YFS Resummation in SHERPA

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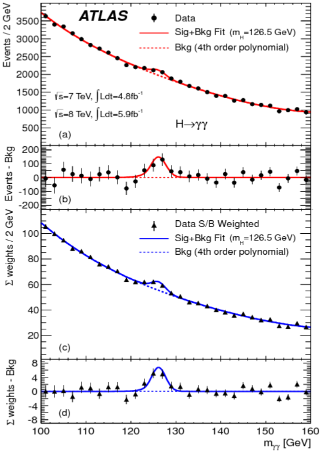
Introduction

1. Motivation
2. YFS Resummation
3. Validation and Results
4. Conclusion

Motivation



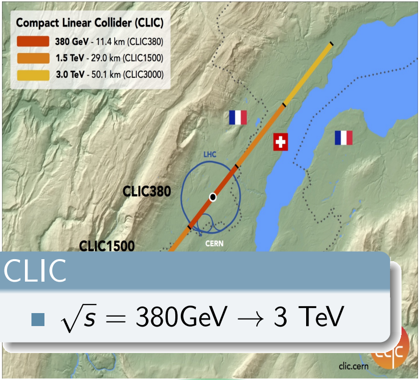
- For precision physics a lepton-lepton collider is desirable
- The discovery of Higgs Boson tells us at what energy to run
- High precision understanding of the Higgs Boson requires below 1% uncertainties in the couplings

Motivation

We must not forget that an e^+e^- will precisely measure EW observables

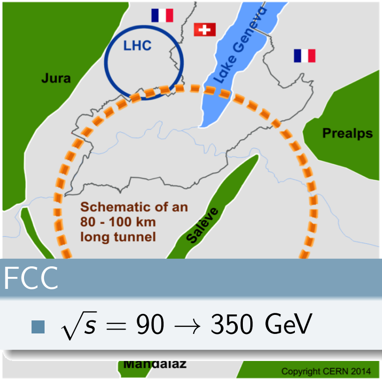
Observable	Where from	Present (LEP)	FCC stat.	FCC syst	$\frac{\text{Now}}{\text{FCC}}$	Challenge
M_Z [MeV]	Z linesh.	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3	QED Corrections
Γ_Z [MeV]	Z linesh.	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2	QED Corrections
$R_l^Z = \Gamma_h/\Gamma_l$	$\sigma(M_Z)$	$20.767 \pm 0.025\{0.012\}$	$6 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	12	QED Corrections
N_ν	$\sigma(M_Z)$	$2.984 \pm 0.008\{0.006\}$	$5 \cdot 10^{-6}$	$1 \cdot 10^{-3}$	6	Bhabha scattering (QED)
M_W [MeV]	ADLO	$80376 \pm 33\{6\}$	0.5	0.3	12	QED Corrections
$A_{FB,\mu}^{M_Z \pm 3.5\mathrm{GeV}}$	$\frac{d\sigma}{d\cos\theta}$	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100	

