Dark Blobs

Exponentially Large Composite Dark Matter

Dorota M Grabowska

CERN

Dark Matter Mass Landscape

Allowable mass range covers ~ 90 orders of magnitude

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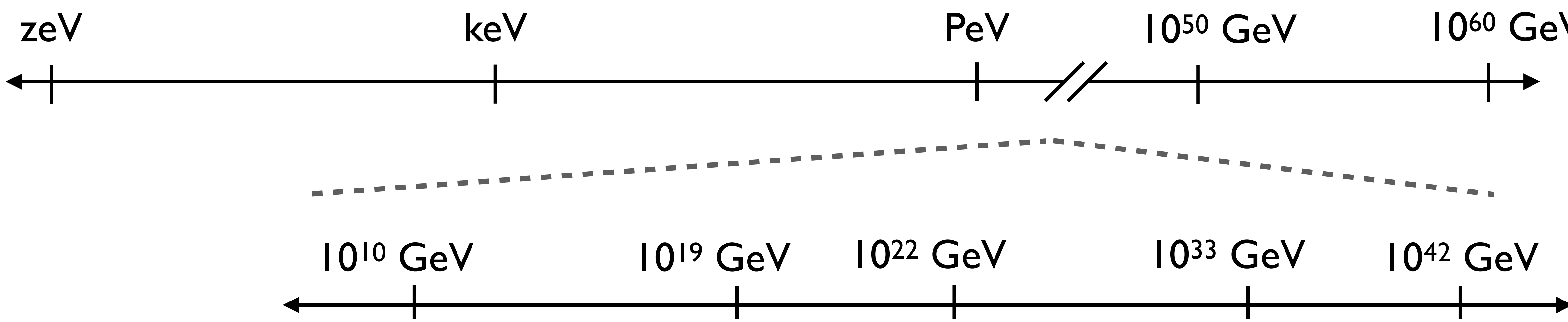
Ultra Light	Particle	Macroscopic
Axions, ALPS, etc	WIMPs, Hidden Sector, Asymmetric, etc	Primordial Black Holes, MACHOS, etc
Semi-Classical Field	*Quantum Mechanics*	*General Relativity*

Dark Matter Mass Landscape

Allowable mass range covers ~ 90 orders of magnitude



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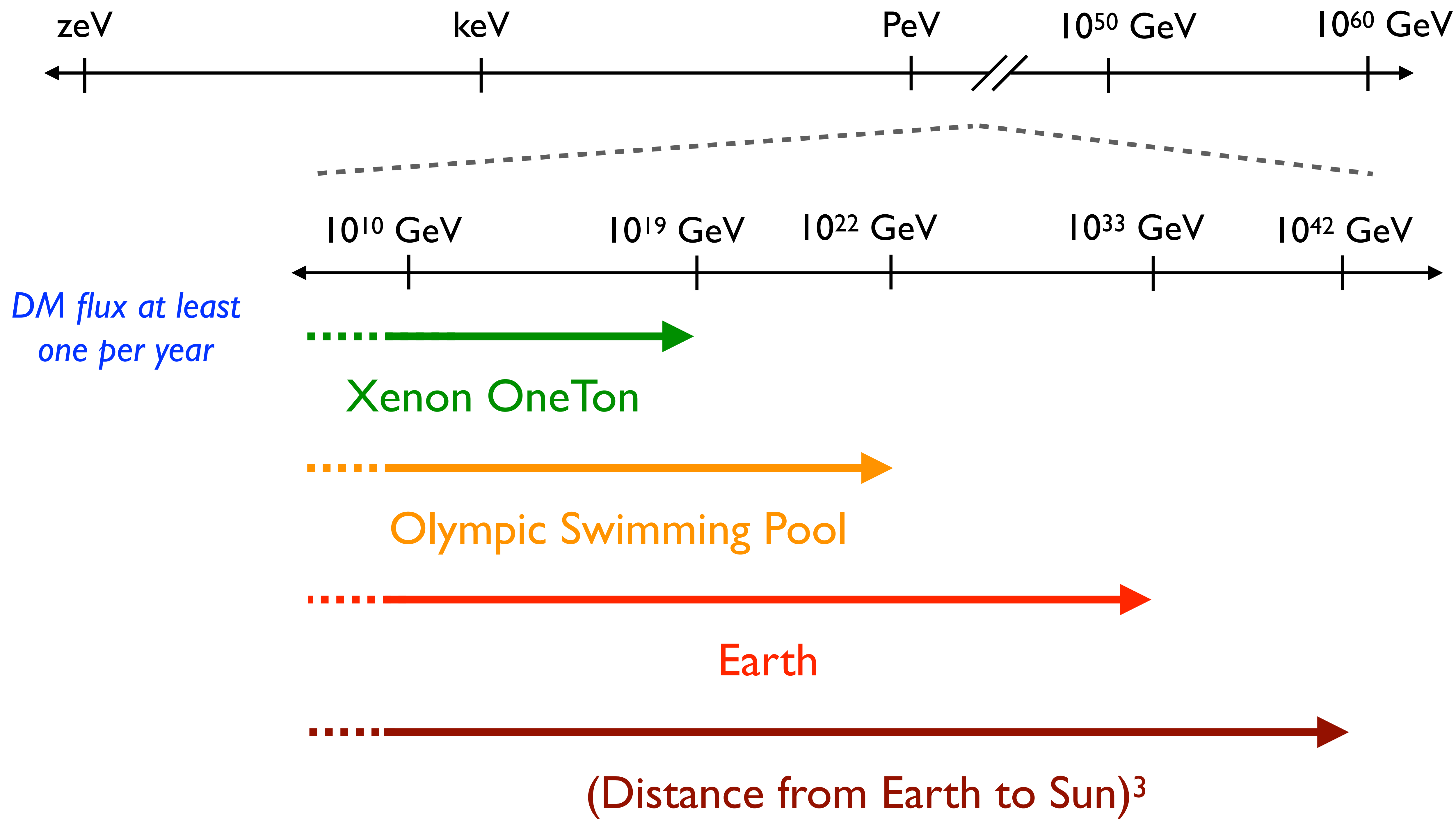


The Plan

1. Interesting paradigm for heavy dark matter

2. Several Detection Strategies

The Plan

1. Interesting paradigm for heavy dark matter

   Take Away: *Bound states in strongly interacting dark sectors have masses that span ~40 orders of magnitude*

2. Several Detection Strategies

   Take Away: *Need multi-prong experimental and observational approach*

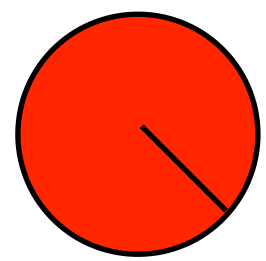
Strongly Interacting Dark Sector

Example: Dark Nuclear Matter

General Properties

- Theory confines at energy scale Λ_χ
- Spectrum contains massive particle with $m_\chi \sim \Lambda_\chi$

"Dark Nucleon"



$r_\chi \sim 1/\Lambda_\chi$

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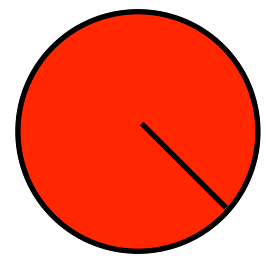
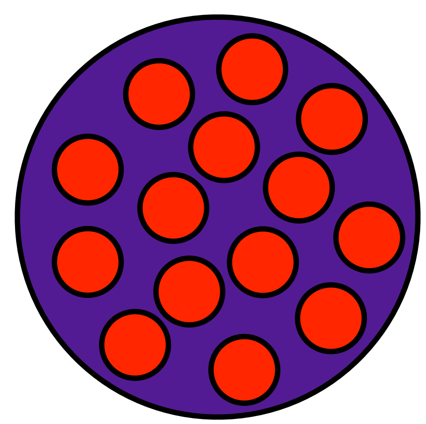


$r_\chi \sim 1/\Lambda_\chi$

- Massive particles form bound states with $M_X \sim N_X \Lambda_\chi$

"Dark Nucleus"



$R_X \sim N_X^{1/3}/\Lambda_\chi$

- Relic abundance set by dark baryon asymmetry

Maximum Size of Dark Nucleus

Semi-empirical mass formula

Treat dark nucleus as drop of liquid to determine how binding energy depends on number of constituents

$$E_{\text{Bind}} \sim \alpha_V N_X - \alpha_S N_X^{2/3} - \alpha_C N_X^{5/3}$$

Maximum Size of Dark Nucleus

Semi-empirical mass formula

Treat dark nucleus as drop of liquid to determine how binding energy depends on number of constituents

$$E_{\text{Bind}} \sim \underbrace{\alpha_V N_X}_{\text{Short Range Strong Interaction}} - \underbrace{\alpha_S N_X^{2/3}}_{\text{Surface Correction}} - \underbrace{\alpha_C N_X^{5/3}}_{\text{Long Range Weak Interaction}}$$

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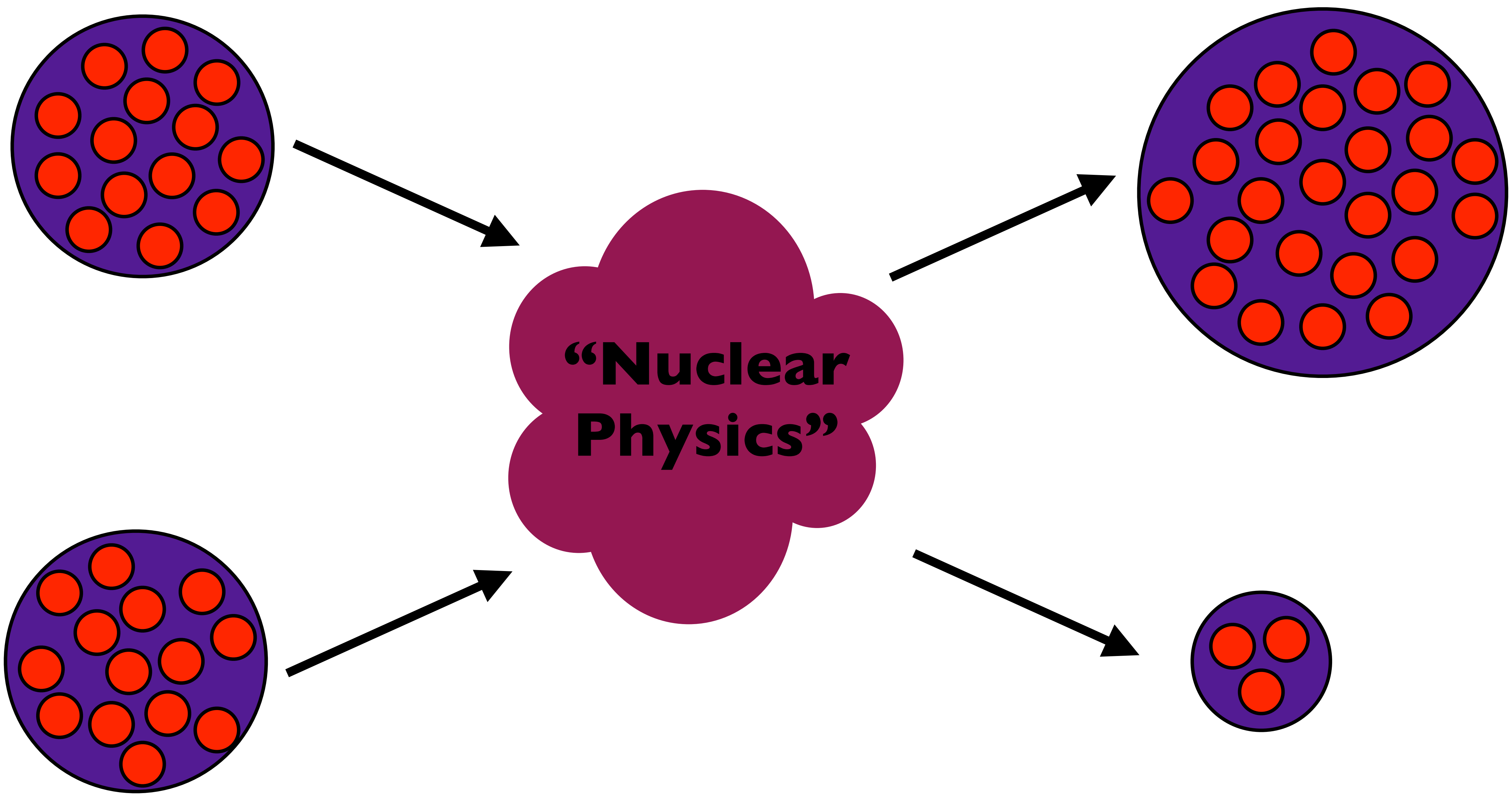
- Assume no long range force to destabilize dark nuclei

Binding energy unbounded from above!

Early Universe Formation

Big Bang Dark Nucleosynthesis

Dark nuclei form via fusion processes in the Early Universe



* Krnjaic & Sigurdson, ’14, Hardy et al, ’14, Gresham, Lou & Zurek, ’17

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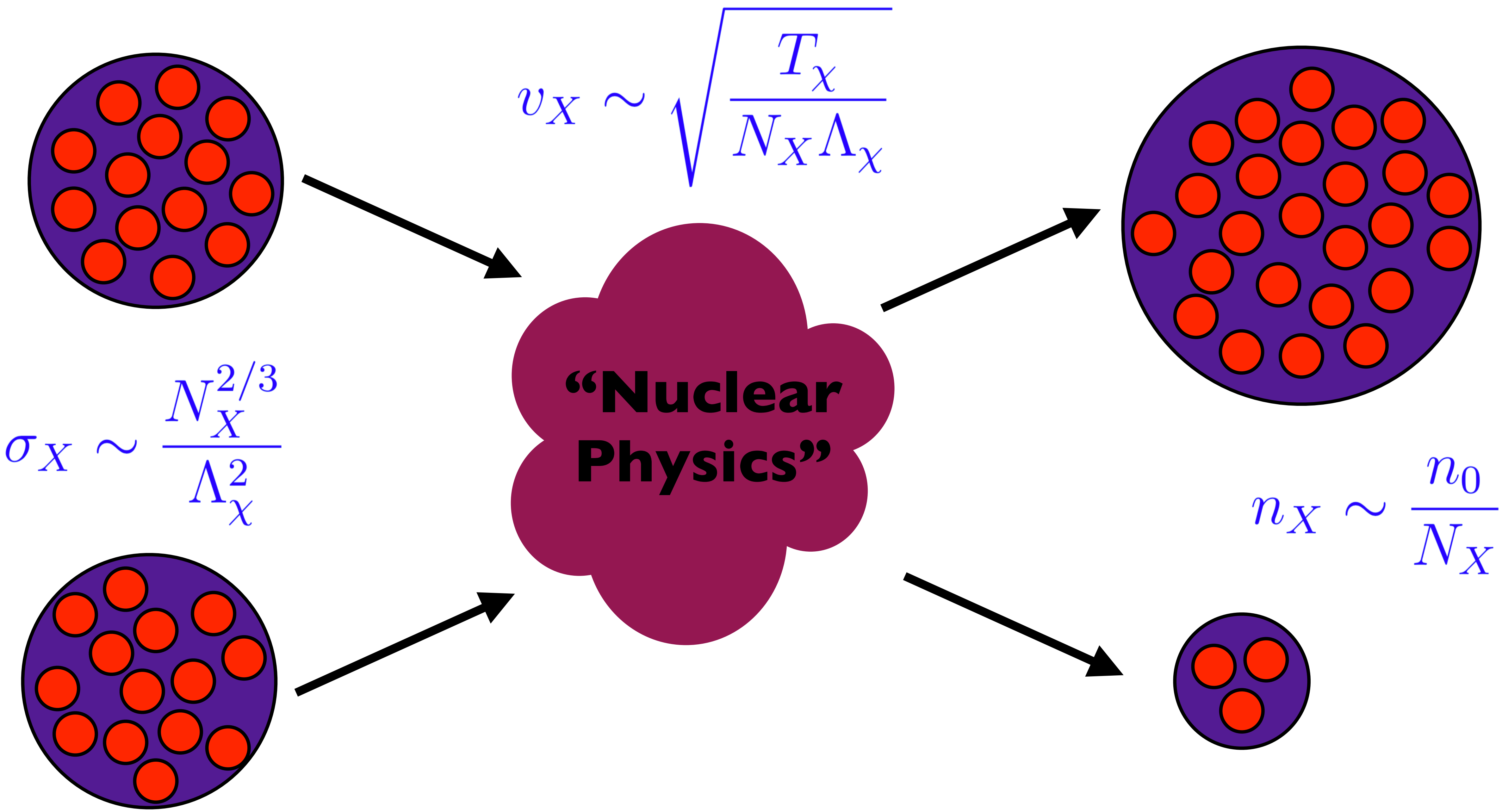

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Dark BBN

Fusion in Early Universe

Dark nuclei size is limited by how long fusion lasts during early Universe

- Compare Fusion rate to Hubble rate for rough estimate

$$\sigma_X \sim \frac{N_X^{2/3}}{\Lambda_\chi^2} \qquad v_X \sim \sqrt{\frac{T_\chi}{N_X \Lambda_\chi}} \qquad n_X \sim \frac{n_0}{N_X}$$

