Local Anomaly Cancellation in Rank Extensions of the SM

by

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- Anomaly cancellation in the SM
- $U(1)$ gauge theory warm-up
- $su(3) \oplus su(2) \oplus u(1) \oplus u(1)$
- Neutral current $B-$anomalies
- Third Family Hypercharge Model

Cambridge Pheno Working Group

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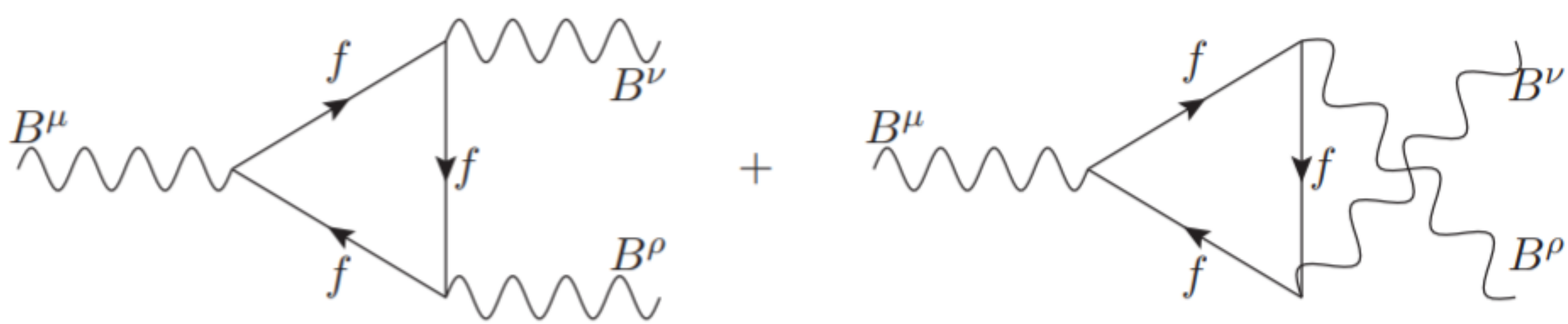
Quantum Field Theory
Anomalies in the SM



$$A \equiv \sum_{LH\ \ f_i} Y_i^3 - \sum_{RH\ \ f_i} Y_i^3$$

Also, replace two B fields by gravitons, gluons or $SU(2)$ W bosons. From now on, write all fields as left-handed.

$$Y^3: \qquad 0 = \sum_{j=1}^{3} \left(6Q_j^3 + 3U_j^3 + 3D_j^3 + 2L_j^3 + E_j^3\right),$$

$$3^2 Y: \qquad 0 = \sum_{j=1}^{3} \left(2Q_j + U_j + D_j\right),$$

$$2^2 Y: \qquad 0 = \sum_{j=1}^{3} \left(3Q_j + L_j\right),$$

$$\text{grav}^2 Y: \qquad 0 = \sum_{j=1}^{3} \left(6Q_j + 3U_j + 3D_j + 2L_j + E_j\right).$$

Note that there's no 'take one family'

Hypercharge anomaly cancellation

Deforming the SM to $SU(3)\times SU(2)\times\mathbb{R}_Y$, and allowing the hypercharges Y of the chiral fermionic fields to float, the combination of gauge ACC and gravitational ACC implies that the hypercharges must be quantised[1] (i.e. that ratios of hypercharges of different chiral fermions are commensurate). Conversely, if the hypercharges are quantised but otherwise free, the gauge ACC implies the gravitational ACC [2].

[1] Weinberg, *The Quantum Theory of Fields (1995), Cambridge University Press*

[2] Lohitsiri and Tong, `arXiv:1907.00514`

A Warm Up: $U(1)$

Pioneering solution to ACCs: Costa, Dobrescu, Fox, arxiv:1905.13729. n chiral fermions with charges z_i:

$$\begin{aligned} z_1^3 + \ldots + z_n^3 &= 0, \\ z_1 + \ldots + z_n &= 0. \end{aligned} \tag{1}$$

Given 2 solutions $\underline{x}$, $\underline{y}$, construct a third by "merger"

$$\{\underline{x}\} \oplus \{\underline{y}\} := \left(\sum_{i=1}^{n} x_i y_i^2 \right) \{\underline{x}\} - \left(\sum_{i=1}^{n} x_i^2 y_i \right) \{\underline{y}\}.$$

Want to find suitably general solutions $\underline{x}$, $\underline{y}$.

Example: even n

$$\{\underline{x}\} = \{l_1, k_1, \ldots, k_m, -l_1, -k_1, \ldots, -k_m\}$$
$$\{\underline{y}\} = \{0, 0, l_1, \ldots, l_m, -l_1, \ldots, -l_m\},$$
$$m = n/2 - 1 \geq 2, \qquad 1 \leq i \leq m$$

$\{\underline{x}\}$ and $\{\underline{y}\}$ are each vector-like solutions but it turns out that $\{\underline{x}\} \oplus \{\underline{y}\}$ is a new chiral solution.

$\{\underline{x}\} \oplus \{\underline{y}\}$ parameterises all solutions up to permutations. There is a *similar story* for odd n.

Geometric Understanding

In BCA, Gripaios, Tooby-Smith, `arXiv:1912.04804`, we provide a geometric understanding of this. First, note that each solution in $\mathbb{Q}$ is equivalent to one in $\mathbb{Z}$ by clearing denominators. Using gravitational anomaly cancellation, eliminate z_n to obtain the homogeneous cubic

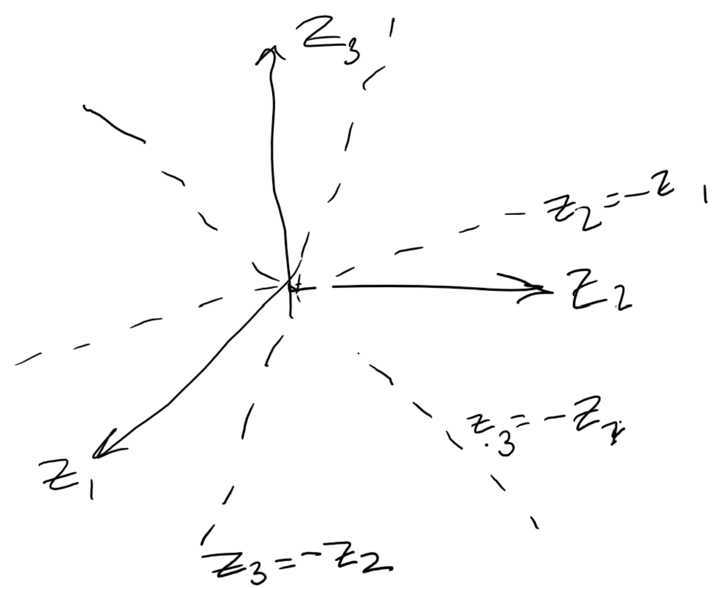
$$\sum_{i=1}^{n-1} z_i^3 - \left(\sum_{i=1}^{n-1} z_i\right)^3 = 0$$

defining a cubic hypersurface in $\mathbb{Q}^{n-1}$.

Special Surface

In fact, our cubic hypersurface is rather special: no purely cubic terms in any one variable: (add perms)

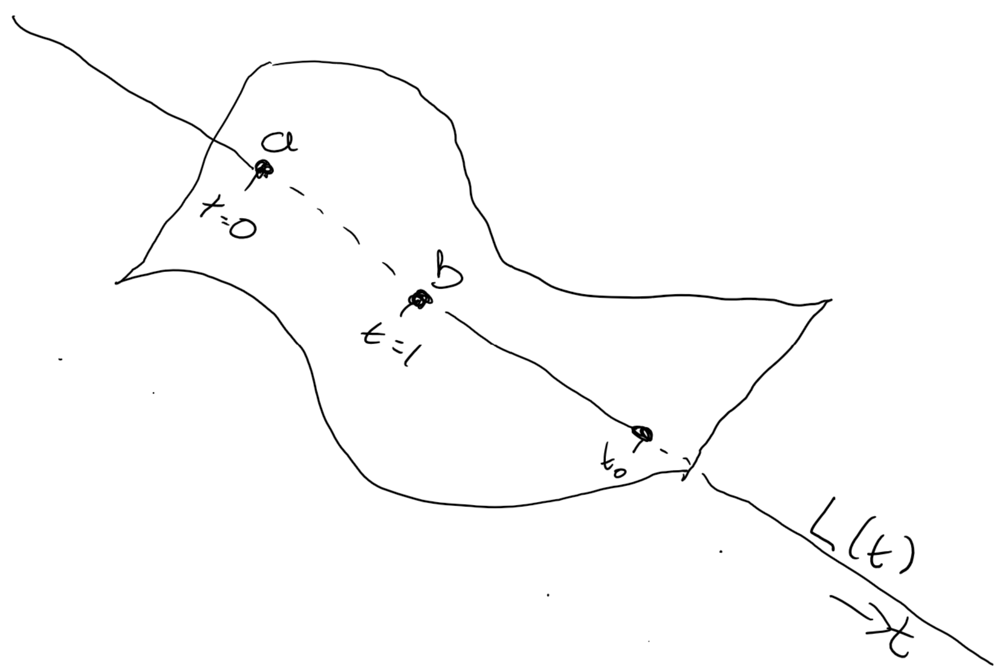
$n = 3$: $\underline{z} = [-a : 0 : a]$, ie three lines $z_3 = -z_1$, $z_2 = 0$



$n = 4$: $\underline{z} = [-x : -y : x : y]$, $x, y \in \mathbb{Q}$ ie three planes

$n > 4$: The Method of Chords[3]

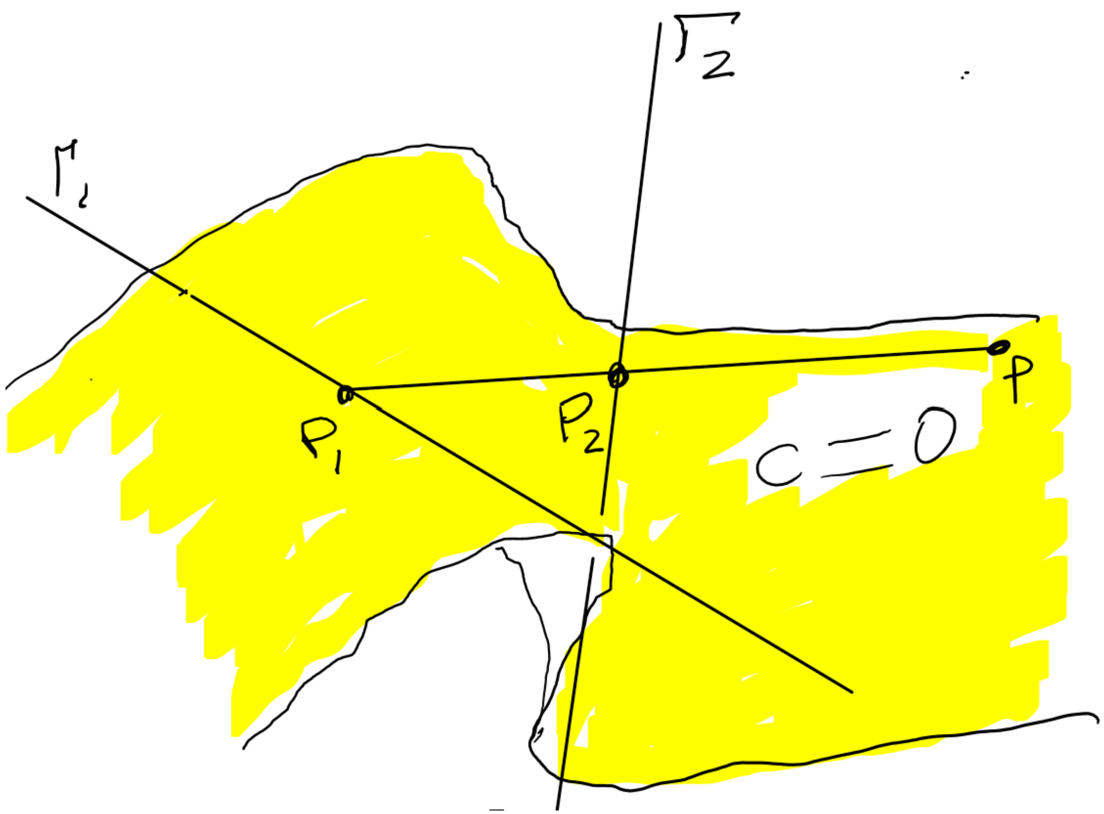
Rational cubic $c(z_1, z_2, z_3) = 0$. Put a line through 2 known intersections a, b: $L(t) = a + t(b - a)$. Along line, $c(L(t)) = kt(t-1)(t-t_0)$, where $k, t_0 \in \mathbb{Q}$.



[3]Newton, Fermat, C17th

Mordell's Theorem[4]

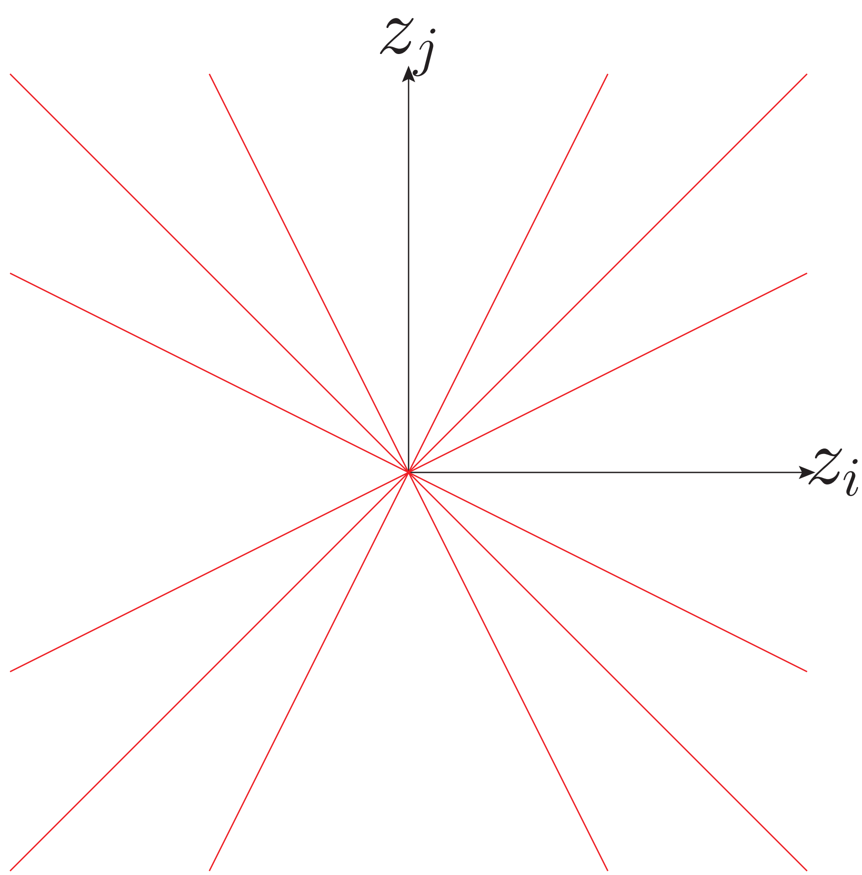
Skew Γ_1, Γ_2 in $c = 0$ $\Rightarrow$all rational points on c can be found this way.



[4]Mordell (1969) *Diophantine Equations*

Projective space

All solutions where z_i differ by a common multiple are physically equivalent so it's in $P\mathbb{Q}^{n-2}$:



In $\mathbf{P}\mathbb{Q}^{n-2}$

Because $\mathbb{Q}$ is a field, geometry works in $\mathrm{P}\mathbb{Q}^{n-2}$, as does the Method of Chords and Mordell's Theorem. We extend Mordell's theorem to an arbitrary cubic hypersurface X in $\mathrm{P}\mathbb{Q}^{n-2}$: Γ_1, Γ_2 are disjoint planes of dimensions $(n-3)/2$ (if n is odd). Every $p \in X$ lies on a chord joining a point in Γ_1 to one in Γ_2.

The merger is exactly this construction. The initial vector-like solutions parameterise Γ_1, Γ_2.

Rank++

Extend SM Lie algebra to $su(3)\oplus su(2)\oplus u(1)\oplus u(1)$: our analysis will cover extensions for which this is a subgroup eg non-abelian extensions.
Motivations include unification, models of dark matter, $(g-2)_\mu$, axions, fermion mass hierarchies, Z' explanations for apparent disagreements with SM predictions in measurements of $B-$meson decays.

$$3^2X:\ \ 0=\sum_{j=1}^{3}(2Q_j+U_j+D_j)\,,$$

$$2^2X:\ \ 0=\sum_{j=1}^{3}(3Q_j+L_j)\,,$$

$$Y^2X:\ \ 0=\sum_{j=1}^{3}(Q_j+8U_j+2D_j+3L_j+6E_j)\,,$$

$$YX^2:\ \ 0=\sum_{j=1}^{3}\left(Q_j^2-2U_j^2+D_j^2-L_j^2+E_j^2\right)\,,$$

$$\mathsf{grav}^2X:\ \ J:=\sum_{i=1}^{n}x_i=-\sum_{j=1}^{3}(6Q_j+3U_j+3D_j+2L_j+E_j),$$

$$X^3:\ \ M+J^3:=\sum_{i=1}^{n}x_i^3=-\sum_{j=1}^{3}\left(6Q_j^3+3U_j^3+3D_j^3+2L_j^3+E_j^3\right).$$

1. Find solutions for SM fermions charges from first 4

2. Apply $GL(3,\mathbb{Z})$ transformation to species F:
   $F_+ := F_1 + F_2 + F_3$, $F_\alpha := F_1 - F_2$, $F_\beta := F_2 + F_3$.

3. Linear equations become
   $D_+ = -2Q_+ - U_+,\ L_+ = -3Q_+,\ E_+ = 2Q_+ - U_+.$

4. Quadratic is a solveable homogeneous diophantine equation of degree 2 in the 12-tuple

$$X := (Q_+, U_+, Q_\alpha, Q_\beta, U_\alpha, U_\beta, D_\alpha, D_\beta, L_\alpha, L_\beta, E_\alpha, E_\beta).$$

$X^T H X = 0$ defines hypersurface $\Gamma \in P\mathbb{Q}^{11}$.

$$H = \begin{pmatrix} 0 & 0 & -2 & -4 & 0 & 0 & 4 & 8 & -6 & 0 & -4 & -8 \\ & 0 & 0 & 0 & 4 & 8 & 2 & 4 & 0 & 0 & 2 & 4 \\ & & 2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & -4 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & -12 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & & 2 & 3 & 0 & 0 & 0 & 0 \\ & & & & & & & 6 & 0 & 0 & 0 & 0 \\ & & & & & & & & -2 & 3 & 0 & 0 \\ & & & & & & & & & 6 & 0 & 0 \\ & & & & & & & & & & 2 & 3 \\ & & & & & & & & & & & 6 \end{pmatrix}.$$

Quadratic

$$X^T H X = 0$$

Consider lines $L = \alpha\tilde{X} + \beta R$ through a known solution $\tilde{X} \in P\mathbb{Q}^{11}$, where $R \in P\mathbb{Q}^{11}$, and $[\alpha : \beta] \in P\mathbb{Q}^1$: (eg $\tilde{X}$ has all zero except $Q_\alpha = L_\alpha = 1$)

$$\beta(2R^T H\tilde{X}\alpha + R^T HR\beta) = 0.$$

Using same trick as before

$$X = (R^T HR)\tilde{X} - 2(R^T H\tilde{X})R.$$

Solution In Detail

$$Q_\alpha = 2R_{Q_\alpha}\Lambda + \Sigma, \qquad L_\alpha = 2R_{L_\alpha}\Lambda + \Sigma,$$

where $\Sigma = R^T H R$ and

$$\Lambda = (8_{R_{Q+}} + 2R_{L_\alpha} + 3R_{L_\beta} - 2R_{Q_\alpha} - 3R_{Q_\beta}).$$

All other charges X are $2R_X\Lambda$, where $R_X \in \mathbb{Z}$.

$$R := \{R_{Q_+}, R_{U_+}, R_{Q_\alpha}, R_{Q_\beta}, R_{U_\alpha}, R_{U_\beta}, R_{D_\alpha}, R_{D_\beta}, R_{L_\alpha}, R_{L_\beta}, R_{E_\alpha}, R_{E_\beta}\}.$$

Then, invert the $GL(3, \mathbb{Z})$.

SM Singlets

Adding n SM singlets with $U(1)$ charges decouples the last two equations. Results:

- We can always find a full solution for $n \geq 5$, eg: $(M/6 \in \mathbb{Z})$ $\{M/6+1,\ M/6-1,\ -M/6,\ -M/6, J\}$
- For lower n, we give restrictions on M, J for when a solution exists.

