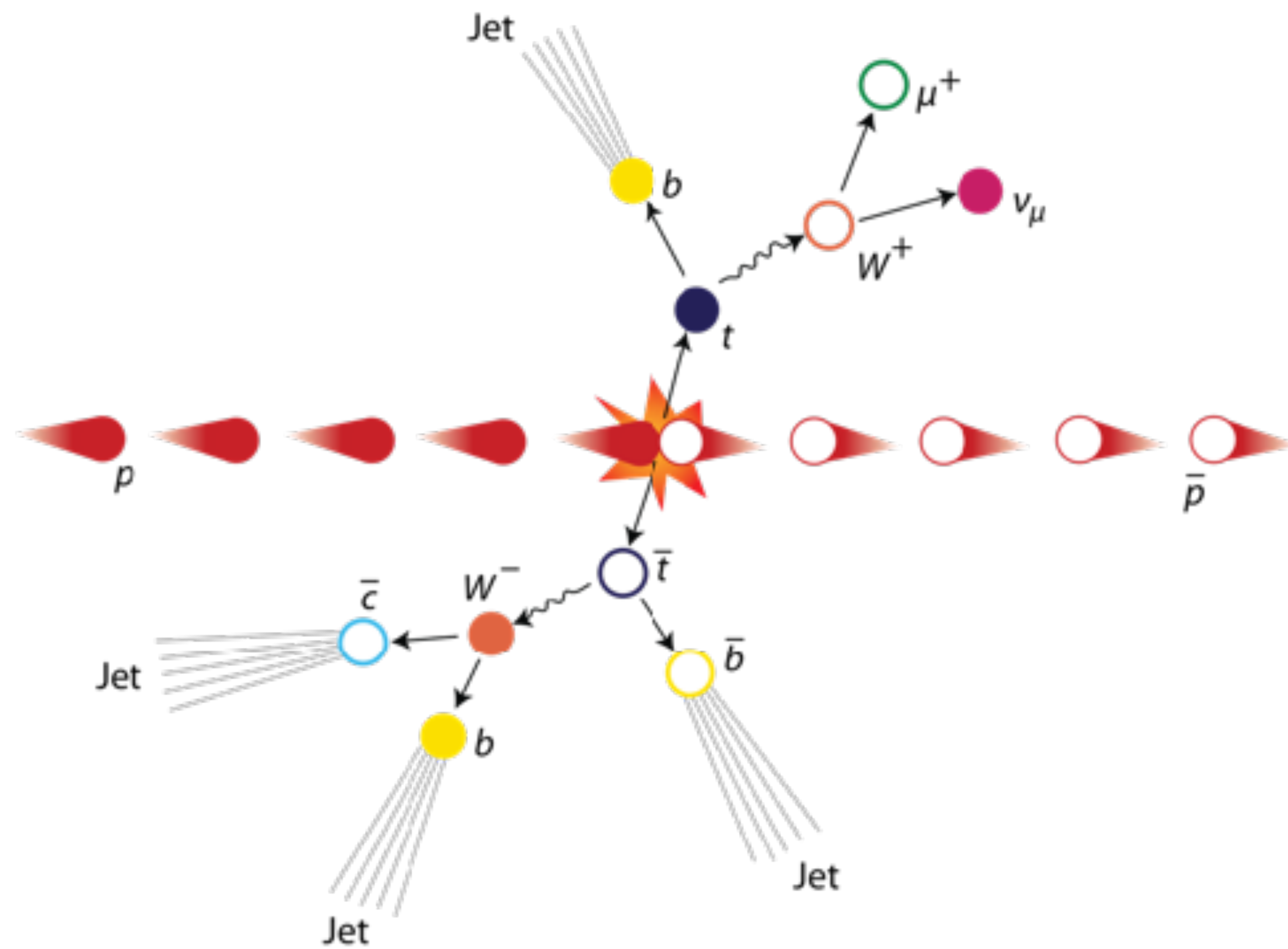
Influence on mass extraction: normalized distr.