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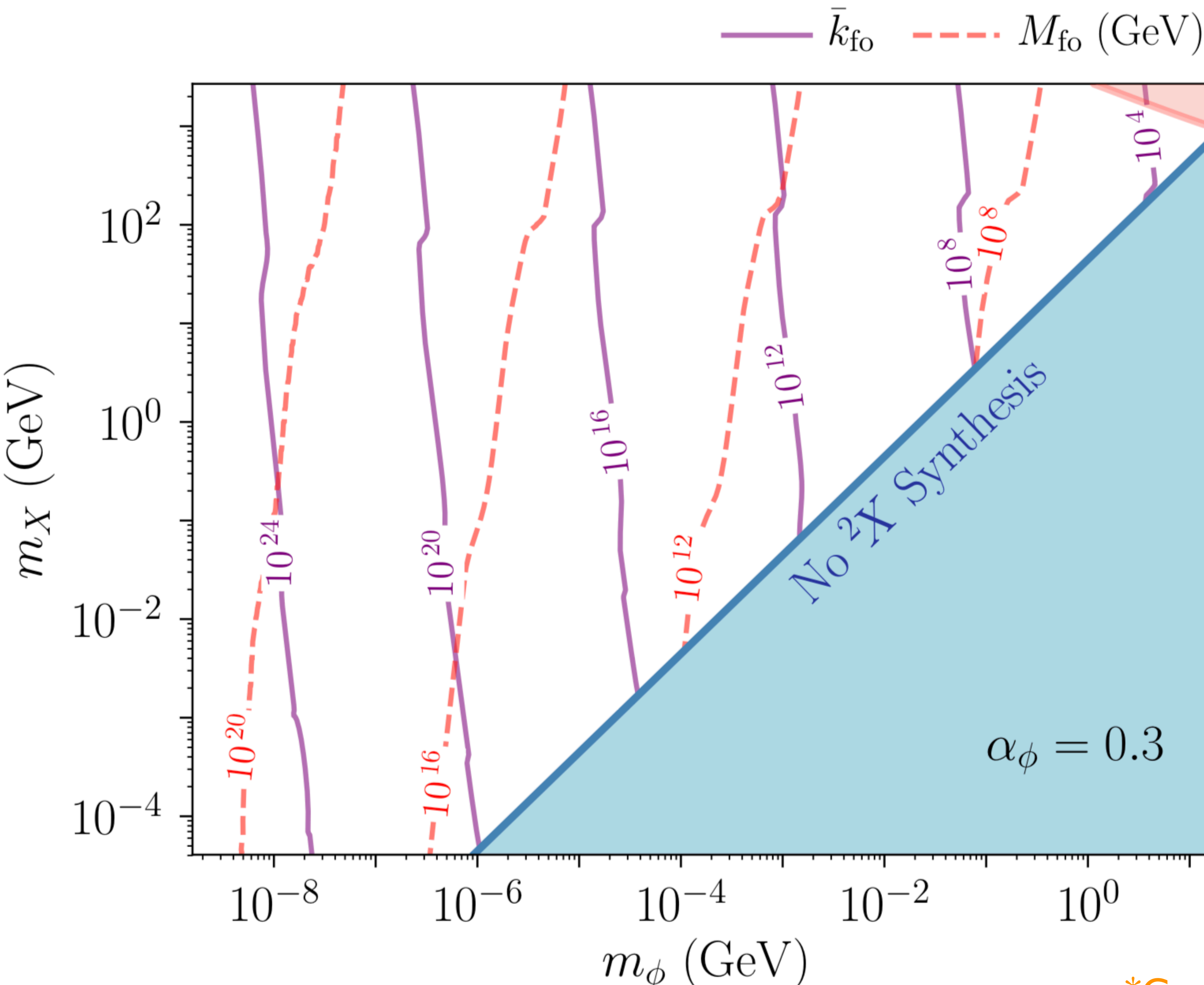
- Generically find exponentially large states

$$M_X \sim 10^{15}\,\mathrm{GeV} \left(\frac{g^*(T)}{10.2}\right)^{3/5} \left(\frac{100\,\mathrm{MeV}}{\Lambda_\chi}\right)^{16/5} \left(\frac{T}{\mathrm{MeV}}\right)^{9/5} \left(\frac{T_\chi}{T}\right)^{3/5}$$

Dark Nuclear Matter

"Dark Nuggets"



*Gresham, Lou and Zurek, '17

Detection

Standard Model Couplings

Nucleon Coupling

General Idea: Assume Dark Matter couples to Standard Model nucleons via mediator of mass m_ϕ

$$\mathcal{L} \supset \frac{1}{2} m_\phi^2 \phi^2 + g_\chi \phi \bar{\chi} \chi + g_n \phi \bar{n} n$$

There are multiple constraints on g_n and g_χ depending on the mass of the mediator

Detection strategy depends heavily on the mass of mediator

Mediator Mass

Three Regimes

Heavy

$m_\varphi R_X >> 1$

Light

$m_\varphi R_X << 1$

Ultralight

Macroscopic λ_φ

Mediator Mass

Three Regimes

Heavy	**Light**	**Ultralight**
$m_\varphi R_X >> 1$	$m_\varphi R_X << 1$	Macroscopic λ_φ

Quantum Mechanical Scattering

Classical Scattering

Form Factor Effects

DM Sources Potential

Mediator Mass

Three Regimes

Heavy	**Light**	**Ultralight**
$m_\varphi R_X >> 1$	$m_\varphi R_X << 1$	Macroscopic λ_φ
Quantum Mechanical Scattering	Quantum Mechanical Scattering	Classical Scattering
Form Factor Effects	Form Factor Effects	DM Sources Potential
High Momentum Dominated	Low Momentum Enhancement	Need Precision Experiments

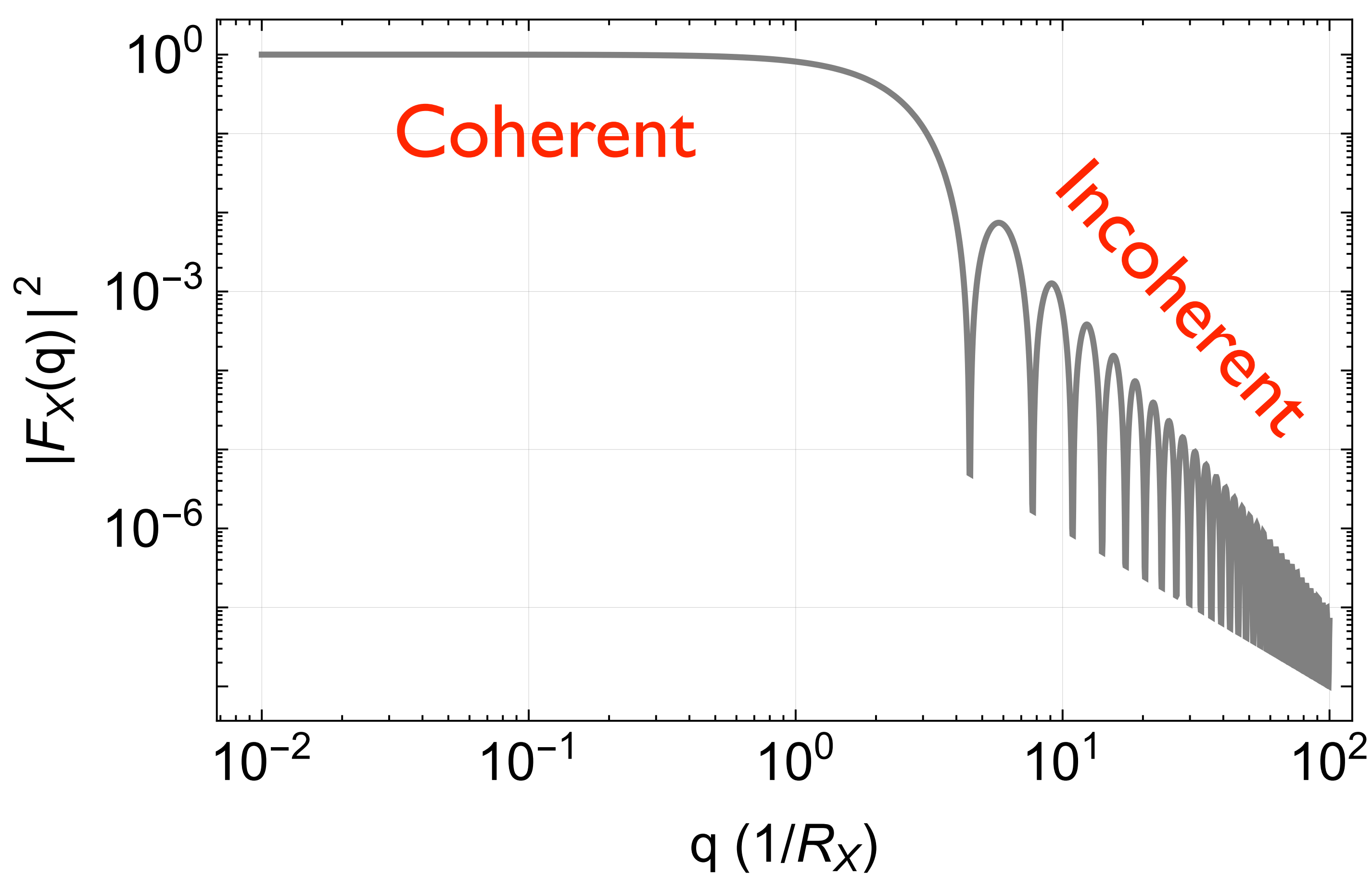
Short Range Mediator

Dark Matter Form Factor

Finite Size effects

General Idea: Form factors “encode” the deviation of scattering amplitudes from the point particle limit

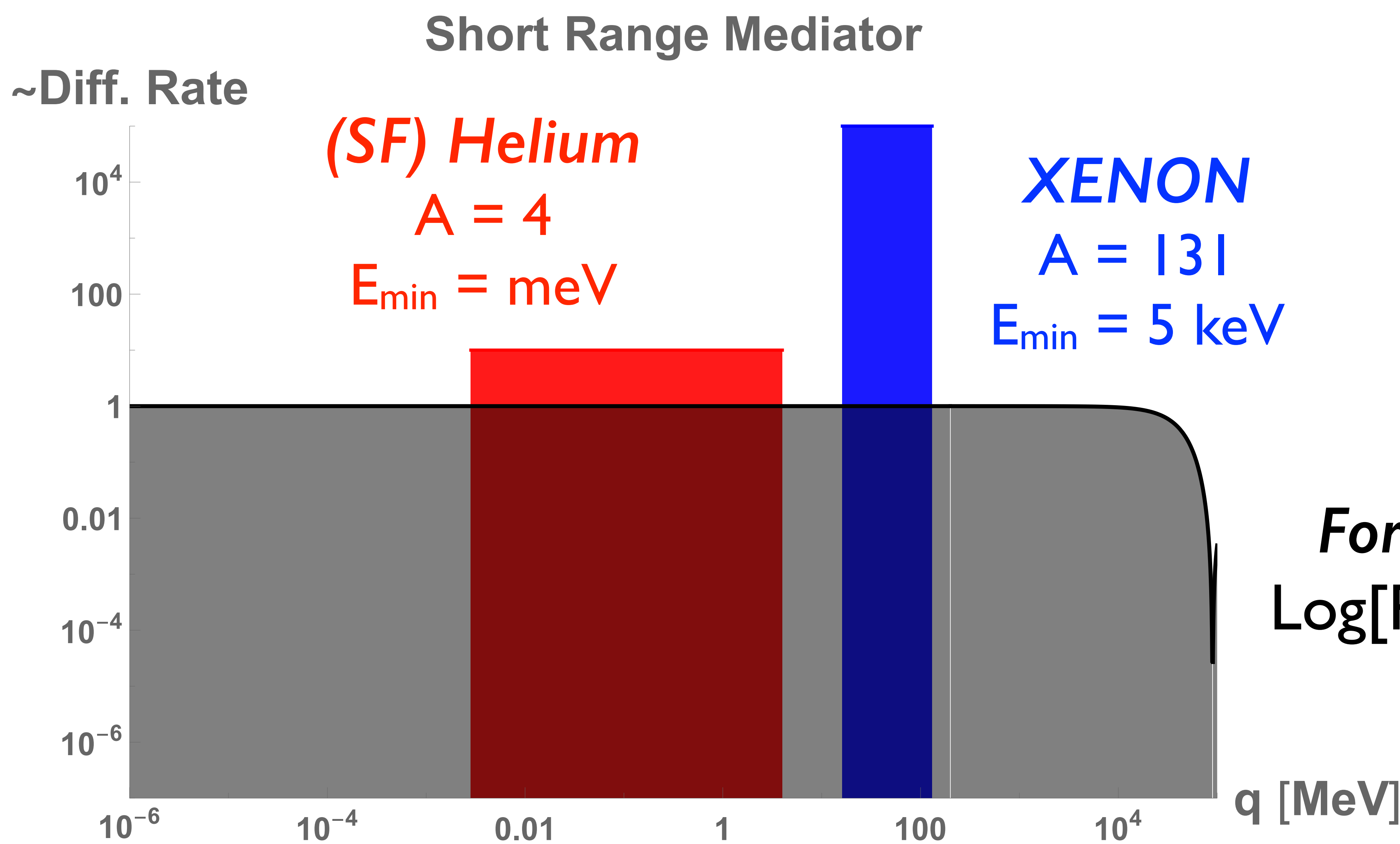
$$F_X(q) = 3\frac{\sin(qR_X) - qR_X\cos(qR_X)}{(qR_X)^3}$$



Form Factor Suppression

DM Finite Size effects

General Idea: Scan through DM radius to show effect on scattering rate



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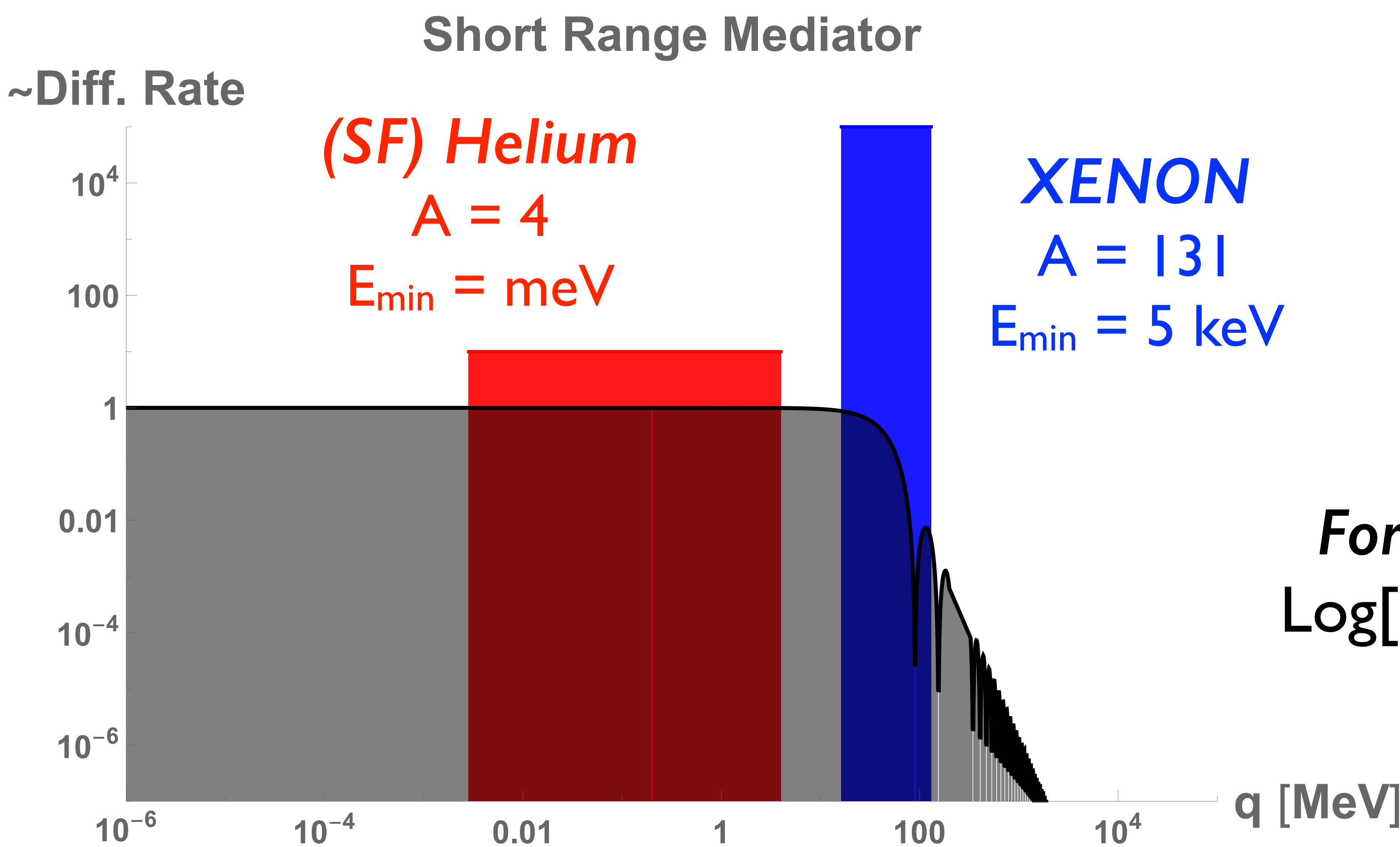