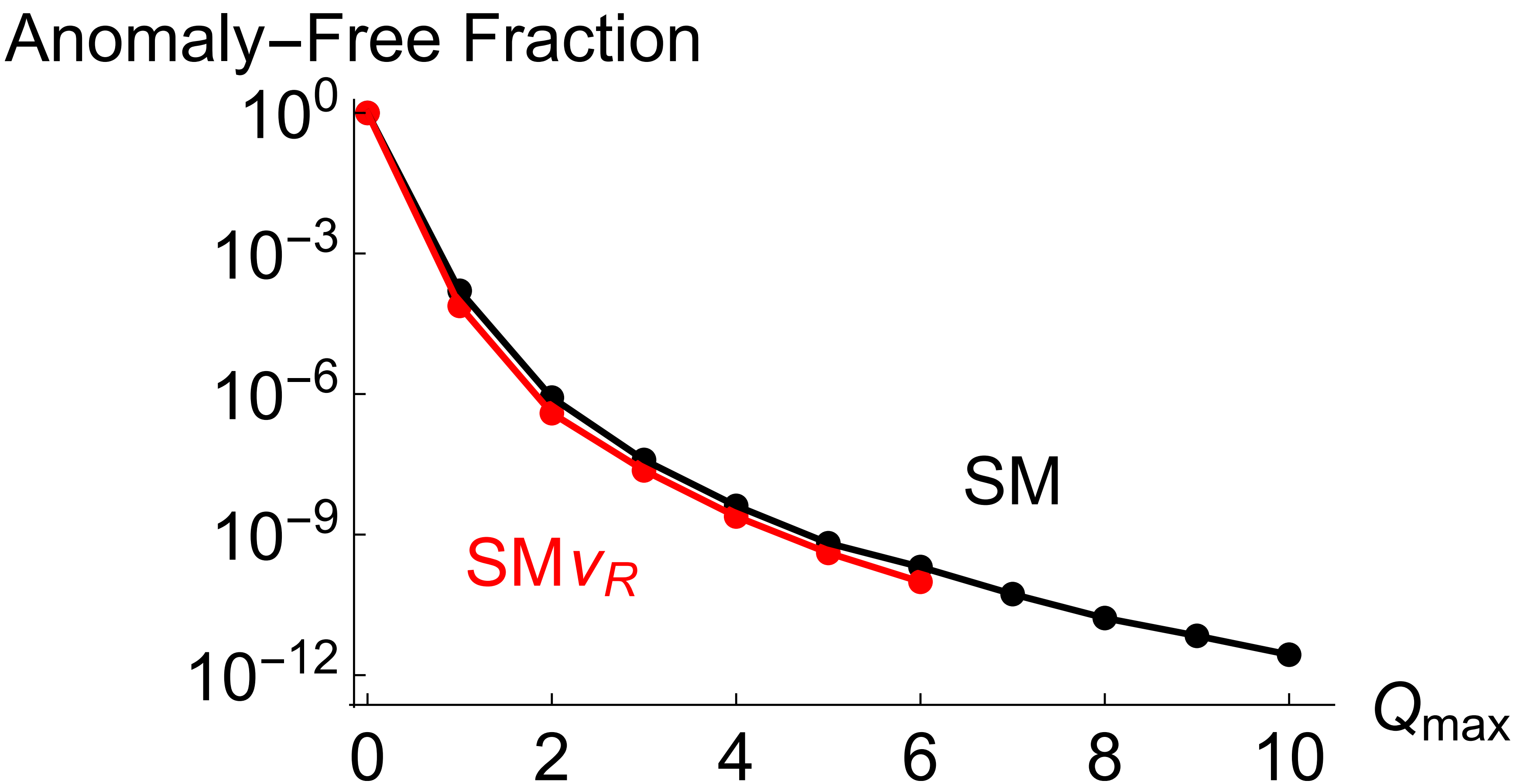
However, annoyingly, we only have a partial solution for the full 6 equations together.

Summary of Solutions

For $n = 0$, we begin with 15 SM charges and 4 anomaly equations reduce these to an 11-dimensional quadratic surface of solutions, extending out to infinity, but becoming more sparse further from the origin.

To find solutions for fixed $n \leq 3$ and charges between -10 and 10, we did a numerical scan: BCA, Davighi, Melville, `arXiv:1812.04602`.

An Anomaly-Free Atlas is available for public use:
`http://doi.org/10.5281/zenodo.1478085`



Anomaly–Free Fraction
10^{0}
10^{-3}
10^{-6}
10^{-9}
10^{-12}
SM
SMν_R
0
2
4
6
8
10
Q_{max}

$Q_{\max}$	**Solutions**	Symmetry	Quadratics	Cubics	Time/sec
1	**8**	8	32	8	0.0
2	**22**	14	1861	161	0.0
3	**82**	32	23288	1061	0.0
4	**251**	56	303949	7757	0.0
5	**626**	114	1966248	35430	0.0
6	**1983**	144	11470333	143171	0.2
7	**3902**	252	46471312	454767	0.6
8	**7068**	336	176496916	1311965	2.2
9	**14354**	492	539687692	3310802	6.7
10	**23800**	582	1580566538	7795283	20

Solutions: $n = 0$

Q	Q	Q	ν	ν	ν	e	e	e	u	u	u	L	L	L	d	d	d
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	-1	0	1
0	0	0	0	0	0	-1	0	1	0	0	0	-1	0	1	0	0	0
0	0	0	0	0	0	-1	0	1	-1	0	1	0	0	0	-1	0	1
-1	0	1	0	0	0	0	0	0	0	0	0	-1	0	1	0	0	0
-1	0	1	0	0	0	0	0	0	-1	0	1	0	0	0	-1	0	1
-1	0	1	0	0	0	-1	0	1	-1	0	1	0	0	0	0	0	0
-1	0	1	0	0	0	-1	0	1	-1	0	1	-1	0	1	-1	0	1

eg: $Q_{max} = 1.$ Charges within a species are listed in *increasing order*.

Known Solutions

Model	Q	Q	Q	ν	ν	ν	e	e	e	u	u	u	L	L	L	d	d	d
$L_\mu - L_\tau$	0	0	0	-1	0	1	-1	0	1	0	0	0	-1	0	1	0	0	0
TFHM	-1	0	0	0	0	0	0	0	6	-4	0	0	0	0	3	0	0	2
$B_3 - L_3$	-1	0	0	0	0	3	0	0	3	-1	0	0	0	0	3	-1	0	0

Caveat?

Anomalies can be cancelled by a Wess-Zumino term, a higher dimension $\mathcal{L}$ operator of topological origin. These can eg be obtained by integrating out heavy states: relevant for singlets.

Generic ones are hard to generate whilst making the relevant heavy states heavy from $U(1)$ spontaneous breakdown.

Other Constraints

Consider perturbativity:

$$\frac{d\ln g}{d\ln\mu} = \frac{g^2\sum_{i\in\chi\cup V} X_i^2}{24\pi^2} < 1$$

$$\Leftrightarrow g < \frac{2\pi\sqrt{6}}{\sqrt{\sum_{i\in\chi\cup V} X_i^2}}.$$

Growing evidence that weakest force in a consistent theory must be gravity. Take field of mass largest Q/m ratio, *WGC* $\Rightarrow$

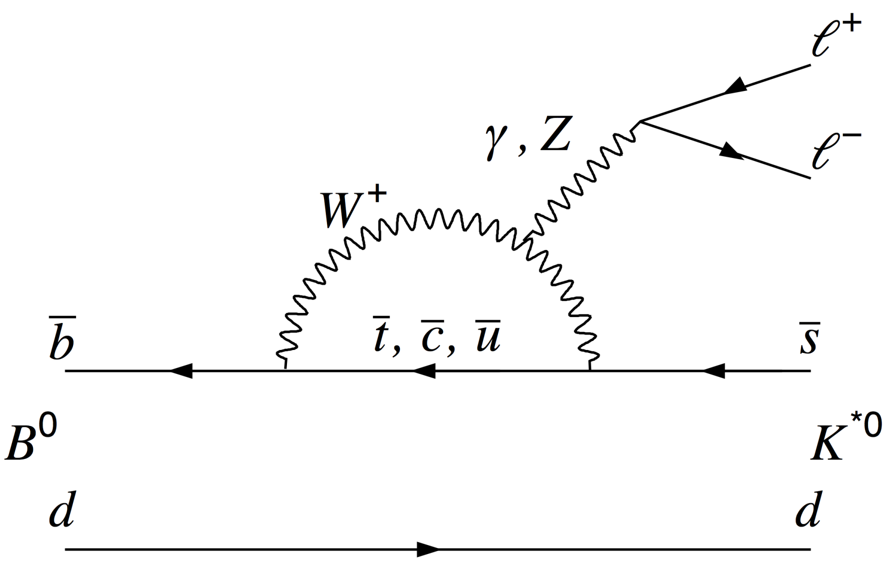
$$g > \frac{m}{QM_P}.$$

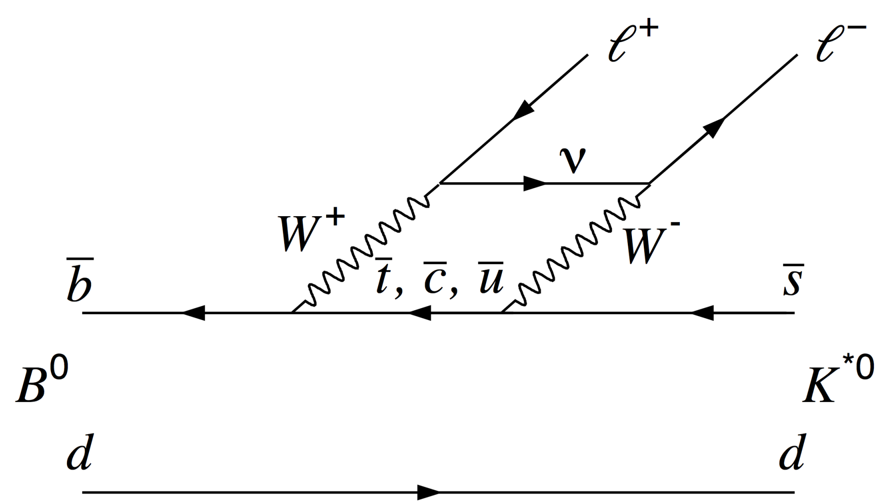
Strange b Activity

$R_K^{(*)}$ in Standard Model

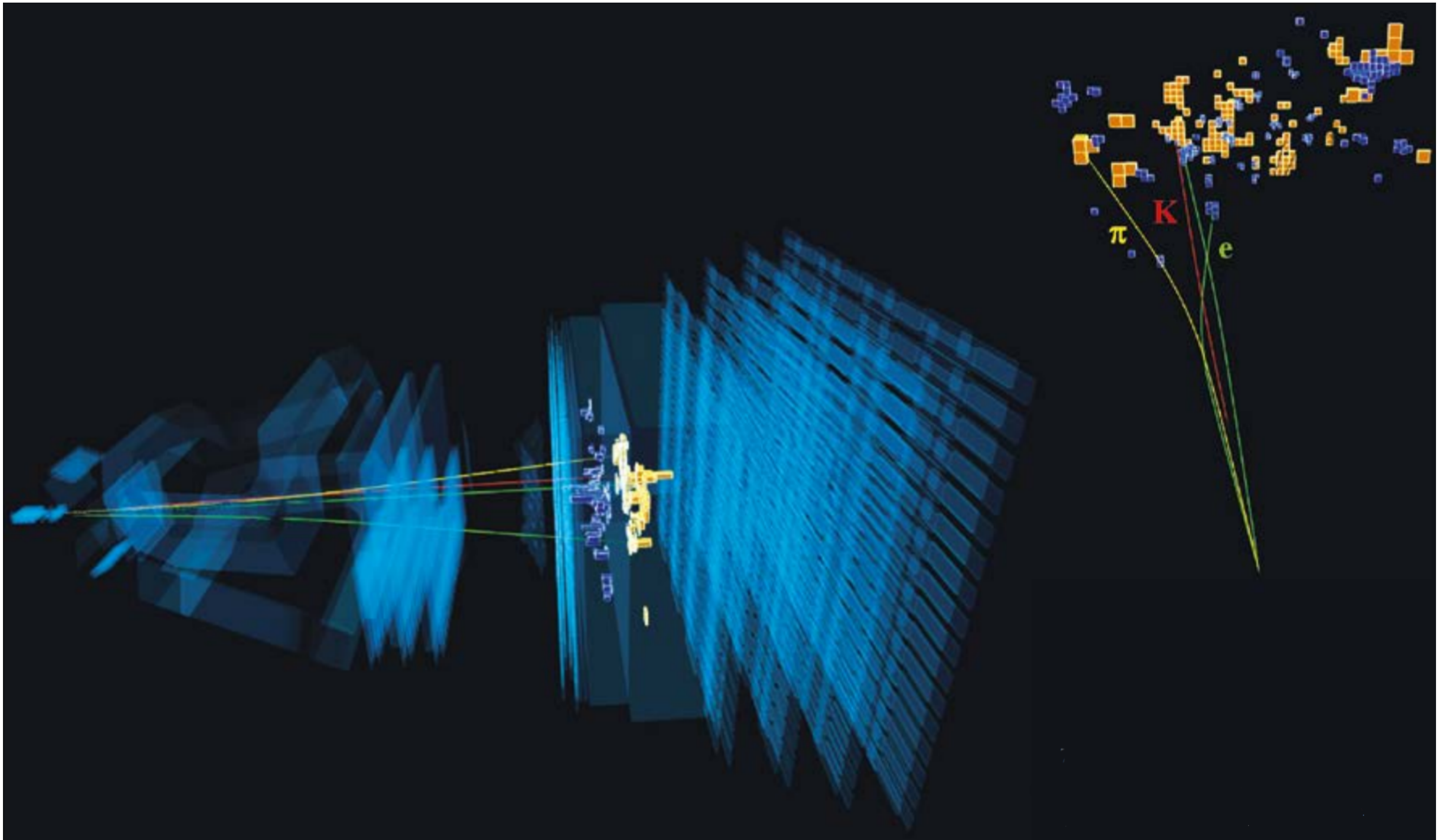
$$R_K = \frac{BR(B \to K\mu^+\mu^-)}{BR(B \to Ke^+e^-)}, \qquad R_{K^*} = \frac{BR(B \to K^*\mu^+\mu^-)}{BR(B \to K^*e^+e^-)}.$$

These are rare decays (each BR$\sim \mathcal{O}(10^{-7})$) because they are absent at tree level in SM.





LHCb $B^0 \to K^{0*}e^+e^-$ Event[5]

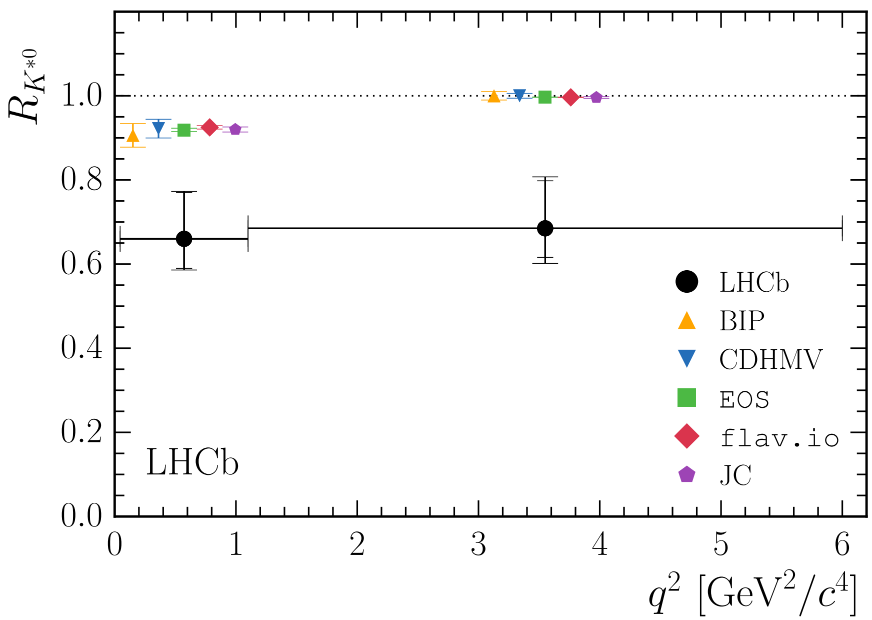


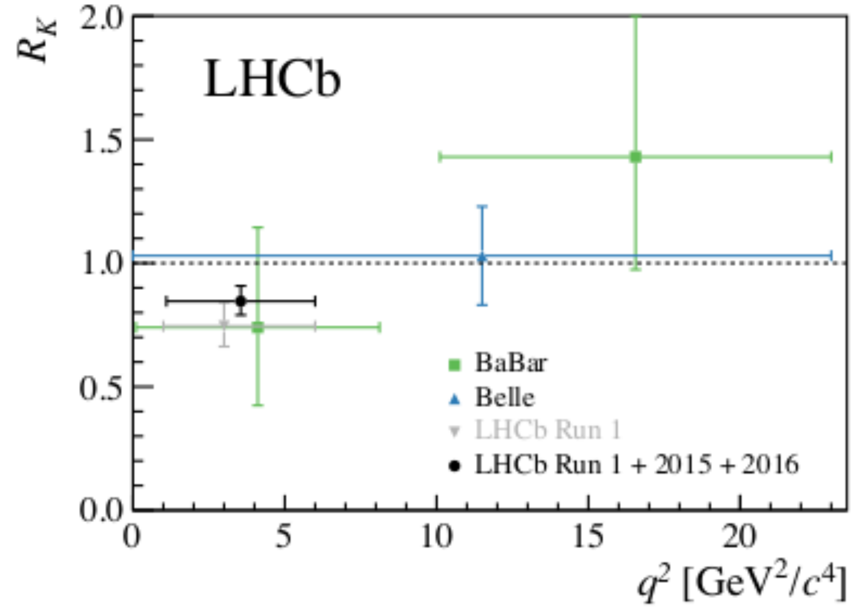
[5] Picture from CERN Courier April 2018

$R_{K^{(*)}}$

LHCb results: $q^2 = m_{ll}^2$.

	q^2/GeV^2	SM	LHCb 3 fb^{-1}	σ
R_K	$[1, 6]$	1.00 ± 0.01	0.846 ± 0.06	2.5
R_{K^*}	$[0.045, 1.1]$	0.91 ± 0.03	$0.66^{+0.11}_{-0.07}$	2.2
R_{K^*}	$[1.1, 6]$	1.00 ± 0.01	$0.69^{+0.11}_{-0.07}$	2.5

