Swampland Phenomenology

John March-Russell
Oxford University & PI

A big subject… This talk based on

Part I:

Tristan Daus, Arthur Hebecker, Sascha Leonhardt, JMR

Towards a Swampland Global Symmetry Conjecture using Weak Gravity, arXiv:2002.02456

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Part II:

Rudin Petrossian-Byrne, JMR

QCD, Flavor, and the de Sitter Swampland, arXiv:XXX to appear

and

Rudin Petrossian-Byrne, JMR

The Standard Model and the de Sitter Swampland, arXiv:XXX in preparation

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The idea that not all effective QFTs remain consistent when embedded in a theory with gravity

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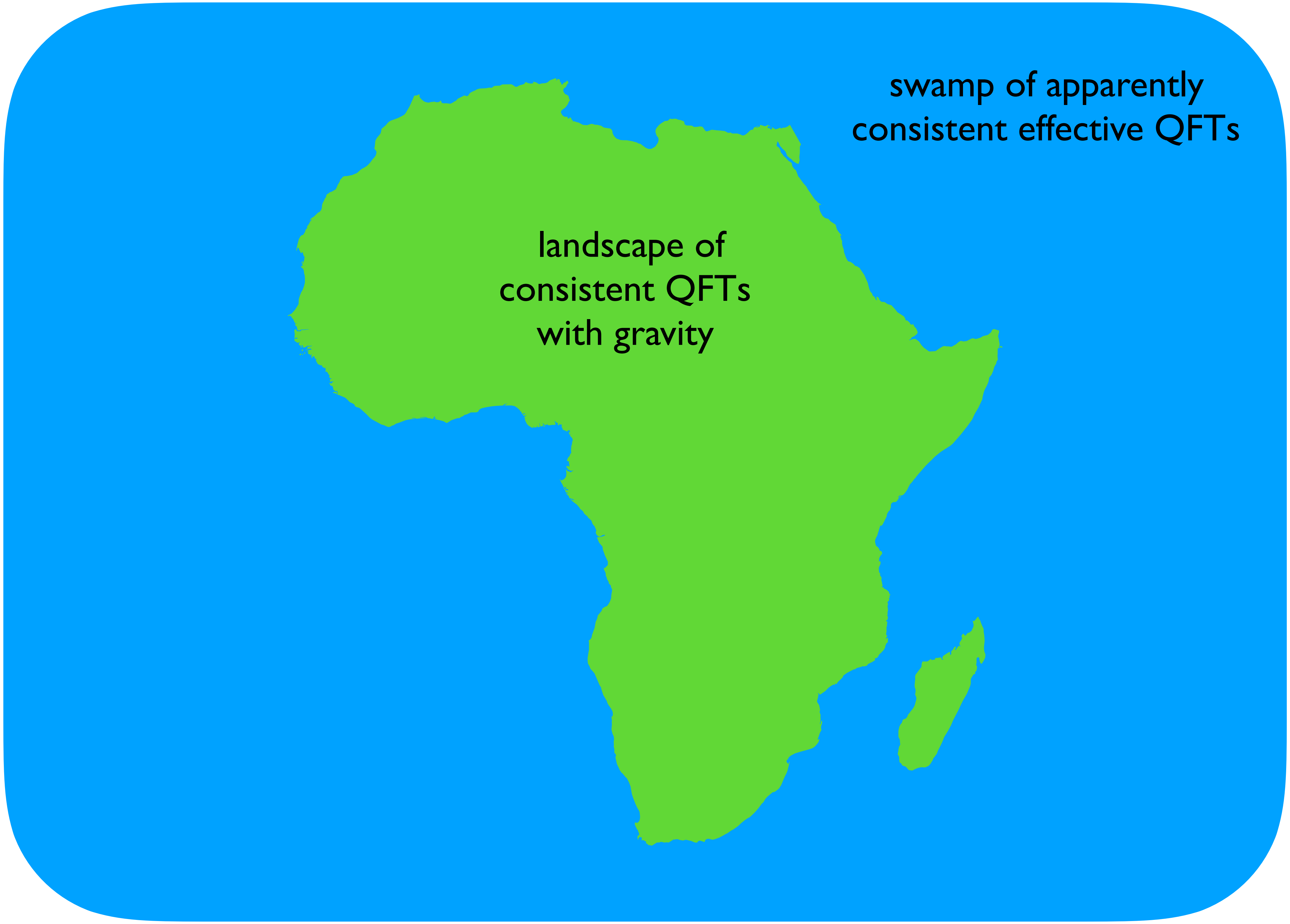
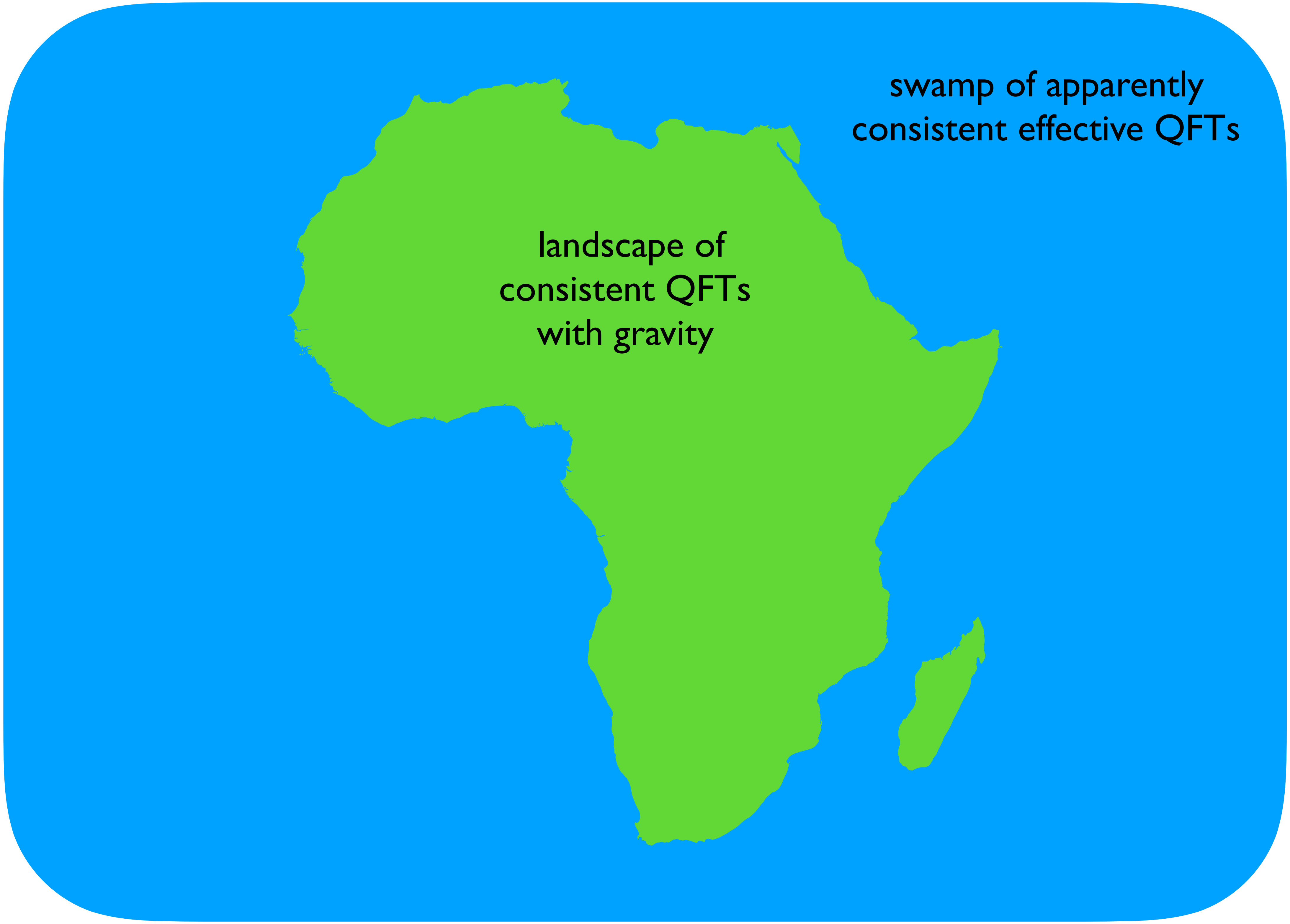
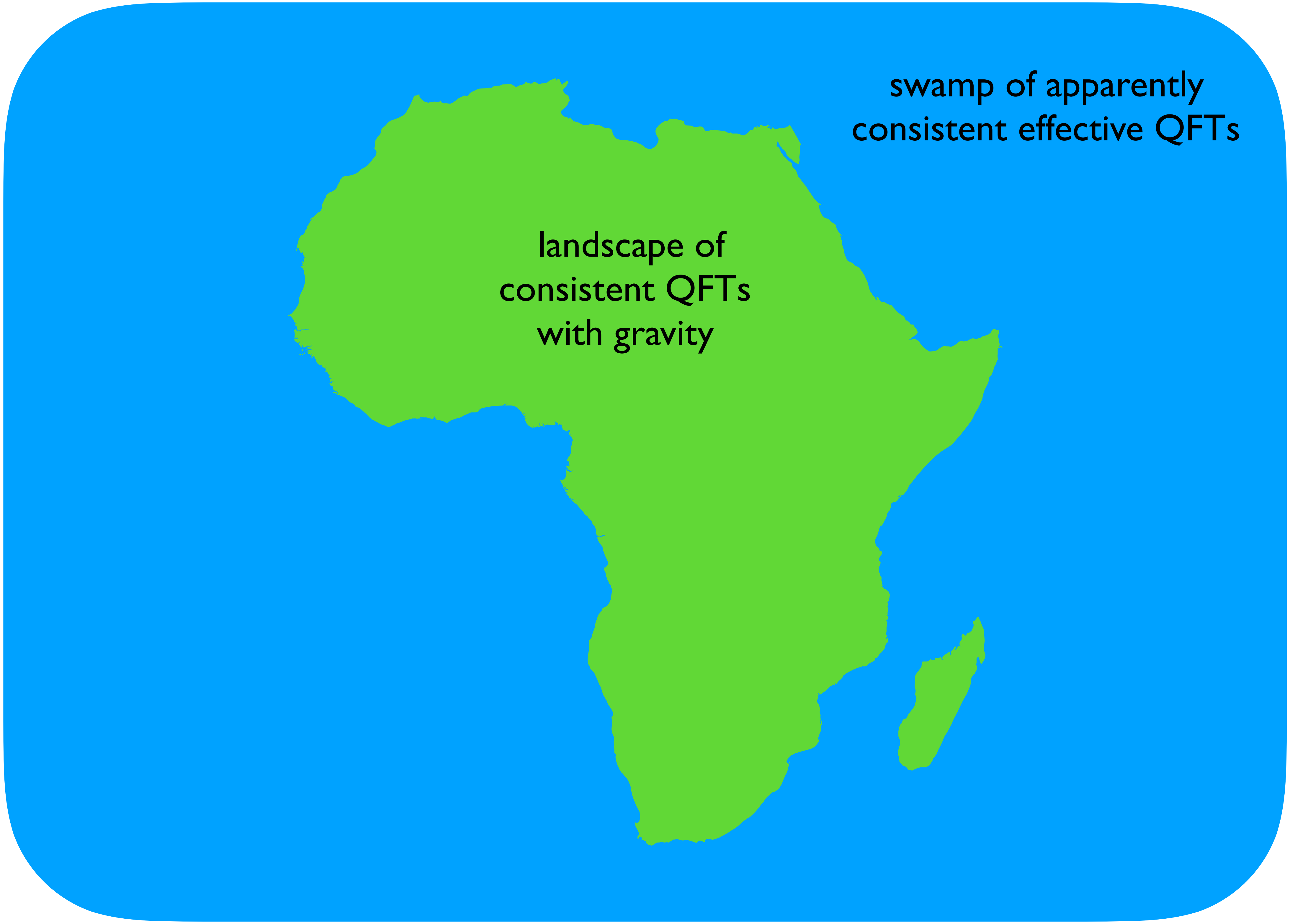
Good if we have restrictions on possible low-energy EFTs!