Form Factor

$\mathrm{Log}[R_X/R_n] = 1$

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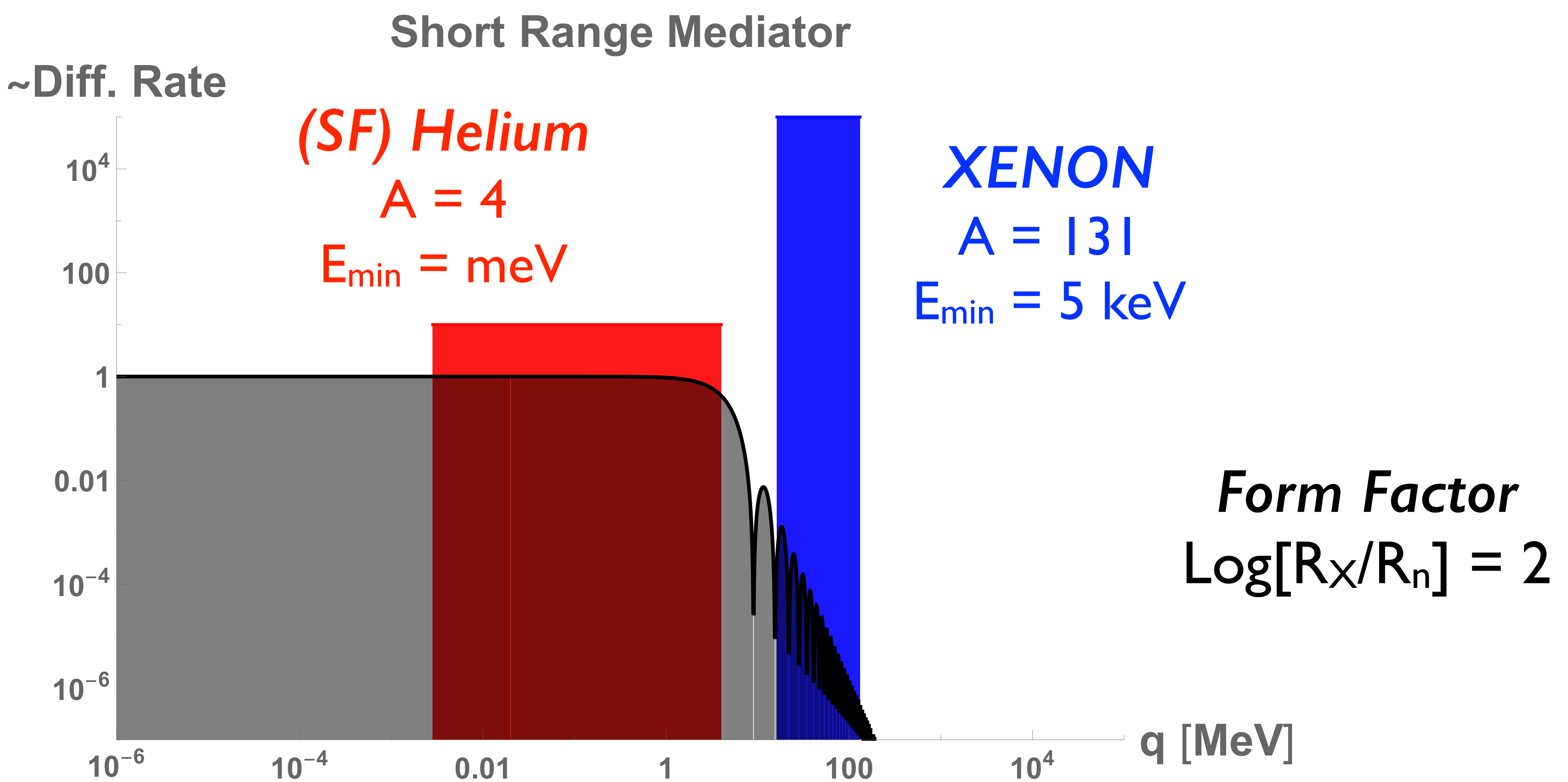

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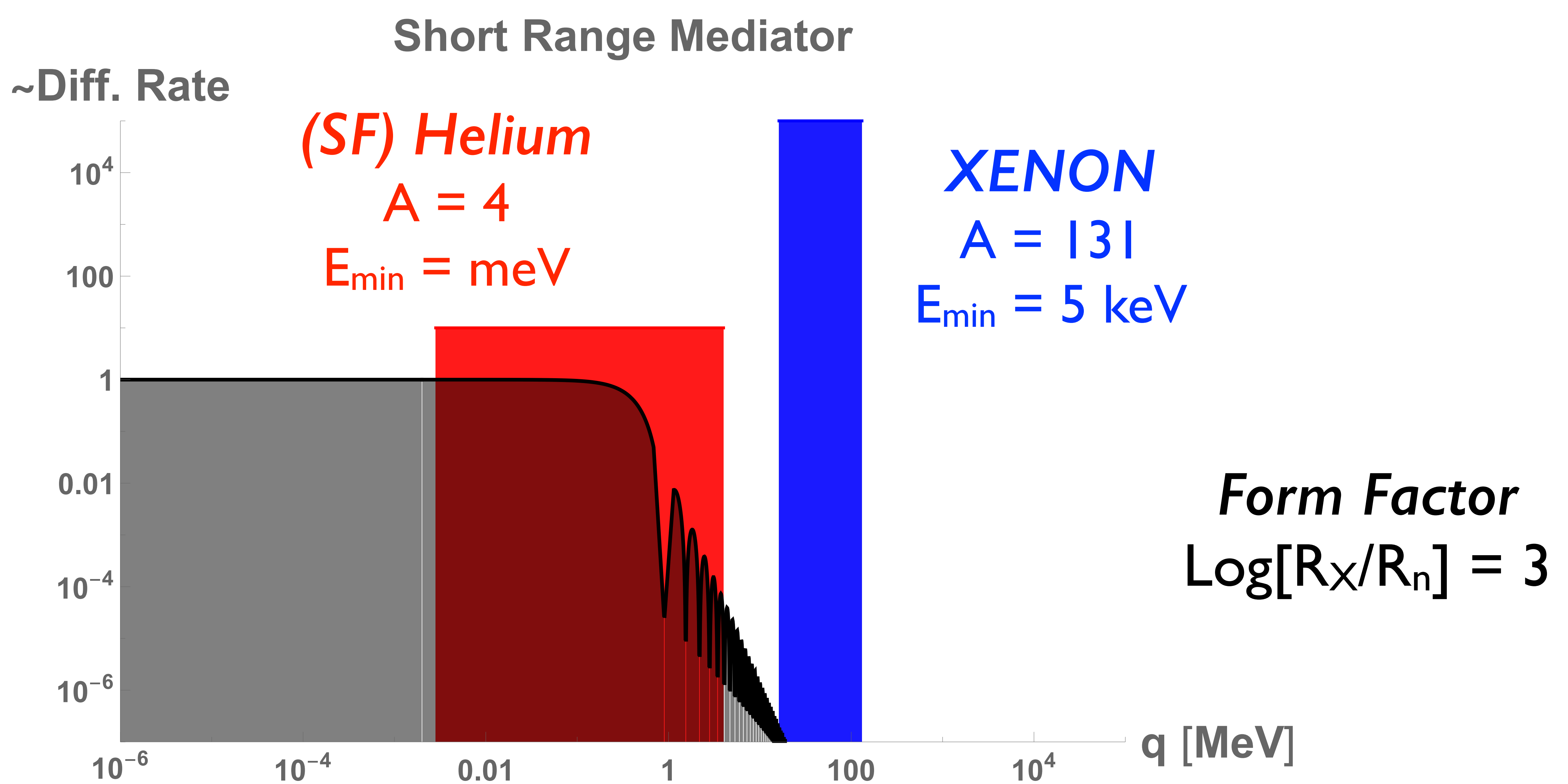

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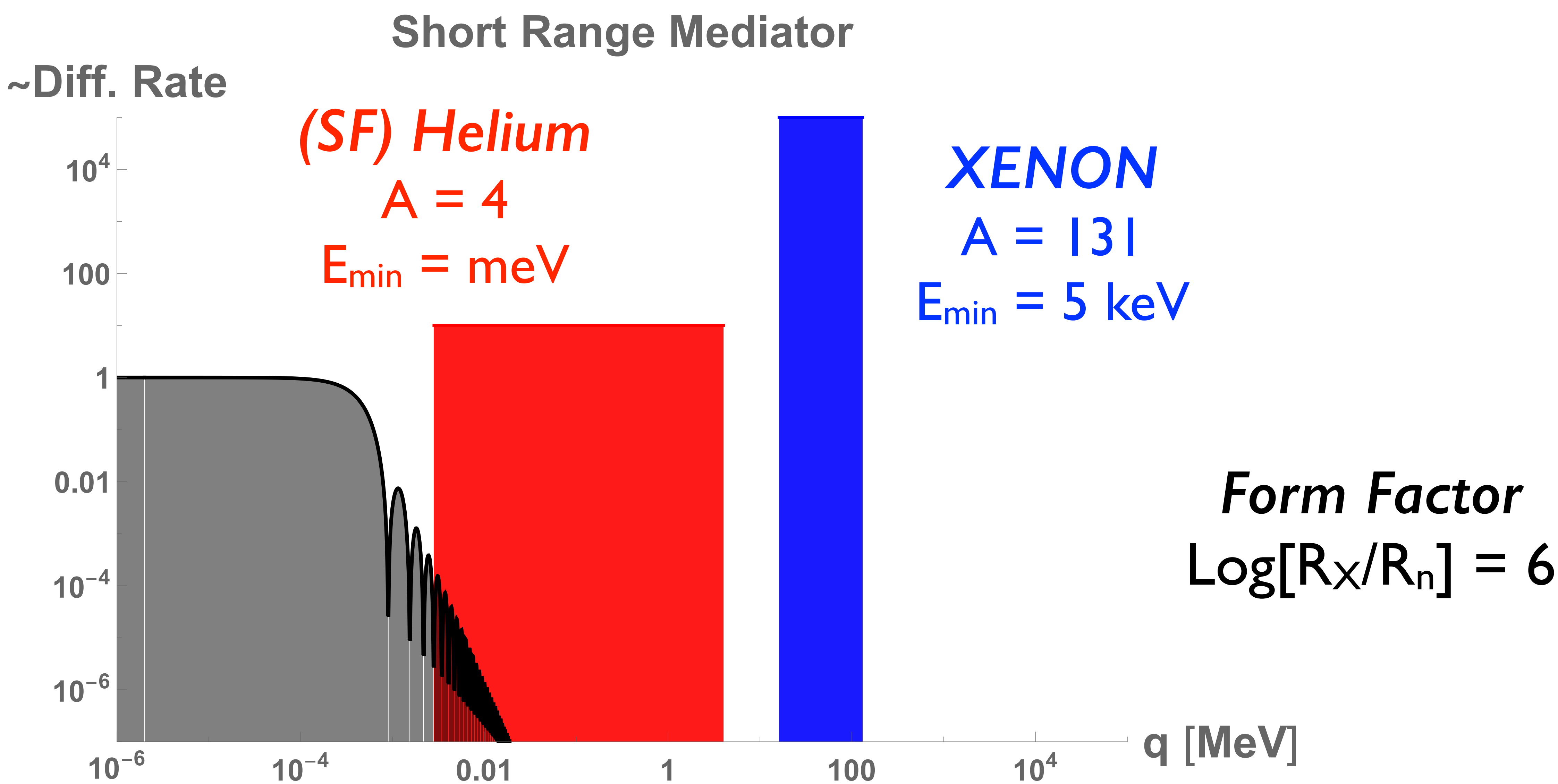
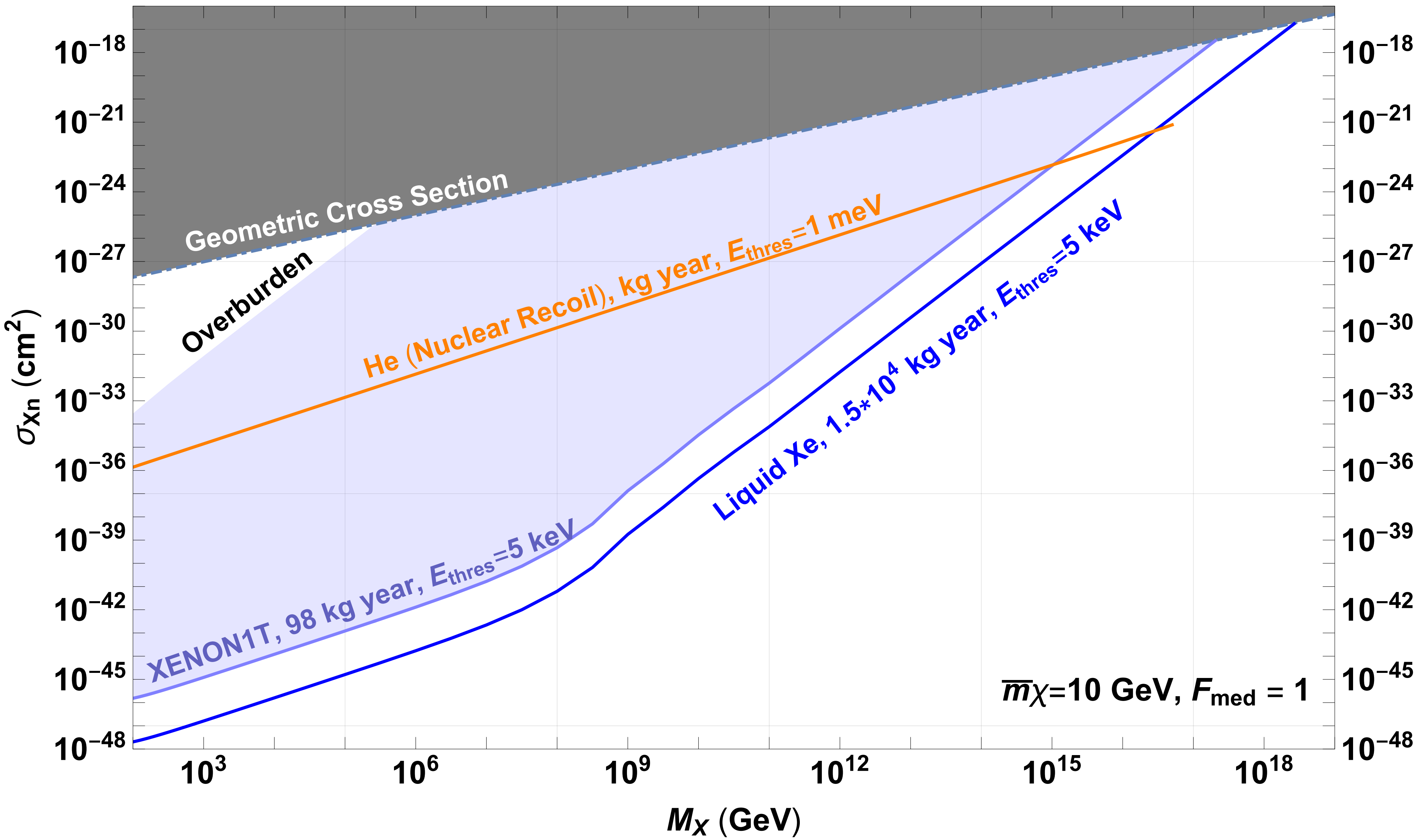


Heavy Mediator

Mediator Mass: 10 GeV



* Constraint on g_n: LHC

*Coskuner, DMG, Knapen & Zurek '18

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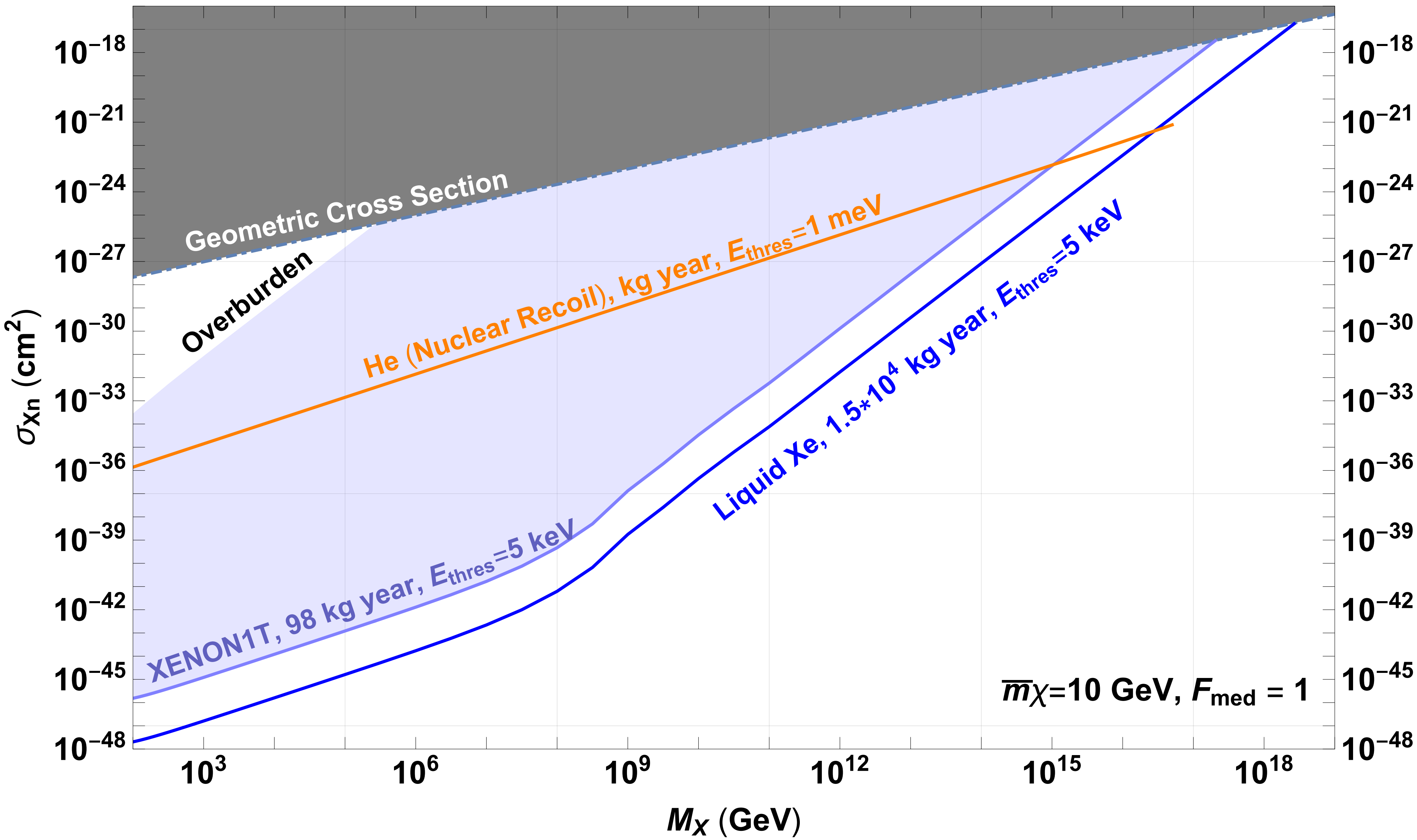
** Happy to show other experimental constraints **