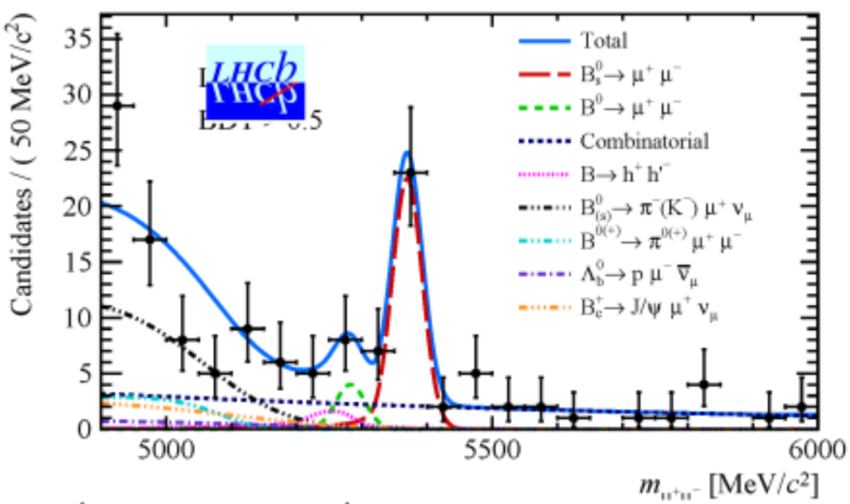


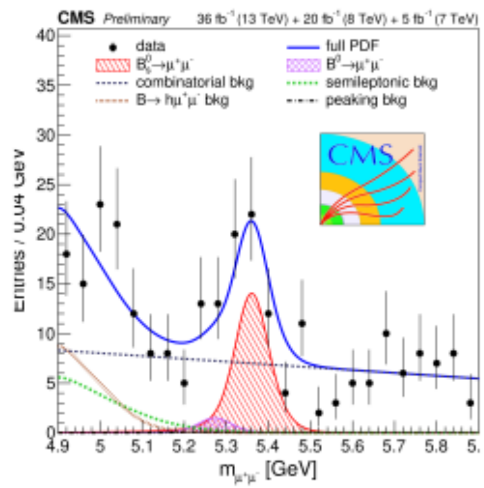


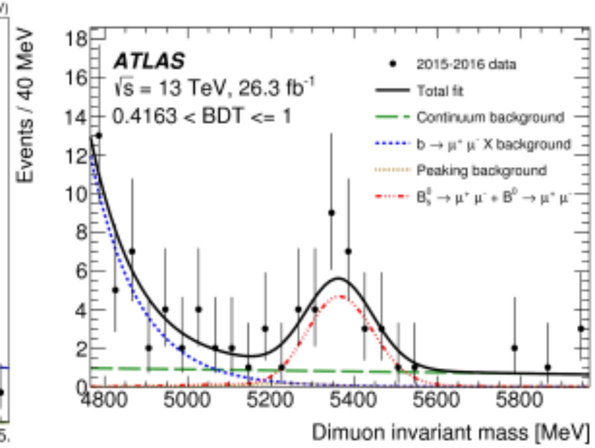
$B_s \to \mu^+\mu^-$

Lattice QCD provides important input to[6]

$$BR(B_s \to \mu\mu)_{SM} = (3.65 \pm 0.23) \times 10^{-9},$$



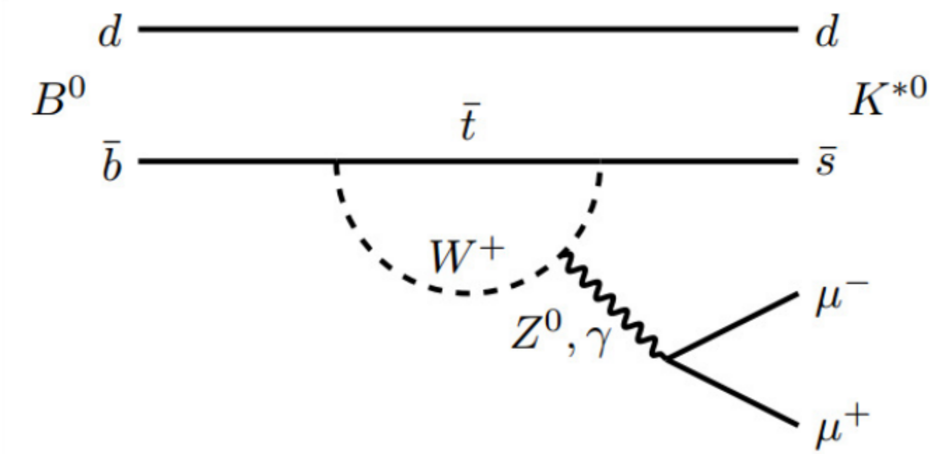


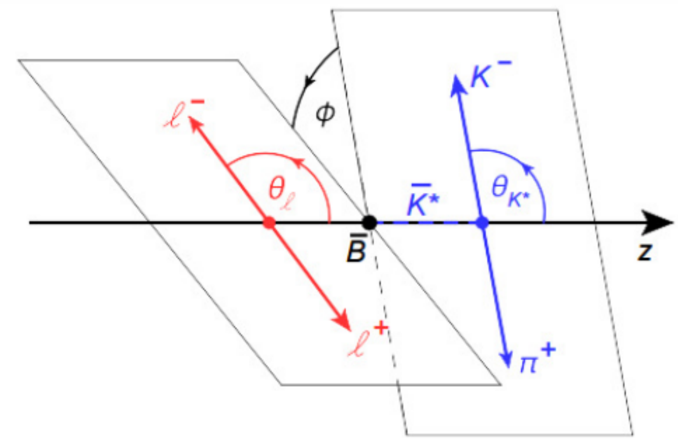


$B\left(B_s \to \mu^+\mu^-\right)$ $\quad \left(3.0 \pm 0.6^{+0.3}_{-0.2}\right) \times 10^{-9}$ $\quad \left(2.9^{+0.7}_{-0.6} \pm 0.2\right) \times 10^{-9}$ $\quad \left(2.8^{+0.8}_{-0.7}\right) \times 10^{-9}$

[6] Bobeth et al, 1311.0903

$B^0 \to K^{*0}(\to K^+\pi^-)\mu^+\mu^-$

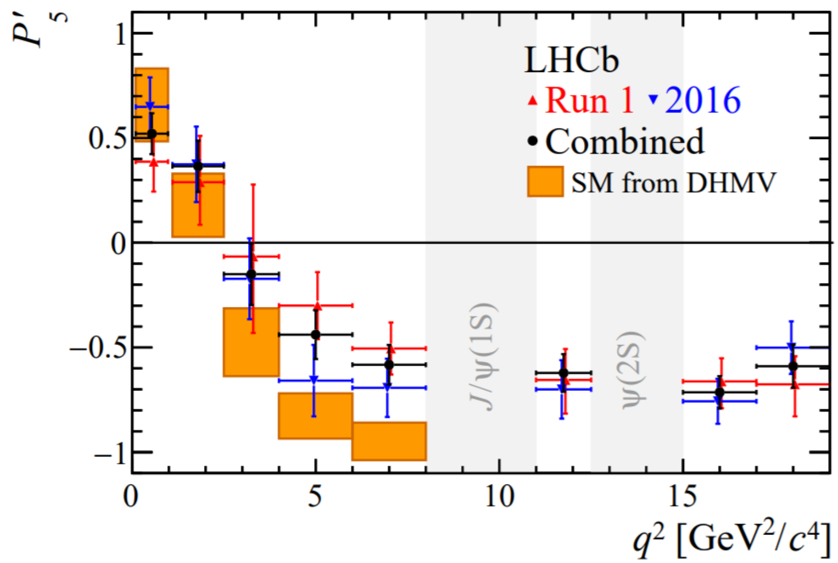


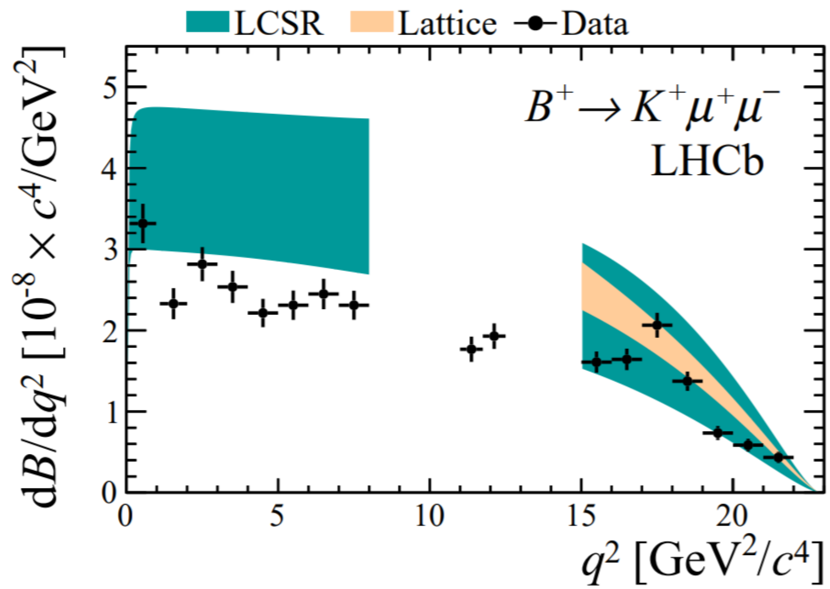


Decay fully described by three helicity angles $\vec{\Omega} = (\theta_\ell, \theta_K, \phi)$ and $q^2 = m^2_{\mu\mu}$

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\vec{\Omega}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_\mathrm{L})\sin^2\theta_K + F_\mathrm{L}\cos^2\theta_K + \tfrac{1}{4}(1-F_\mathrm{L})\sin^2\theta_K\cos 2\theta_\ell$$
$$- F_\mathrm{L}\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi$$
$$+ S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_K\sin\theta_\ell\cos\phi$$
$$+ \tfrac{4}{3}A_\mathrm{FB}\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$
$$+ S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

P_5'

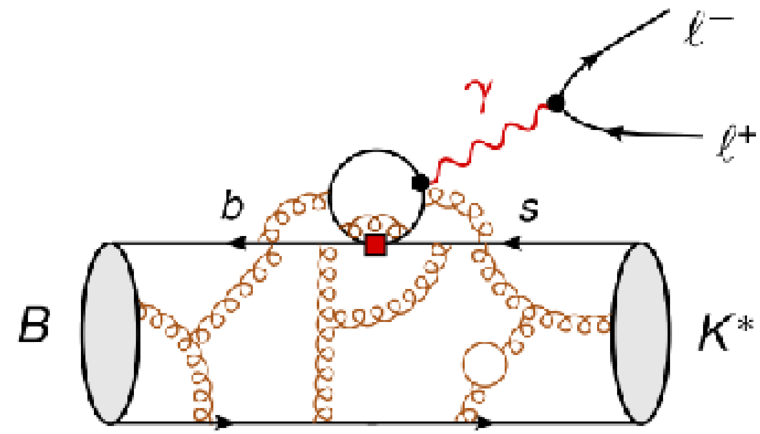




$P_5' = S_5/\sqrt{F_L(1-F_L)}$, leading form factor uncertainties cancel 2003.04831

Hadronic Uncertainties

- Hadronic effects like charm loop are photon-mediated $\Rightarrow$ vector-like coupling to leptons just like C_9



- How to disentangle NP $\leftrightarrow$ QCD?
  - Hadronic effect can have different q^2 dependence
  - Hadronic effect is lepton flavour universal ($\to R_K$!)

Wilson Coefficients $\bar{c}^l_{ij}$

In SM, can form an EFT since $m_B \ll M_W$:

$$\begin{aligned}
\mathcal{O}^l_{ij} &= (\bar{s}\gamma^\mu P_i b)(\bar{l}\gamma_\mu P_j l)\,. \\
\mathcal{L}_{\rm eff} &\supset \sum_{l=e,\mu,\tau}\sum_{i=L,R}\sum_{j=L,R}\frac{c^l_{ij}}{\Lambda^2_{l,ij}}\mathcal{O}^l_{ij}\,, \\
&= \sum_{l=e,\mu,\tau} V_{tb}V^*_{ts}\frac{\alpha}{4\pi v^2}\left(\bar{c}^l_{LL}\mathcal{O}^l_{LL}+\bar{c}^l_{LR}\mathcal{O}^l_{LR}\right. \\
&\quad \left.+\bar{c}^l_{RL}\mathcal{O}^l_{RL}+\bar{c}^l_{RR}\mathcal{O}^l_{RR}\right) \\
\Rightarrow \bar{c}^l_{ij} &= (36\ \mathrm{TeV}/\Lambda)^2 c^l_{ij}.
\end{aligned}$$

$c^l_{ij} \sim \pm\mathcal{O}(1)$ all predicted by weak interactions in SM.

Which Ones Work?

Options *for a single* *BSM* *operator*:

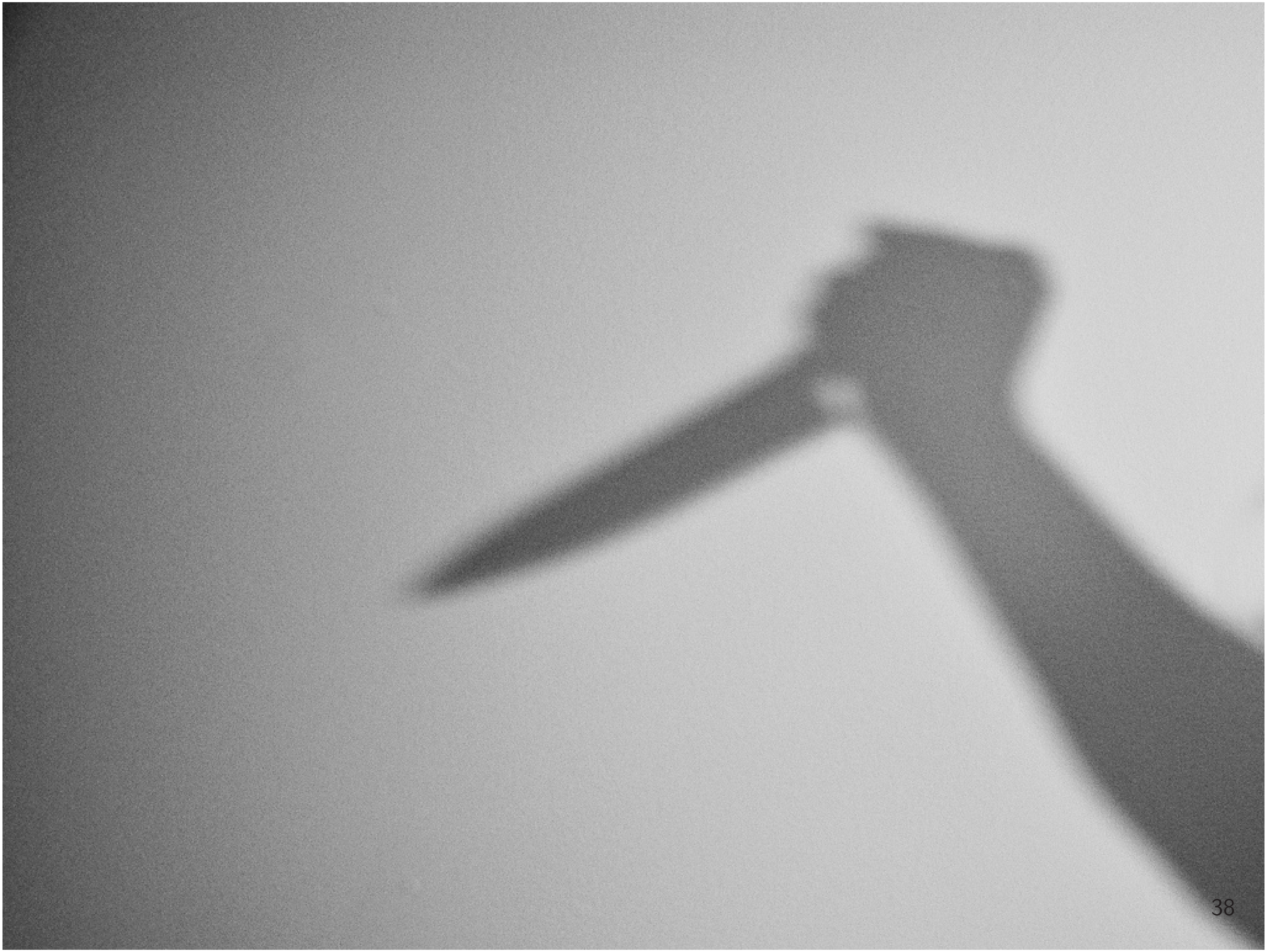
- $\bar{c}^e_{ij}$ operators fine for $R_{K^{(*)}}$ but are disfavoured by global fits including other observables.
- $\bar{c}^\mu_{LR}$ disfavoured: predicts *enhancement* in both R_K and R_{K^*}
- $\bar{c}^\mu_{RR}$, $\bar{c}^\mu_{RL}$ disfavoured: they pull R_K and R_{K^*} in *opposite directions*.
- $\bar{c}^\mu_{LL} = -1.06$ fits well globally[7].

[7] D'Amico et al, 1704.05438; Aebischer et al 1903.10434.

Statistics[8]

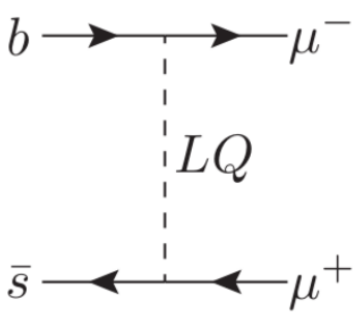
	$\bar{c}^{\mu}_{LL}$	$\sqrt{\chi^2_{SM} - \chi^2_{best}}$
clean	-1.33 ± 0.34	4.1
dirty	-1.33 ± 0.32	4.6
all	-1.06 ± 0.16	6.5
	$C^{\mu}_{9} = (\bar{c}^{\mu}_{LL} + \bar{c}^{\mu}_{LR})/2$	$\sqrt{\chi^2_{SM} - \chi^2_{best}}$
clean	-1.51 ± 0.46	3.9
dirty	-1.15 ± 0.17	5.5
all	-0.95 ± 0.15	5.8

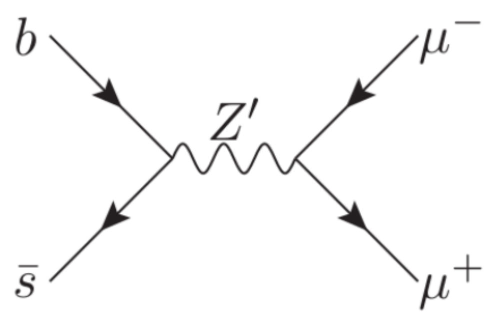
[8] 'clean' (R_K, R_{K^*}, $B_s \to \mu\mu$) and 'dirty' (P_5', $B \to \phi\mu\mu$+100 others). D'Amico, Nardecchia, Panci, Sannino, Strumia, Torre, Urbano 1704.05438; Aebischer, Altmanshoffer, Guadagnoli, Reboud, Stangl, Straub, 1903.10434. SM $p-$value around 3σ for NCBAs.

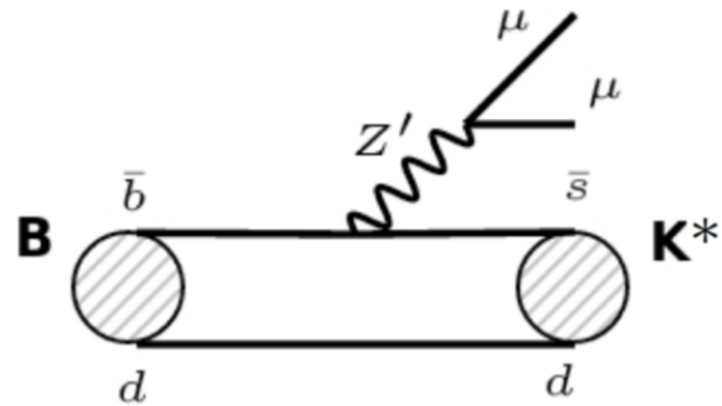


Simplified Models for c_{LL}^{μ}

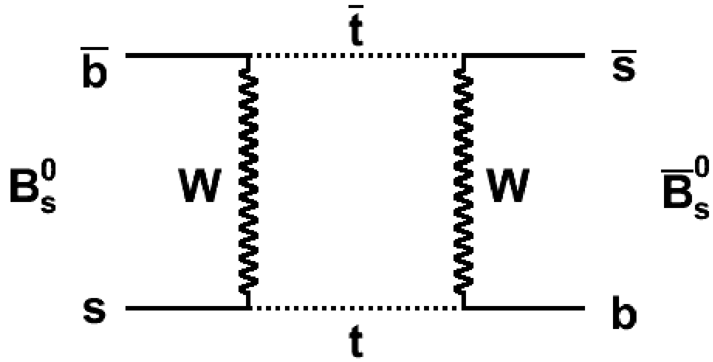
At tree-level, we have:

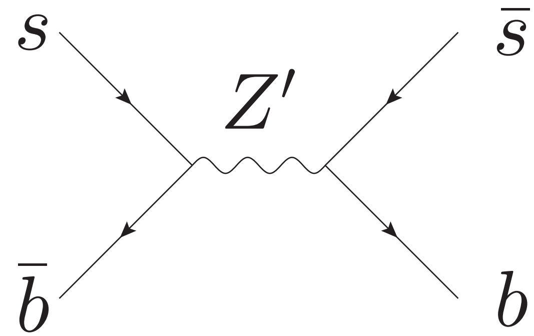






$B_s - \bar{B}_s$ Mixing





$$\bar{g}_L^{sb} \lesssim \frac{M_{Z'}}{194\ \text{TeV}}$$

from QCD sum rules and lattice[9]

[9] King, Lenz, Rauh, `arXiv:1904.00940`

$Z' \to \mu\mu$ ATLAS 13 TeV 139 fb^{-1}

ATLAS analysis: look for two track-based isolated μ, $p_T > 30$ GeV. One reconstructed primary vertex. Keep only highest scalar sum p_T pair[10]

$$m^2_{\mu_1\mu_2} = (p_1^\mu + p_2^\mu)\left(p_{1\mu} + p_{2\mu}\right)$$

CMS also have released[11] a similar 36 fb^{-1} analysis.