swamp of apparently consistent effective QFTs
landscape of consistent QFTs with gravity



Major question #1 what are landscape membership criteria?



Major question #1 what are landscape membership criteria?

#2 how large is the landscape?

(How much "tuning" is allowed?)

At present most of the suggested criteria take the form of conjectures, often with unknown O(1) constants appearing, and having varying levels of support

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There is an ever-growing interconnected list

i) No global symmetries
ii) Weak gravity conjecture (WGC)
iii) Compactness and completeness
iv) Swampland de Sitter
v) Distance conjecture
vi) ....

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Major question #3 what impact does the Swampland Program have on our understanding of the *Standard Model (if any)*?

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Natural pheno question:

What is the bound on how *high-quality* an *approximate* GS can be, and how precisely does the violation occur?

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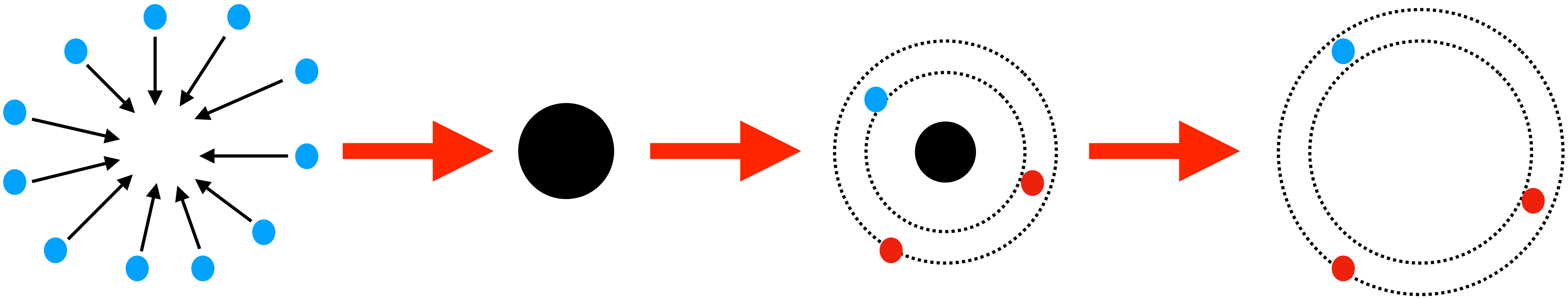

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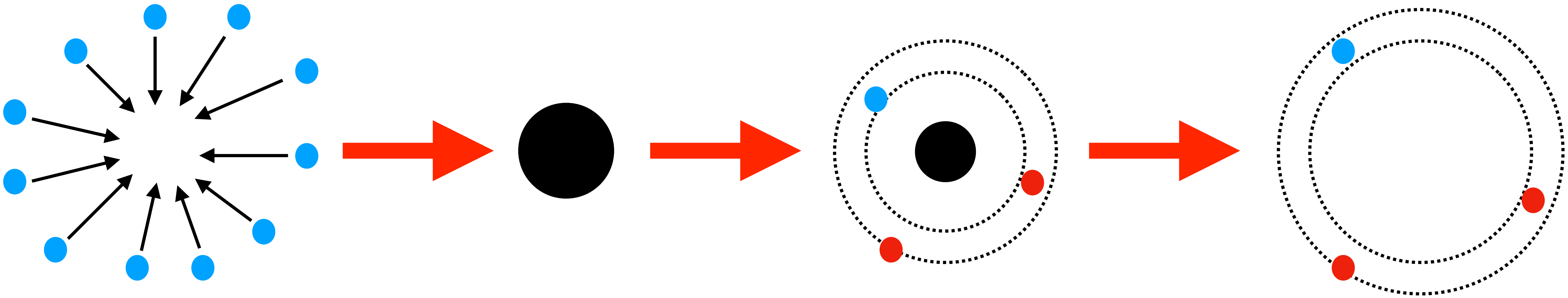