Table: Adapted from Arxiv:1903.09895



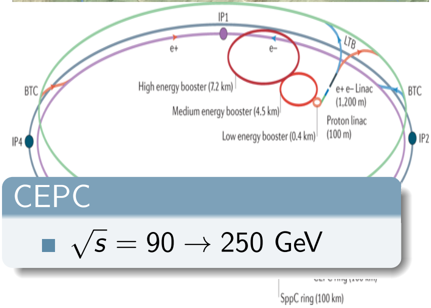
CLIC

- $\sqrt{s} = 380\text{GeV} \rightarrow 3\ \text{TeV}$



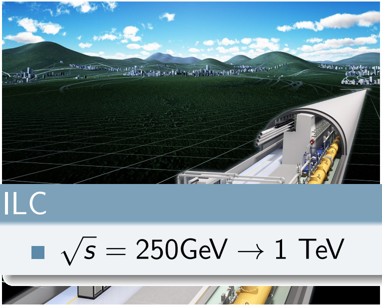
FCC

- $\sqrt{s} = 90 \rightarrow 350\ \text{GeV}$



CEPC

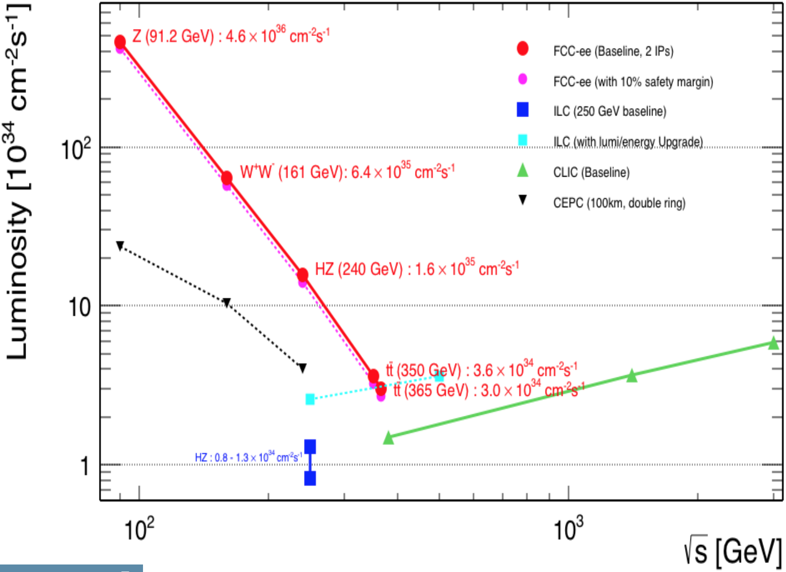
- $\sqrt{s} = 90 \rightarrow 250\ \text{GeV}$



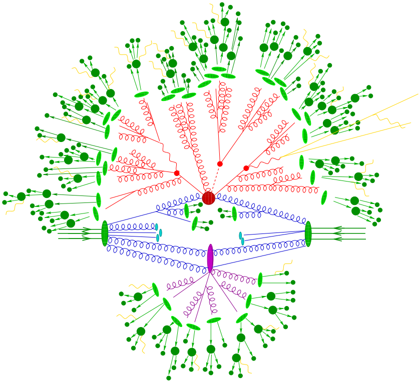
ILC

- $\sqrt{s} = 250\text{GeV} \rightarrow 1\ \text{TeV}$

Linear vs Circular



- Hard Interaction
  - LO, NLO QCD/EW
- Radiative Corrections
  - Catani-Seymour based PS, Dire,
  - YFS QED Resummation
- Multiple Interactions
  - Sjöstrand-Zijl mode
- Hadronization
  - Cluster hadronization model
- Hadron Decays
  - Phase space or EFTs,YFS QED corrections



QED Corrections at the Z Pole

QED corrections (ISR) can be parametrised as,

$$\gamma_{n,m} = \left(\frac{\alpha}{\pi}\right)^n \left(2\ln\left(\frac{m_Z^2}{m_e^2}\right)\right)^m \quad 0 \leq m \leq n$$

- For future lepton collider $\gamma \leq 10^{-5} \rightarrow$ need to include corrections up to $\mathcal{O}\left(\alpha^3, L_e^3\right), \quad L_e = \ln\left(\frac{s}{m_e}\right)$
- Soft and Collinear photons need to be resummed

- The Problem: Emission of soft and/or Collinear photons which can cause divergences
- Solution: Calculate the Cross Section for $e^+e^- \rightarrow X + n_\gamma$

$$\sigma = \frac{1}{flux} \int \sum_{i=0}^{n_R} d\Phi_q d\Phi_{k_i} (2\pi)^4 \, \delta \left(P - \sum q_f - \sum k_i \right) \left| \sum_{n_V=0}^{\infty} \mathcal{M}_{n_R}^{n_V + \frac{1}{2} n_R} \right|^2$$

Cross Section Calculation

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YFS Virtual Corrections

- Lets first consider virtual photon corrections in the soft limit

$$\mathcal{M}_0^0 = \underbrace{M_0^0}_{\text{Born ME}}$$

$$\mathcal{M}_0^1 = \alpha B M_0^0 + M_0^1$$

$$\mathcal{M}_0^1 = \frac{(\alpha B)^2}{2} M_0^0 + \alpha B M_0^1 + M_0^2$$

$$\ldots$$

$$\mathcal{M}_0^{n_V} = \sum_{r=0}^{n_V} M_0^{n_V} \frac{(\alpha B)^r}{r\,!}$$

where B is the virtual infrared factor.

$$B = 2\alpha \Re \int \frac{d^4k}{k^2} \frac{i}{(2\pi)^2} \left(\frac{2p_1 - k}{2kp_1 - k^2} - \frac{2p_2 - k}{2kp_2 - k^2} \right)^2$$

Summing to infinity yields,

$$\sum_{n_V=0}^{\infty} \mathcal{M}_0^{n_V} = e^{\alpha B} \sum_{n_V=0}^{\infty} M_0^{n_V}$$