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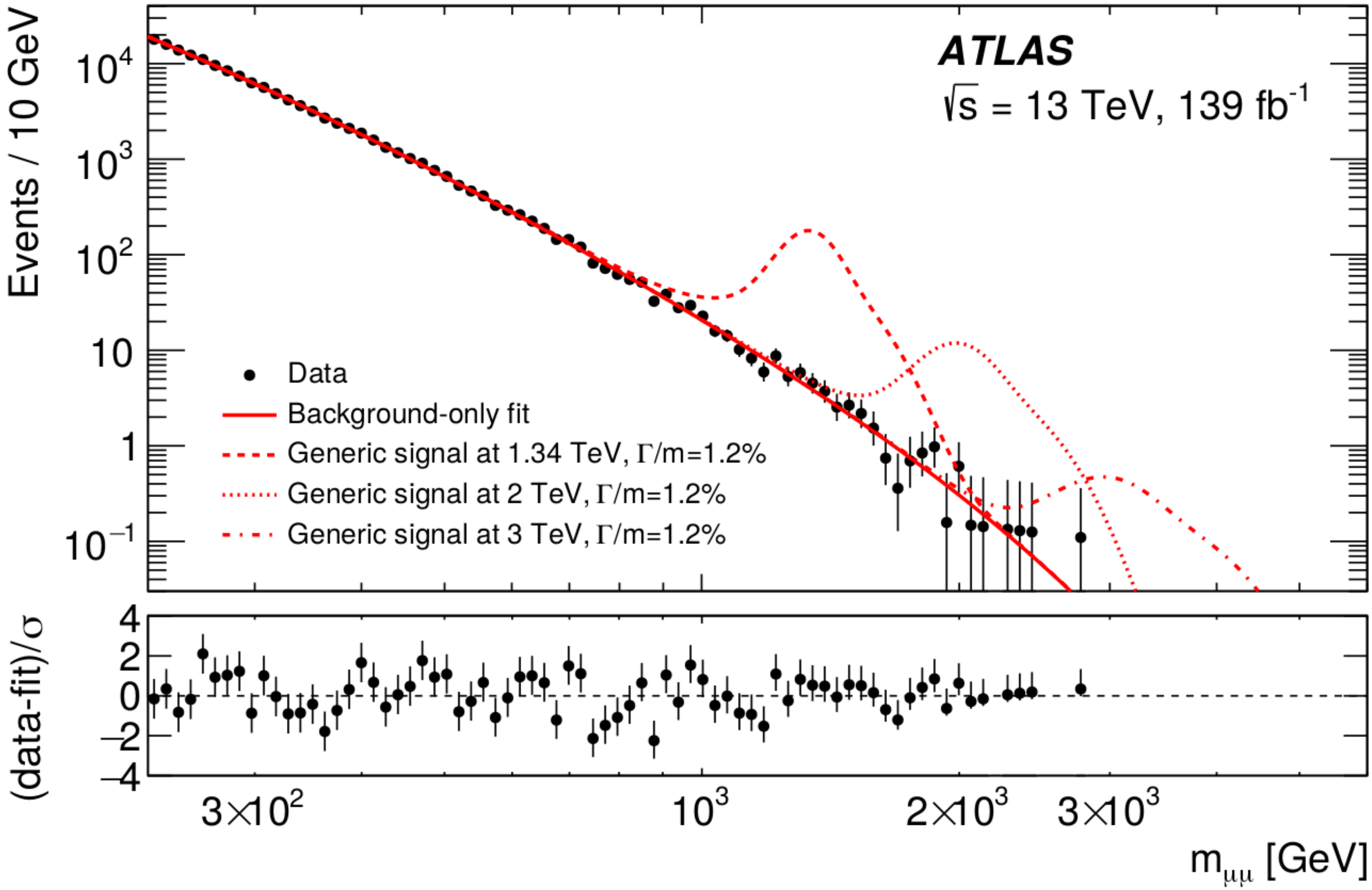
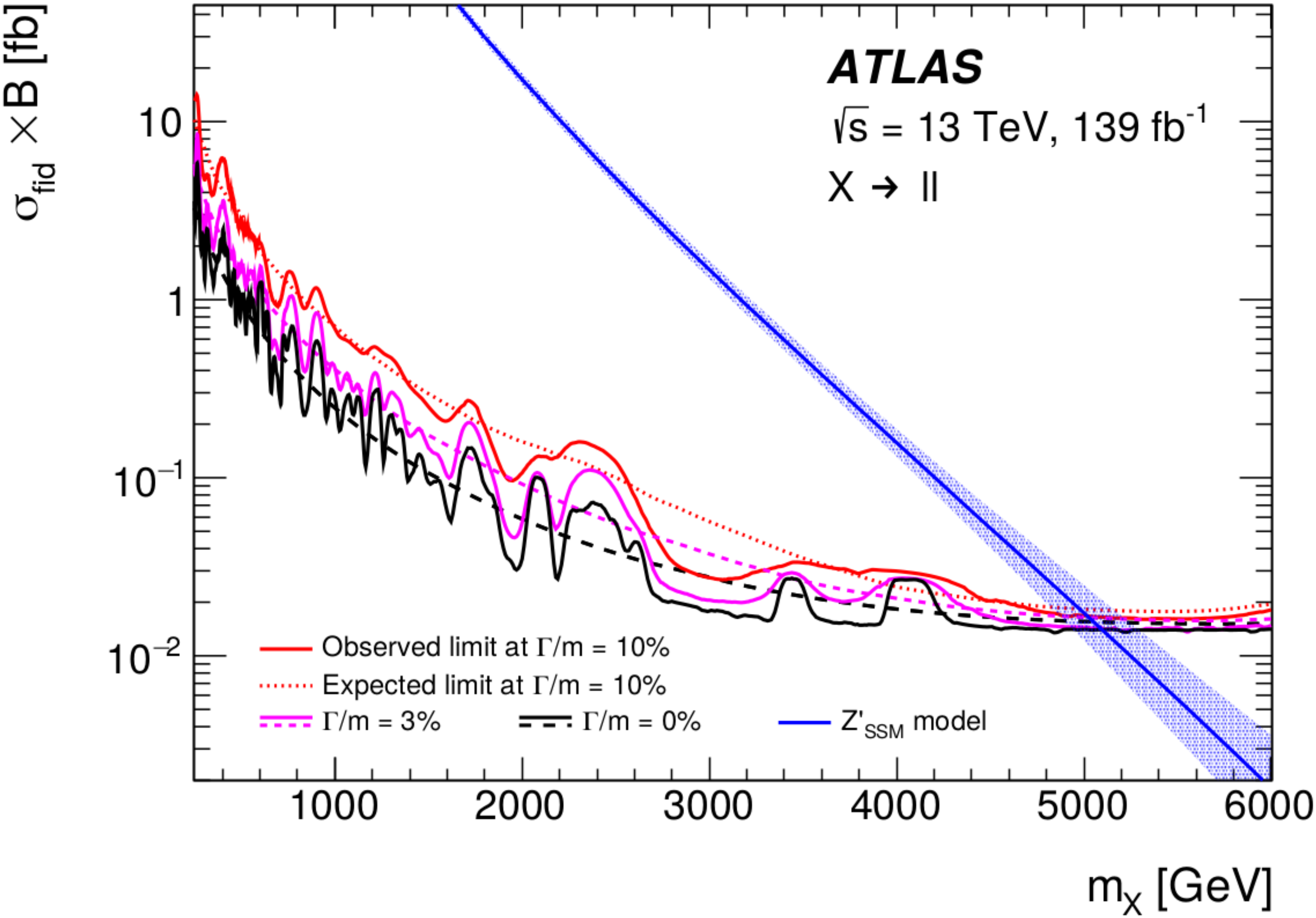
[10] 1903.06248

[11] 1803.06292



ATLAS
$\sqrt{s}$ = 13 TeV, 139 fb^{-1}
Data
Background-only fit
Generic signal at 1.34 TeV, Γ/m=1.2%
Generic signal at 2 TeV, Γ/m=1.2%
Generic signal at 3 TeV, Γ/m=1.2%
Events / 10 GeV
(data-fit)/σ
$m_{\mu\mu}$ [GeV]

ATLAS l^+l^- limits



1903.06248

During the 1990s

We wanted to be the Grand Architects, searching for **the** string model to rule them all

During the 2010s

We are happy with **any** beyond the Standard Model roof

A Model

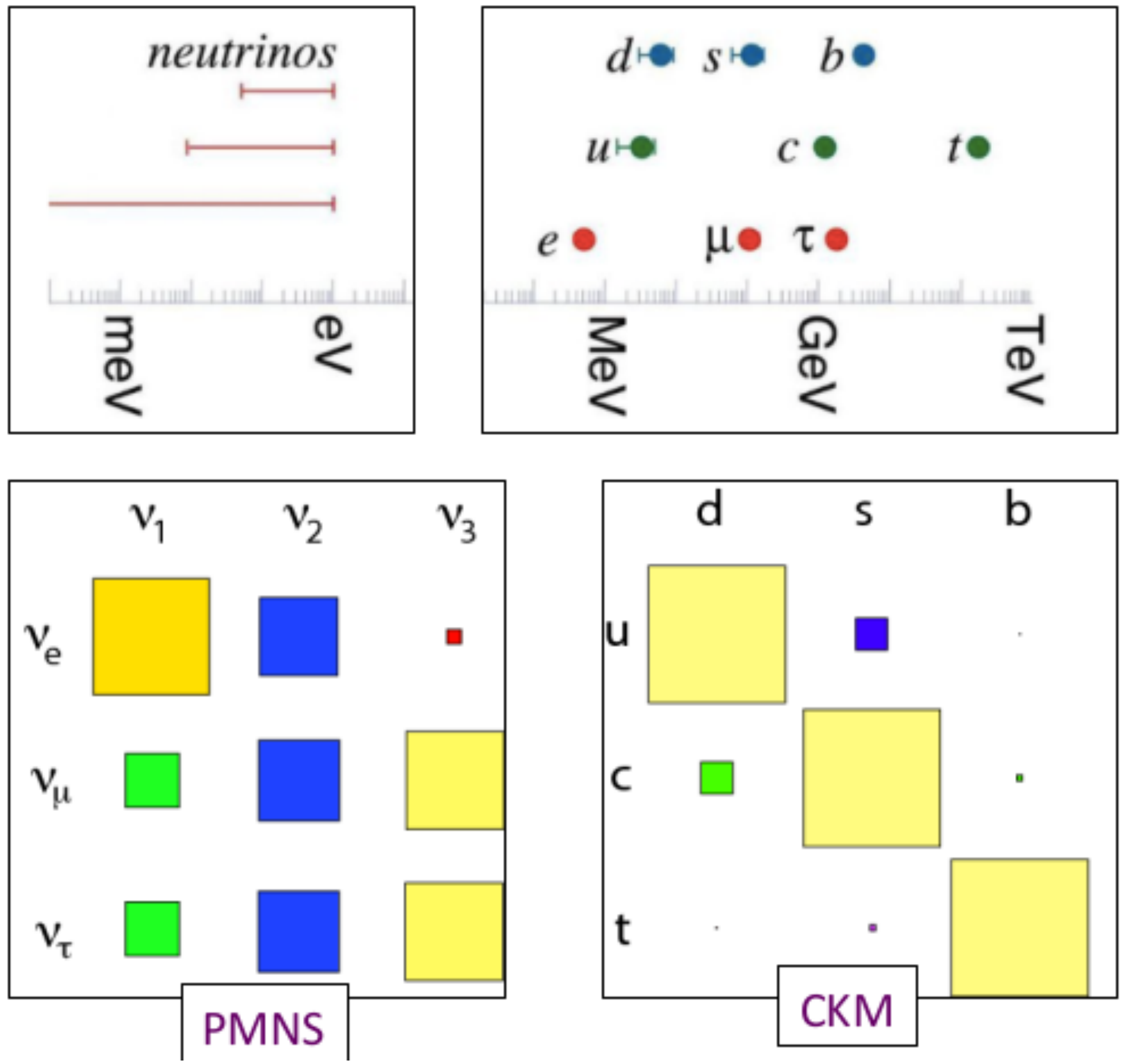
BCA, Davighi, arXiv:1809.01158: Add complex SM singlet scalar θ and gauged $U(1)_F$:

$$
\begin{array}{c}
SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_F \\
\Big\downarrow \langle \theta \rangle \sim \text{Several TeV} \\
SU(3) \times SU(2)_L \times U(1)_Y \\
\Big\downarrow \langle H \rangle \sim 246 \text{ GeV} \\
SU(3) \times U(1)_{em}
\end{array}
$$

- SM fermion content
- anomaly cancellation
- 0 F charges for first two generations

The Flavour Problem

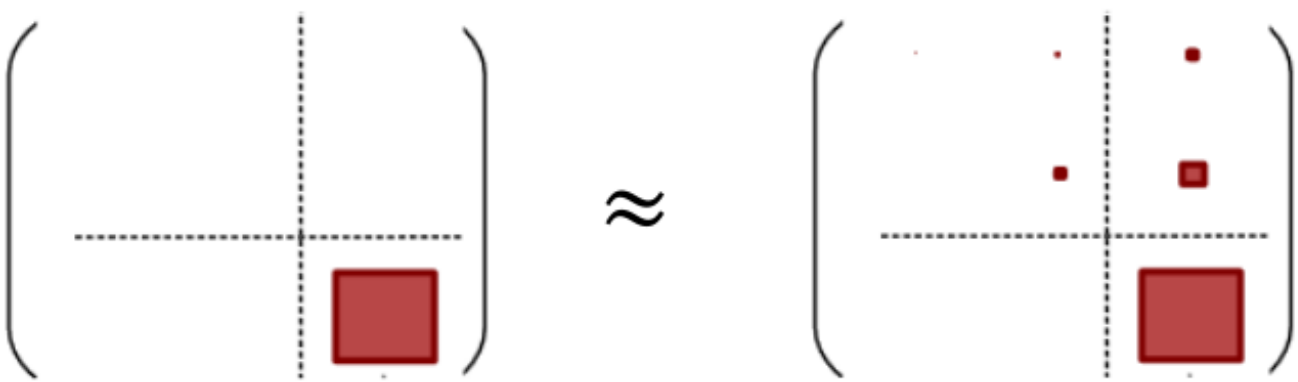
The Flavour Problem



Unique Solution

$$\boxed{\begin{array}{cccc} F_{Q'_i}=0 & F_{u_{R'_i}}=0 & F_{d_{R'_i}}=0 & F_{L'_i}=0 \\ F_{e_{R'_i}}=0 & F_H=-1/2 & F_{Q'_3}=1/6 & F_{u'_{R3}}=2/3 \\ F_{d'_{R3}}=-1/3 & F_{L'_3}=-1/2 & F_{e'_{R3}}=-1 & F_\theta \neq 0 \end{array}}$$

$$\mathcal{L} = Y_t \overline{Q_3'}_L H t'_R + Y_b \overline{Q'_3}_L H^c b'_R + Y_\tau \overline{L_3'}_L H^c \tau'_R + H.c.,$$



Yukawa Advantages

- First two families massless at renormalisable level
- Their masses and fermion mixings generated by small non-renormalisable operators

This explains the hierarchical heaviness of the third family and small CKM angles

$Z - X$ mixing

Because $F_H = -1/2$, $Z - X$ mix:

$$\mathcal{M}_N^2 = \frac{v^2}{4} \begin{pmatrix} g'^2 & -gg' & g'g_F \\ -gg' & g^2 & -gg_F \\ g'g_F & -gg_F & g_F^2(1+4F_\theta^2 r^2) \end{pmatrix} \begin{array}{c} -B_\mu \\ -W_\mu^3 \\ -X_\mu \end{array}$$

- $v \approx 246$ GeV is SM Higgs VEV
- $g_F = U(1)_F$ gauge coupling
- $r \equiv v_F/v \gg 1$, where $v_F = \langle\theta\rangle$
- F_θ is F *charge* of θ field

$Z - X$ mixing angle

$$\sin\alpha_z \approx \frac{g_F}{\sqrt{g^2 + g'^2}} \left(\frac{M_Z}{M'_Z}\right)^2 \ll 1.$$

This gives small non-flavour universal couplings to the Z boson propotional to g_F and:

$$Z_\mu = \cos\alpha_z \left(-\sin\theta_w B_\mu + \cos\theta_w W_\mu^3\right) + \sin\alpha_z X_\mu,$$

$$
\begin{aligned}
\mathcal{L}_{X\psi} \;=\; g_F & \left(\frac{1}{6}\overline{\mathbf{u_L}}\Lambda^{(u_L)}\gamma^\rho\mathbf{u_L} + \frac{1}{6}\overline{\mathbf{d_L}}\Lambda^{(d_L)}\gamma^\rho\mathbf{d_L} - \right. \\
& \frac{1}{2}\overline{\mathbf{n_L}}\Lambda^{(n_L)}\gamma^\rho\mathbf{n_L} - \frac{1}{2}\overline{\mathbf{e_L}}\Lambda^{(e_L)}\gamma^\rho\mathbf{e_L} + \\
& \frac{2}{3}\overline{\mathbf{u_R}}\Lambda^{(u_R)}\gamma^\rho\mathbf{u_R} - \\
& \left.\frac{1}{3}\overline{\mathbf{d_R}}\Lambda^{(d_R)}\gamma^\rho\mathbf{d_R} - \overline{\mathbf{e_R}}\Lambda^{(e_R)}\gamma^\rho\mathbf{e_R}\right) Z'_\rho,
\end{aligned}
$$

$$
\Lambda^{(I)} \quad \equiv \quad V_I^\dagger \xi V_I, \qquad \xi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}
$$

Z' couplings, $I \in \{u_L, d_L, e_L, \nu_L, u_R, d_R, e_R\}$

A simple limiting case

$$V_{u_R} = V_{d_R} = V_{e_R} = 1$$

for simplicity and the ease of passing bounds.

$$V_{d_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{sb} & -\sin\theta_{sb} \\ 0 & \sin\theta_{sb} & \cos\theta_{sb} \end{pmatrix}, \qquad V_{e_L} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$\Rightarrow V_{u_L} = V_{d_L} V_{CKM}^{\dagger}$ and $V_{\nu_L} = V_{e_L} U_{PMNS}^{\dagger}$.

Important Z' Couplings

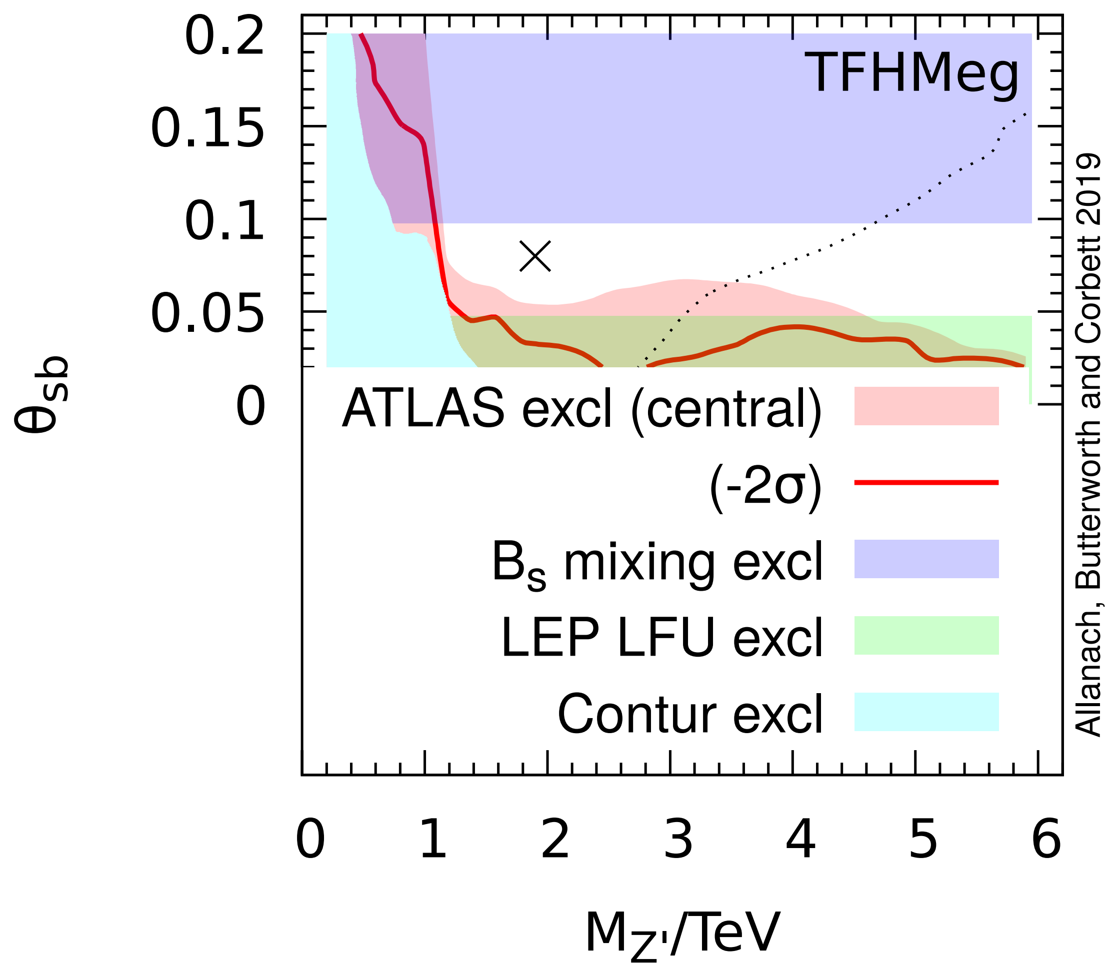
$$g_F \left[\frac{1}{6}\overline{\mathbf{d_L}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sin^2\theta_{sb} & \frac{1}{2}\sin 2\theta_{sb} \\ 0 & \frac{1}{2}\sin 2\theta_{sb} & \cos^2\theta_{sb} \end{pmatrix} \not{Z}' \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \right.$$

$$\left. -\frac{1}{2}\overline{\mathbf{e_L}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \not{Z}' \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \right]$$

Put $|\theta_{sb}| \sim \mathcal{O}(|V_{ts}|) = 0.04$, so $|g_{\mu\mu}| \gg |g_{bs}|$, which helps us survive $B_s - \overline{B_s}$ constraint.