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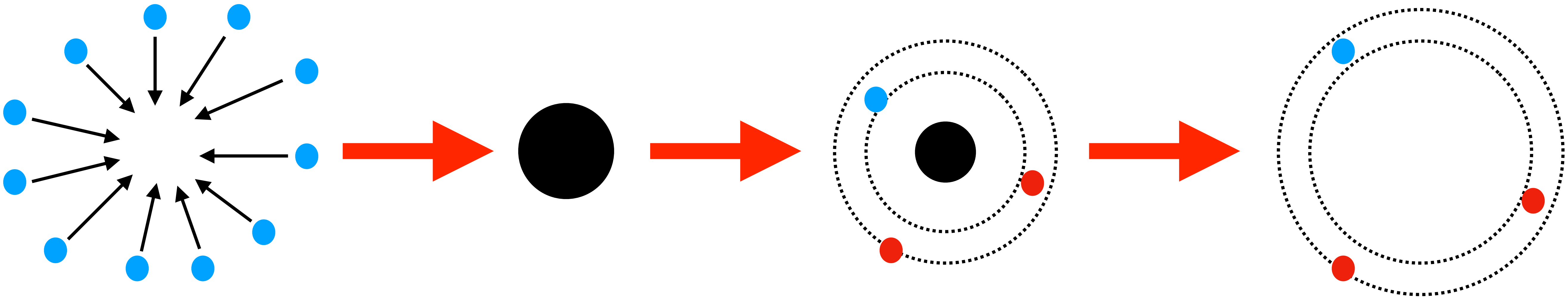

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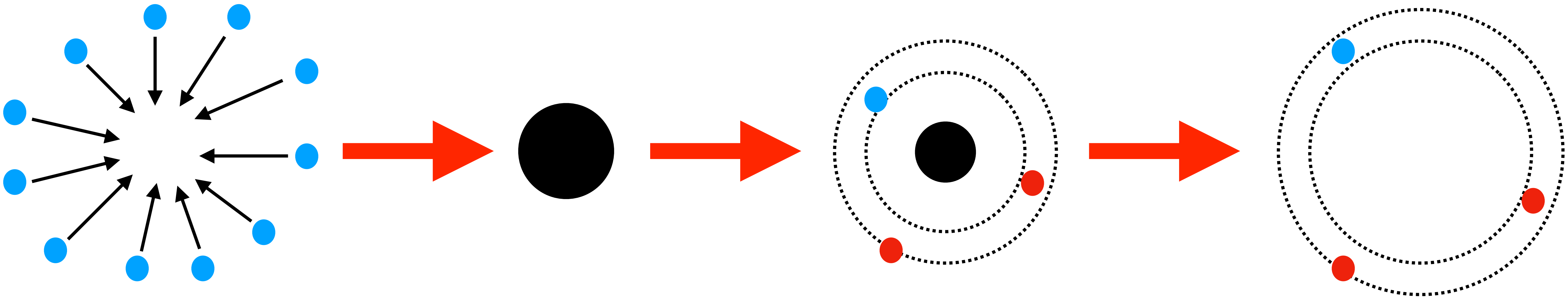

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Very stimulating but (to me) aspects are mysterious

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They can only appear as boundaries of p-branes charged under A_{p+1}

$$S \supset \int_{B_p} A_{p+1} + \int_{\partial B_p} A_p$$

only this combination is invariant under $\delta A_{p+1} = \mathrm{d}\chi_p,\ \delta A_p = -\chi_p$