Due to the Abelian nature of QED this can be generalised to n_R real photons,

$$\sum_{n_V=0}^{\infty} \left| \mathcal{M}_{n_R}^{n_V+\frac{1}{2}n_R} \right|^2 = e^{2\alpha B} \sum_{n_V=0}^{\infty} \left| M_{n_R}^{n_V+\frac{1}{2}n_R} \right|^2$$

YFS Real Emissions

For a single photon emission we have,

$$\frac{1}{2\left(2\pi\right)^3}\sum_{n_V=0}^{\infty}\left|M_1^{n_V+\frac{1}{2}}\right|^2=\tilde{S}\left(k\right)\left|M_0^{n_V+\frac{1}{2}}\right|^2+\sum_{n_V=0}^{\infty}\tilde{\beta}_1^{n_V+1}\left(k\right)$$

- Factorisation of real emissions occurs at the amplitude squared level
- Eikonal term $\tilde{S}\left(k\right)=-\frac{\alpha}{4\pi^2}\left(\frac{p_1}{p_1k}-\frac{p_2}{p_2k}\right)^2$
- $\tilde{\beta}_{n_R}^{n_V+n_R}$ complete IR finite squared matrix element for born process plus n_V virtual and n_R photons
- Note abbreviation used in this talk

$$\tilde{\beta}_{n_R}=\sum_{n_V=0}^{\infty}\tilde{\beta}_{n_R}^{n_V+\frac{1}{2}n_R}$$

YFS Real Emissions

For n_R photons summed over all virtual contributions,

$$\left(\frac{1}{2(2\pi)^3}\right)^{n_R}\left|\sum_{n_V=0}^{\infty} M_{n_R}^{n_V+\frac{1}{2}n_R}\right|^2$$

$$= \tilde{\beta}_0 \prod_{i=1}^{n_R}\left[\tilde{S}(k_i)\right] + \sum_{i=1}^{n_R}\left[\frac{\tilde{\beta}_1(k_i)}{\tilde{S}(k_i)}\right]\prod_{j=1}^{n_R}\left[\tilde{S}(k_j)\right]$$

$$+ \sum_{\substack{i,j=1\\ i\neq j}}^{n_R}\left[\frac{\tilde{\beta}_2(k_i,k_j)}{\tilde{S}(k_i)\tilde{S}(k_j)}\right]\prod_{l=1}^{n_R}\left[\tilde{S}(k_l)\right] + \ldots$$

$$+ \sum_{i=1}^{n_R}\left[\tilde{\beta}_{n_R-1}(k_1,\ldots,k_{i-1},k_{i+1},\ldots,k_{n_R})\,\tilde{S}(k_i)\right] + \tilde{\beta}_{n_R}(k_1,\ldots,k_{n_R})$$

This gives us our cross section

$$\sigma = \sum_{n=0}^{\infty} \frac{1}{n!} \int d\Phi_f \; e^{2\alpha B + 2\alpha \tilde{B}} \prod_{j=1}^{n} \tilde{S}\,(k_j)\,\theta(\Omega; k_j) \left[\tilde{\beta}_0\,(p_1, p_2; q_1, \cdots, q_{n'}) + \sum_{j=1}^{n} \frac{\tilde{\beta}_1\,(p_1, p_2; q_1, \cdots, q_{n'}; , k_j)}{S(k_j)} + \sum_{\substack{j,l=1 \\ j \neq l}}^{n} \frac{\tilde{\beta}_2\,(p_1, p_2; q_1, \cdots, q_{n'}; k_j, k_l)}{S(k_j) S(k_l)} + \cdots \right]$$

The exponentiation of real emissions gives a factor

$$\tilde{B} = -\frac{1}{8\pi^2} \int \frac{d^3 k}{k^0} \Theta(\Omega, k) \left(\frac{p_1}{p_1 k} - \frac{p_2}{p_2 k} \right)^2$$

$$
\begin{aligned}
\tilde{\beta}_0^0 &= M_0^0 M_0^{0*} \\
\tilde{\beta}_0^1 &= M_0^1 M_0^{0*} + M_0^0 M_1^{0*} \\
\tilde{\beta}_1^1 &= \frac{1}{2\left(2\pi\right)^3} M_0^{\frac{1}{2}} M_0^{\frac{1}{2}*} - \tilde{S}\left(k\right) M_0^0 M_0^{0*} = \frac{1}{2\left(2\pi\right)^3} M_0^{\frac{1}{2}} M_0^{\frac{1}{2}*} - \tilde{S}\left(k\right) \tilde{\beta}_0^0
\end{aligned}
$$

