

# B-meson production in $t_{t\bar{t}}$ events at NNLO in QCD

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Based on work in progress with:  
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# Some “Prehistory”

- ✓ This talk is a great example about the value of the Top20xx Workshops 😊
- ✓ Last year (at Top2019) a new results by ATLAS was presented:

“Determination of the top quark mass with soft muons”

ATLAS CERN-THESIS-2020-105 (2020)

- ✓ After extensive and very useful discussions with ATLAS and CMS people it became clear that such a measurement is perspective but needs improved modeling from the theory side.
- ✓ So we offered to help and started the effort I’ll describe in the following...

P.S. The subject of the talk is much broader than the (Indico) title indicates 😊

# What is b-fragmentation?

## ✓ What is b-fragmentation?

- The non-perturbative process of formation of B-flavored hadrons (most of the time mesons) from QCD partons.

## ✓ It is process independent as long as QCD factorization applies, i.e. there is a hard scale. Examples:

- For open B production:  $P_T$  of the B
- B production in top decay:  $m_{\text{top}}$
- B production at LEP (Z-pole):  $m_Z$

## ✓ How do we describe b-fragmentation? Two approaches:

- Fragmentation as used in MC event generators: uses a model that is tuned to data which 'decides' how at low scales the partons resulting from the shower are organized into hadrons. This is the case for heavy as well as light hadrons.
- **Analytic approach (used in this work)**: the transition  $b \rightarrow B$  at low energy is described in terms of an explicit non-perturbative function "Fragmentation function". This function is the exact equivalent of pdfs in the description of collider processes. It is also:
  - Non-perturbative
  - Process independent
  - Extracted from experiment (typically  $e^+e^-$ ; will discuss LHC later on)

## What is b-fragmentation?

- ✓ The essence of the non-perturbative aspect was understood long ago (late 1970's)

Kartvelishvili, Likhoded, Petrov (1978)  
Peterson, Schlatter, Schmitt, Zerwas (1983)  
...

- ✓ The heavy-flavored hadron B is produced at a scale  $\mu \sim m_B$  by the non-perturbative fragmentation of the b-quark  $D_{b \rightarrow B}(z)$ .
  - Here z is the fraction of b's momentum carried by B:  $p_B = z p_b$ .
- ✓ The description of b-quark production down to scales  $\mu \sim m_B$  can be described in perturbation theory. The modern framework was laid down 30 years ago as is known as the ***Perturbative Fragmentation Function*** approach

Mele, Nason (1991)
- ✓ The idea is based on factorization and properly accounts for the separation of process-independent and process-dependent corrections as well as short- from long-distance physics.

## PFF formalism

- ✓ In the Perturbative Fragmentation Function approach an observable for a meson B can schematically be written as:

Mele, Nason (1991)

$$O_B \left( \frac{Q^2}{m^2}, \alpha_S \right) = O_b \left( \frac{Q^2}{m^2}, \alpha_S \right) \otimes D_{b \rightarrow B}^{\text{np}}(z)$$

Non-perturbative.  
Fit from data

- $m$ : mass of the b-quark
- $Q \gg m$ : a hard scale that depends on the process

$$O_b \left( \frac{Q^2}{m^2}, \alpha_S \right) = \sum_{i,j=\text{all massless partons}} C_i \left( \frac{Q^2}{\mu_F^2}, \alpha_S \right) \otimes E_{i \rightarrow j} \left( \frac{\mu_F^2}{\mu_0^2}, \alpha_S \right) \otimes D_{i \rightarrow b} \left( \frac{\mu_0^2}{m^2}, \alpha_S \right)$$

Perturbative cross-section  
for producing any parton 'i'

DGLAP evolution: for a parton 'i'  
produced at a high scale  $\mu_F$  to  
evolve via radiation to a parton 'j'  
at to low scale  $\mu_0 \sim m$

PFF: perturbative splitting of  
the parton 'j' into a heavy  
quark b. This happens at a  
low (perturbative) scale  
 $\mu_0 \sim m$

## PFF formalism: main features

- ✓ The formalism applies to the small-mass limit  $m \ll Q$ .
  - ✓ Power corrections  $(m/Q)^n$  are neglected
  - ✓ All logarithmic terms  $\text{Log}^n(m/Q)$  are included correctly, as well as mass independent terms
  - ✓ At large  $Q$  (example: large  $P_T$ ) the formalism correctly resums all collinear logs
  - ✓ It is not applicable at small  $Q$  (i.e. small  $P_T$ ) where missing corrections  $m/Q \sim 1$  become sizable
    - For this regime one needs to supplement the PFF predictions with dedicated fixed order calculations that contain all mass dependence (but no resummation).
- ✓ This matching is the basis for the so called FONLL approach at NLO

Cacciari, Greco, Nason, Oleari,... (1990's)

# Heavy flavor production: past work

✓ Many FONLL applications exist:

- $e^+e^-$
  - Hadron colliders (remember the b-saga at the Tevatron?)
  - DIS
  - Top decay
- Many new pheno applications of this technology; latest one: gluon pdf from LHCb data

Cacciari, Mangano, Nason (2015)

- Works on heavy flavor production by other groups

Binnewies, Kniehl, Kramer (1997 - )  
Kniehl, Kramer, Schienbein, Spiesberger (2004 - )  
Kniehl, Kramer, Moosavi Nejad (2012)