Similar behavior for [300,400] GeV range

Reconstructed vs true top quarks

In reality we can only reconstruct top quarks from decay products and extra radiations

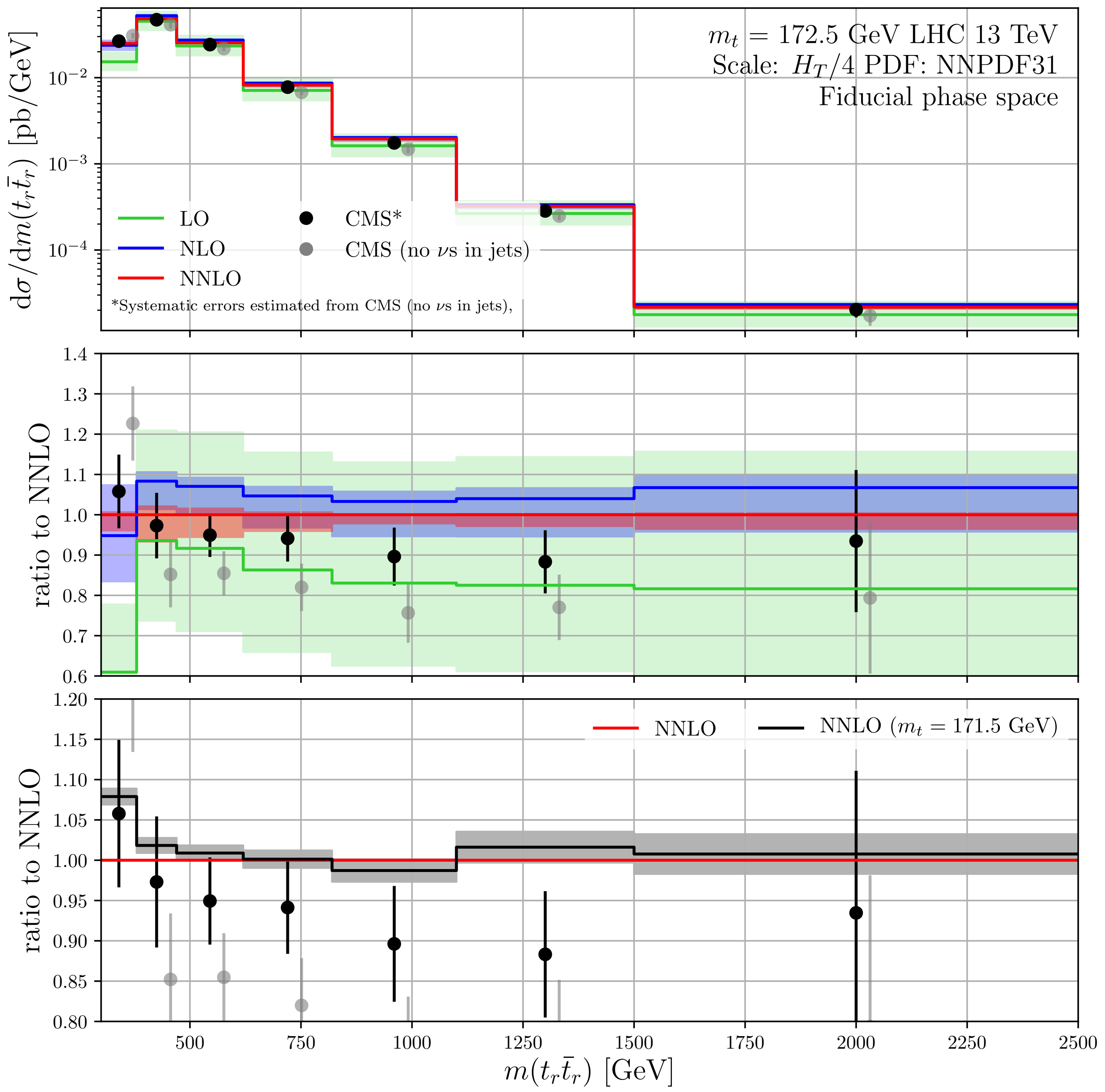


In our study we have worked with stable top quarks, therefore we can only compare with parton-level data (unfolded)