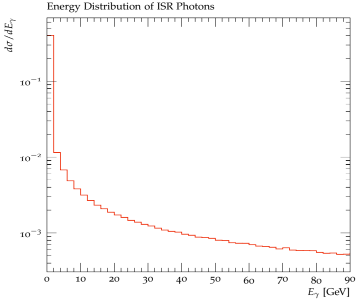
- These are the infrared subtracted squared matrix elements read, up to $\mathcal{O}\left(\alpha\right)$
- They have been implemented in SHERPA up to $\mathcal{O}\left(\alpha^2\right)$ for ISR

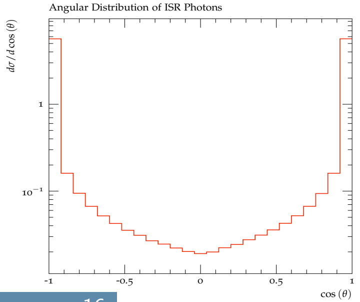
The Algorithm

1. First the phasespace is rearrange by a change of variable $v = 1 - \frac{s'}{s}$
2. v is then generated by standard Monte-Carlo Methods e.g Importance Sampling
3. If $v < \epsilon$ no photons are generate. For $v \geq \epsilon$ the multiplicity is generated according to a Poissonian distribution with average $\mu = \frac{2\alpha}{\pi} \log(\frac{s}{m_e^2}) \log(\frac{v}{\epsilon})$
4. The n photon momenta are distributed according to the Eikonal $\tilde{S}(k)$ and then rescaled to ensure 4-momentum conservation
5. The perturbative parts can be calculated separately using ME generator
6. Give everything to the master equation to calculate the cross section

The Power of YFS

- $\tilde{\beta}$ are infrared finite and are calculated perturbatively *order by order*
  - Non trivial for other methods e.g Structure function
- Photons are explicitly created
  - Allows us to calculate properties of soft photons e.g Energy, Angular distribution
  - Allows for decays e.g $\gamma \to e^+e^-$





Validation

- For LEP there were dedicated MC codes that implemented YFS for specific processes
  - KKMC Jadach et.al, ARXIV:9912214
  - KoralWW Jadach et.al, ARXIV:0012094
  - YFSWW Jadach et.al, ARXIV:0012094

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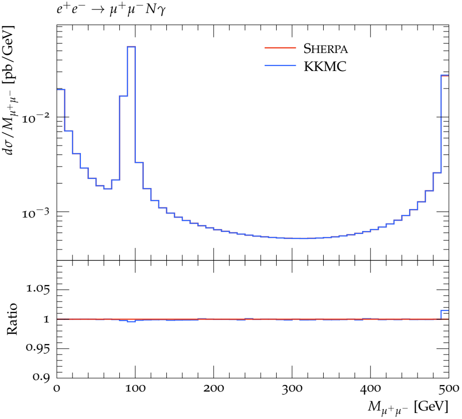
Validation

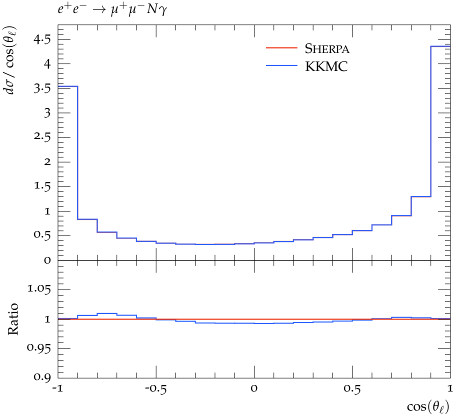
YFS $\mathcal{O}(\alpha^0)$	KKMC 500 GeV	Sherpa 500 GeV
$e^+e^- \to \mu^+\mu^-$	1.748(3) pb	1.746(3) pb
$e^+e^- \to \nu_\mu\bar{\nu}_\mu$	1.6235(8) pb	1.626(3) pb
$e^+e^- \to \nu_\tau\bar{\nu}_\tau$	1.6235(8) pb	1.626(3) pb
$e^+e^- \to$ jj	19.94(2) pb	19.98(8) pb

YFS $\mathcal{O}(\alpha^1)$	KKMC 500 GeV	Sherpa 500 GeV
$e^+e^- \to \mu^+\mu^-$	1.024(3) pb	1.020(3) pb
$e^+e^- \to \nu_\mu\bar{\nu}_\mu$	0.8072(7) pb	0.808(2) pb
$e^+e^- \to \nu_\tau\bar{\nu}_\tau$	0.8072(7) pb	0.808(2) pb
$e^+e^- \to$ jj	10.54(1) pb	10.21(4) pb