- The only existing NNLO application is in QED: electron spectrum in muon decay

Anastasiou, Melnikov, Petriello (2005)

# PFF formalism: “new” developments (i.e. beyond NLO)

- ✓ How to go beyond NLO?
- ✓ A number of ingredients are needed at NNLO (all already known):
  - PFF @ NNLO  
Melnikov, Mitov (2004)
  - Time-like NNLO splitting functions  
Mitov, Moch, Vogt (2006)  
Almasy, Moch, Vogt (2012)
  - $e^+e^-$  coefficient functions (for fits of the non-perturbative FF)  
Rijken, van Neerven (1996)  
Mitov, Moch (2006)
  - Fits of FF at NNLO (no applications yet!)  
Fickinger, Fleming, Kim, Mereghetti (2016)  
Salajegheh, Nejad, Khanpour, Kniehl, Soleymaninia (2019)
- ✓ The only missing piece [by far the most complicated one!] for a complete NNLO application are the coefficient functions at NNLO
  - This talk: First NNLO application:  $b$ -production in  $t\bar{t}_{\text{bar}}$  production at the LHC



# NNLO corrections to B production in $t\bar{t}$ -production

- ✓  $t\bar{t}_{\text{bar}}$  is the natural first application in view of top mass applications (mainly using  $m_{B\ell}$ )  
Kharchilava (1999)  
ATLAS CERN-THESIS-2020-105 (2020)
- ✓ Past work at NLO
  - Top decay  
Cacciari, Corcella, Mitov (2001)  
Corcella, Drollinger (2005)  
Corcella, Mescia (2009)  
Moosavi Nejad, Soleymaninia, Khorramian, Maktoubian, Balali, Abbaspour (2012 - )  
Kniehl, Kramer, Moosavi Nejad (2012)
  - Top production and decay in the NWA  
Biswas, Melnikov, Schulze (2010)
- ✓ In this work we:
  1. Produce NNLO fragmentation functions using previous NP FF extractions  
Cacciari, Nason, Oleari (2005)  
Fickinger, Fleming, Kim, Mereghetti (2016)
  2. Calculate NNLO corrections to massless parton production in  $t\bar{t}_{\text{bar}}$  production and decay
    - Use the STRIPPER framework Czakon (2010) with additional subtraction in final state
    - Upon convoluting (2.) with our FF (1.) we have a fully differential MC calculation for one B-hadron + anything else. Extension to other processes is straightforward.

## A note on the fragmentation function

✓ As a basis for our study we take the fragmentation function from

- In principle it is ideal for us since it is: Fickinger, Fleming, Kim, Mereghetti (2016)
    1. Extracted at NNLO (from  $e^+e^-$ )
    2. Soft gluon resummation (using SCET) at NNLL
    3. LHAPDF grids available from authors upon request
  - In practice there are some differences which do not allow us to reuse them directly:
    1. Only b-initiated contribution included (specifically, the Non-Singlet  $b-b_{\text{bar}}$ )
    2. DGLAP evolution is affected by soft-gluon resummation/matching. Does not match the factorization scale dependence of our partonic calculation.
- ✓ Our resolution: some ambiguity remains; we build two “variants” of it and compare them
1. Construct FF set with “correct” DGLAP evolution: take their  $b \rightarrow B$  FF, evaluate it at low scale (this maintains soft-gluon resummation for PFF), then evolve it upwards ourselves with standard DGLAP. Include all other partonic contributions  $i \rightarrow B$ .
  2. Reuse their resummed NS component at all scales and supplement it with the other partonic components evolved with “correct” DGLAP evolution.

## A note on the fragmentation function

- ✓ We have also produced LHAPDF grids based on the NLO NP FF from

Cacciari, Nason, Oleari (2005)

1. Implemented our own NLL soft-gluon resummation on top
2. Added NNLO DGLAP
3. Added all partonic channels

- In principle the perturbative part is equivalent to the one constructed recently in

Ridolfi, Ubiali, Zaro (2019)