Reconstructed vs true top quarks

A recent study for reconstructed tops in the di-lepton channel

Czakon, Mitov, Poncelet: 2008.11133



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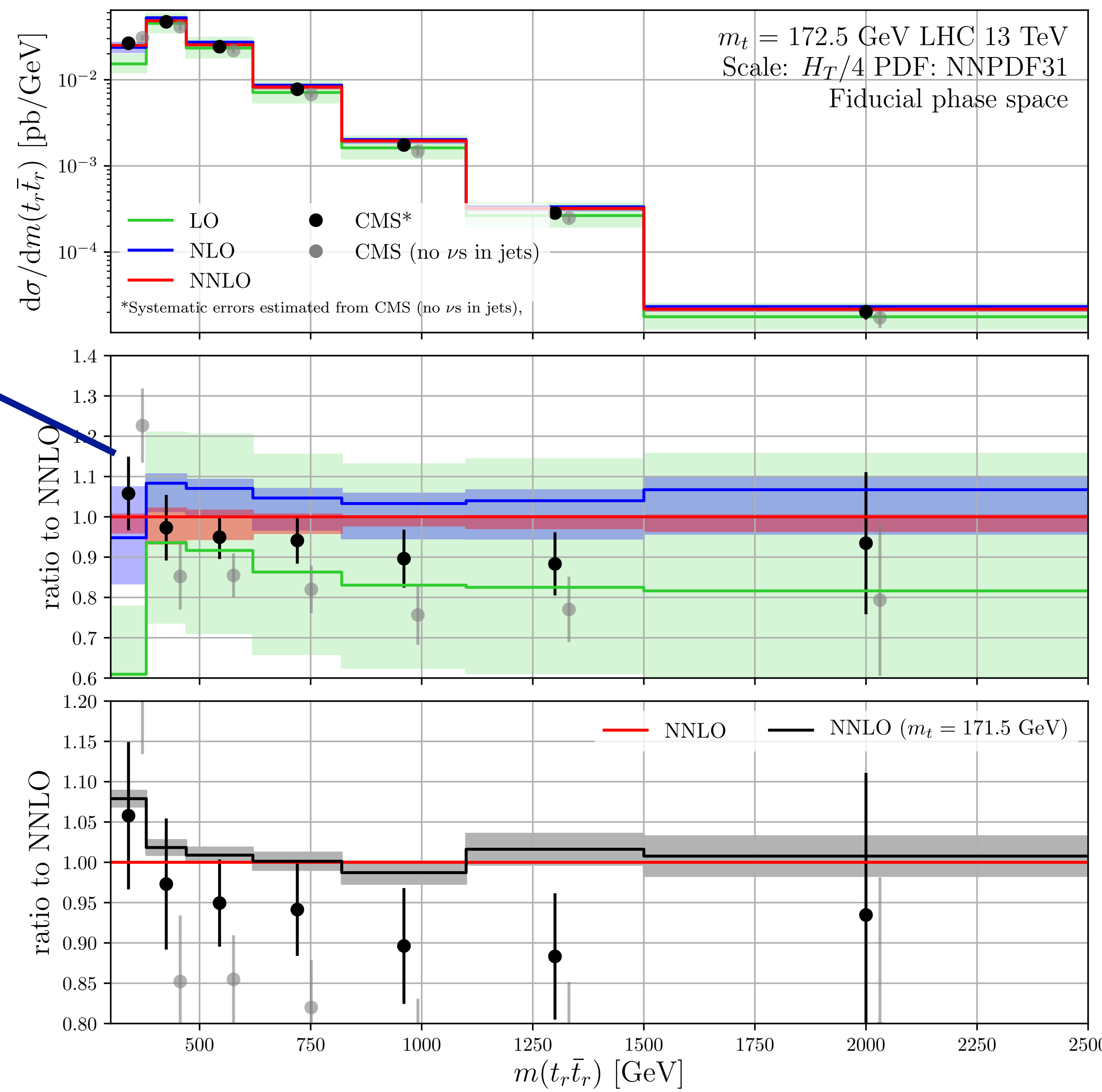