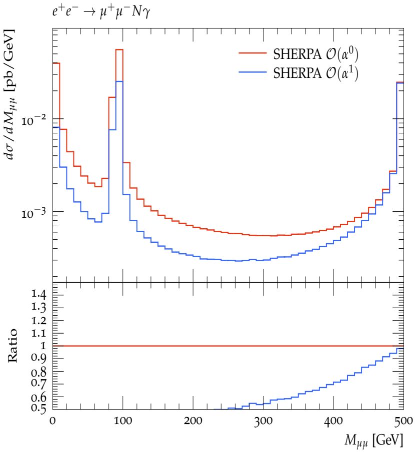
YFS $\mathcal{O}(\alpha^2)$	KKMC 500 GeV	Sherpa 500 GeV
$e^+e^- \to \mu^+\mu^-$	1.080(3) pb	1.082(6) pb
$e^+e^- \to \nu_\mu\bar{\nu}_\mu$	0.8604(7) pb	0.860(3) pb
$e^+e^- \to \nu_\tau\bar{\nu}_\tau$	0.8604(7) pb	0.860(3) pb
$e^+e^- \to$ jj	11.18(2) pb	11.10(8) pb

Differential Distributions



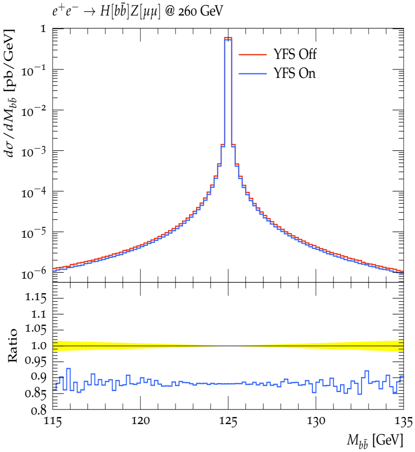


Differential Distribution



- The large deviation between α^0 and higher order is expected and is in agreement with semi-analytical approximation
- $\frac{\mathcal{O}(\alpha^1)}{\mathcal{O}(\alpha^0)} \approx 0.5$ at the z-pole
- $\frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha^1)} \approx 0.98$ at the z-pole

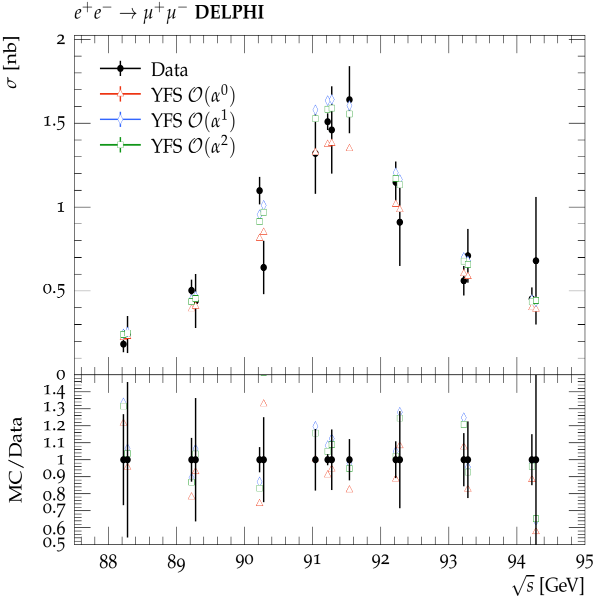
Higgs-Strahlung



- At the Higgs pole ISR contributes a $\approx 10\%$ effect

	ZH-Pole 240 GeV
$\mathcal{O}(\alpha^0)$	201.46 fb
$\mathcal{O}(\alpha)$	234.59 fb
$\mathcal{O}(\alpha^2)$	233.23 fb