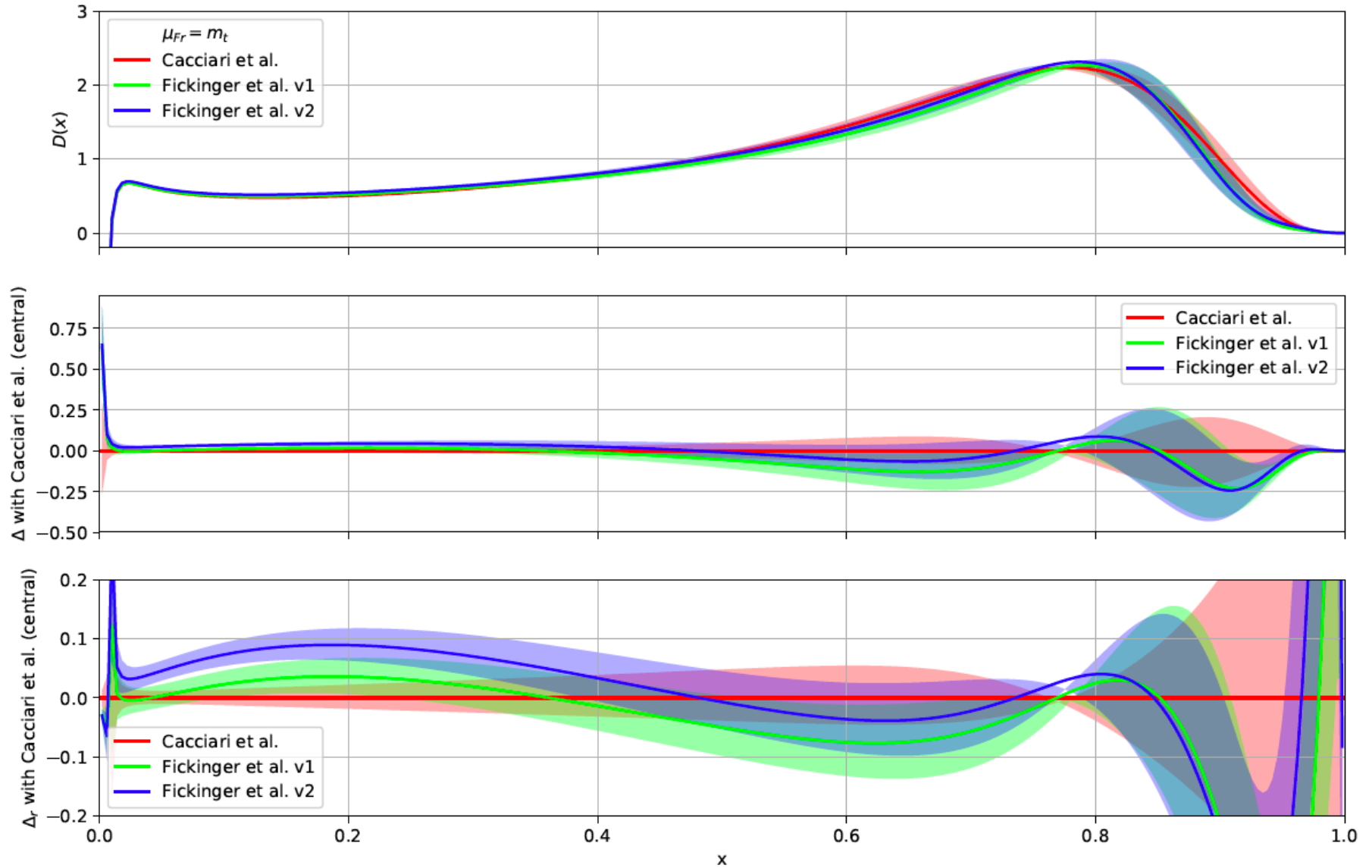
- ✓ We have not utilized the Non-Perturbative FF extracted in

Salajegheh, Nejad, Khanpour, Kniehl, Soleymaninia (2019)

- it does not utilize the Perturbative Fragmentation Function framework and is therefore not directly compatible with our calculation

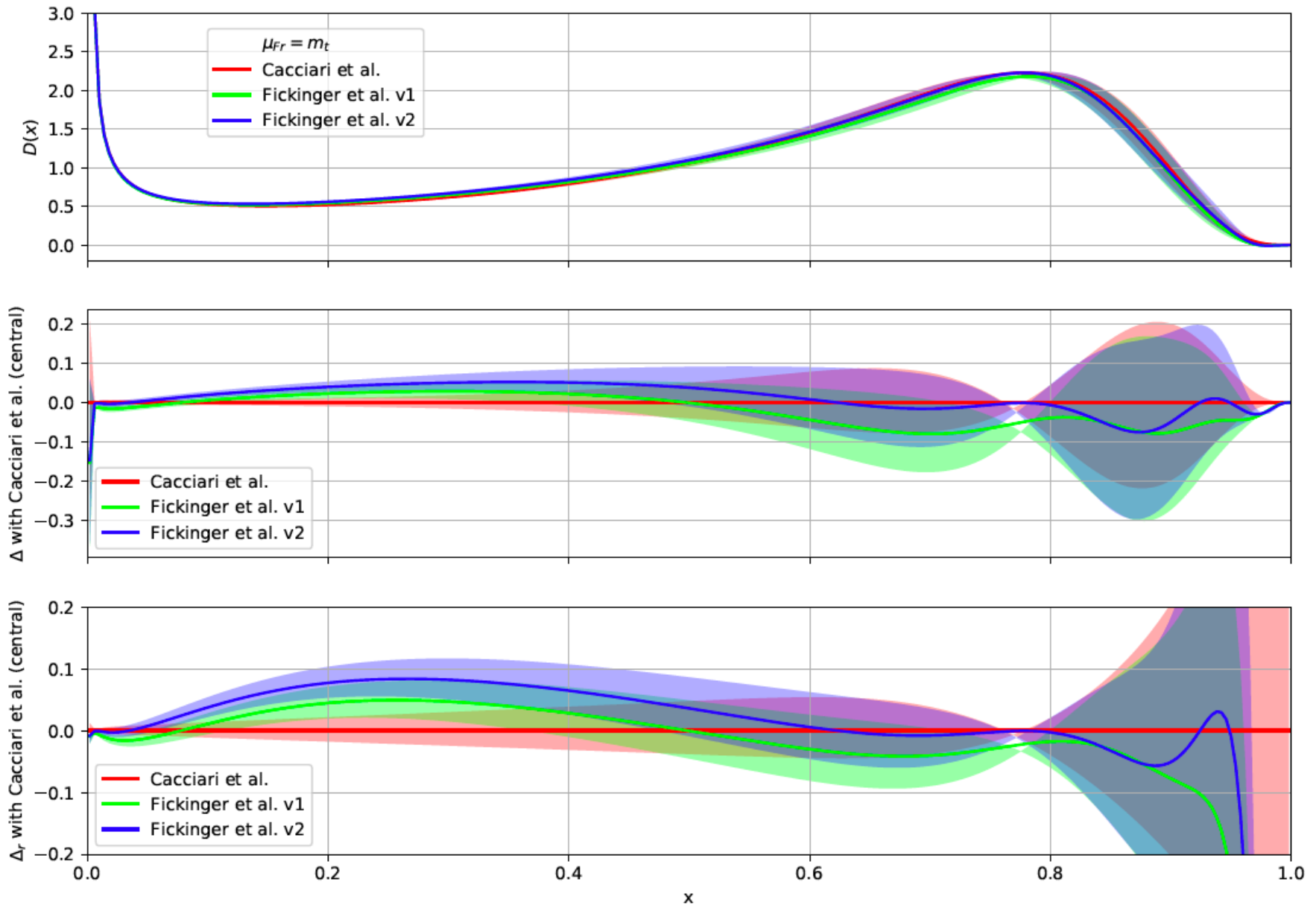
# Comparison of the fragmentation functions