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Long Range Mediator

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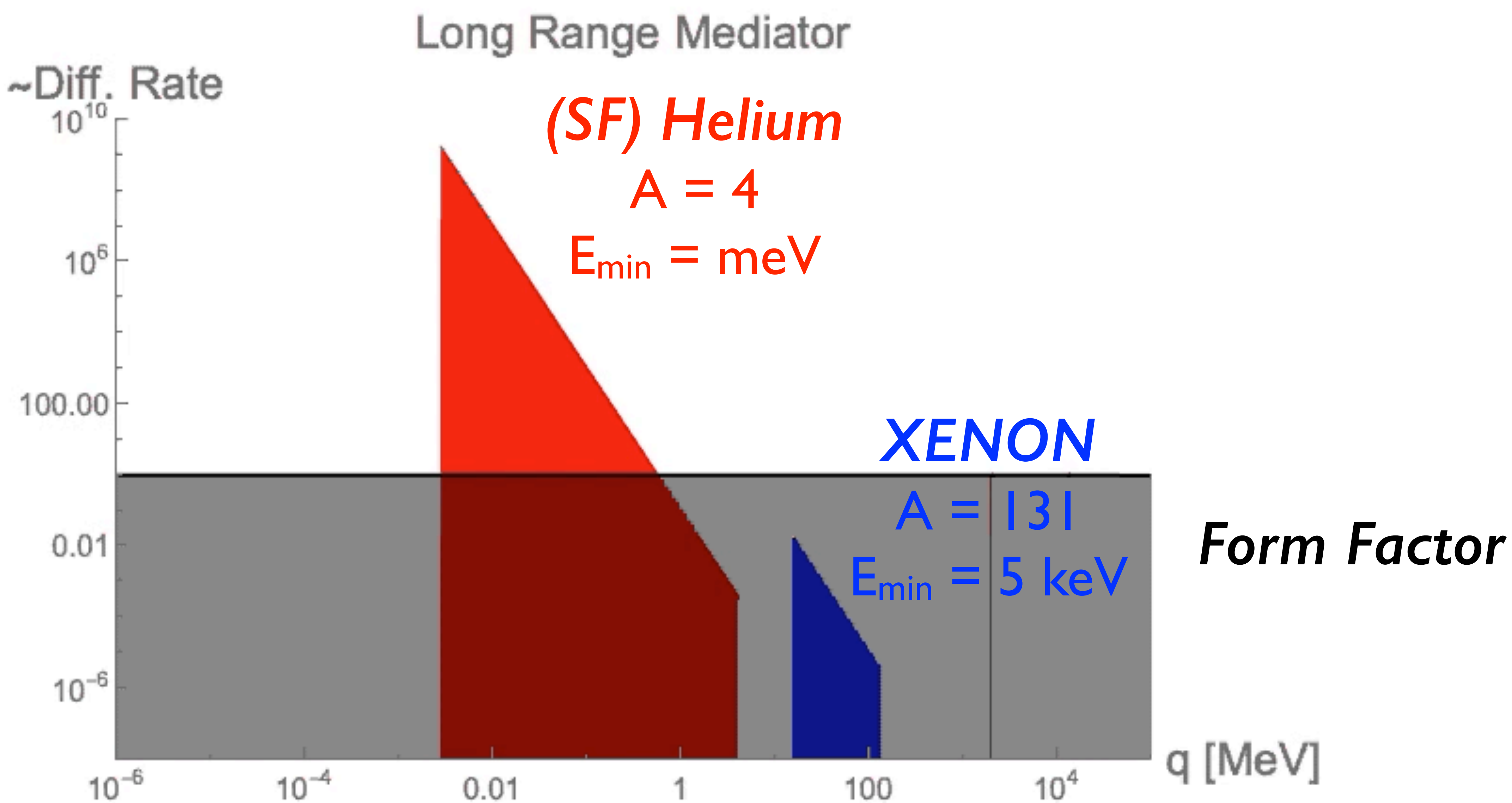
DM Finite Size effects

General Idea: Light mediator differential cross section has additional support at low momentum transfer



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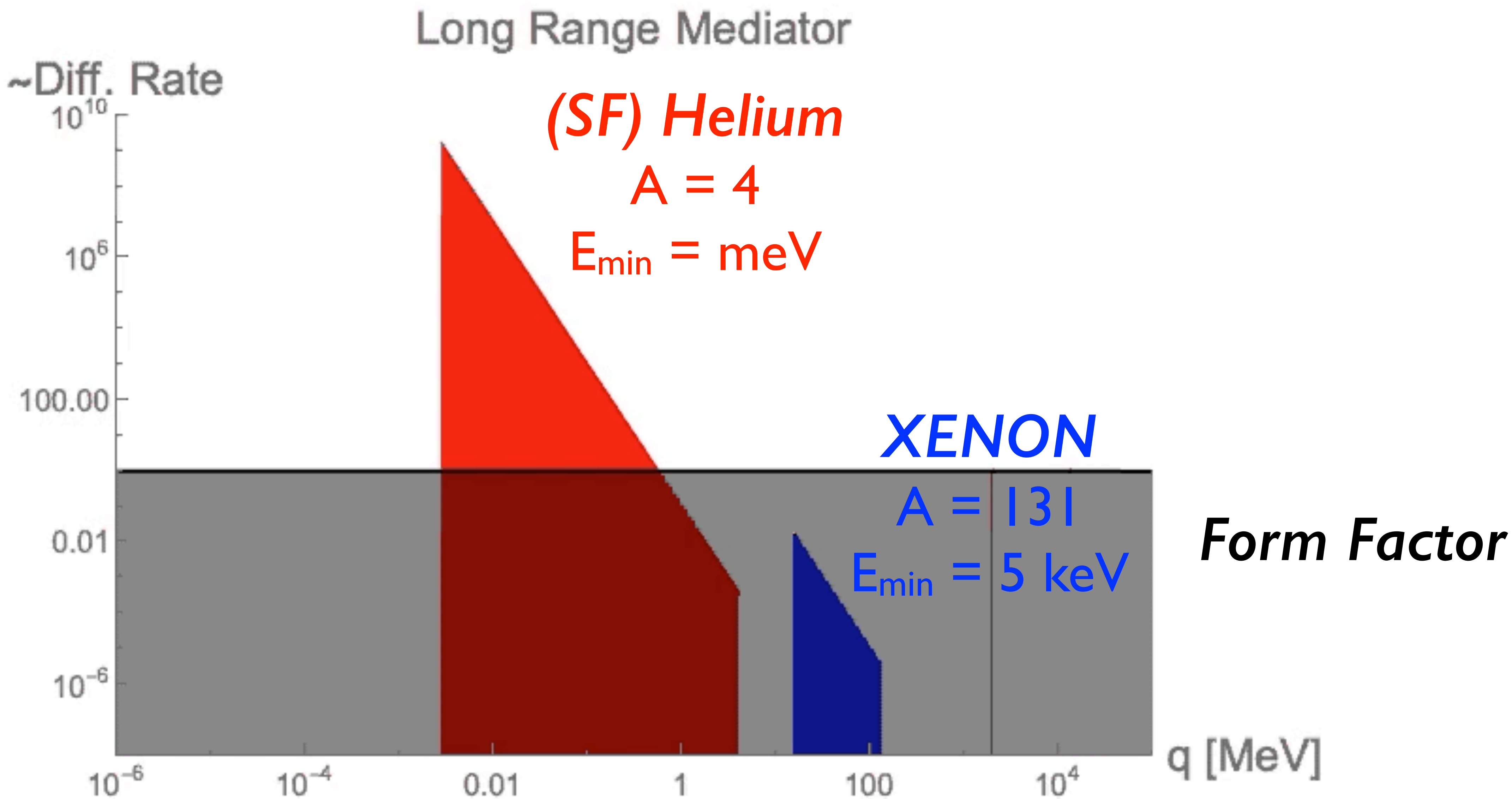
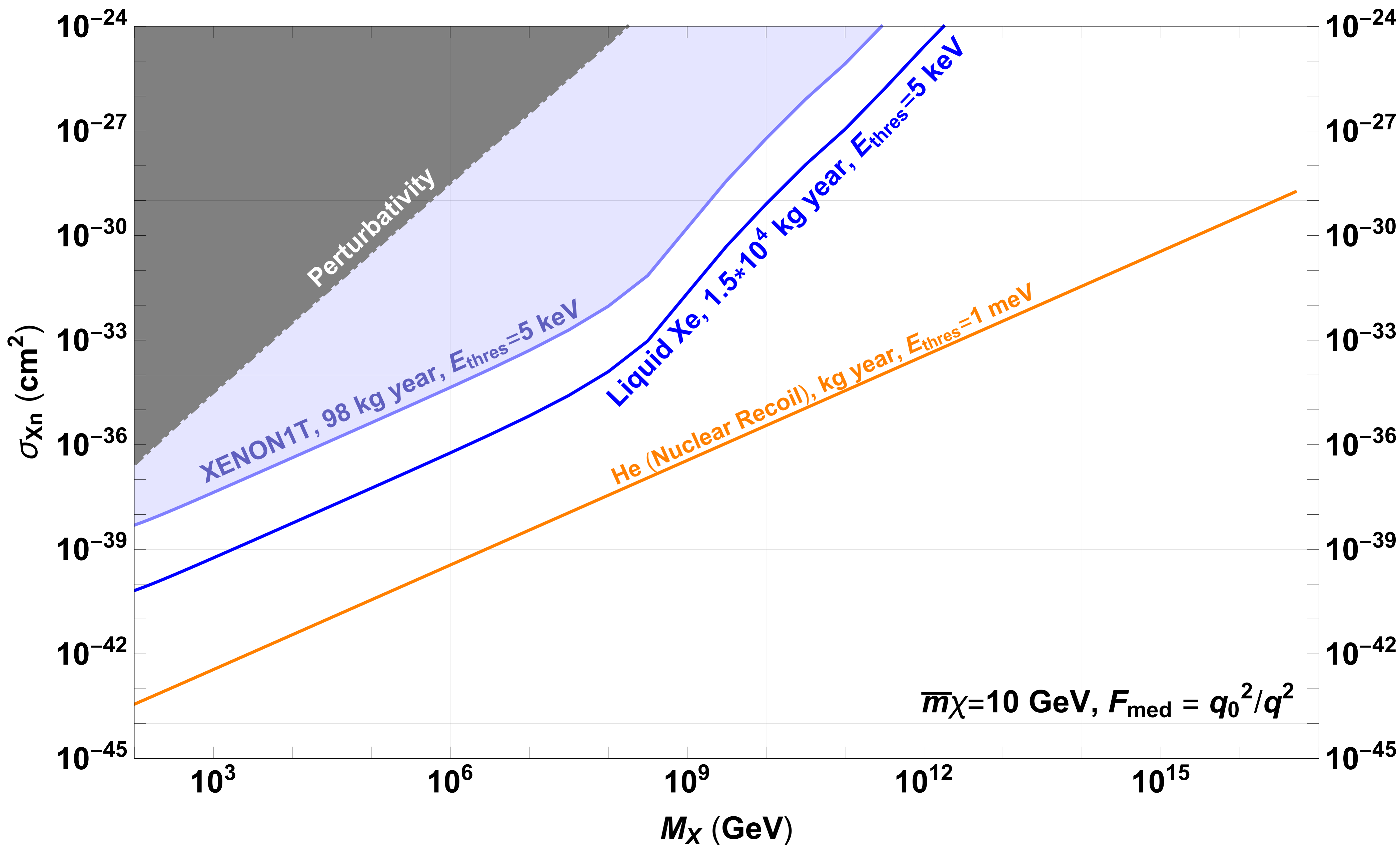


Light Mediator

Mediator Mass: eV



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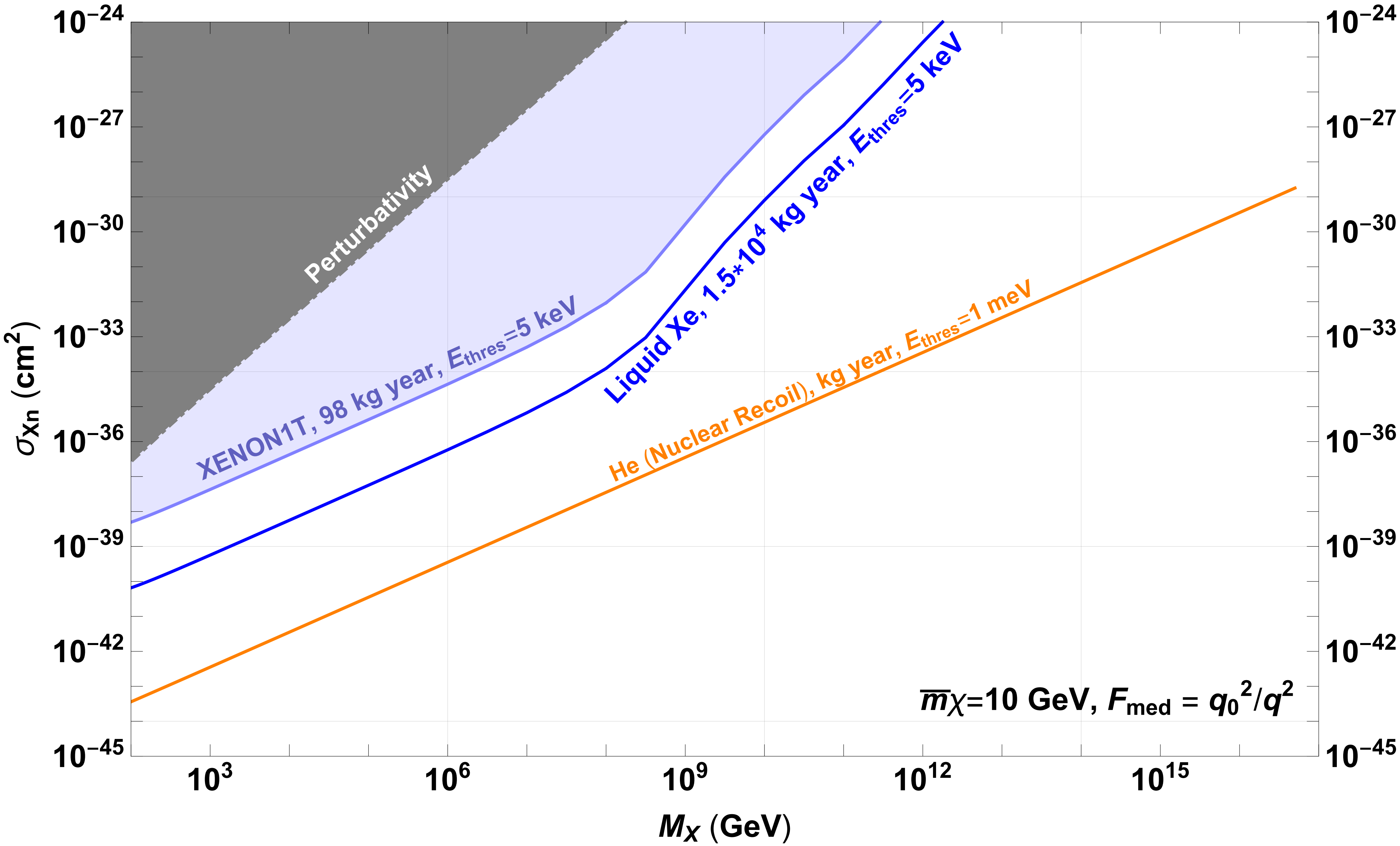

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Extremely Long Range Mediator