$c_{LL} = g_F^2 \sin 2\theta_{sb}/(24M_{Z'}^2)$.

$$g_F \propto M_{Z'}/\sqrt{\sin 2\theta_{bs}}$$



Example Case Predictions

Mode	BR	Mode	BR	Mode	BR
$t\bar{t}$	0.42	$b\bar{b}$	0.12	$\nu\bar{\nu}'$	0.08
$\mu^+\mu^-$	0.08	$\tau^+\tau^-$	0.30	other $f_i f_j$	$\sim \mathcal{O}(10^{-4})$

LEP LFU

$$g_F^2 \left(\frac{M_Z}{M_{Z'}}\right)^2 \leq 0.004 \Rightarrow g_F \leq \frac{M_{Z'}}{1.3\ \text{TeV}}.$$

It's worth chasing $BR(B \to K^{(*)}\tau^\pm\tau^\mp)$.

Conclusion

We have a partial solution to the full set of anomaly equations for SM rank extensions.

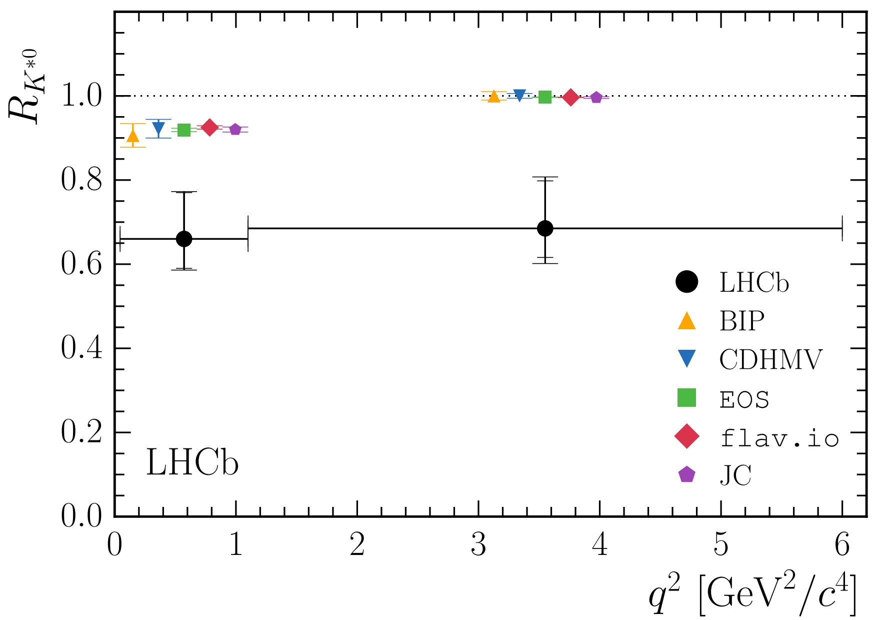
The answers to the questions raised by the neutral current $B-$anomalies may provide a direct experimental probe into the flavour problem.

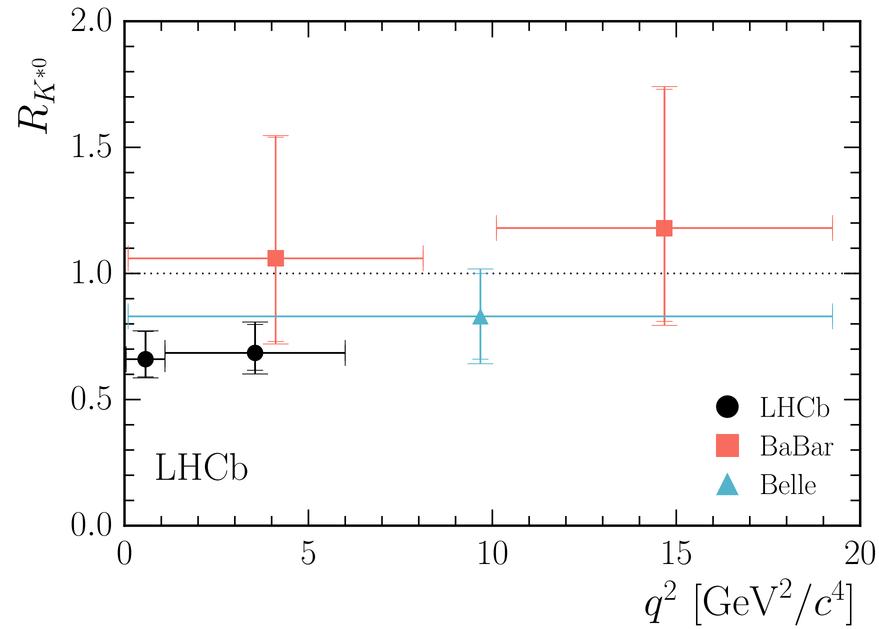
Backup

$R_{K^{(*)}}$ pre Moriond 2019

LHCb results from 7 and 8 TeV: $q^2 = m_{ll}^2$.

	q^2/GeV^2	SM	LHCb 3 fb^{-1}	σ
R_K	$[1, 6]$	1.00 ± 0.01	$0.745^{+0.090}_{-0.074}$	2.6
R_{K^*}	$[0.045, 1.1]$	0.91 ± 0.03	$0.66^{+0.11}_{-0.07}$	2.2
R_{K^*}	$[1.1, 6]$	1.00 ± 0.01	$0.69^{+0.11}_{-0.07}$	2.5

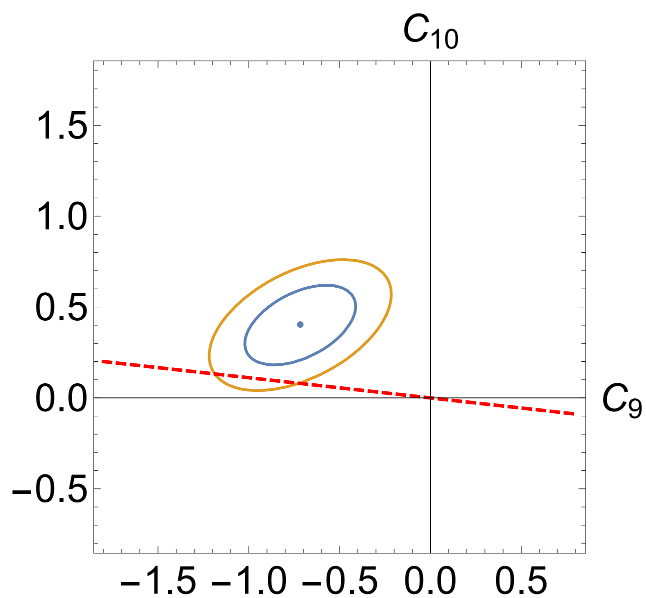


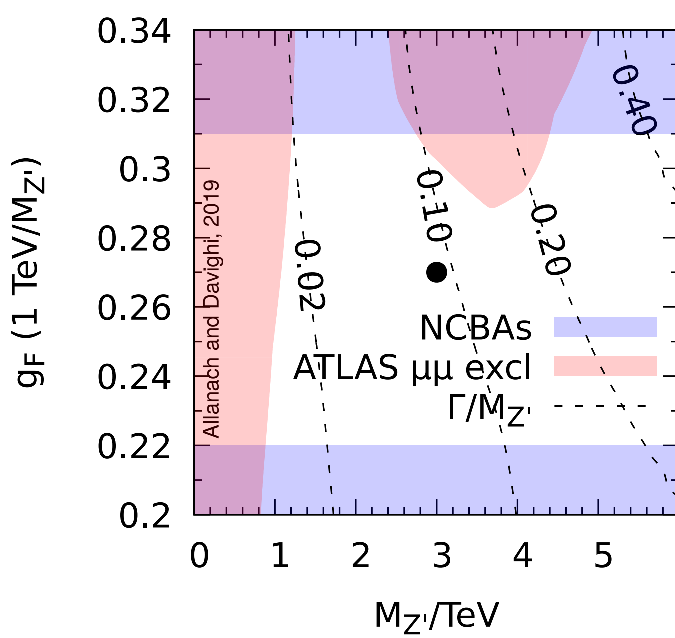


Deformed TFHM

$$\boxed{\begin{array}{llll} F_{Q'_i}=0 & F_{u_{R'_i}}=0 & F_{d_{R'_i}}=0 & F_H=-1/2 \\ F_{e_{R'_1}}=0 & F_{e_{R'_2}}=2/3 & F_{e_{R'_3}}=-5/3 & \\ F_{L'_1}=0 & F_{L'_2}=5/6 & F_{L'_3}=-4/3 & \\ F_{Q'_3}=1/6 & F_{u'_{R3}}=2/3 & F_{d'_{R3}}=-1/3 & F_\theta\neq 0 \end{array}}$$

$$\mathcal{L}=Y_t\overline{Q_3'}_L H t'_R + Y_b\overline{Q'_{3L}}H^c b'_R + H.c.,$$





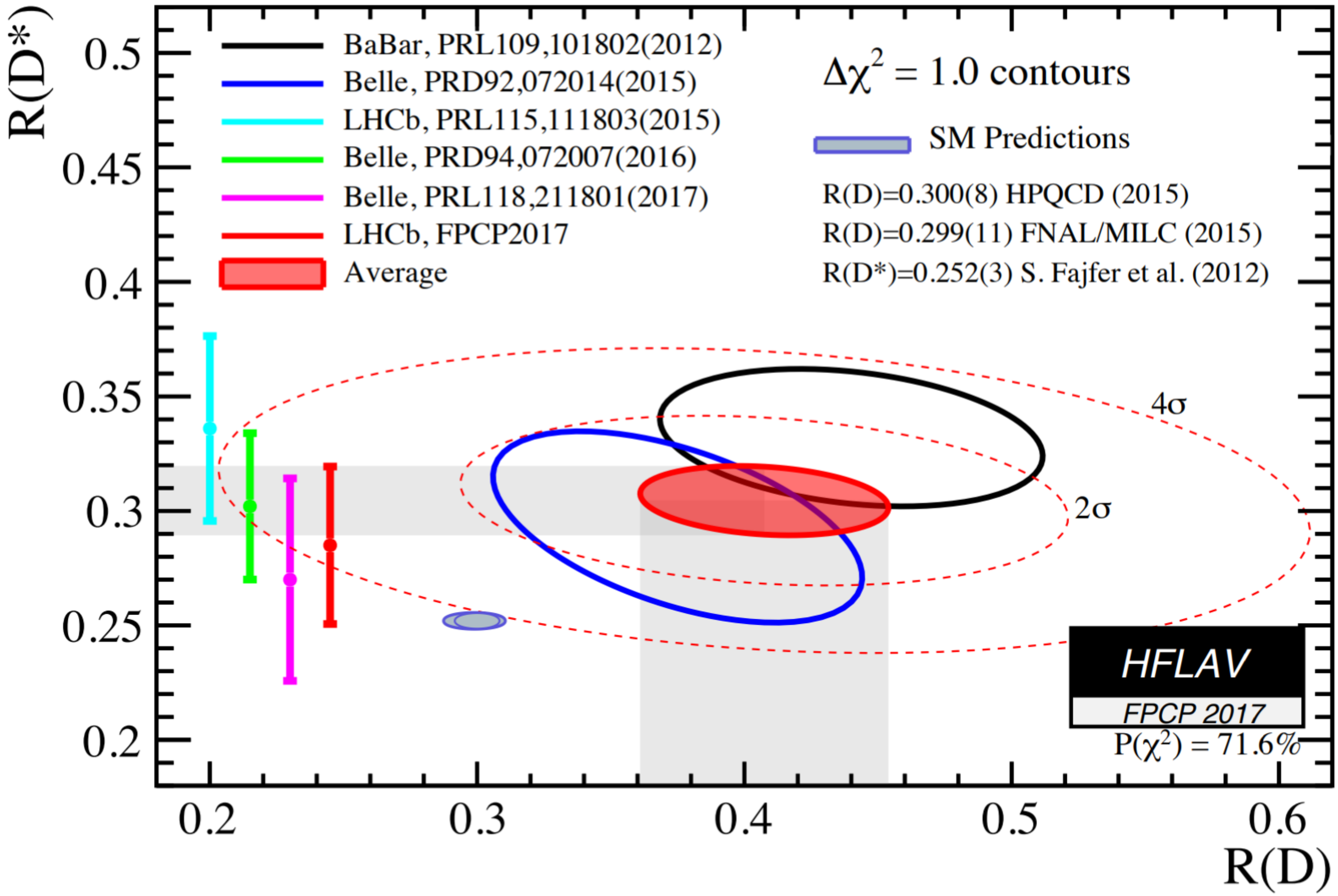
Invisible Width of Z Boson

$\Gamma^{(\mathrm{exp})}_{\mathrm{inv}} = 499.0 \pm 1.5$ MeV, whereas $\Gamma^{(\mathrm{SM})}_{\mathrm{inv}} = 501.44$ MeV.

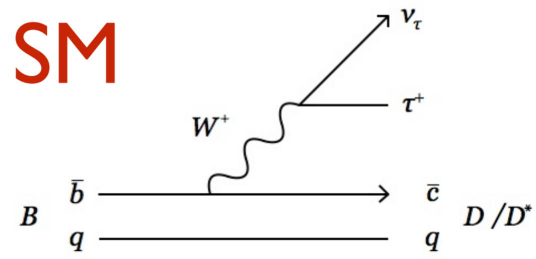
$$\Rightarrow \Delta\Gamma^{(\mathrm{exp})} = \Gamma^{(\mathrm{exp})}_{\mathrm{inv}} - \Gamma^{(\mathrm{SM})}_{\mathrm{inv}} = -2.5 \pm 1.5 \text{ MeV}.$$

$$\begin{aligned}
\mathcal{L}_{\bar{\nu}\nu Z} \;&=\; -\frac{g}{2\cos\theta_w}\overline{\nu'_{Le}}\not{Z} P_L \nu'_{Le} \\
&\quad -\overline{\nu'_{L\mu}}\left(\frac{g}{2\cos\theta_w} + \frac{5}{6} g_F \sin\alpha_z\right)\not{Z}\nu'_{L\mu} \\
&\quad -\overline{\nu'_{L\tau}}\left(\frac{g}{2\cos\theta_w} - \frac{8}{6} g_F \sin\alpha_z\right)\not{Z}\nu'_{L\tau}.
\end{aligned}$$

$$R_{D^{(*)}} = BR(B^- \to D^{(*)}\tau\nu)/BR(B^- \to D^{(*)}\mu\nu)$$



$R_{D^{(*)}}$: BSM Explanation



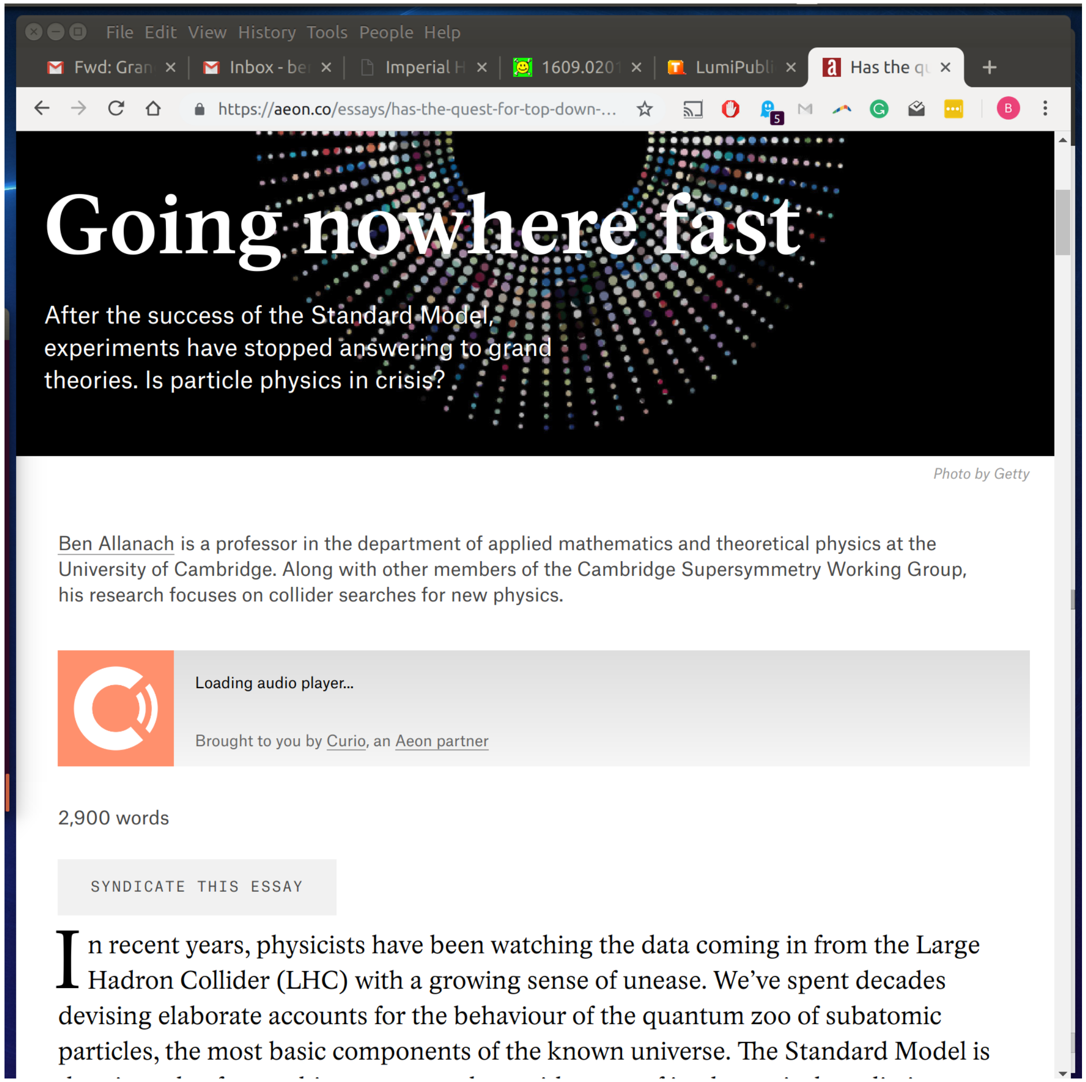
... has to compete with

$$\mathcal{L}_{eff} = -\frac{2}{\Lambda^2}\left(\bar{c}_L\gamma^\mu b_L\right)\left(\bar{\tau}_L\gamma_\mu\nu_{\tau L}\right) + H.c.$$

$$\Lambda = 3.4\ \mathrm{TeV}$$

A factor 10 lower than required for $R_{K^{(*)}} \Rightarrow$ *different explanation?*

PMP$\Rightarrow$we ignore $R_{D^{(*)}}$.



Going nowhere fast

After the success of the Standard Model, experiments have stopped answering to grand theories. Is particle physics in crisis?

Photo by Getty

Ben Allanach is a professor in the department of applied mathematics and theoretical physics at the University of Cambridge. Along with other members of the Cambridge Supersymmetry Working Group, his research focuses on collider searches for new physics.