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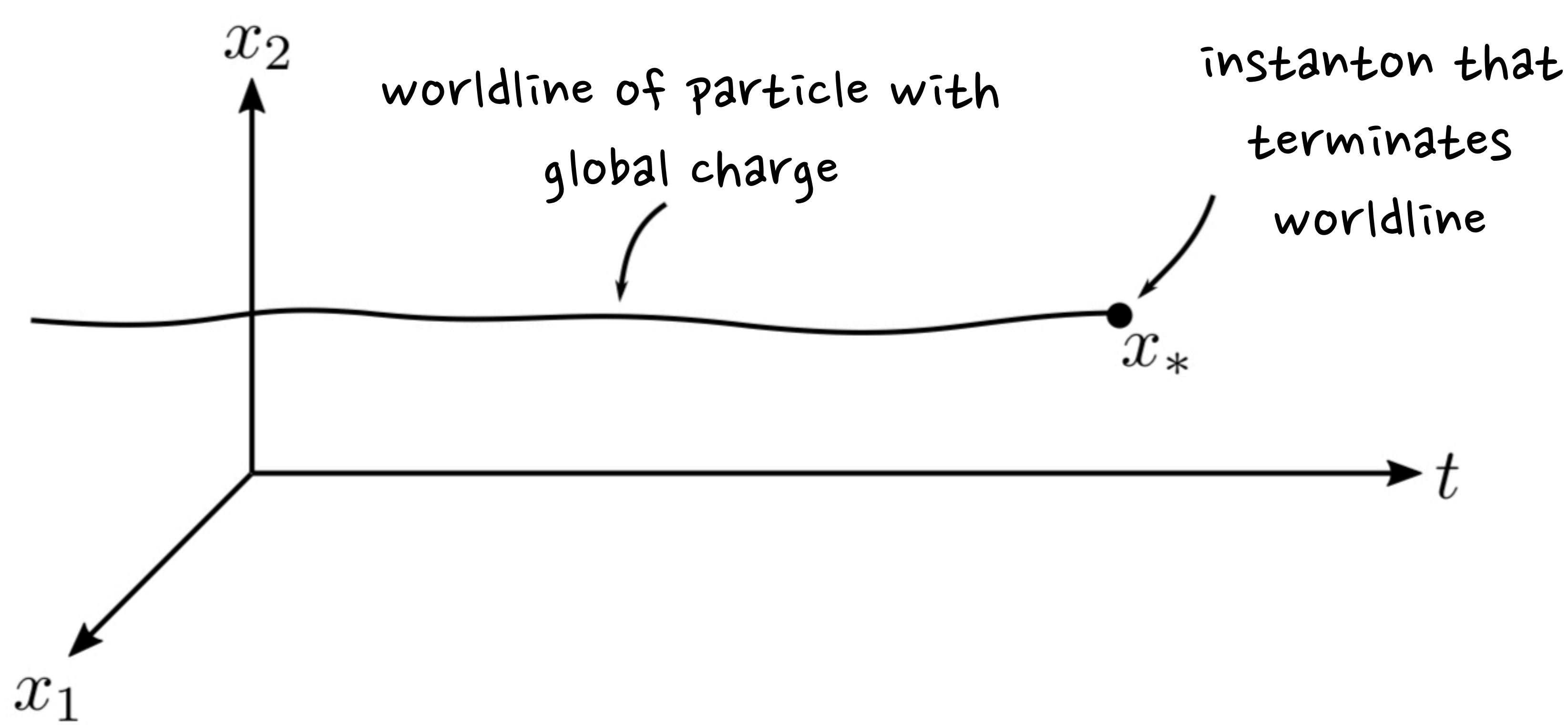
Crucially, though, the "(p-1)-brane" a "-1-brane" is now an *instanton* that is the boundary of the particle worldline



This corresponds (for a fixed instanton position x_*) to the term

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It follows that the operator induced in the EFT by the instanton sum is of general form

instanton action

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Primary question: is there a bound on the instanton action?

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Together gives lower bound on the coeff of the $\Delta N = \pm 1$ operators

$$\exp(-S_I) = \exp\left(-c\frac{M_{\text{pl}}^2}{\Lambda^2}\right)$$

where $c \sim \mathcal{O}(1)$

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5) Lots of explicit examples satisfy this bound (see paper...)

Part II

Swampland de Sitter Conjecture

One of the most striking of the conjectured constraints is the refined swampland de Sitter conjecture (SdSC) for the potential $V(\{\phi_i\})$

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Potentials with metastable de Sitter vacua are in the swampland, as are regions of field space that are too flat for V>0

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Here will make the claim that plausibly SdSC limits flavour structure of quarks and maybe even sheds new light on the hierarchy problem

Metastable States of QCD with Light Quarks

Well known that for N>2 light quarks the Chiral Lagrangian predicts metastable states (Witten, 1980; Creutz, 1995; Smilga, 1999;...)

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Consider SU(3) QCD with N light quarks, so that theory is still confining and chiral symmetry breaking occurs

$$SU(N)_L \times SU(N)_R \to SU(N)_V$$

Low-energy physics determined by pNGB Chiral Lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4}\,\mathrm{Tr}\left(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\right) - B_0\,\mathrm{Tr}\left(e^{-i\bar{\theta}/N} M_q^\dagger \Sigma + e^{i\bar{\theta}/N} \Sigma^\dagger M_q\right)$$

NXN light quark mass matrix
(diagonal without loss of generality)

$$\Sigma(x) = \exp(2i\pi^a(x)T^a/f_\pi) \in SU(N).$$

Parameters (real) determining local vacuum structure are

$$M_q = \mathrm{Diag}(m_1, m_2, \ldots, m_N), \text{ with } m_1 \geq m_2 \geq \ldots \geq m_N$$