Mediator Coupling Constraints

Dark Sector Constraints

Scalar Coupling g_χ Bound: Stability of Dark Nuclei

- Fermi degeneracy pressure must balance self-energy due to long range attractive interaction*

$$g_\chi \lesssim N_X^{-1/3}$$

* Akin to gravitational collapse

Mediator Coupling Constraints

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Scalar Coupling g_χ Bound: Stability of Dark Nuclei

- Fermi degeneracy pressure must balance self-energy due to long range attractive interaction*

$$g_\chi \lesssim N_X^{-1/3}$$

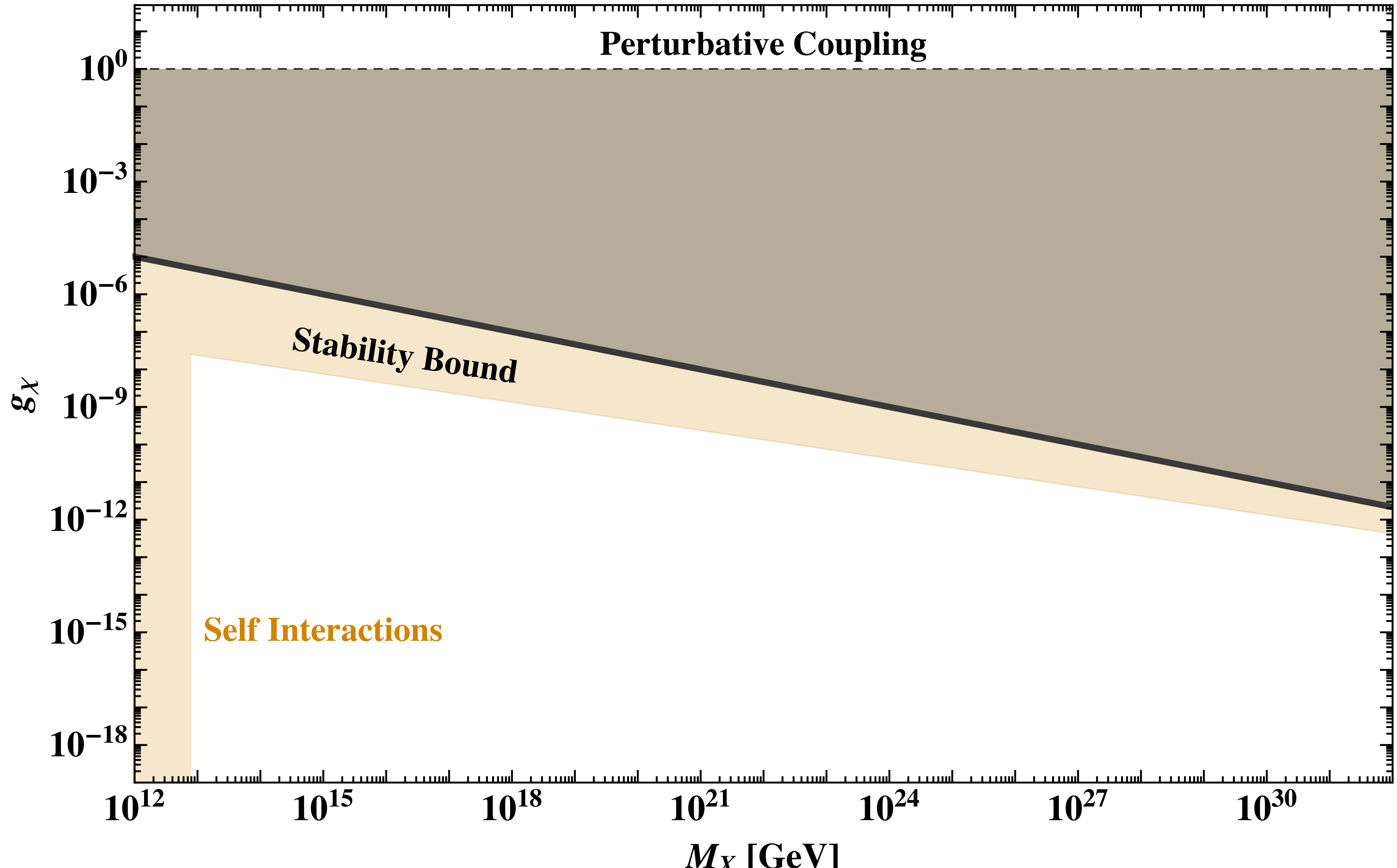
* Akin to gravitational collapse

- Additional repulsive interactions does not help as dark nuclei become unbound

$$E_{\text{Bind}} \sim \underbrace{\alpha_V}_{\sim \Lambda_\chi} N_X - \alpha_S N_X^{2/3} - \underbrace{\alpha_C}_{\sim g_\chi^2 \Lambda_\chi} N_X^{5/3}$$

Ultralight Mediator

Λ_x = MeV, λ = 200 km, $g_n \sim 10^{-23}$

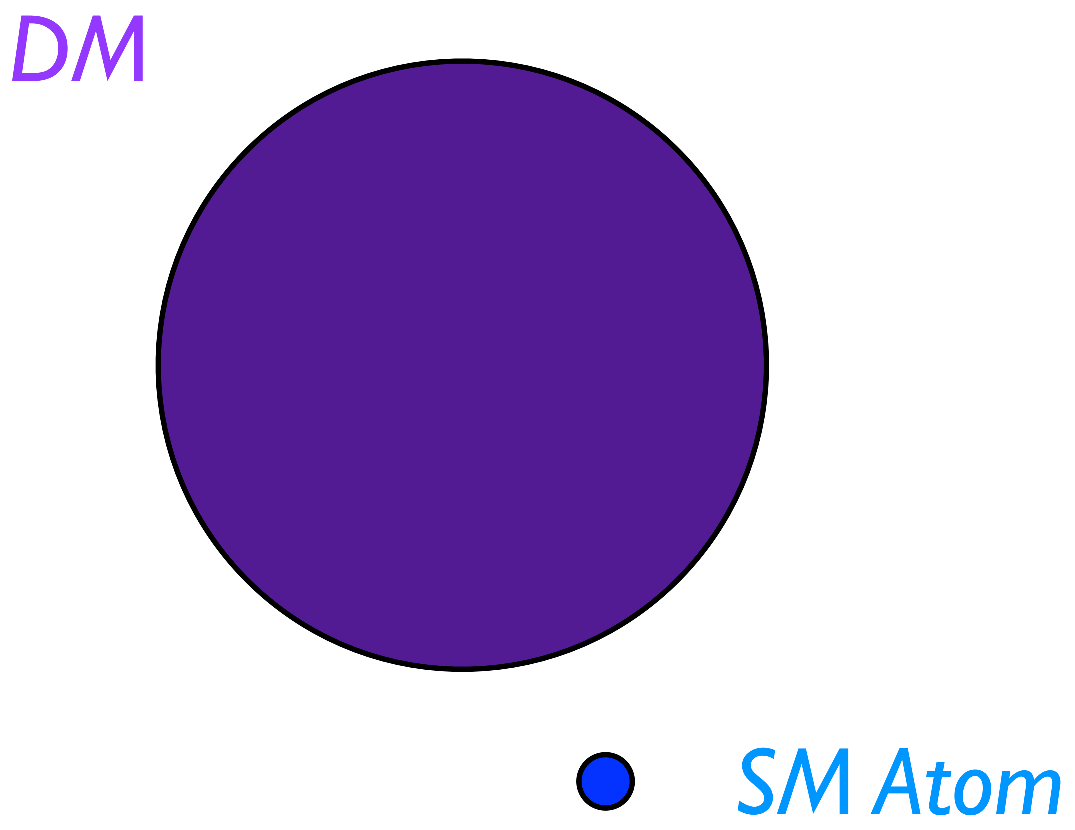


DMG, Melia and Rajendran '18

Ionization and Scintillation Signals

Standard Dark Matter Search

General Idea: momentum transfer during collision between blob and ***single*** SM atom



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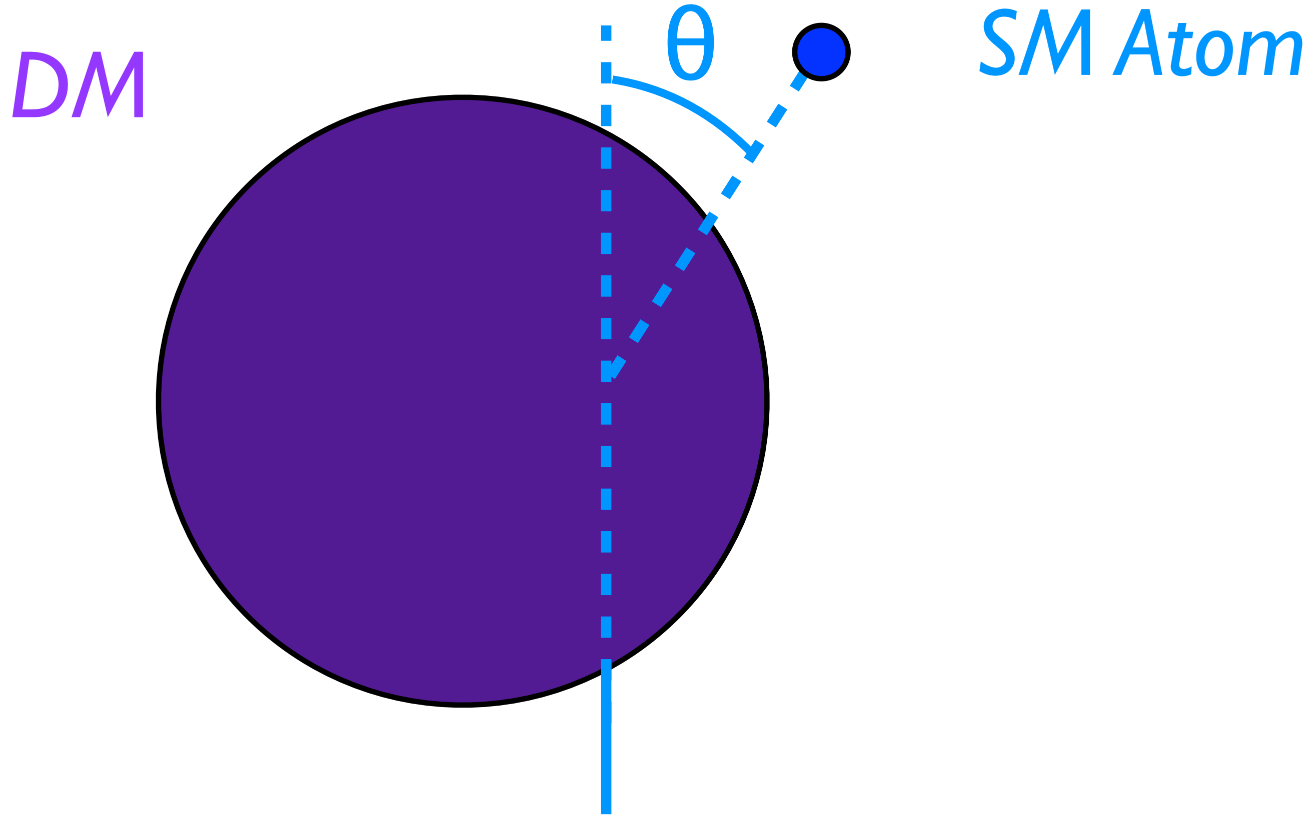

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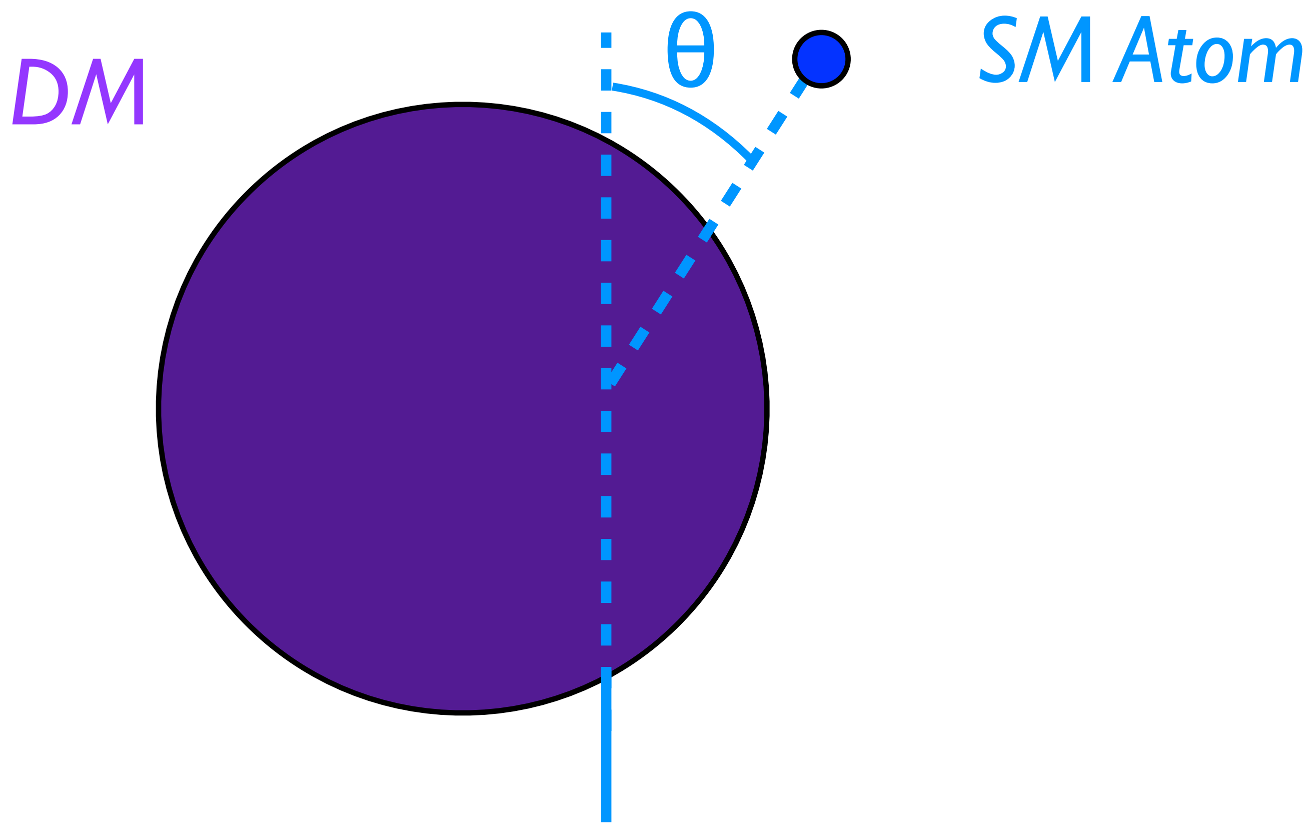
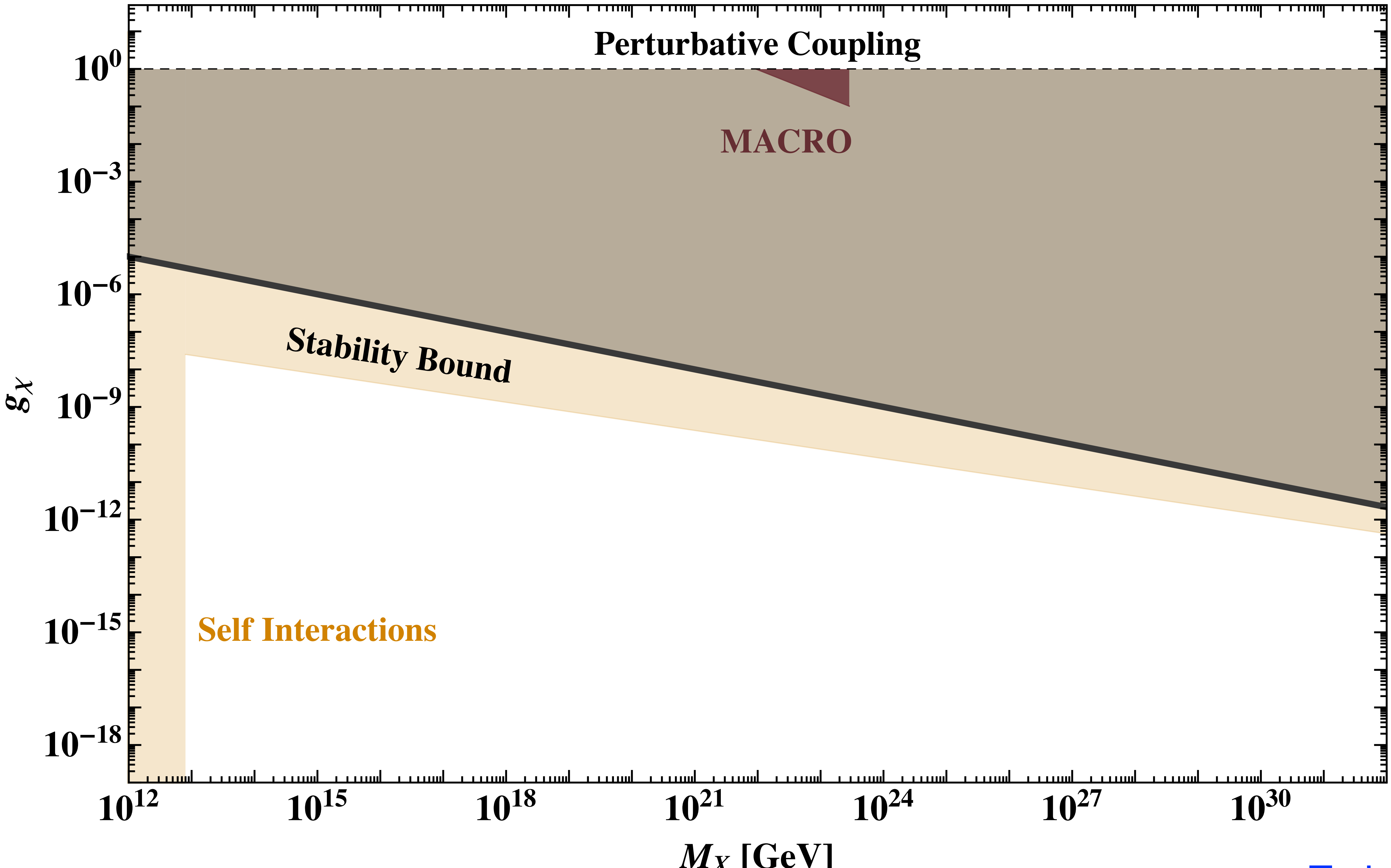