- ✓ NLO comparison of our 3 fragmentation functions (bands: FF uncertainty)



# Comparison of the fragmentation functions

- ✓ NNLO comparison of our 3 fragmentation functions (bands: FF uncertainty)



## A note on the fragmentation function

✓ Few comments on the extracted FF's:

- Our predictions are for  $B+B_{\text{bar}}$
- Besides the differences in their definitions, the 3 FF's are compatible
- The low- $x$  region is not very relevant, especially here, since we do not include power corrections in  $m_b$
- The end-result strongly depends on its consistent implementation. In particular soft-gluon resummation at large  $x$ :
  - An application of a FF should match its extraction!
- Heavy-flavor FF's is a great place for practicing all the bells and whistles of soft-gluon resummation.
- An old issue: the available LEP data used for b-FF's is not separated into different hadron species, i.e. the predictions are for averaged B-production.
  - Can we use the LHC data to start producing new fits on b-fragmentation? More later.

## B-production in top decay at NNLO

✓ First application: B-production in the decay of unpolarized top quark

✓ Some defaults:

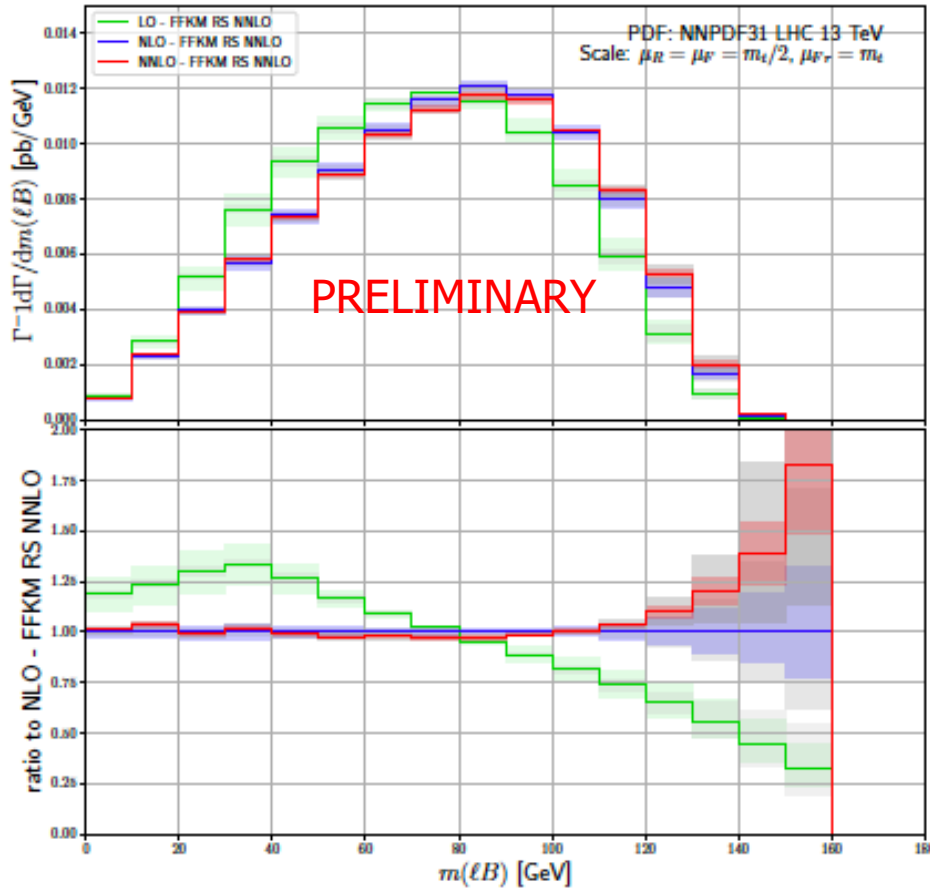
- $\mu_{\text{Fr}} = m_t$
- $\alpha_s(m_Z) = 0.118$
- Scale uncertainty: standard 7-point variation
- $m_t = 172.5 \text{ GeV}$