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Here $(\phi_i + \overline{\theta}/N) f_\pi \equiv \langle \pi^i \rangle$ are a useful re-combination of the vev's of the SU(N) Cartan sub-algebra pions, satisfying constraint

$$\phi_1 + \cdots + \phi_N + \overline{\theta} = 0 \mod 2\pi$$

Thus form of potential determining critical points of the theory is

$$V(\phi_i) = -B_0 \sum_i^N m_i \cos \phi_i$$

subject to the constraint $\phi_1 + \cdots + \phi_N + \theta = 0 \mod 2\pi$

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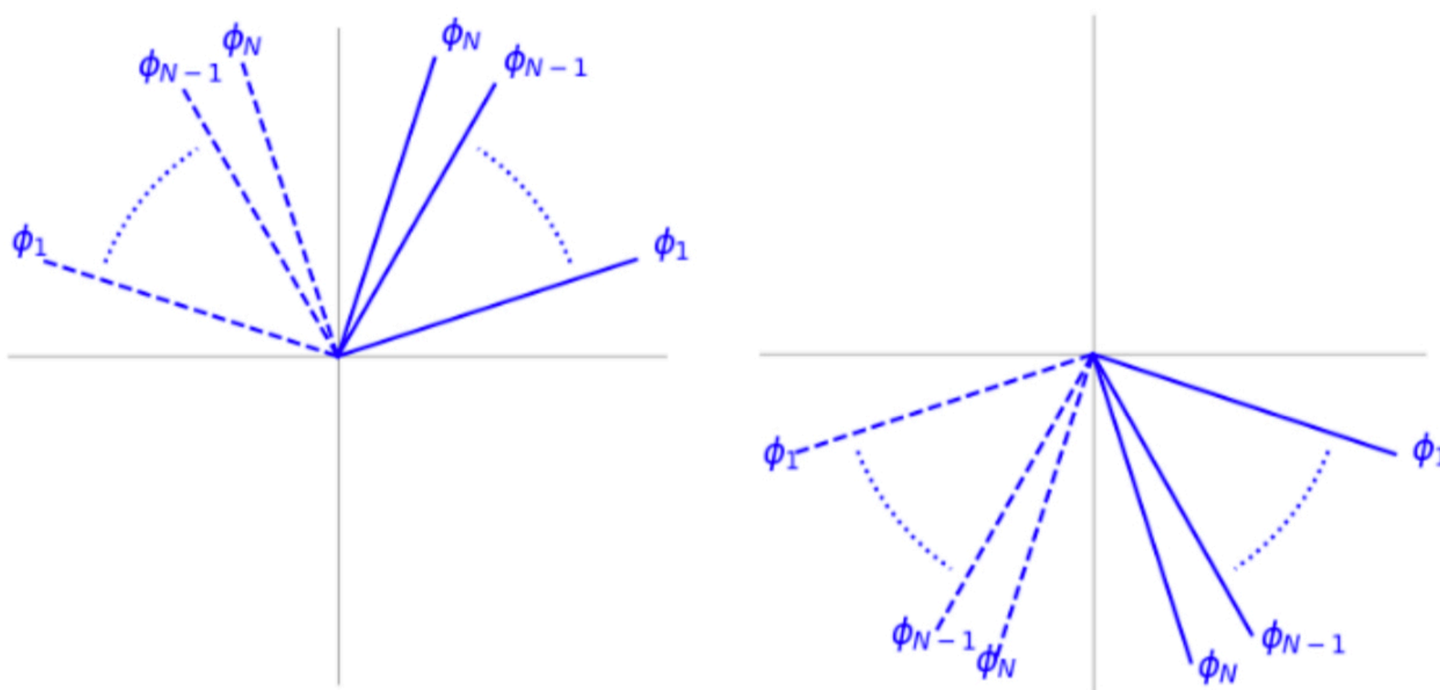
can understand critical points geometrically - 'fan diagrams' (see paper...)



(a) *Critical point*

Hessian matrix around these critical points is

$$H_{ij} = \delta_{ij} m_i \cos\phi_i + m_N \cos\phi_N$$

so a necessary condition for positive definiteness is

$$\cos\phi_i > 0 \quad \forall_{i<N}$$

Can straightforwardly study nature of critical points as quark mass ratios and topological angle vary...