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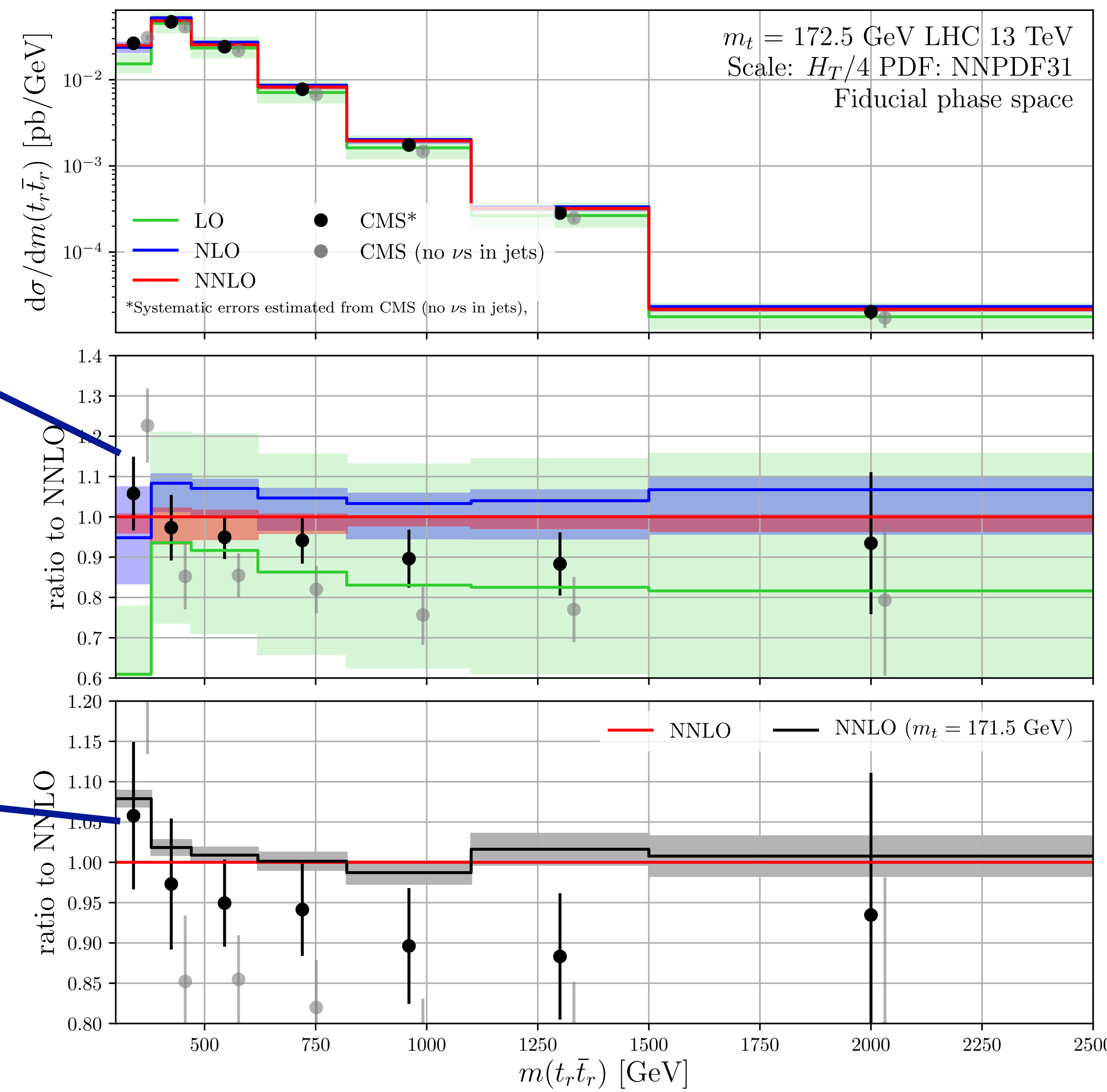
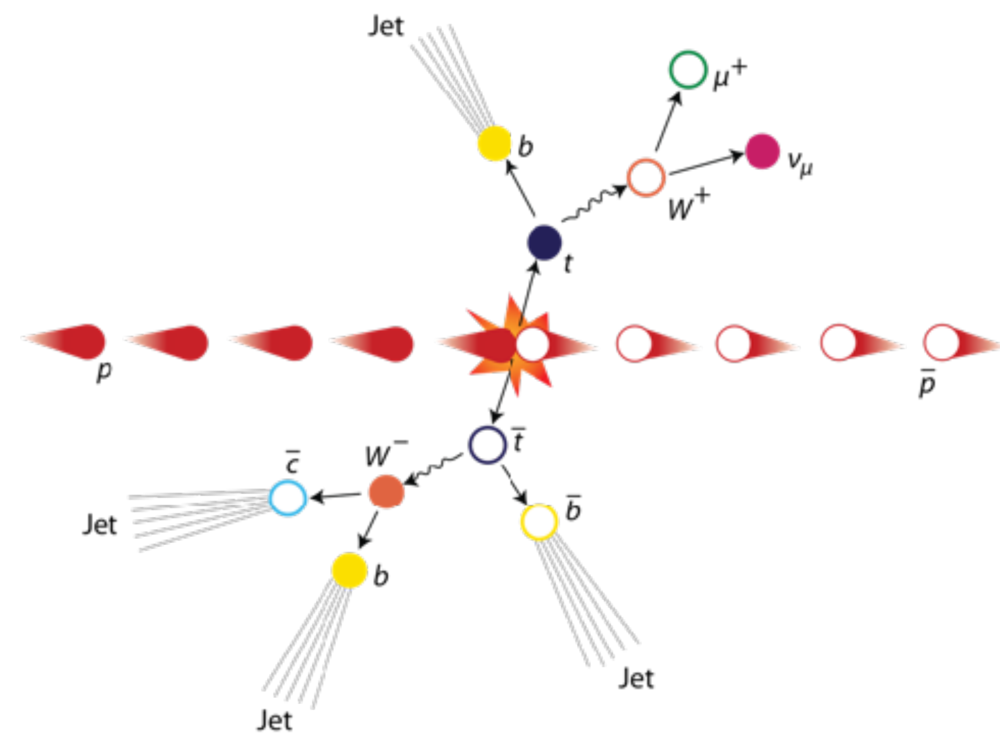


Data still slightly favors lower mass…

What about adding Coulomb resummation for reconstructed tops?