- Only small angle scattering due to weak coupling
- Ionization occurs if mom. transfer above ~100 keV

Ionization/Scintillation

Λ_x = MeV, λ = 200 km, $g_n \sim 10^{-23}$



DMG, Melia and Rajendran '18

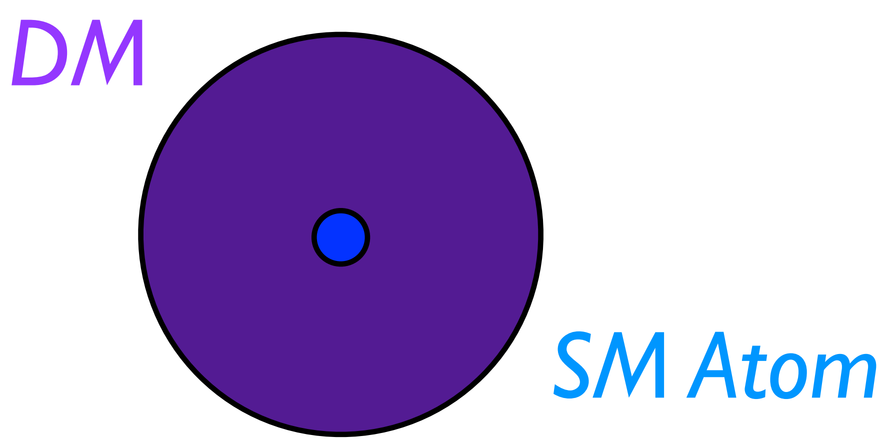
Tank: $(500\ \mathrm{m})^3$

Heat Deposition

DM with large radius

General Idea: Composite DM deposits large amounts of energy without necessarily causing ionization/scintillation

- Large DM radius allows multiple SM atoms to experience significant change in momentum



Heading: Heat Deposition

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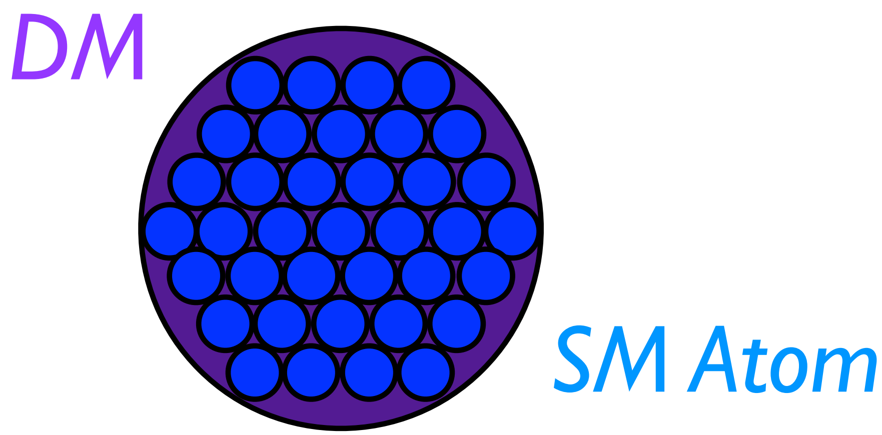

$$\Delta E_{\mathrm{Tot}} \sim \left(\frac{R_X}{R_0}\right)^2 \Delta E^{\mathrm{Max}}_{\mathrm{Single}}$$

Relies on "guaranteed hit"

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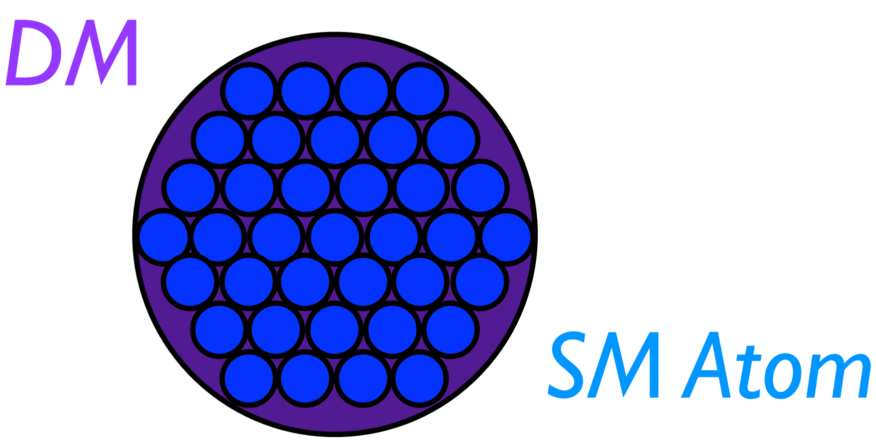


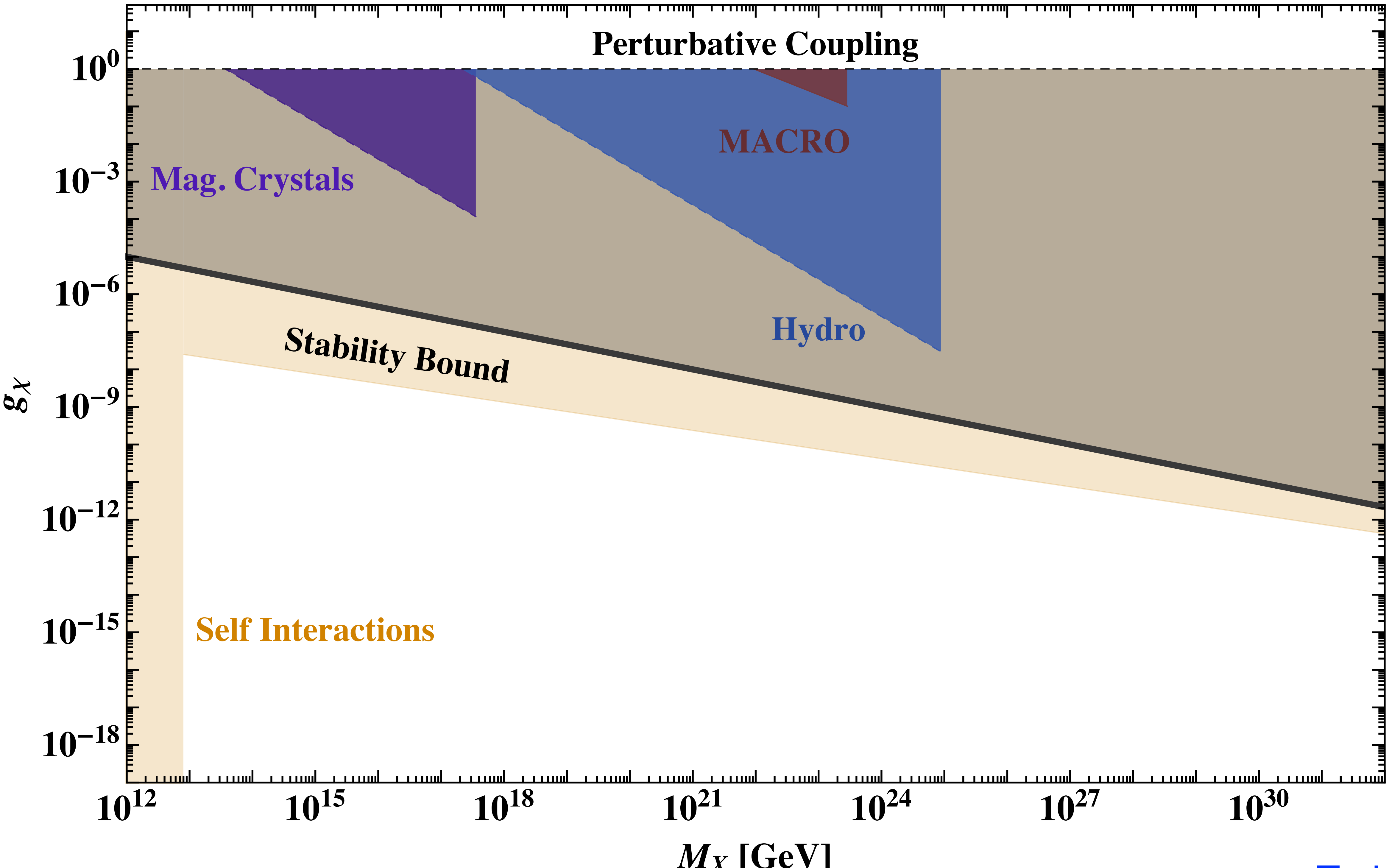
$$\Delta E_{\text{Tot}} \sim \left(\frac{R_X}{R_0}\right)^2 \Delta E_{\text{Single}}^{\text{Max}}$$

Relies on "guaranteed hit"

- **Example:** Hydrophones in tank of water are sensitive to energy deposition of ~ 10 keV/Angstrom

Energy Deposition

Λ_x = MeV, λ = 200 km, $g_n \sim 10^{-23}$



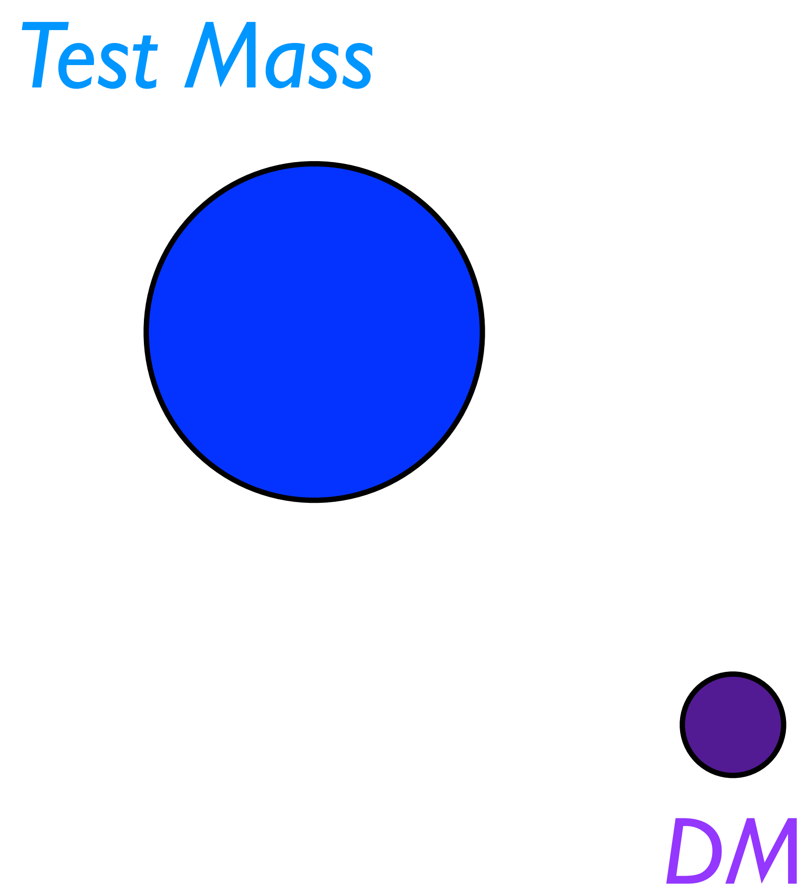
DMG, Melia and Rajendran '18

Tank: (500 m)^3

Acceleration

Free hanging test mass

General Idea: Passing DM induces motion in test mass



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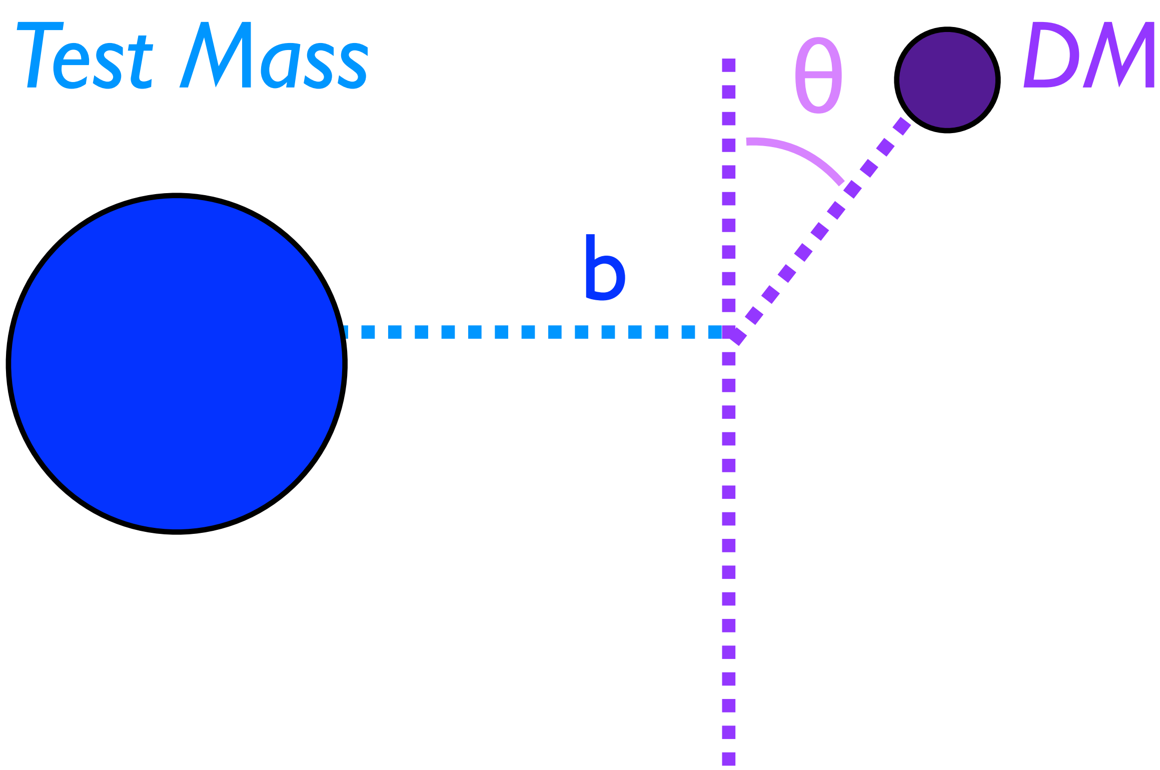