• For applications in top-pair production we will use:

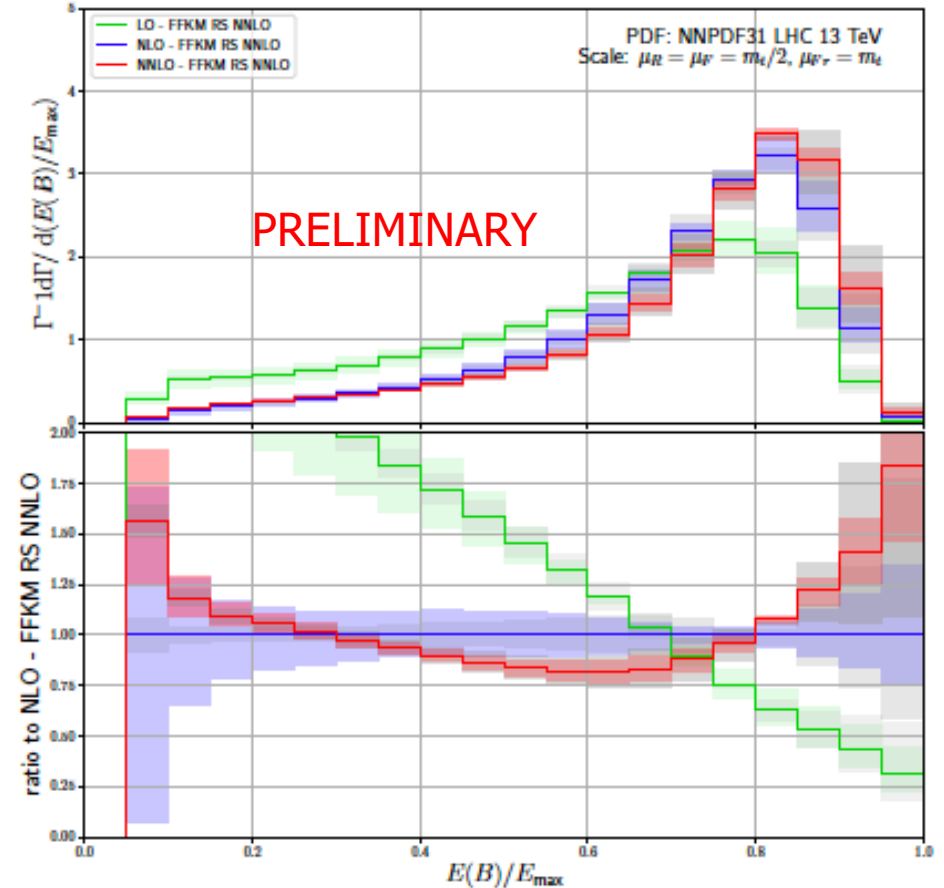
- $\mu_F = \mu_R = m_t/2$
- NNPDF3.1 pdf set
- B mesons inside b-jets:
  - all jets are defined as light flavorless jets.
  - We plot the distribution of the jet in which the B is found.
  - This makes sense since in the perturbative calculation b-quark (and  $b_{\text{bar}}$ ) enter on equal footing with all other partons.

# B-production in top decay at NNLO

- ✓ First application: B-production in the decay of unpolarized top quark
  - Color bands: scale variation (7 point)
  - Gray band: FF variation



Relevant for  $m_t$  studies



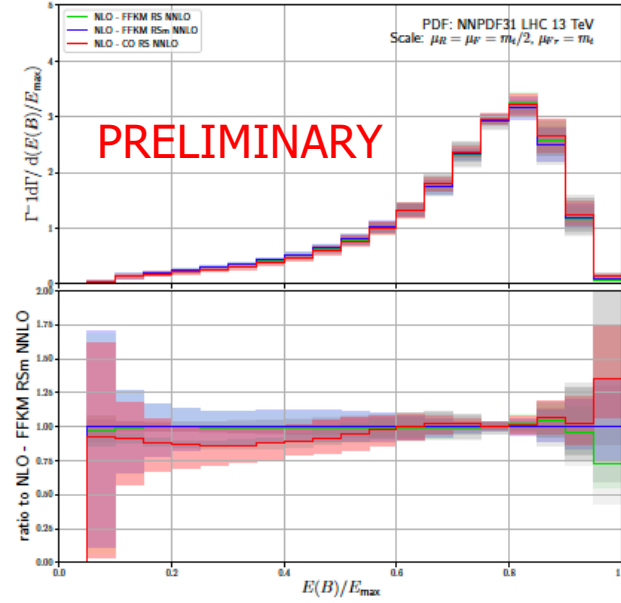
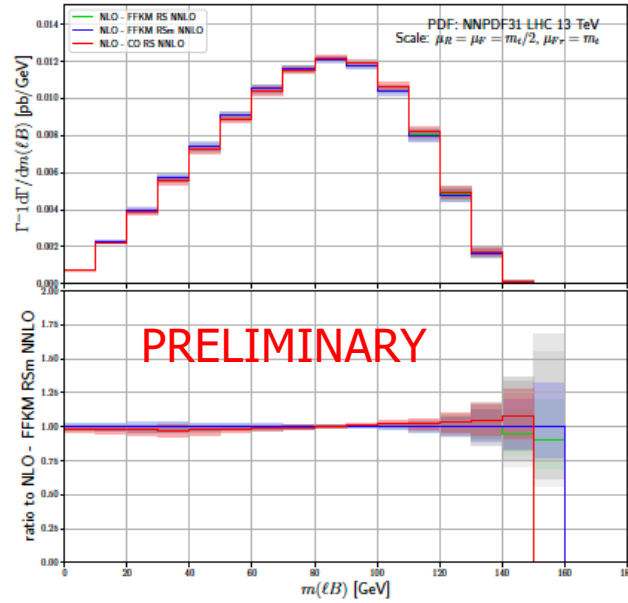
If it could be measured it would have been an ideal place to extract B-fragmentation functions