Reconstructed vs true top quarks

Another issue in the reconstruction/unfolding: there are two b-jets and two W's



Reconstructed vs true top quarks

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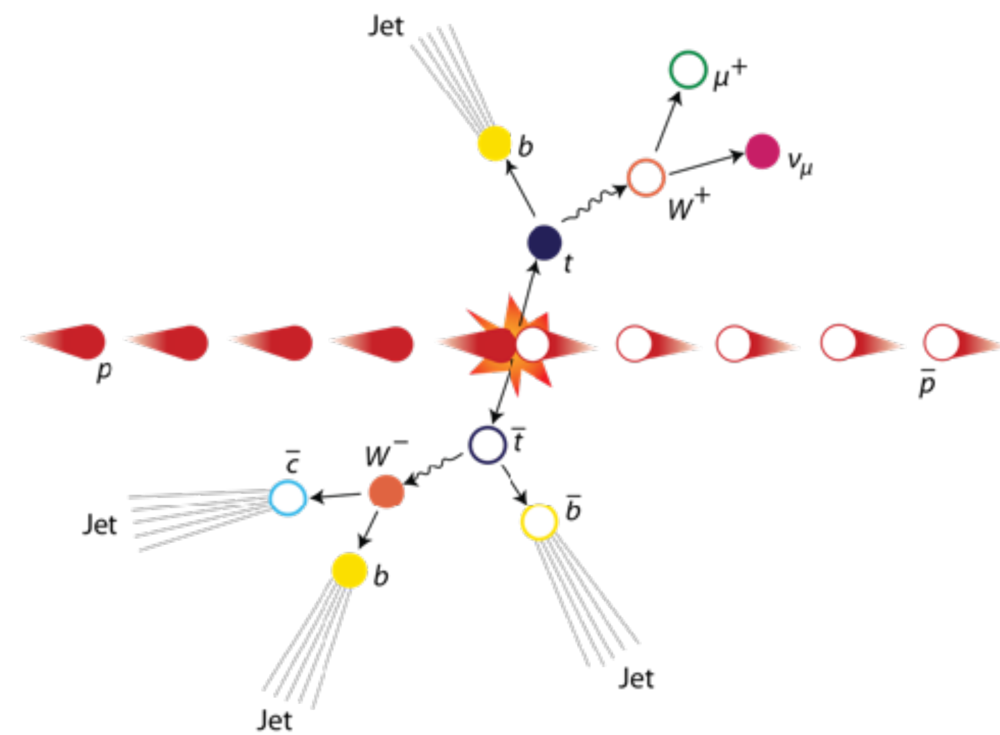
Usually one chooses to minimize