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2,900 words

SYNDICATE THIS ESSAY

In recent years, physicists have been watching the data coming in from the Large Hadron Collider (LHC) with a growing sense of unease. We've spent decades devising elaborate accounts for the behaviour of the quantum zoo of subatomic particles, the most basic components of the known universe. The Standard Model is

Other conclusions

- The answers to the questions raise by $R_{K^{(*)}}$ may provide a direct experimental probe into the flavour problem.
- Focused on *tree-level* explanations of $R_{K^{(*)}}$ as they are usually harder to discover: Z' and leptoquarks.
- News on $R_K^{(*)}$ expected *in 2019*. At the current central value, Belle II can reach 5σ by mid 2021. LHCb's R_{K^*} would be close to[12] 5σ by 2020.
- *$R_{K^{(*)}} \Rightarrow$ HL-LHC, HE-LHC and FCC-hh*

[12] Albrecht *et al*, 1709.10308

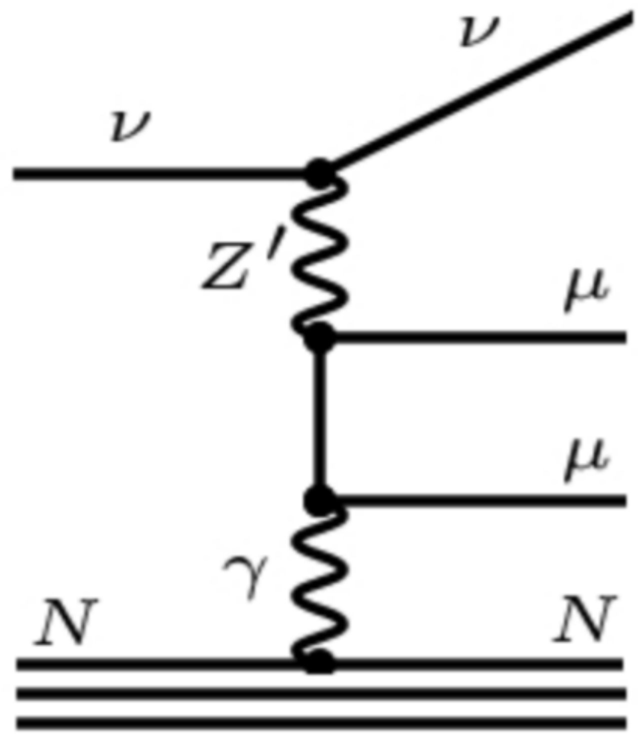
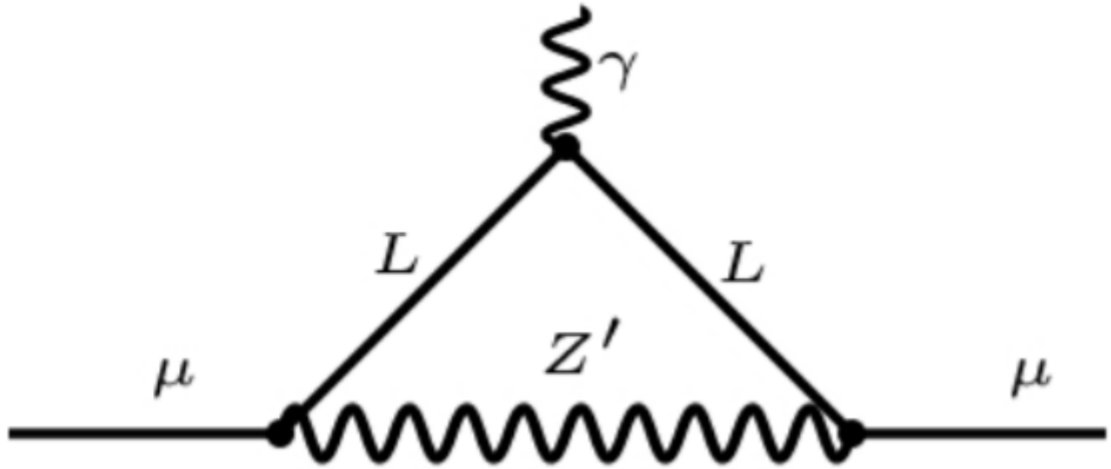
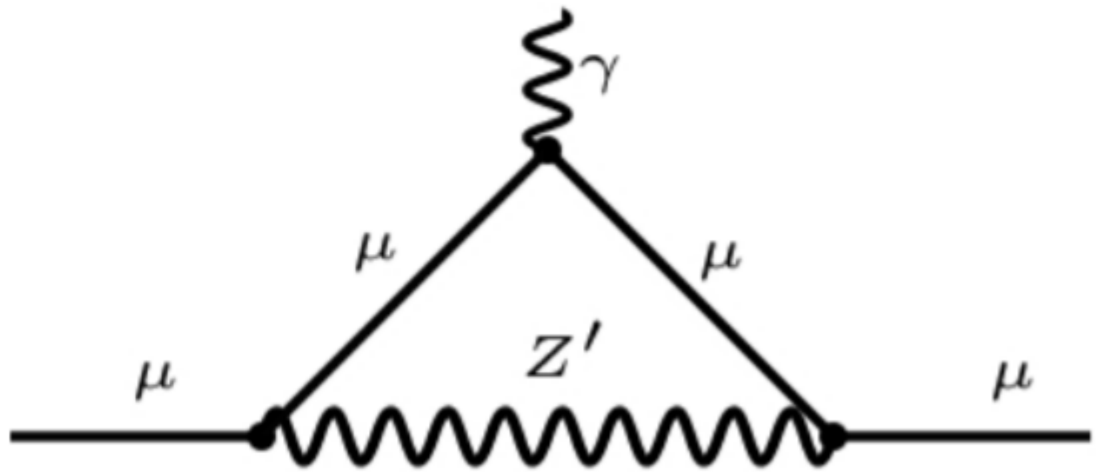


FIG. 10. Neutrino trident process that leads to constraints on the Z^{μ} coupling strength to neutrinos-muons, namely $M_{Z'}/g_{v\mu} \gtrsim 750$ GeV.



γ
L
L
Z'
μ
μ

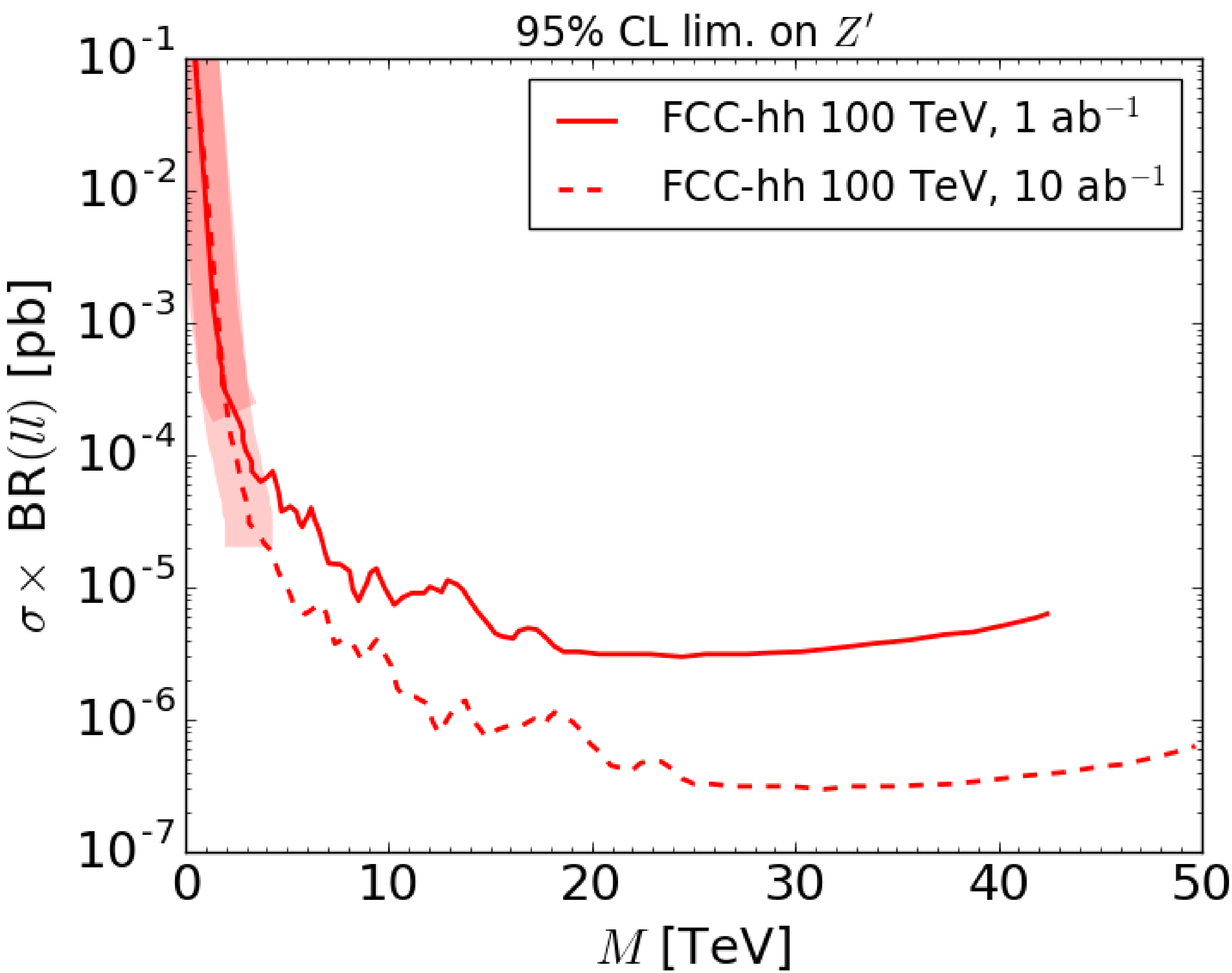


γ
μ
μ
Z'
μ
μ

$Q_{\max}$	**Solutions**	Symmetry	Quadratics	Cubics	Time/sec
1	**38**	16	144	38	0.0
2	**358**	48	31439	2829	0.0
3	**4116**	154	1571716	69421	0.1
4	**24552**	338	34761022	932736	0.6
5	**111152**	796	442549238	7993169	6.8
6	**435305**	1218	3813718154	49541883	56

SM + 3 ν_R: number of solutions etc

13 TeV ATLAS 3.2 fb^{-1} $\mu\mu$



Neutrino Masses

At dimension 5:

$$\mathcal{L}_{SS} = \frac{1}{2M}({L'_3}^T H^c)(L'_3 H^c),$$

but if we add RH neutrinos, then integrate them out

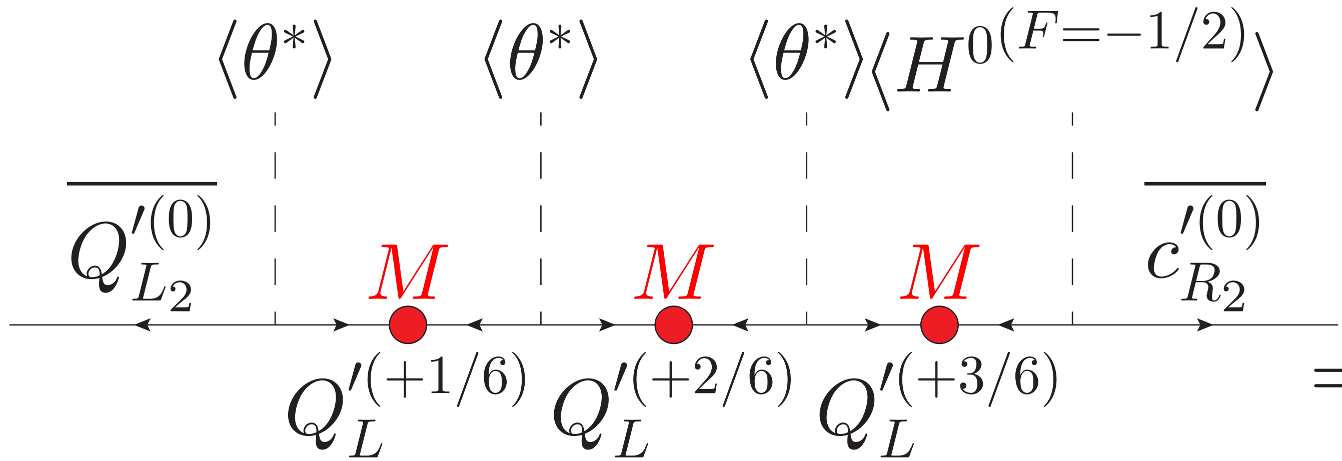
$$\mathcal{L}_{SS} = 1/2 \sum_{ij} (L'_i H^c)(M^{-1})_{ij}(L'_j H^c),$$

where now $(M^{-1})_{ij}$ may well have a non-trivial structure. If $(M^{-1})_{ij}$ are of same order, large PMNS mixing results.

Froggatt Neilsen Mechanism[13]

A means of generating the non-renormalisable Yukawa terms, e.g. $F_\theta = 1/6$:

$$Y_c \overline{Q'_{L2}}^{(F=0)} H^{(F=-1/2)} {c'_R}^{(F=0)} \sim \mathcal{O}\left[\left(\frac{\langle\theta\rangle}{M}\right)^3 \overline{Q'_{L2}} H c'_R\right]$$



eg $\left(\frac{\langle\theta\rangle}{M}\right) \sim 0.2$

$\Rightarrow Y_c/Y_t \sim 1/100$

[13] C Froggatt and H Neilsen, NPB **147** (1979) 277

LQ Models

Scalar[14] $S_3 = (\bar{3},\ 3,\ 1/3)$ of $SU(2) \times SU(2)_L \times U(1)_Y$:

$$\mathcal{L} = \ldots + y_{3b\mu} Q_3 L_2 S_3 + y_{3s\mu} Q_2 L_2 S_3 + y_q Q Q S_3^\dagger + \text{h.c.}$$

Vector $V_1 = (\bar{3}, 1, 2/3)$ or $V_3 = (3, 3, 2/3)$

$$\mathcal{L} = \ldots + y_3' V_3^\mu \bar{Q} \gamma_\mu L + y_1 V_1^\mu \bar{Q} \gamma_\mu L + y_1' V_1^\mu \bar{d} \gamma_\mu l + \text{h.c.}$$

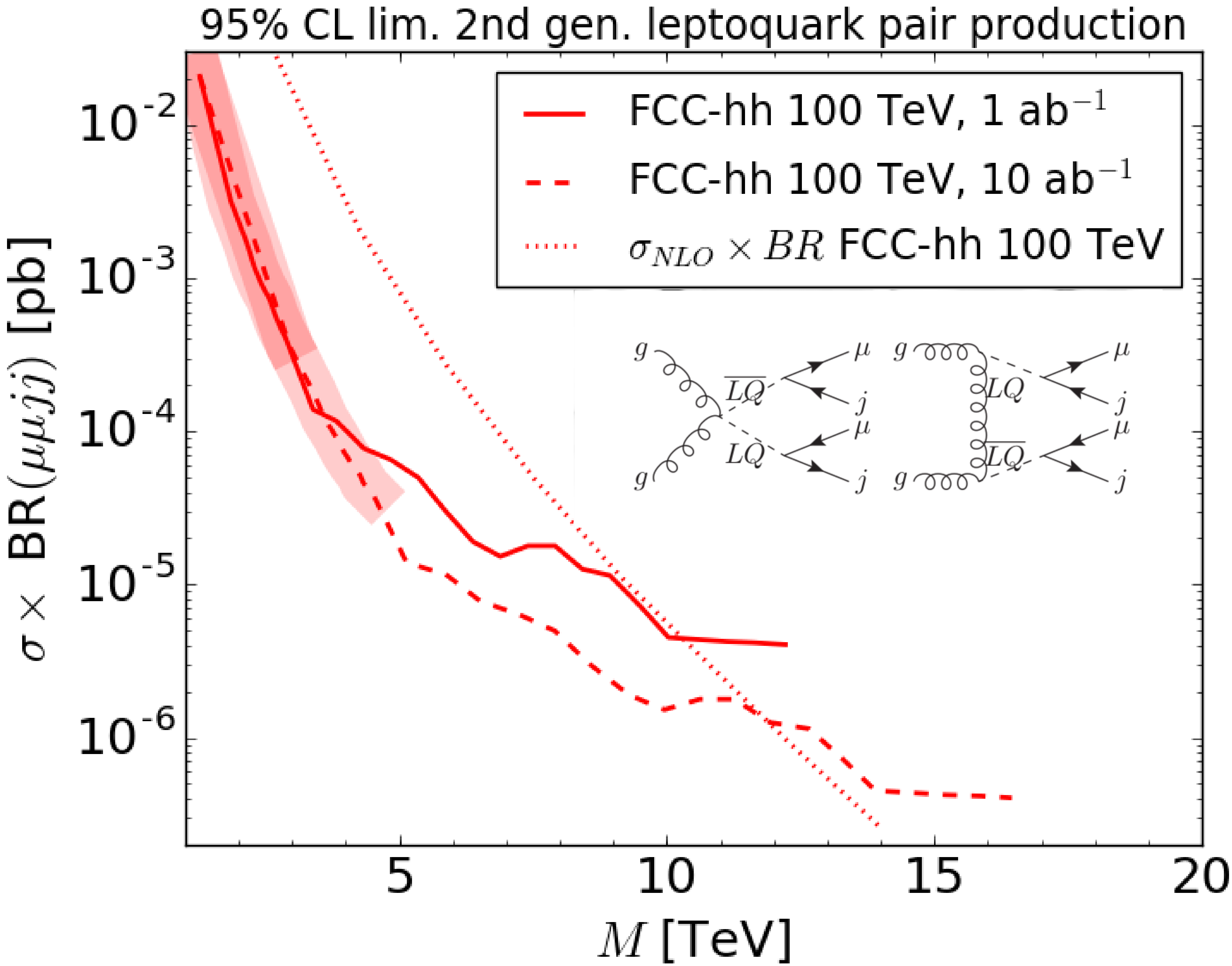
$$\Rightarrow \bar{c}_{LL}^\mu = \kappa \frac{4\pi v^2}{\alpha_{\text{EM}} V_{tb} V_{ts}^*} \frac{y_{3b\mu}^* y_{3s\mu}}{M^2}.$$

$\kappa = 1, -1, -1$ and $y = y_3, y_1, y_3'$ for S_3, V_1, V_3.

[14] Capdevila *et al* 1704.05340, Hiller and Hisandzic 1704.05444, D'Amico *et al* 1704.05438.

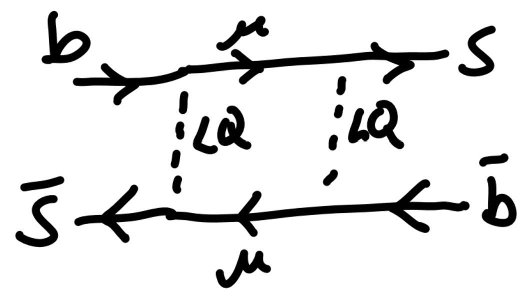
CMS 8 TeV 20fb^{-1} 2nd gen

CMS-PAS-EXO-12-042: $M > 1.07$ TeV.



Other Constraints On LQs

Note that the extrapolation is very rough for pair production. Fix $M = 2M_{LQ}$, assuming they are produced close to threshold: $\Delta = 0.1$.



$B_s - \bar{B}_s$ mixing is at one-loop:

$$\mathcal{L}_{\bar{b}s\bar{b}s} = k \frac{\left|y_{b\mu} y^*_{s\mu}\right|^2}{32\pi^2 M_{LQ}^2} \left(\bar{b}\gamma_\mu P_L s\right) \left(\bar{s}\gamma^\mu P_L b\right) + \text{h.c.}$$

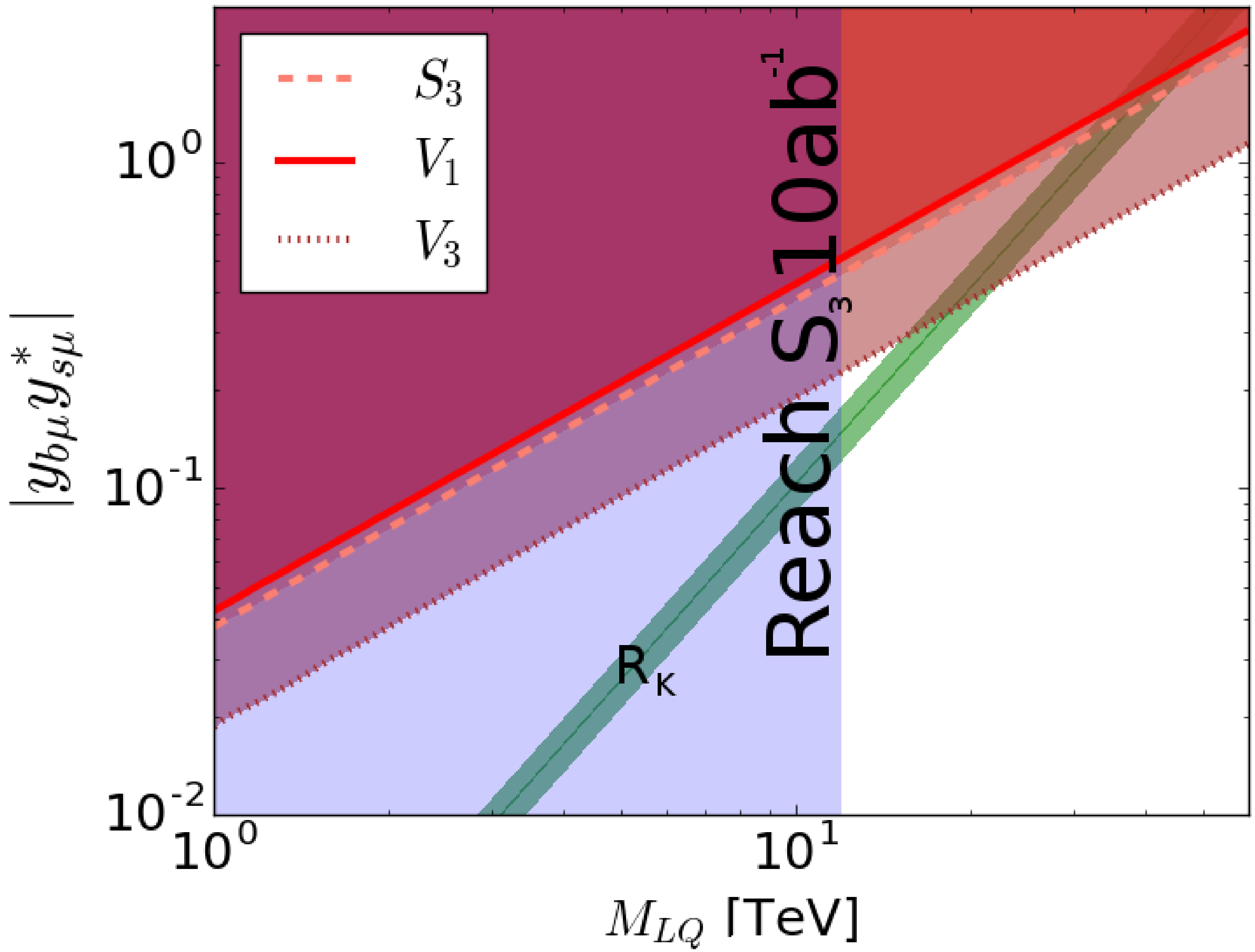
$y = y_3, y_1, y_3'$ and $k = 5, 4, 20$ for S_3, V_1, V_3.
Data $\Rightarrow c_{LL}^{bb} < 1/(210\text{TeV})^2$.