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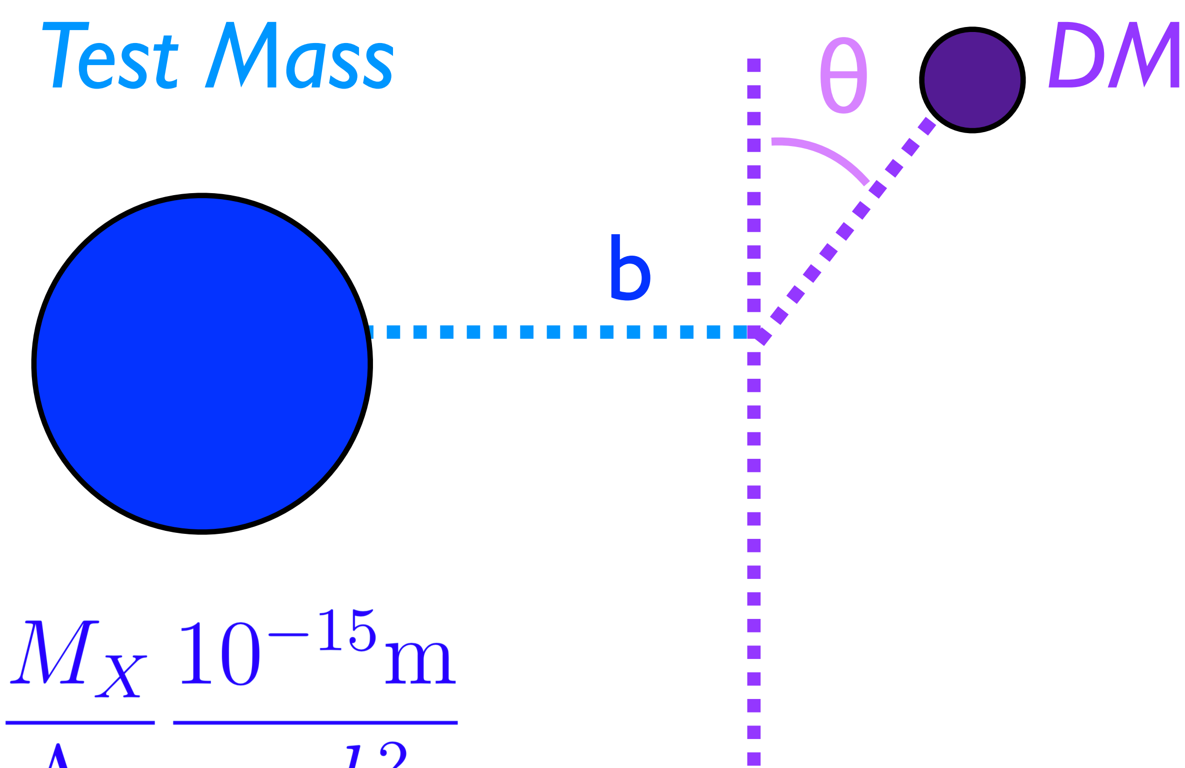
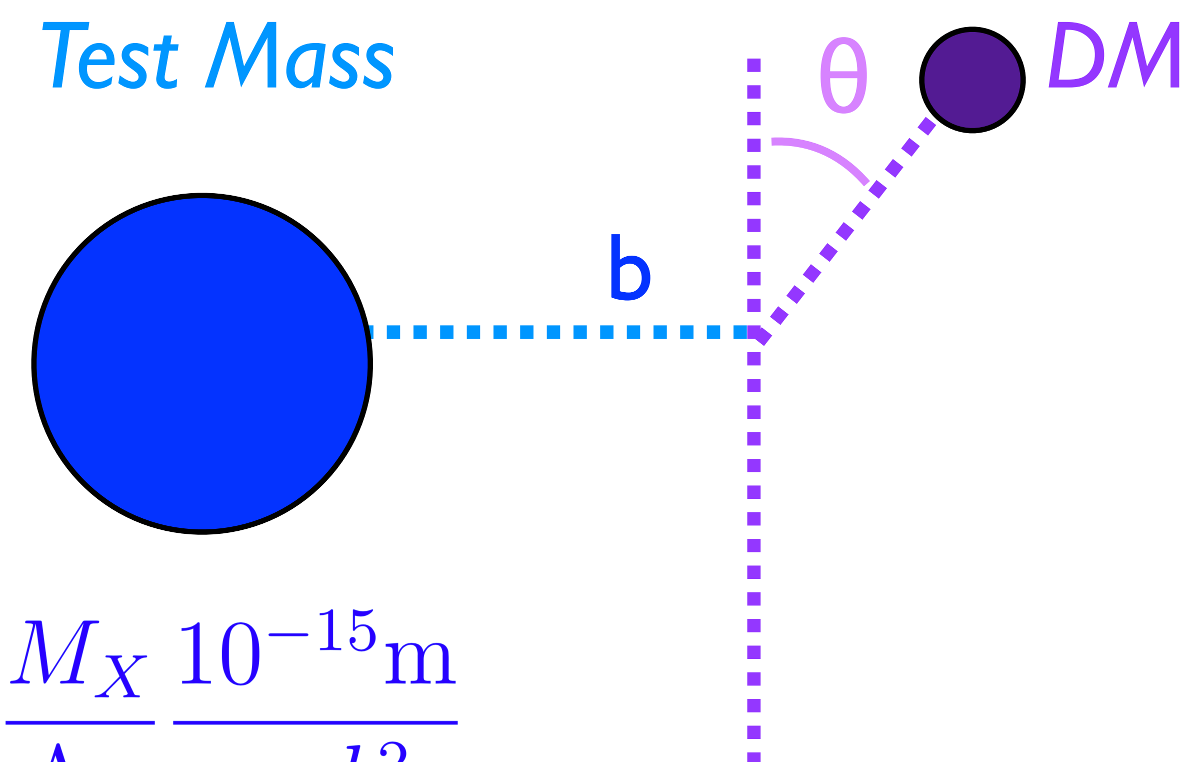


$$h \sim \frac{g_N g_\chi}{v^2} \frac{M_X}{\Lambda_\chi} \frac{10^{-15}\mathrm{m}}{m_\phi b^2}$$

Acceleration

Free hanging test mass

General Idea: Passing DM induces motion in test mass

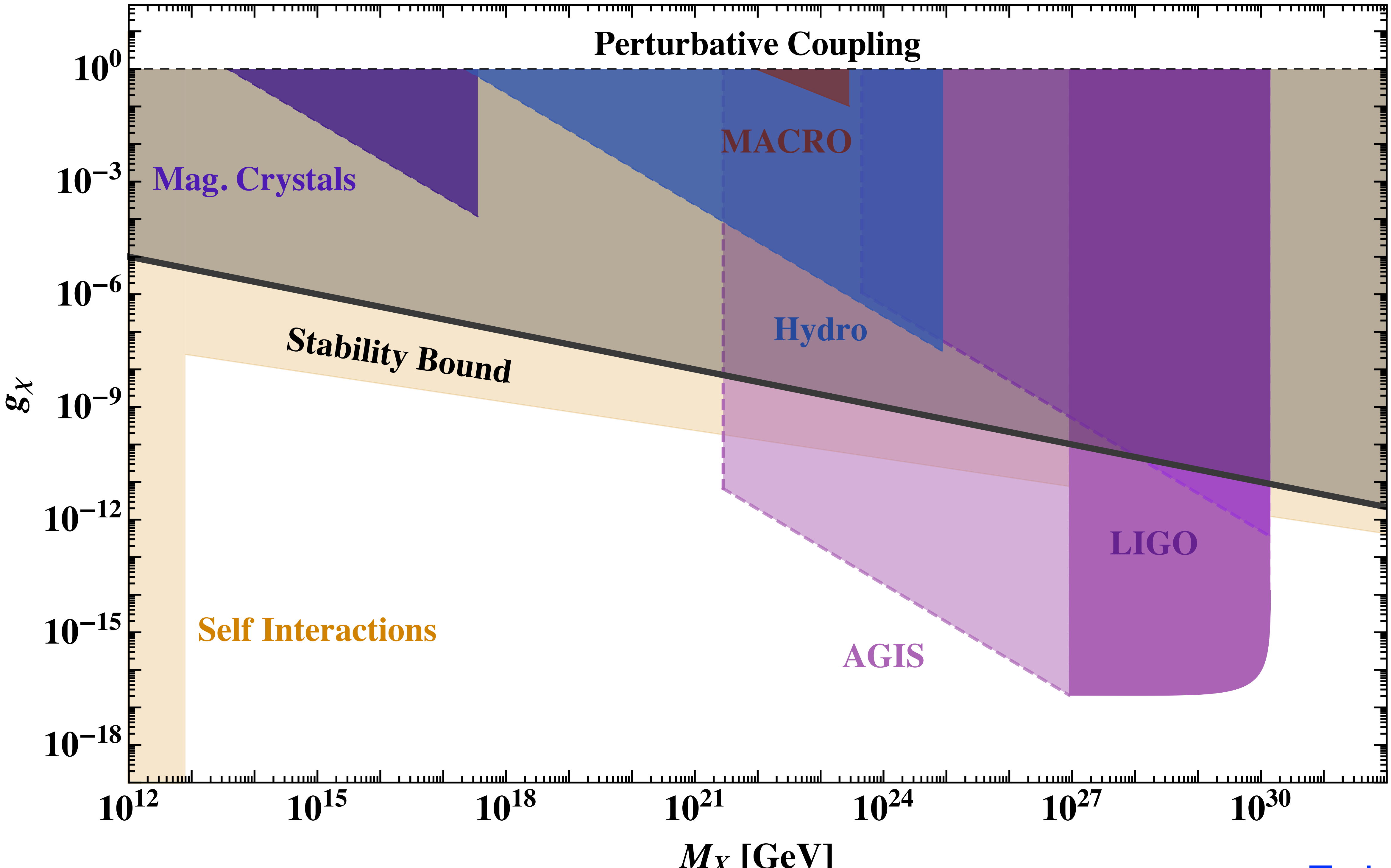


$$h \sim \frac{g_N g_\chi}{v^2} \frac{M_X}{\Lambda_\chi} \frac{10^{-15}\mathrm{m}}{m_\phi b^2}$$

- **Example:** LIGO sensitive to Δx ~ 0.1 fm/Hz$^{1/2}$

Extremely Long Range Mediator

Λ_x = MeV, λ = 200 km, $g_n \sim 10^{-23}$



DMG, Melia and Rajendran '18

Tank: (500 m)^3

Conclusions

Focus: Exponentially large composite dark matter

Take Away: Strongly interacting dark sector can give DM whose mass ranges over 40 orders of magnitude

Take Away: Need multi-prong approach to span the full parameter space of masses and confinement scales

Looking Forward: Mega Dark Matter

Reinvigorated Interest in Composite Dark Matter

Mega Dark Matter Candidates:

- Dark Nuclei, DM Nuggets, Dark Quark Nuggets
- Dark Blobs
- Primordial Black Holes
- Axion/Boson Stars
- Solitons (Q-balls, monopoles, etc)
- *Basically anything composite!!!*

Looking Forward: Mega Dark Matter

Reinvigorated Interest in Composite Dark Matter

Production Side:

- Connections to strongly coupled physics
- Production during Cosmological Inflation, Preheating, Reheating
- Connection to solitons
- Production during cosmological phase transitions

Looking Forward: Mega Dark Matter

Reinvigorated Interest in Composite Dark Matter

Detection Side:

- Terrestrial probes: direct detection, neutrino and alternative detectors
- Gravitational Probes: gravitational waves, lensing, etc.
- Cosmological Probes: CMB anisotropies and BBN
- Astrophysical probes: heating, collisions and indirect detection

If this sounds interesting, please join me, Yang Bai, Joseph Bramante, Andrew Long and Jessica Turner at the Mainz Institute for Theoretical Physics for

MEGA DARK MATTER

Theory and Detection

Summer 2021

CERN Virtual Activities

During this time of self-isolation and working remotely, CERN has moved to being fully virtually while striving to maintain our scientific activities.

Many of our seminars, coffee hours and scientific activities are open to ALL members of the community.

https://theory.cern/th-department-virtual-activities

Of perhaps special interest

* BSM Forum (Thurs., 15h00)
* BSM Coffee (Tues., Thurs., 16h30)
* Lattice Seminar (Thurs. 16h30)
* Pub Quiz (Fri., 21h00)

PUB QUIZ

General Idea: Reproduce trivia night that might occur in a real pub while maintaining social distancing

- Quiz is hosted in main room by one of three pub masters
- Individual teams use Skype to discuss among themselves
- Usually about 60min-90min on Friday evenings
- Open to all members of the community, including family and friends — no physics questions, promise

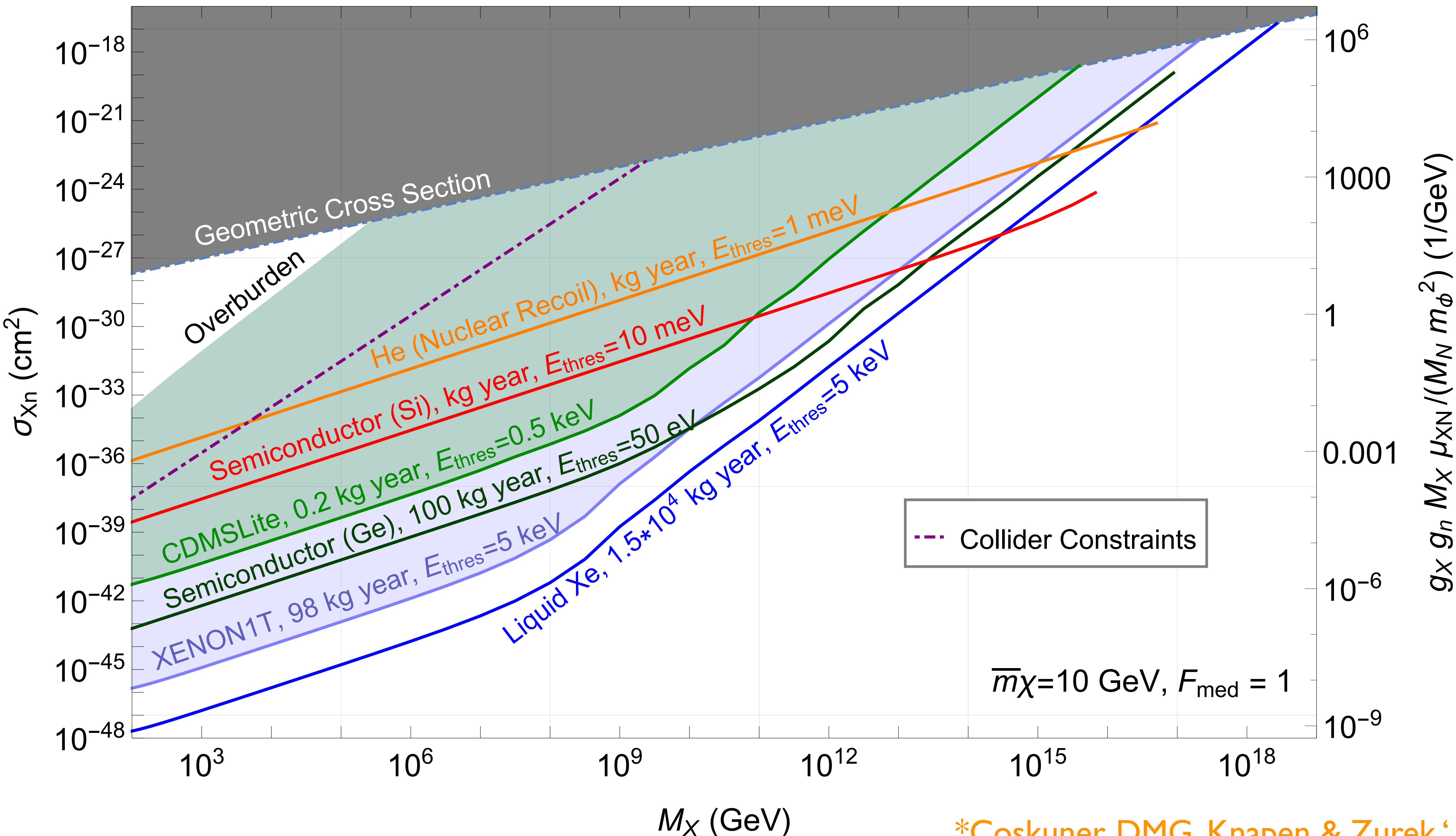
More info, including mailing list sign up at
https://theory.cern/th-department-virtual-activities

HOSTED BY YOURS TRULY TOMORROW!

BACK UP

Heavy Mediator

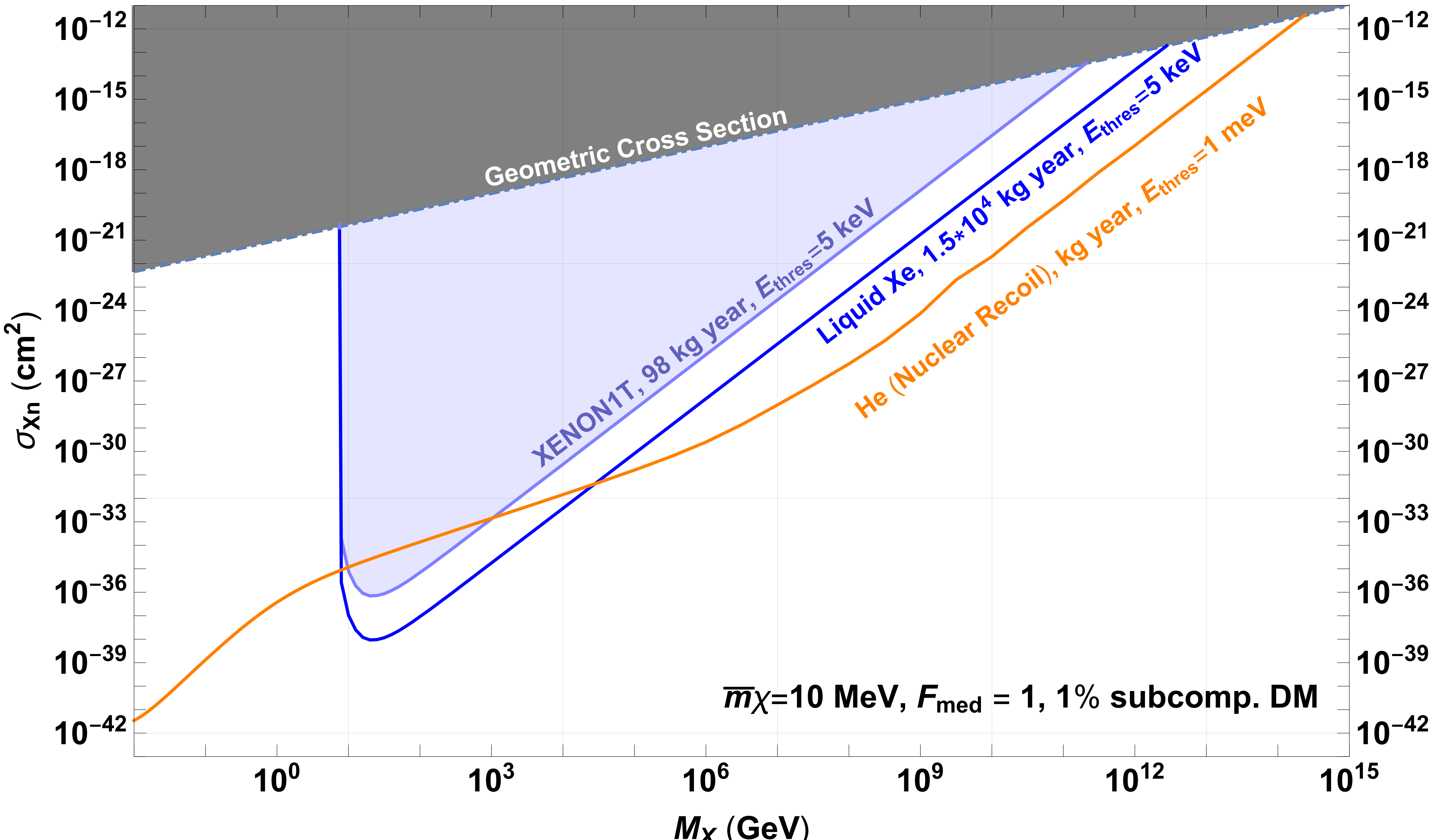
Mediator Mass: 10 GeV



*Coskuner, DMG, Knapen & Zurek '18

Short Range Mediator

m_x = 10 MeV



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(See paper for sensitivities of other detectors)