$$|m_{Wb1} - m_t| + |m_{Wb2} - m_t|$$



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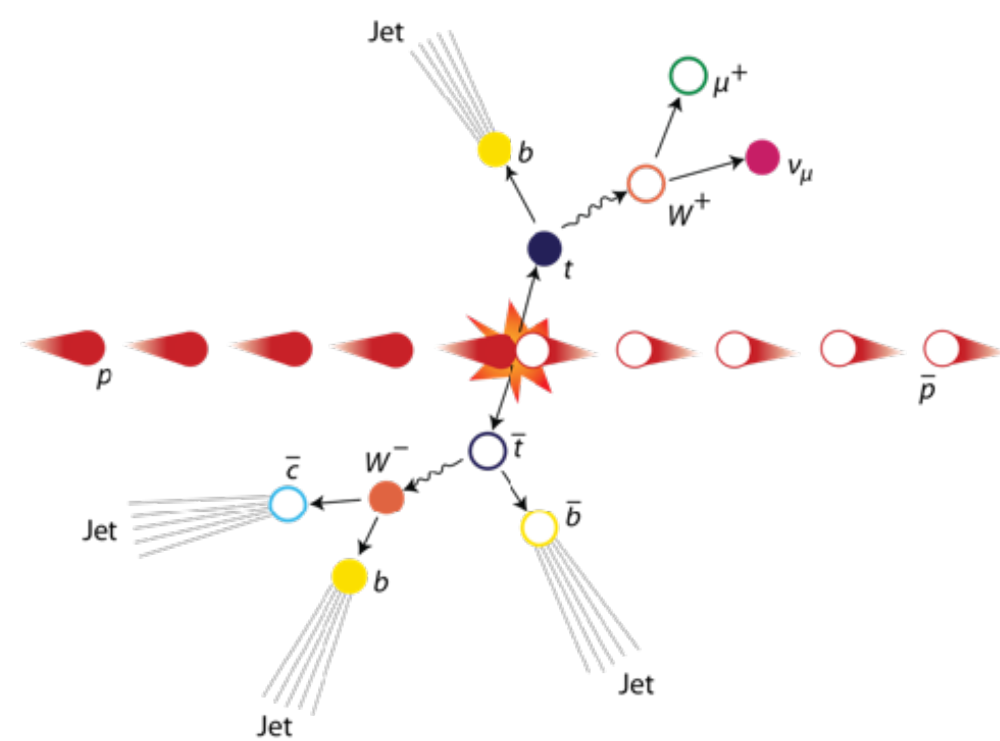

Usually one chooses to minimize

$$|m_{Wb1} - m_t| + |m_{Wb2} - m_t|$$

m_t enters the reconstruction (and also the unfolding procedure)

May have an impact on top quark mass extraction!

Beyond Narrow-Width-Approximation

Ideally one would like to study the invariant mass of $b\bar{b}W^+W^-$ without NWA; difficult when aiming for high precision (NNLO and/or resummation)