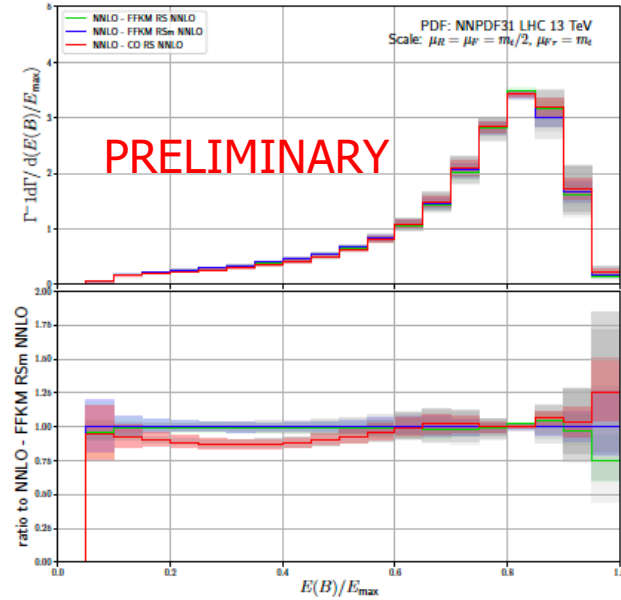
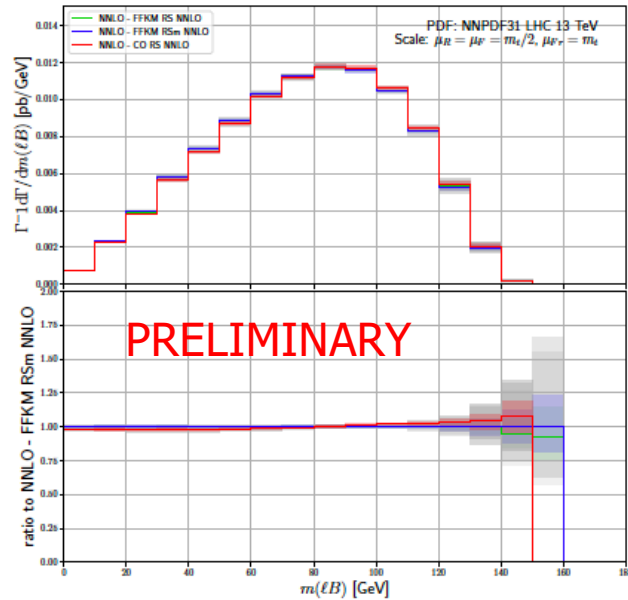
# B-production in top decay at NNLO

- ✓ Comparison between the 3 FF sets. Good potential sensitivity of  $E(B)$  for a fit of FF's

NLO:

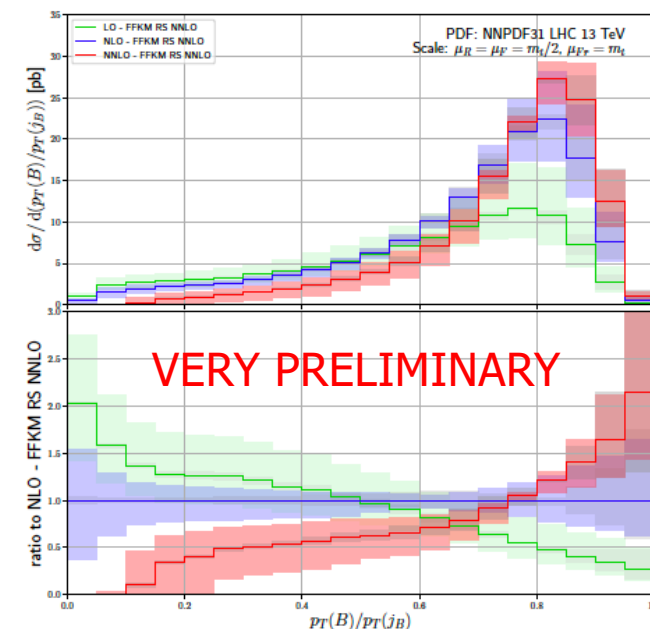
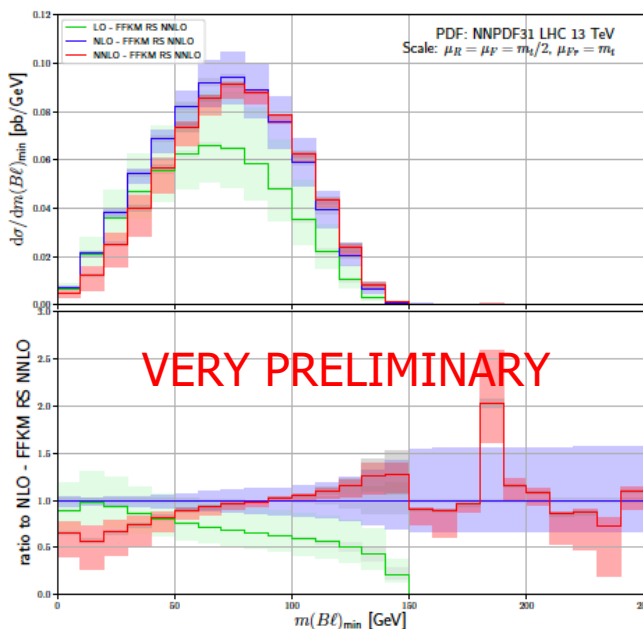
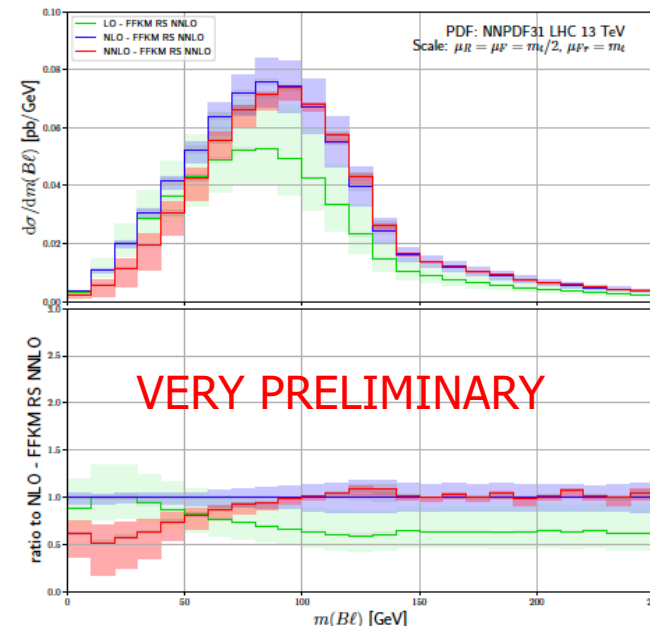
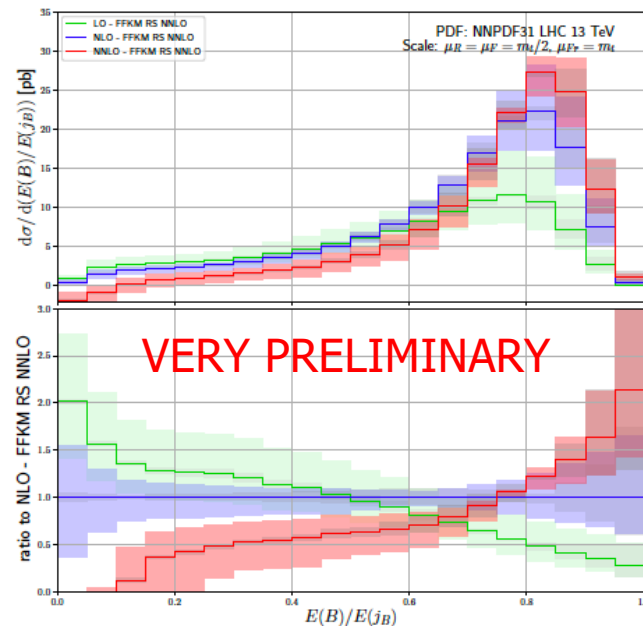


NNLO:



# B-production in $t\bar{t}b\bar{b}$ production + decay at NNLO

- ✓ Our default FF
- ✓ LO, NLO and NNLO
- ✓ Absolute distributions
- ✓ Color bands: 11 point scale variation
- ✓ Gray band: FF variation



Good potential for extracting FF from LHC B-inside-jet data!

All details are still very preliminary!