Short Range Mediator

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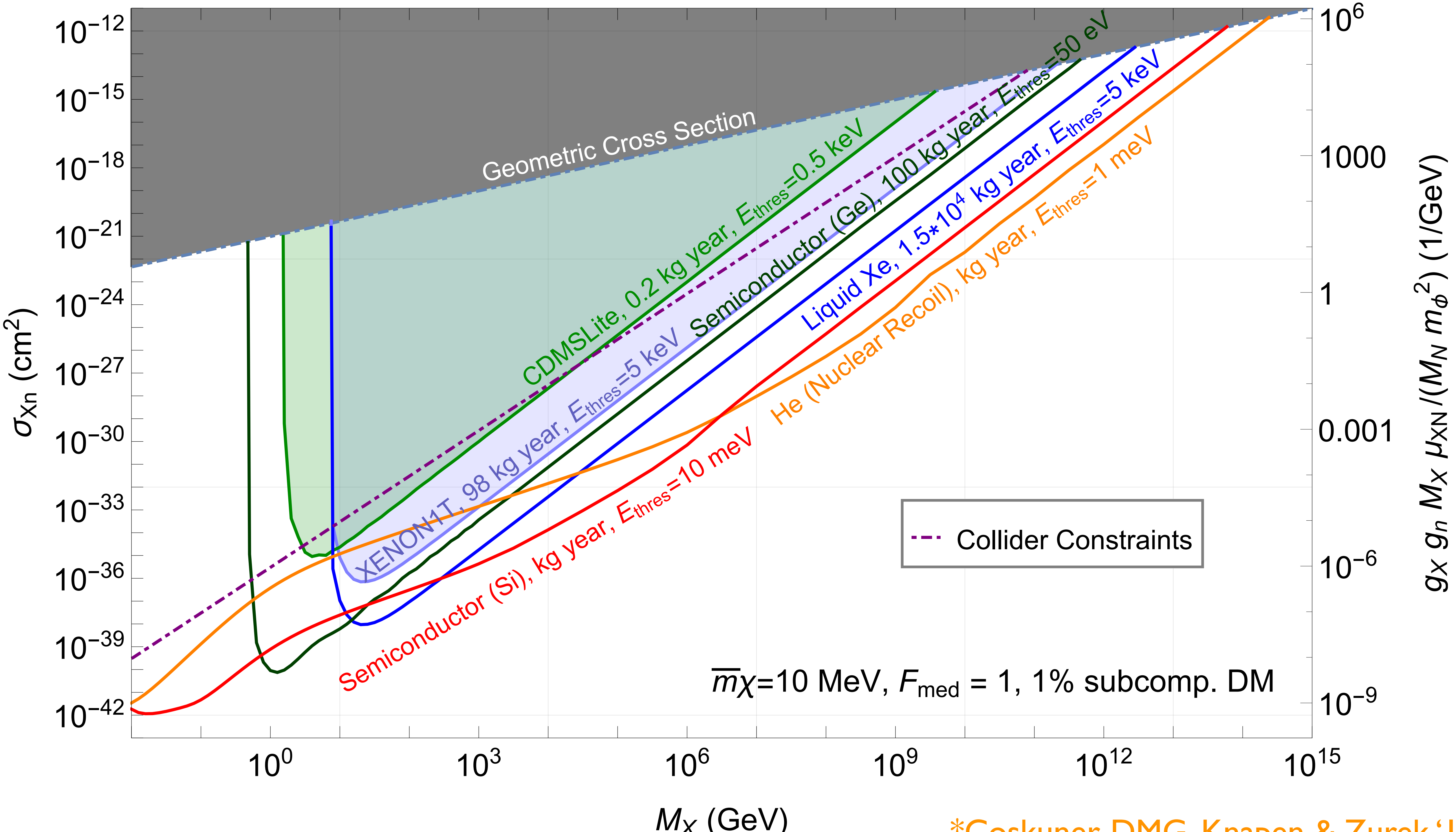

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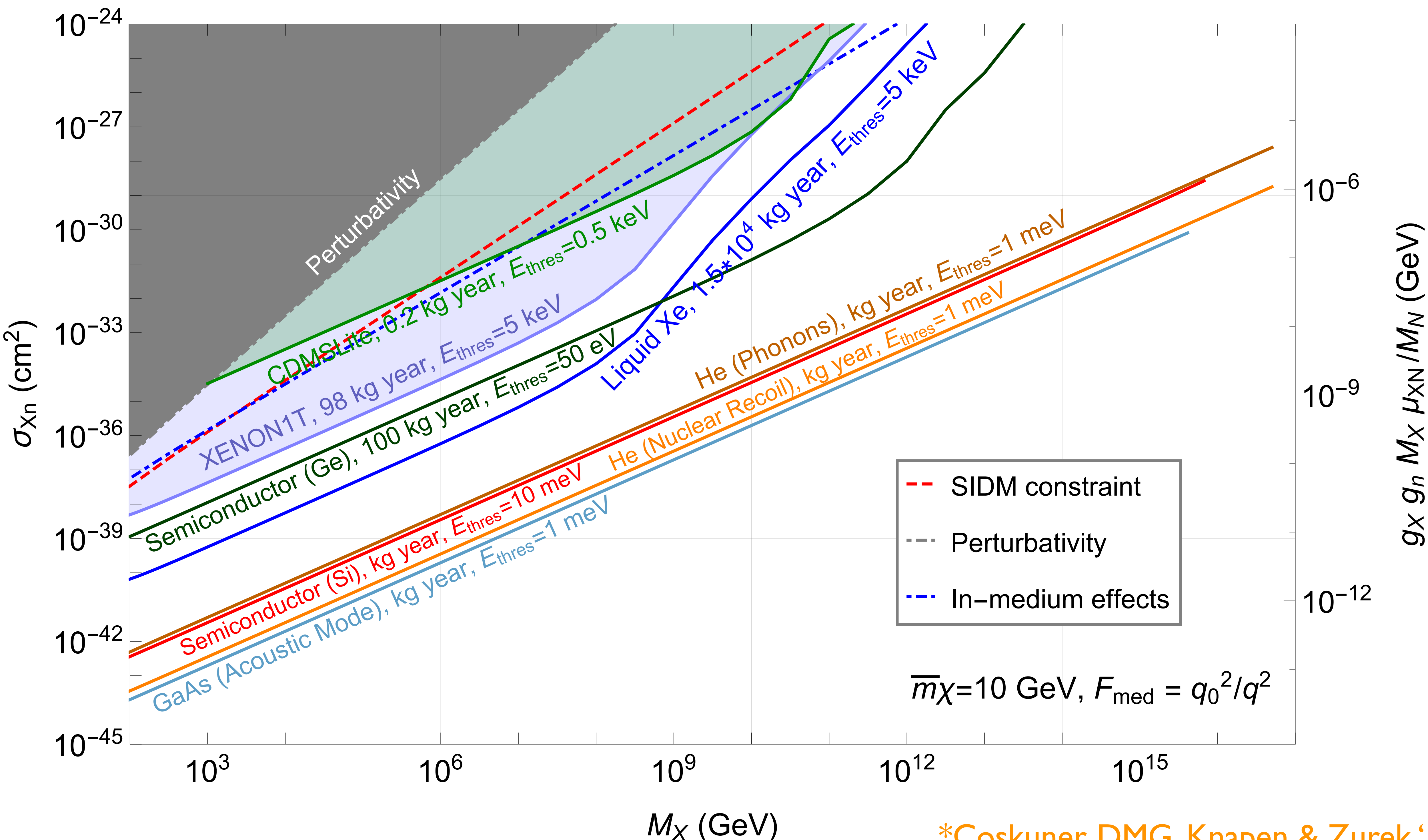
Long Range Mediator

Mediator Mass: eV



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DM Form Factor

Finite Size effects

General Idea: Form factors "encode" the deviation of scattering amplitudes from the point particle limit

$$F_X(\mathbf{q}) = \frac{1}{M_X}\int d^3\mathbf{r}\, e^{i\mathbf{q}\cdot\mathbf{r}} \rho_X^{(\mathrm{ch.})}(\mathbf{r})$$

Charge Density

ASSUME: Uniform charge density inside dark nucleus

$$F_X(q) = 3\frac{\sin(qR_X) - qR_X\cos(qR_X)}{(qR_X)^3}$$

q: Momentum Transfer

DM Form Factor

Finite Size effects

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Charge Density

ASSUME: Uniform charge density inside dark nucleus

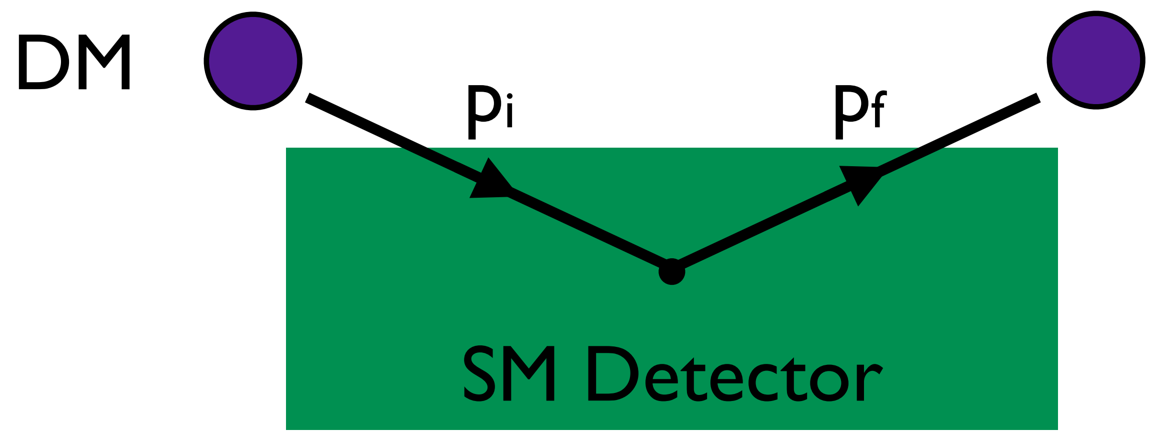
$$F_X(q) = 3\frac{\sin(qR_X) - qR_X\cos(qR_X)}{(qR_X)^3}$$

q: Momentum Transfer

Dark Matter Direct Detection

DM Scattering

General Idea: DM passes through detector, resulting in either nuclei/electron recoil or collective mode excitation



Differential Scattering Rate per Unit Target Mass

$$\frac{dR}{d\omega} = \frac{\rho_X}{M_X}\frac{n_T}{\rho_T}\int d^3\mathbf{v}\, f(\mathbf{v}) \int_{|p_i - p_f|}^{p_i + p_f} dq\, \frac{d^2\Gamma}{d\omega\, dq}$$

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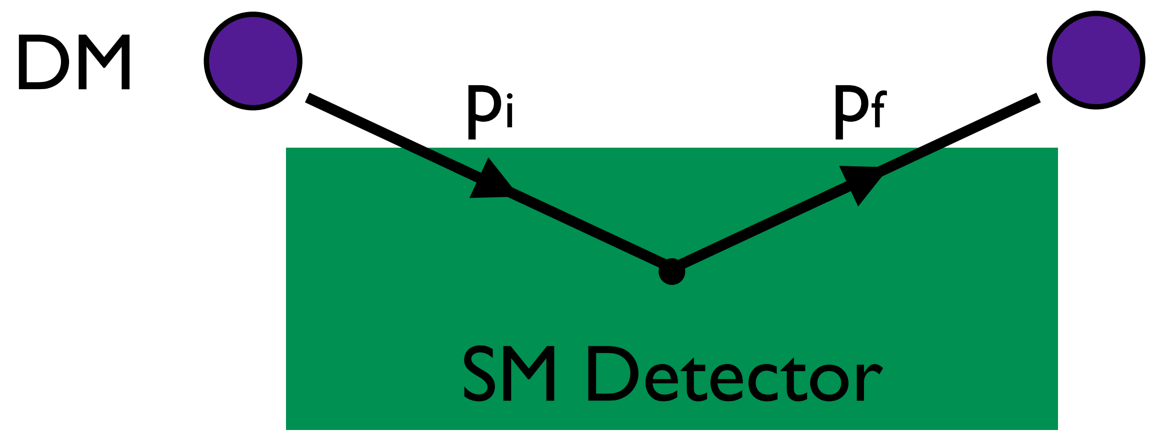

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Target and DM Dependent!

Differential Rate with Form Factor

Key for Direct Detection

General Idea: Factor out contributions to rate that are DM dependent versus detector dependent

$$\frac{dR}{d\omega} = \frac{\rho_X}{M_X}\frac{n_T}{\rho_T}\frac{\overline{\sigma}_0}{2\mu_{Xt}^2}\int dv\,\frac{1}{v}f(v)\int dq\,q\,|F_X(q)|^2\,|F_{\mathrm{med}}(q)|^2\,S(q,\omega)$$

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DM Velocity Dist.

$$\frac{dR}{d\omega} = \boxed{\frac{\rho_X}{M_X}} \frac{n_T}{\rho_T} \frac{\overline{\sigma}_0}{2\mu_{Xt}^2} \int dv \, \frac{1}{v} \boxed{f(v)} \int dq \, q \, |\boxed{F_X(q)}|^2 \, |F_{\text{med}}(q)|^2 \, S(q,\omega)$$

DM Number Density

DM Form Factor

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DM Velocity Dist.

Mediator "Form Factor"

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DM Form Factor

DM Number Density

Target Response

Structure Function S(q, ω) quantifies response of target to injection of momentum, q, and energy, ω

Standard Model Couplings

Nucleon Coupling

General Idea: Dark Matter couples to Standard Model nucleons via mediator of mass m$_\phi$

$$\mathcal{L} \supset \frac{1}{2} m_\phi^2 \phi^2 + g_\chi \phi \bar{\chi} \chi + g_n \phi \bar{n} n$$

Results in spin-independent four fermion interaction with specific mediator "form factor"

$$F_{\mathrm{med}}(q) = \begin{cases} 1 & \text{(heavy mediator)} \\ q_0^2/q^2 & \text{(light mediator)} \end{cases}$$

Nuclear Recoil

Traditional WIMP Search

General Idea: Dark Matter scatters of single nucleus, which then recoils

$$S(q,\omega) = A^2 |F_N(q)|^2 \delta\left(\omega - \frac{q^2}{2m_N}\right)$$

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Form Factor of Nucleus

On-Shell recoil

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Effect of DM's finite radius on detectability depends on how much of scattering phase space is suppressed by form factor

$$q_{\min} = \sqrt{2m_N E_{\text{thres}}} \qquad q_{\max} \sim 2m_N v_0$$

Form Factor Effects

Massive Mediator

General Idea: Effect of form factor depends on how DM radius compares to q_{max} and q_{min}

$$\sigma_{NR}^{\text{short}} \approx \overline{\sigma}_0 \times \begin{cases} A^4 & \frac{1}{R_X} \gtrsim q_{\max} \\ \frac{1}{R_X^2}\left(\frac{A}{m_n v_0}\right)^2 & q_{\max} \gtrsim \frac{1}{R_X} \gtrsim q_{\min} \\ \frac{1}{R_X^4}\frac{A}{m_n^2 v_0^2}\frac{1}{m_n E_{\text{thres}}} & q_{\min} \gtrsim \frac{1}{R_X} \end{cases}$$

- $\overline{\sigma}_0$: Cross section for point-like DM scattering off nucleon

Maximizing target's atomic number is key

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Nuclear Recoil Long Range

Enhancement at Low Momentum

Recall: Scattering due to light mediator is enhanced at low momentum by $1/q^4$

$$\sigma_{NR}^{\text{long}} \approx \overline{\sigma}_0 \times q_0^4 \times \begin{cases} \dfrac{A}{8E_{\text{thres}} v_0^2 m_n^3} & \text{Region I \& II} \\[2ex] \dfrac{3}{64 A R_X^4 v_0^2 m_n^5 E_{\text{thres}}^3} & \text{Region III} \end{cases}$$

$$\sigma_{LA}^{\text{long}} \approx \overline{\sigma}_0 \times q_0^4 \times \frac{A}{m_n^3 v_0^2 c_s} \begin{cases} \dfrac{c_s}{E_{\text{thres}}} & \text{Region I \& II} \\[2ex] \dfrac{9c_s^5}{10 R_X^4 E_{\text{thres}}^5} & \text{Region III} \end{cases}$$

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Minimizing detector's energy threshold is key