Beyond Narrow-Width-Approximation

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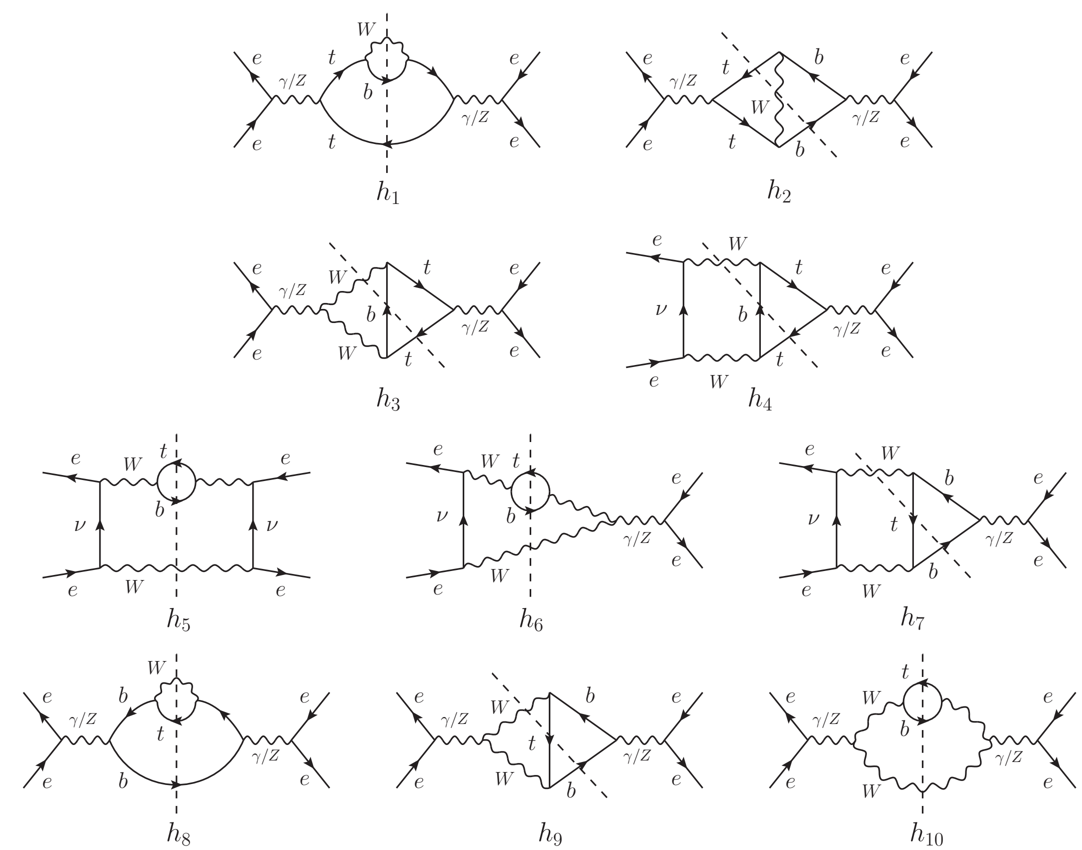
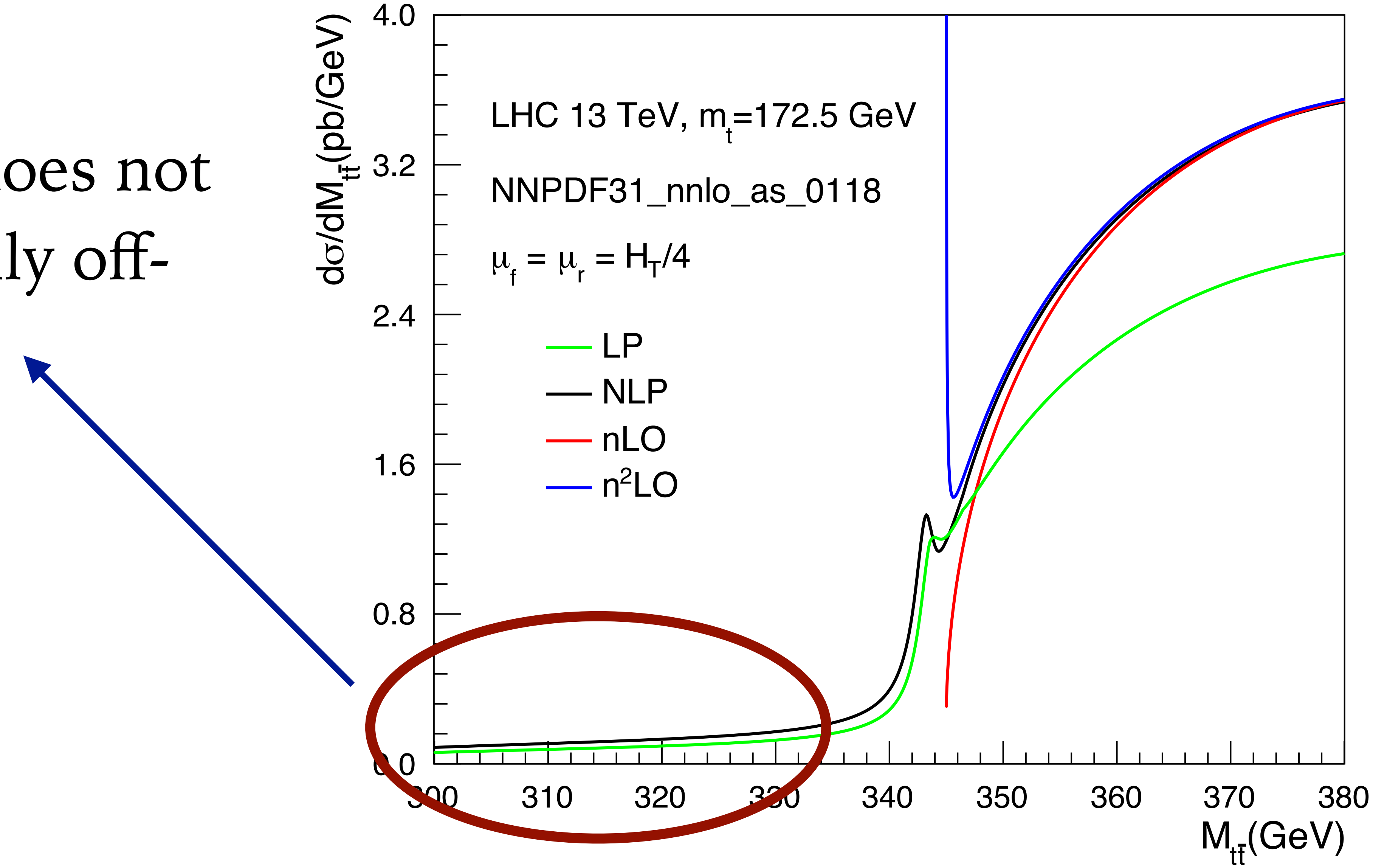
Existing studies mostly restricted to e^+e^- collisions



Beneke et al. 2004; Hoang, Reisser 2004; Beneke et al. 2010; Beneke et al. 2017; Bach et al. 2017; …

Beyond Narrow-Width-Approximation

The NWA also does not work in the highly off-shell region



Need to study Coulomb effects without NWA!