# B-production in $t\bar{t}b\bar{b}$ production + decay at NNLO

Comparison of  
the 3 FF sets

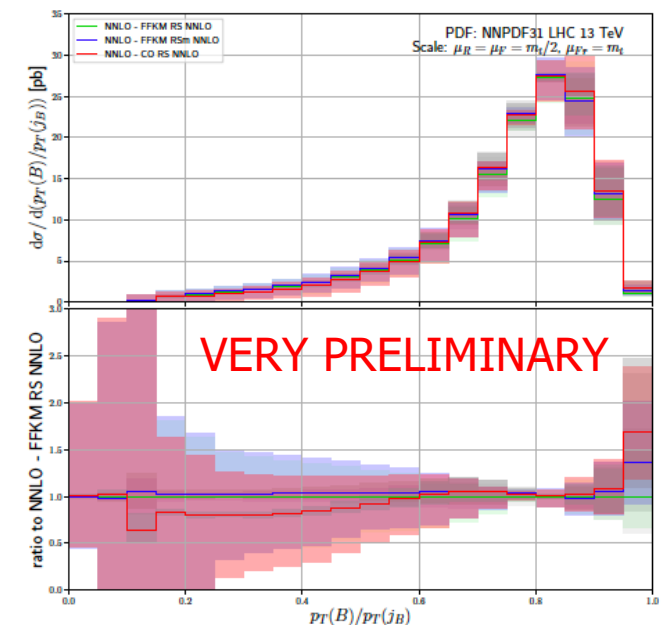
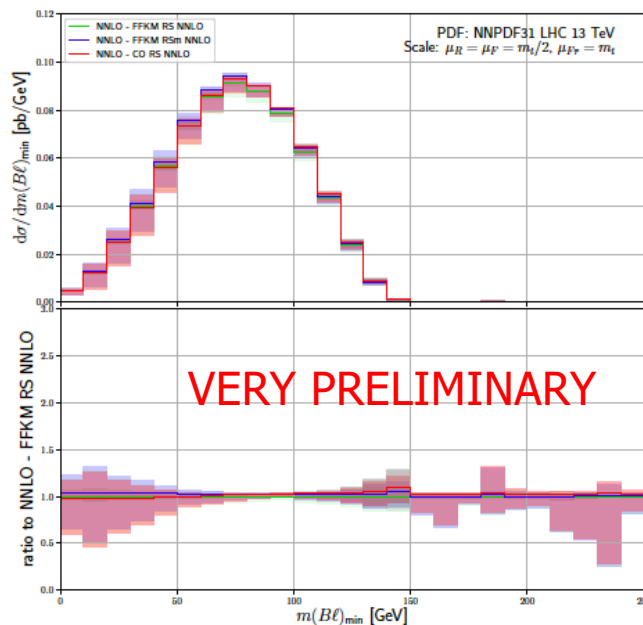
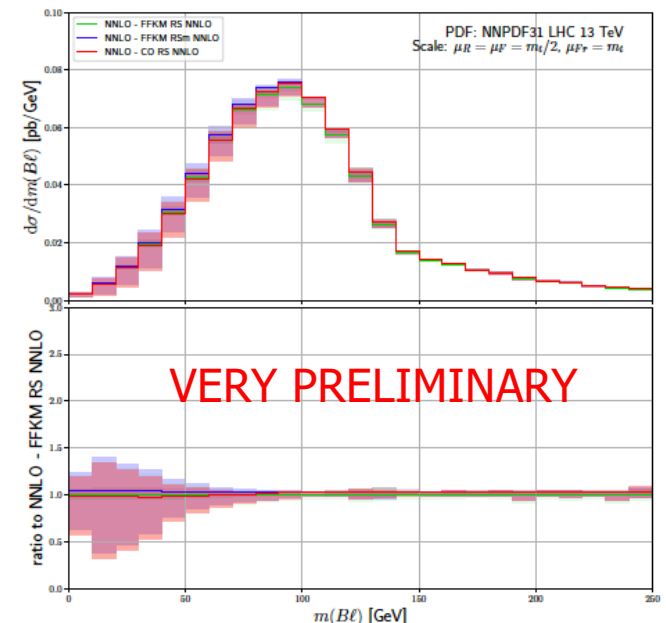
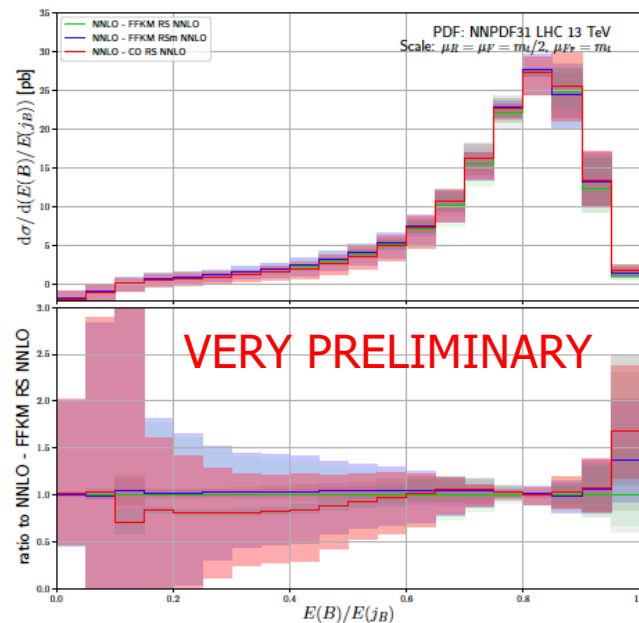
All details are still  
very preliminary!

✓ LO, NLO and NNLO

✓ Absolute  
distributions

✓ Color bands: 11  
point scale  
variation

✓ Gray band: FF  
variation



## Conclusions

- ✓ After a long and illustrious “career” of NLO calculations to open HF production, here we have presented the first-ever calculation of identified B-meson production at hadron colliders in NNLO QCD
- ✓ First application: B-production in  $t\bar{t}$  events
  - Obvious immediate application:  $m_t$  determination
- ✓ Only phenomenological restriction: massless b. In principle this could be dealt with
- ✓ Potential future work:
  - Here we have studied final states with B-mesons. Of more interest are their decays:  $B \rightarrow J/\Psi$  or semileptonic decays. This could be included in principle?
  - B-fragmentation has long been recognized as a potential bottleneck. The LEP data becomes increasingly outdated relative to our current precision needs. Probably it is time to start thinking of using LHC data for this
    - Here we have noticed that using the B-meson-in-jet distribution may be an excellent place for such an extraction
  - Applications beyond  $t\bar{t}$  events
- ✓ Any requests or suggestions?