Mass Constraints: Summary

S_3	41 TeV
V_1	41 TeV
V_3	18 TeV

Upper mass limits for leptoquarks that satisfy neutral current $B-$anomaly fits and B_s-mixing constraints.

8 TeV CMS 20fb^{-1} 2nd gen



Up to 14 TeV LQs with 100 TeV 10 ab^{-1} FCC-hh. $M_{LQ} < 41$ TeV.

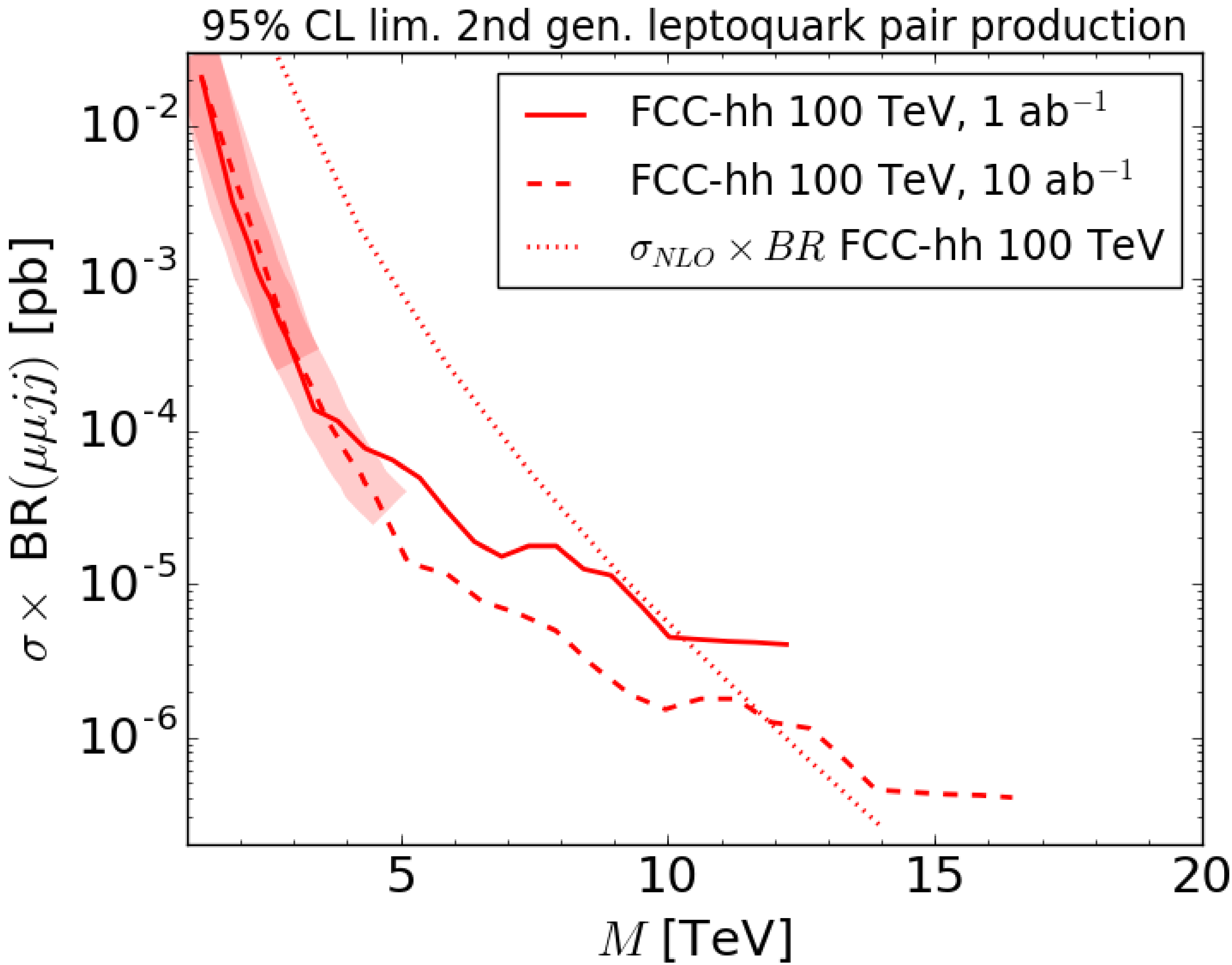
LQ Mass Limits

S_3	41 TeV
V_1	41 TeV
V_3	18 TeV

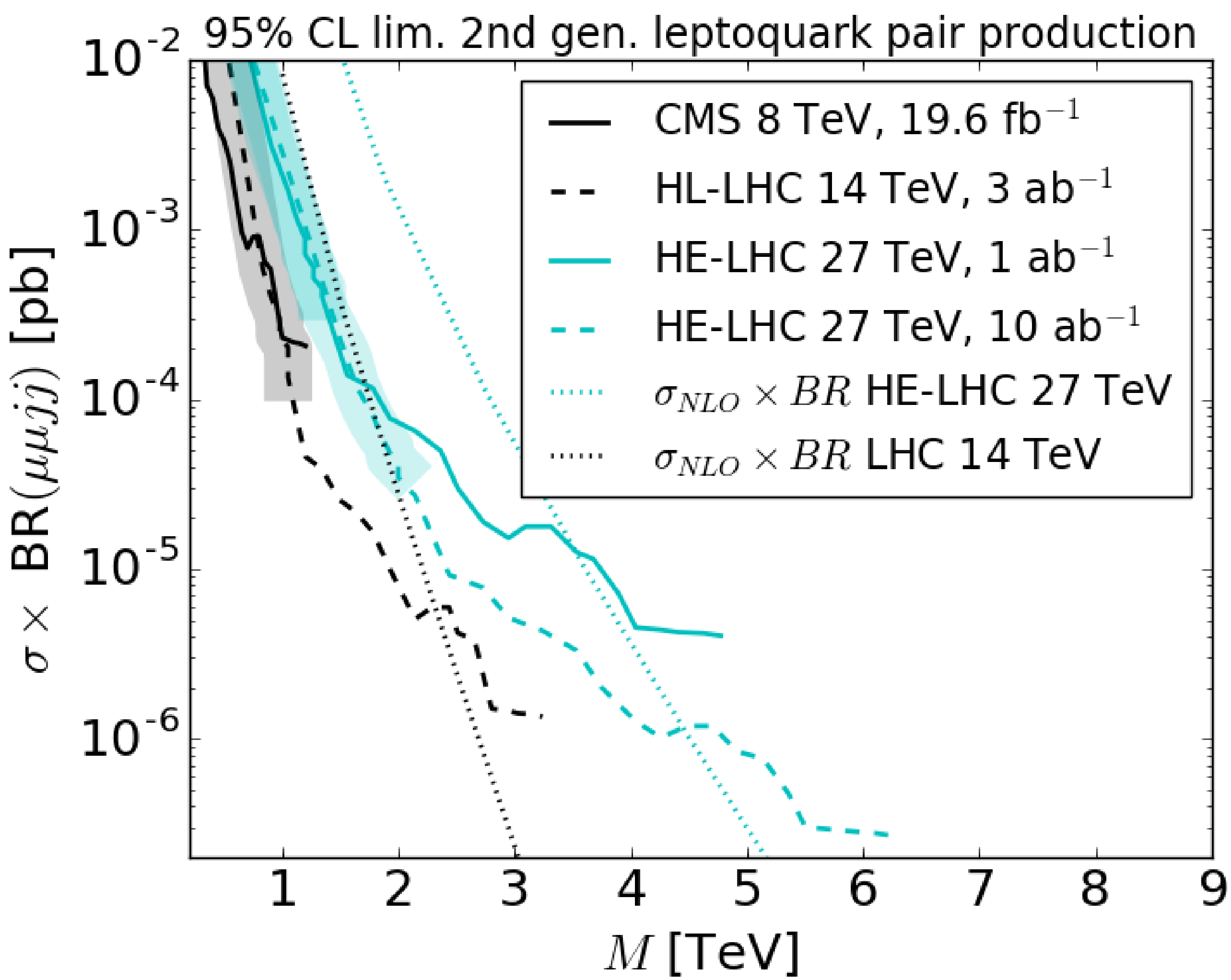
From $B_s - \bar{B}_s$ mixing and fitting $b-$anomalies.

Pair production has a reach up to 12 TeV.

The pair production cross-section is insensitive to the representation of $SU(2)$ in this case.


95% CL lim. 2nd gen. leptoquark pair production
FCC-hh 100 TeV, 1 ab^{-1}
FCC-hh 100 TeV, 10 ab^{-1}
$\sigma_{NLO} \times BR$ FCC-hh 100 TeV
$\sigma \times$ BR$(\mu\mu jj)$ [pb]
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
5
10
15
20
M [TeV]

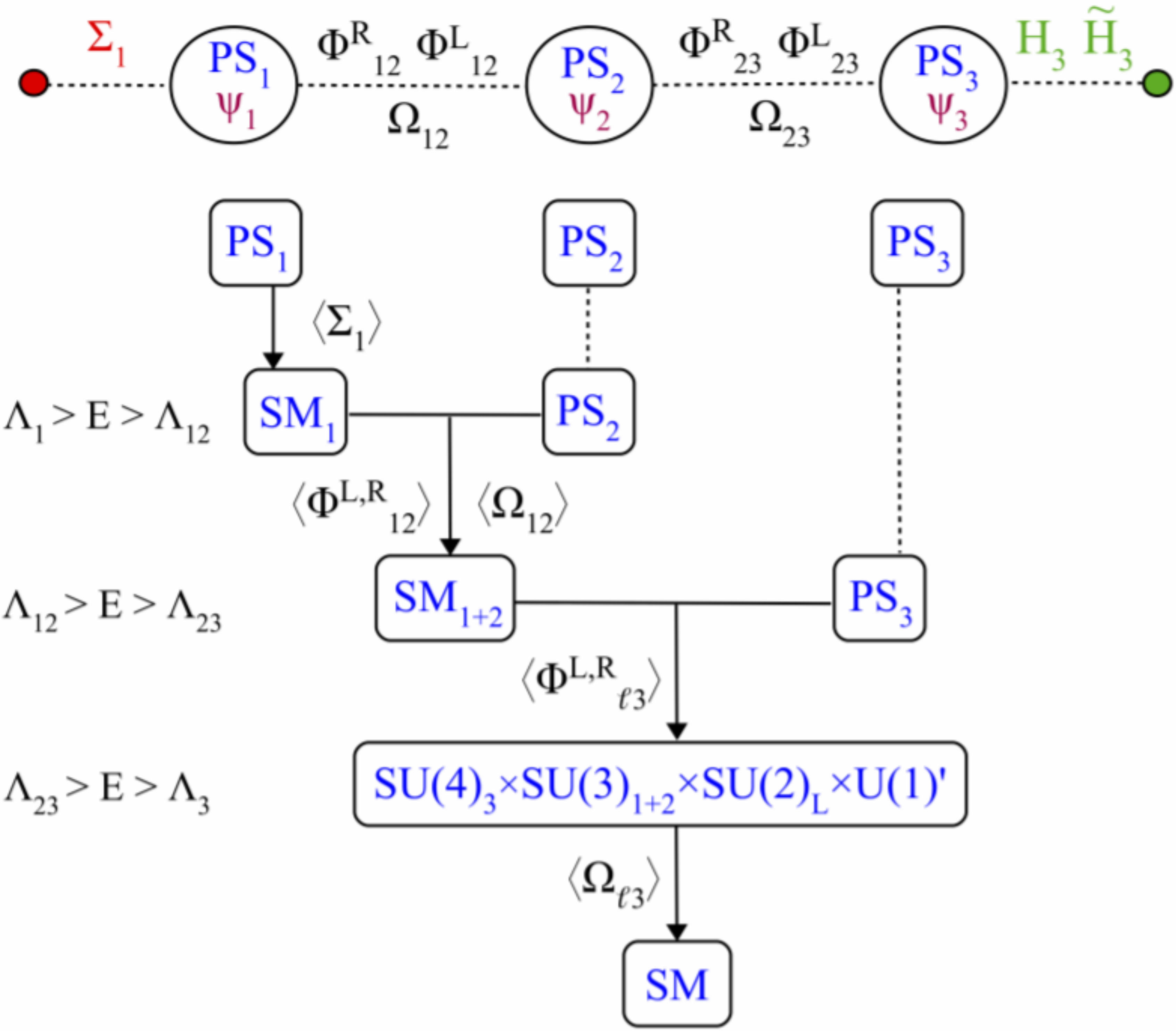
HL-LHC/HE-LHC LQs



Other Flavour Models

Realising[15] the vector LQ solution based on $PS = [SU(4) \times SU(2)_L \times SU(2)_R]^3$. SM-like Higgs lies in third generation PS group, explaining large Yukawas (others come from VEV hierarchies). Get $U(2)_Q \times U(2)_L$ approximate global flavour symmetry.

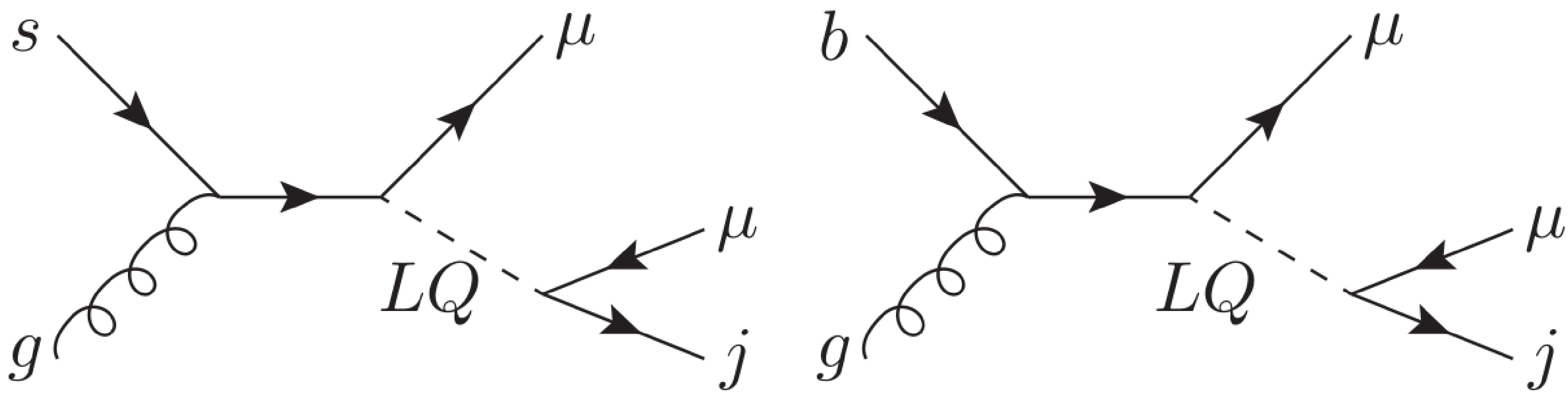
[15] Di Luzio Greljo, Nardecchia arXiv:1708.08450, Bordone, Cornella, Fuentes-Martin, Isidori, arXiv:1712.01368



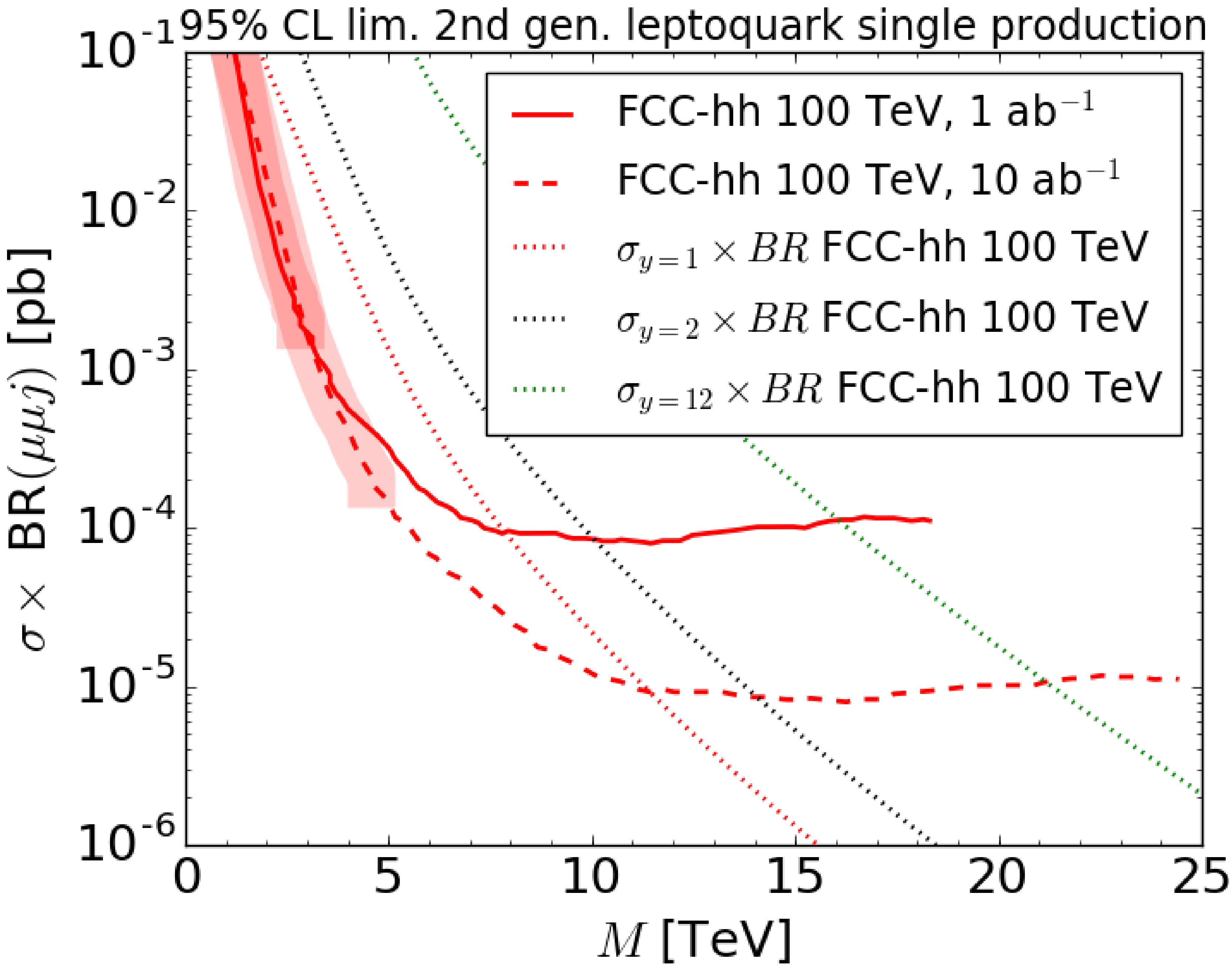
Σ_1
PS_1
ψ_1
$\Phi^R{}_{12}$ $\Phi^L{}_{12}$
Ω_{12}
PS_2
ψ_2
$\Phi^R{}_{23}$ $\Phi^L{}_{23}$
Ω_{23}
PS_3
ψ_3
H_3 $\widetilde{H}_3$
PS_1
PS_2
PS_3
$\langle\Sigma_1\rangle$
$\Lambda_1 > E > \Lambda_{12}$
SM_1
PS_2
$\langle\Phi^{L,R}{}_{12}\rangle$
$\langle\Omega_{12}\rangle$
$\Lambda_{12} > E > \Lambda_{23}$
SM_{1+2}
PS_3
$\langle\Phi^{L,R}{}_{\ell 3}\rangle$
$\Lambda_{23} > E > \Lambda_3$
$SU(4)_3 \times SU(3)_{1+2} \times SU(2)_L \times U(1)'$
$\langle\Omega_{\ell 3}\rangle$
SM

Single Production of LQ

Depends upon LQ coupling as well as LQ mass



Current bound by CMS from 8 TeV 20 fb^{-1}: $M_{LQ} > 660$ GeV for $s\mu$ coupling of 1. We include b as well from `NNPDF2.3LO` ($\alpha_s(M_Z) = 0.119$), re-summing large logs from initial state b. Integrate $\hat{\sigma}$ with `LHAPDF`.