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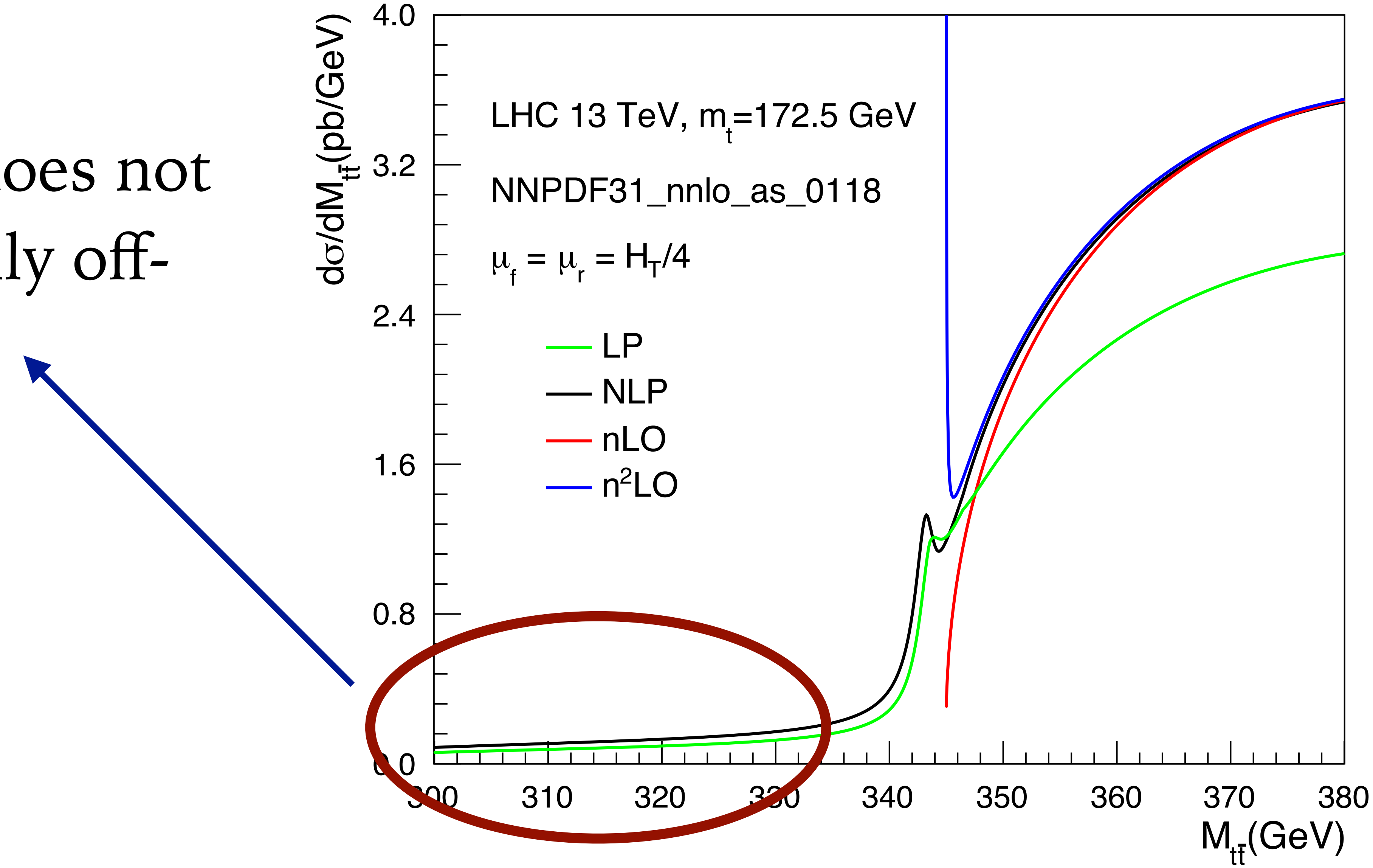

Need to study Coulomb effects without NWA!

Would be interesting to see how this affects top quark mass extraction…

Summary and outlook

- The top quark mass, as an important parameter, needs to measured to high precisions using different methods
- The indirect measurement is highly sensitive to the low-$M_{t\bar{t}}$ threshold region, where various issues may affect the outcome
  - We reanalyzed the Coulomb effects in the threshold region and found that they lead to better compatibilities between the extracted top quark mass and the world average
  - There are also issues in the reconstruction and unfolding which seem to have even bigger impacts
- Needs to assess the Coulomb effects at a more exclusive level in the future