Comparison to Data



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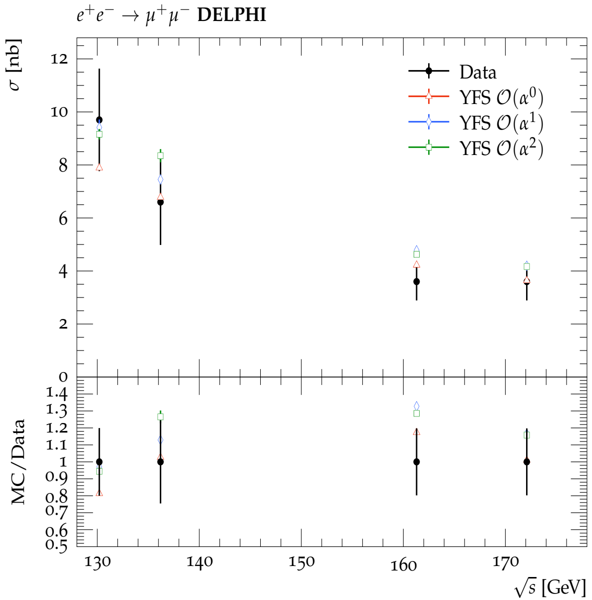

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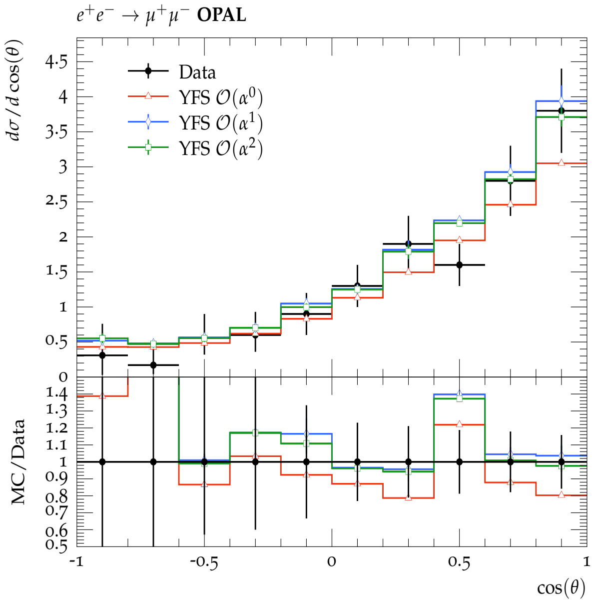


- Interface with other SHERPA modules e.g ME Generator, PS, Hadronization, UFO...
- Fully automated for arbitrary processes
- Higher order corrections have been hard coded for ISR
- YFS has been fully implemented for FSR up to NNLO QCD and $\mathrm{NLO}_{\mathrm{EW}}$ Krauss et.al, ARXIV:1809.10650
- Matching of YFS resummation to $\mathrm{NLO}_{\mathrm{EW}}$ is under-way
  - EW Loops are now provided in full by OPENLOOPS Buccioni et.al,ARXIV:1907.13071
  - Real emission ME provided by COMIX/AMEGIC
  - Main problem is lack of massive leptons in the loops

The SHERPA 2.2 event generator framework

User Inputs

Initial Beams

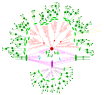
- collider setup
- PDFs (built-in, LHAPDF)
- beam spectra

Parameters/Models

- FeynRules/UFO
- couplings
- masses
- variations
- shower settings
- non-perturbative parameters

Physics Process

- parton level
- perturbative order (QCD/EW)
- selectors
- matching/merging
- partonic decays



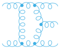
Matrix Elements

Matrix Element Generators

- AMEGIC
- COMIX
- CS subtraction

1-loop Amplitudes

- OpenLoops
- Recola
- GoSam
- BLHA



Parton Showers

CS-Shower (default)

- dipole shower
- fully massive
- QED splittings

DIRE

- hybrid dipole-parton shower algorithm
- fully massive

Matching and Merging

Automated MC@NLO style matching

Multijet-merging algorithms

- based on truncated showers
- tree-level and one-loop matrix elements: MEPS@LO and MEPS@NLO
- approximate electroweak corrections

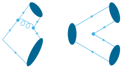
NNLO QCD with parton showers

- selected processes only

Soft Physics

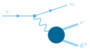
Hadronisation

- AHADIC: a cluster fragmentation model
- interface to Pythia string fragmentation



Hadron Decays

- decay tables for hadronic resonances
- dedicated form-factor models, e.g. τ, B, Λ
- spin correlations
- YFS QED corrections
- partonic channels



Underlying Event

- multiple parton interactions
- beam-remnant colours
- intrinsic transverse momentum

Interfaces/Outputs

Output Formats

- HepMC
- LHEF
- Root Ntuple

Interfaces

- RIVET analyses
- C++/Python ME access
- MCgrid
- integration into ATLAS/CMS

Code/Docu

- HepForge
- GitLab
- online documentation

sherpa.hepforge.org

gitlab.com/sherpa-team/sherpa