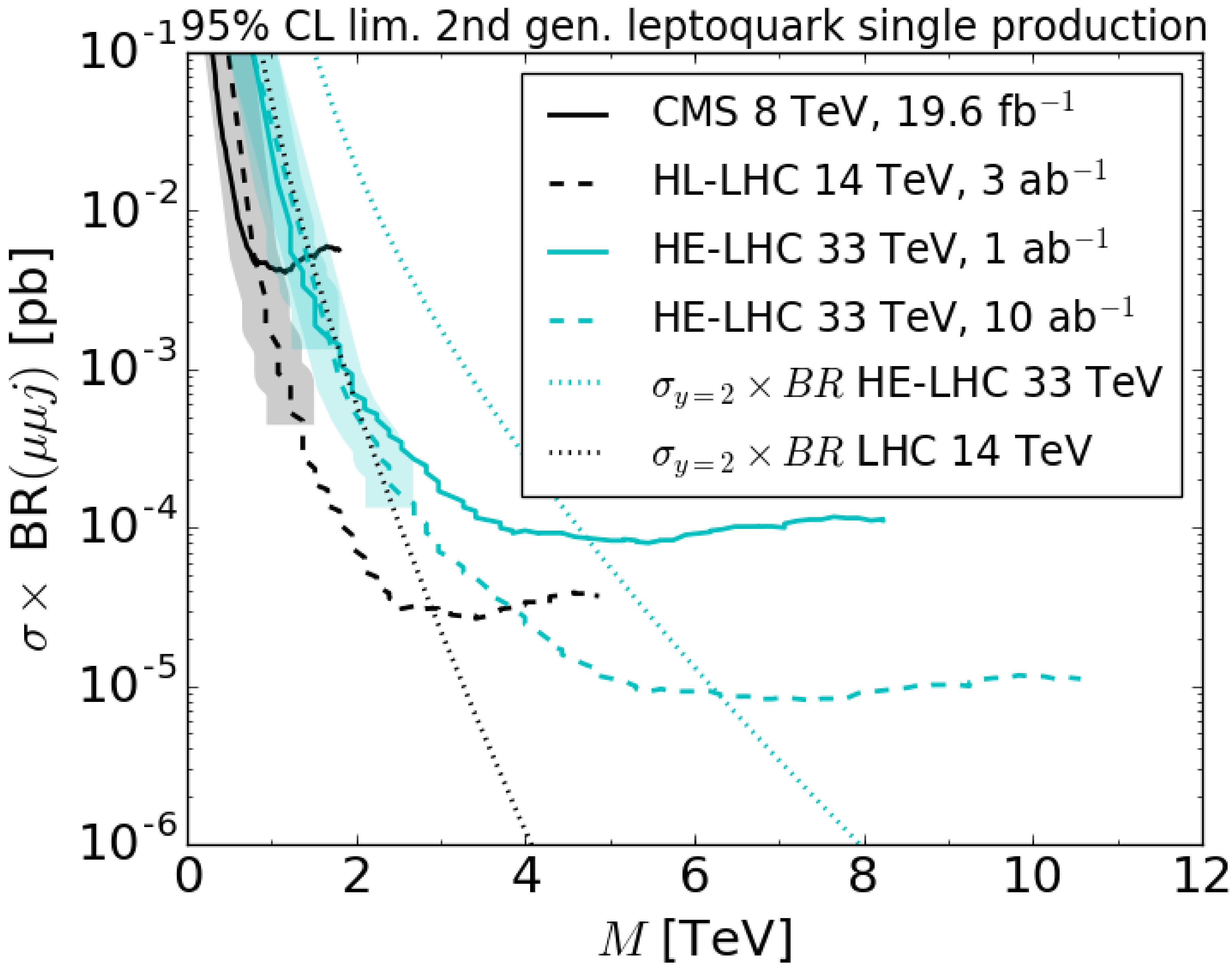
σs for S_3 with $y_{s\mu} = y_{b\mu} = y$.

Single LQ Production σ

$$\hat{\sigma}(qg \to \phi l) = \frac{y^2 \alpha_S}{96 \hat{s}} \left(1 + 6r - 7r^2 + 4r(r+1)\ln r\right) ,$$

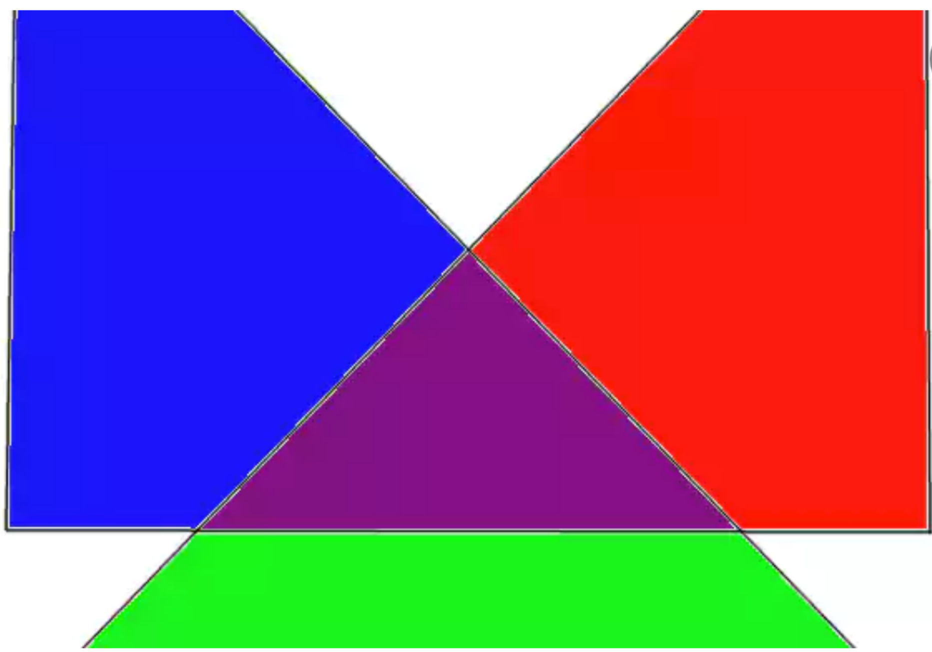
where[16] $r = M_{LQ}^2/\hat{s}$ and we set $y_{s\mu} = y_{b\mu} = y$.

[16] Hewett and Pakvasa, PRD **57** (1988) 3165.


95% CL lim. 2nd gen. leptoquark single production
CMS 8 TeV, 19.6 fb^{-1}
HL-LHC 14 TeV, 3 ab^{-1}
HE-LHC 33 TeV, 1 ab^{-1}
HE-LHC 33 TeV, 10 ab^{-1}
$\sigma_{y=2} \times BR$ HE-LHC 33 TeV
$\sigma_{y=2} \times BR$ LHC 14 TeV
$\sigma \times$ BR$(\mu\mu j)$ [pb]
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0
2
4
6
8
10
12
M [TeV]

Modelling the fourth colour: dispatch from de Moriond

At the particle physics conference, it's clear inconclusive LHCb data are stimulating strange new ideas



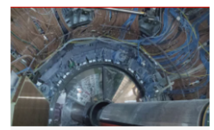
Four colours (or colors?) Photograph: Ben Allanach

Ben Allanach

Sat 17 Mar 2018 10.15 GMT

In the middle of the Rencontres de Moriond particle physics conference in Italy, the scientific talks stopped to allow a standing ovation dedicated to the memory and achievements of my inspirational colleague Stephen Hawking, who we heard had died earlier that day.

The talks quickly resumed, which I think Stephen would have approved of. The most striking thing about the scientific content of the conference this year was that a whole day was dedicated to the weirdness in bottom particles that Tevong You and I wrote about last November. As Marco Nardecchia reviewed in his talk (PDF), bottom particles produced in the LHCb detector in proton collisions are decaying too often in certain particular ways, compared to predictions from the Standard Model of particle physics. Their decay products are coming out with the wrong angles too often compared with predictions, too.



Anomalous bottoms at Cern and the case for a new collider

Read more

We were hoping for an update on the data at the conference: the amount of data has roughly doubled since they were last released, and we need to see the new data to be convinced that something really new is happening in the collisions. I strongly suspect that if the effect is seen in the new data, the theoretical physics community will "go nuts" and we will quickly see the resulting avalanche of papers. If the new data look ordinary, the effect will be forgotten and everyone will move on. Taking such measurements correctly takes care and time, however, and the LHCb experiment didn't release them. We shall have to wait until other conferences later this year for the LHCb to present its analyses of the new data.

There were interesting theory talks on how new forces could explain the strange properties of the bottom particle decays. The full mathematical models look quite baroque: they need a lot of "bells and whistles" in order to pass other experimental tests. But these models prove that it can be done, and they are quite different to what has been proposed before.

One of them even unifies different classes of particle (leptons and quarks), describing the lepton as the "fourth colour" of a quark. We are used to the idea that quarks come in three (otherwise identical) copies: physicists label them red, green and blue to distinguish them. As Javier Fuentes-Martin describe (PDF), once you design the mathematics to make leptons the fourth colour, the existence of a new force-carrying particle with just the correct properties to break up the

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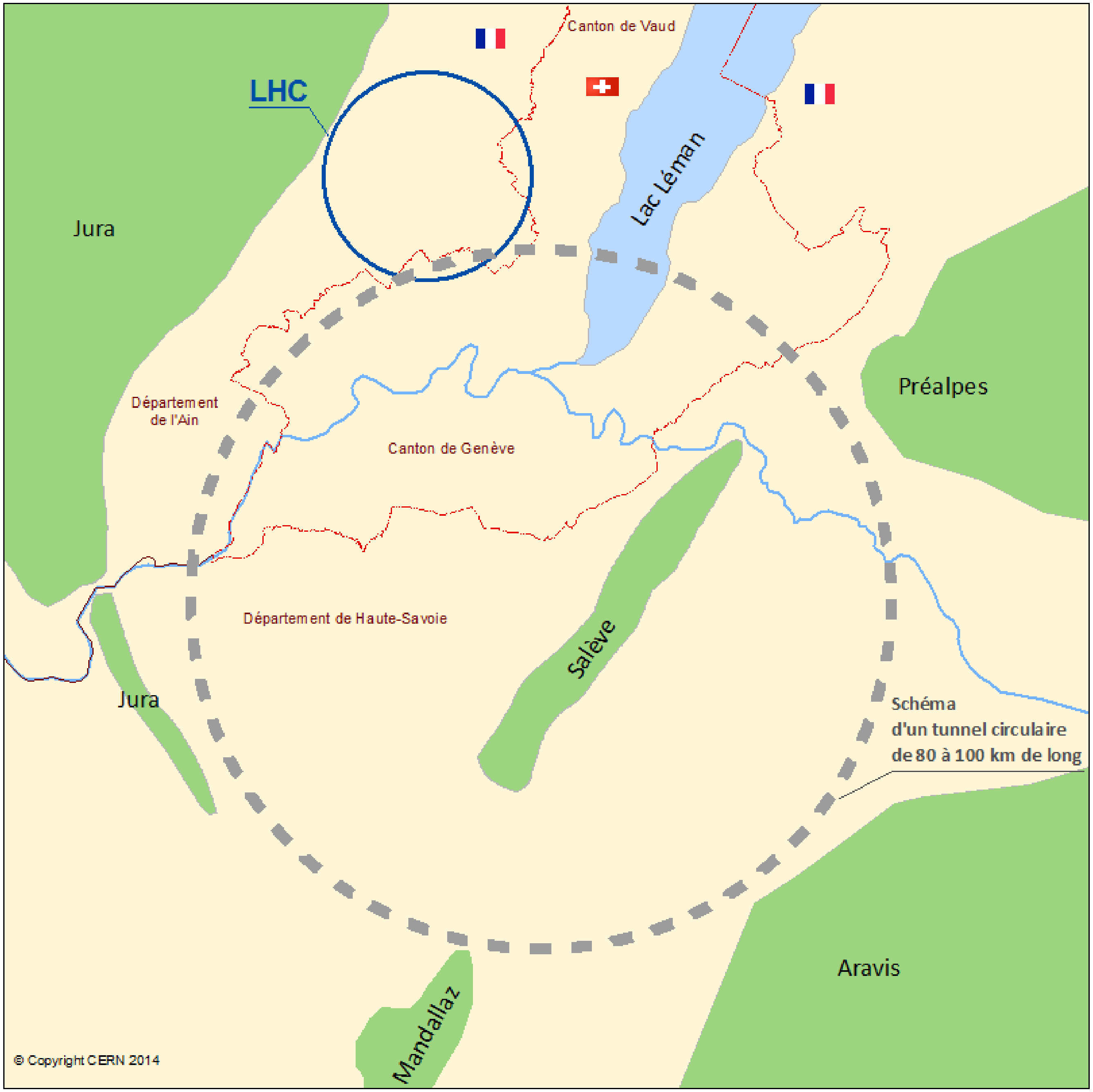
Cern

Cern draws up plans for collider four times the size of Large Hadron

The Future Circular Collider would smash particles together in a tunnel 100km long


LHC
Canton de Vaud
Lac Léman
Jura
Département de l'Ain
Canton de Genève
Préalpes
Département de Haute-Savoie
Salève
Jura
Schéma
d'un tunnel circulaire
de 80 à 100 km de long
Aravis
Mandallaz
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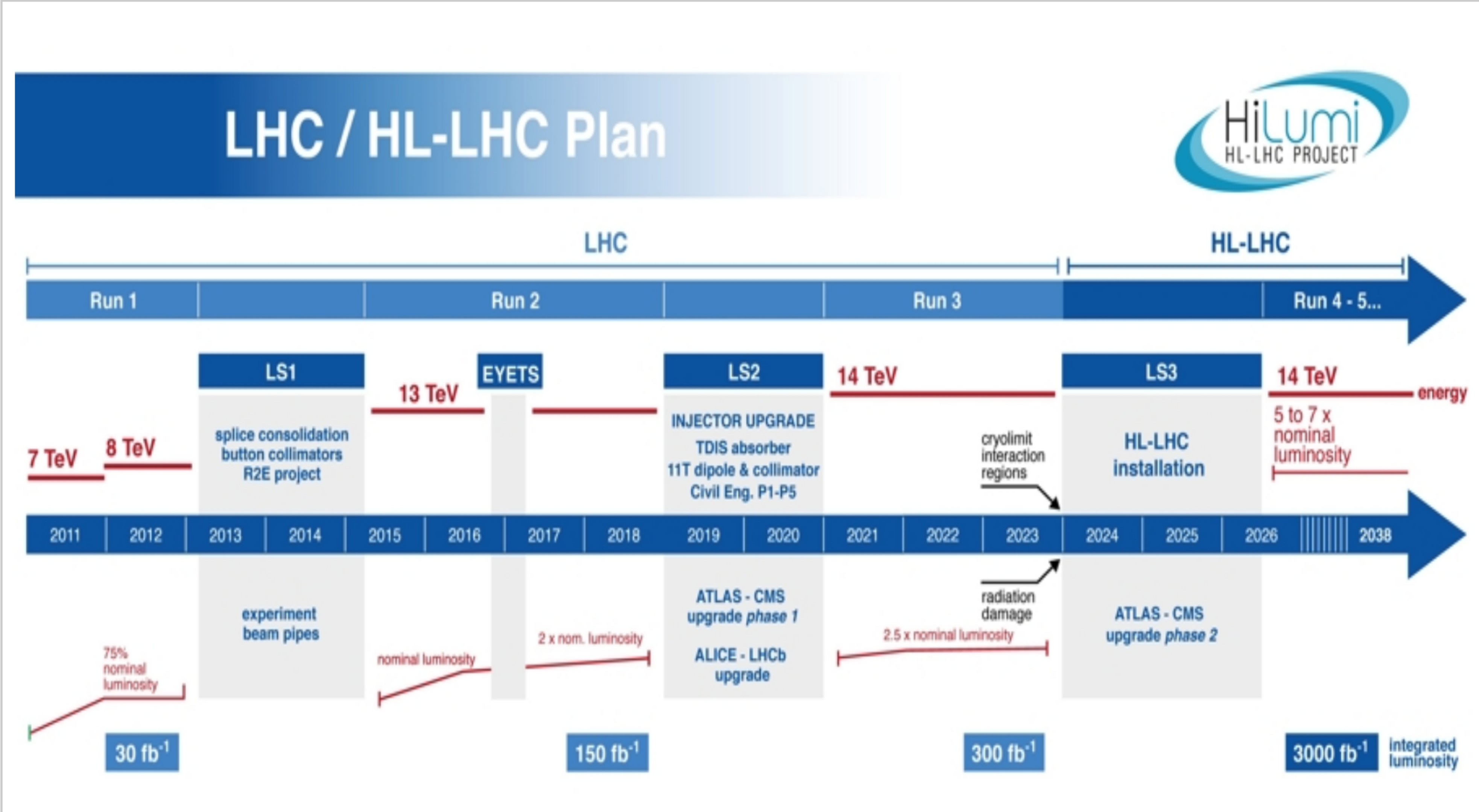
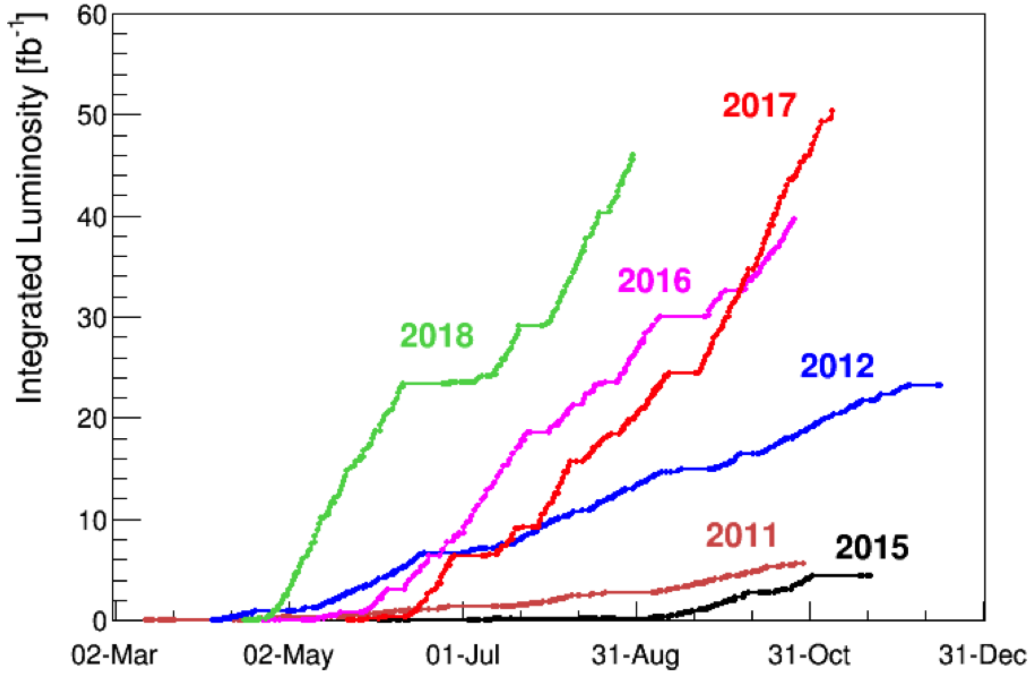

LHC / HL-LHC Plan
HiLumi
HL-LHC PROJECT
LHC
HL-LHC
Run 1
Run 2
Run 3
Run 4 - 5...
LS1
EYETS
LS2
LS3
7 TeV
8 TeV
13 TeV
14 TeV
14 TeV
energy
splice consolidation
button collimators
R2E project
INJECTOR UPGRADE
TDIS absorber
11T dipole & collimator
Civil Eng. P1-P5
cryolimit
interaction
regions
HL-LHC
installation
5 to 7 x
nominal
luminosity
2011
2012
2013
2014
2015
2016
2017
2018
2019
2020
2021
2022
2023
2024
2025
2026
2038
experiment
beam pipes
ATLAS - CMS
upgrade phase 1
ALICE - LHCb
upgrade
radiation
damage
ATLAS - CMS
upgrade phase 2
75%
nominal
luminosity
nominal luminosity
2 x nom. luminosity
2.5 x nominal luminosity
30 fb-1
150 fb-1
300 fb-1
3000 fb-1
integrated
luminosity

LHC Upgrades



High Luminosity (HL) LHC: go to 3000 fb^{-1} (3 ab^{-1}).
High Energy (HE) LHC: Put FCC magnets (16 Tesla rather than 8.33 Tesla) into LHC ring: roughly *twice* collision energy: 27 TeV.

Properties of anomaly-free solutions