Simple case: Metastable states at equal quark masses

A particularly simple case occurs if all masses are equal: then the critical point condition + the necessary condition for positive definiteness + the unitary condition

$$\Longrightarrow \qquad \phi_i = \frac{2\pi n - \bar{\theta}}{N} \equiv \phi \qquad \forall i$$

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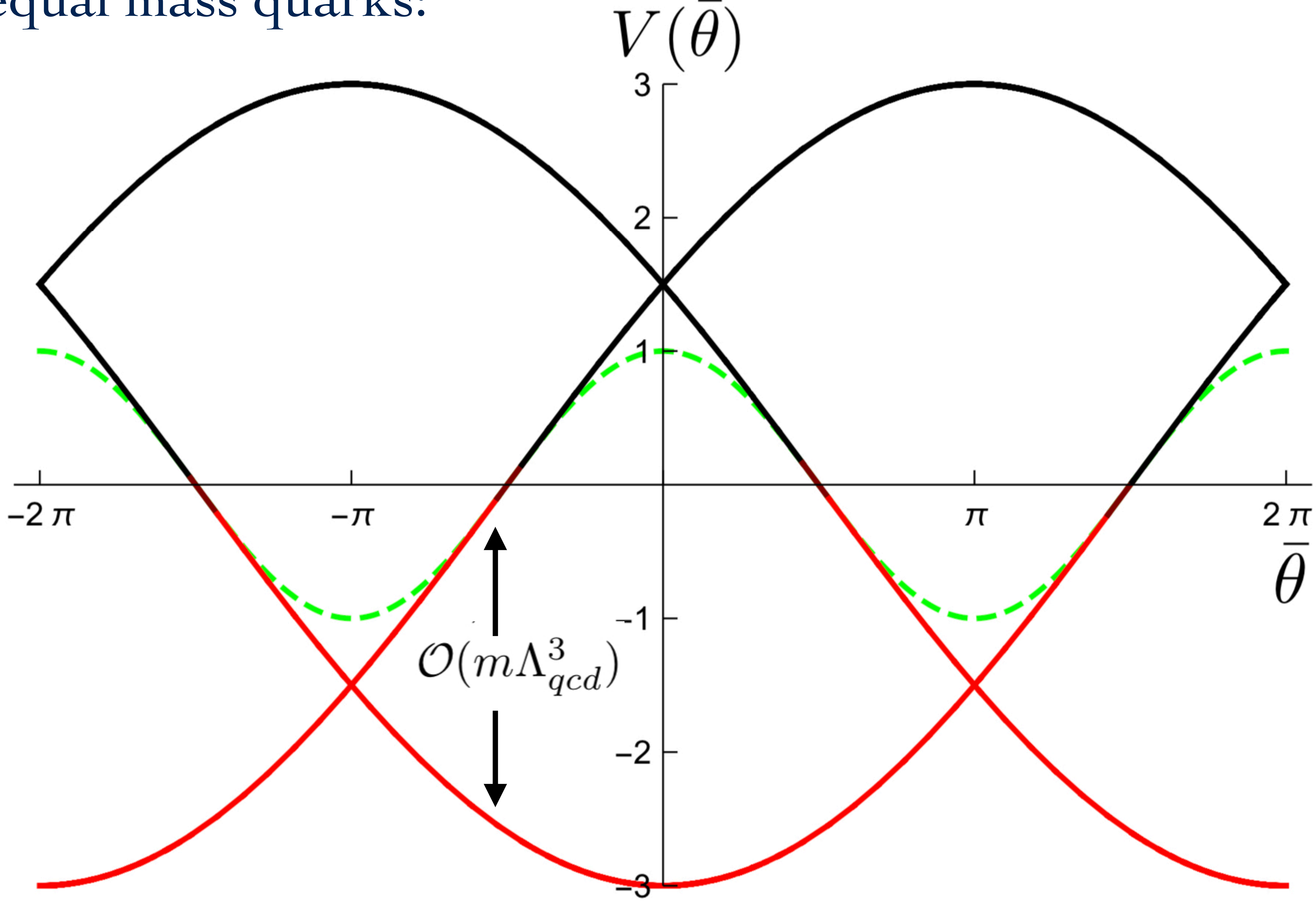
Thus for N>2 there is more than one n in the range, and metastable states exist

Eg, 3 equal mass quarks:



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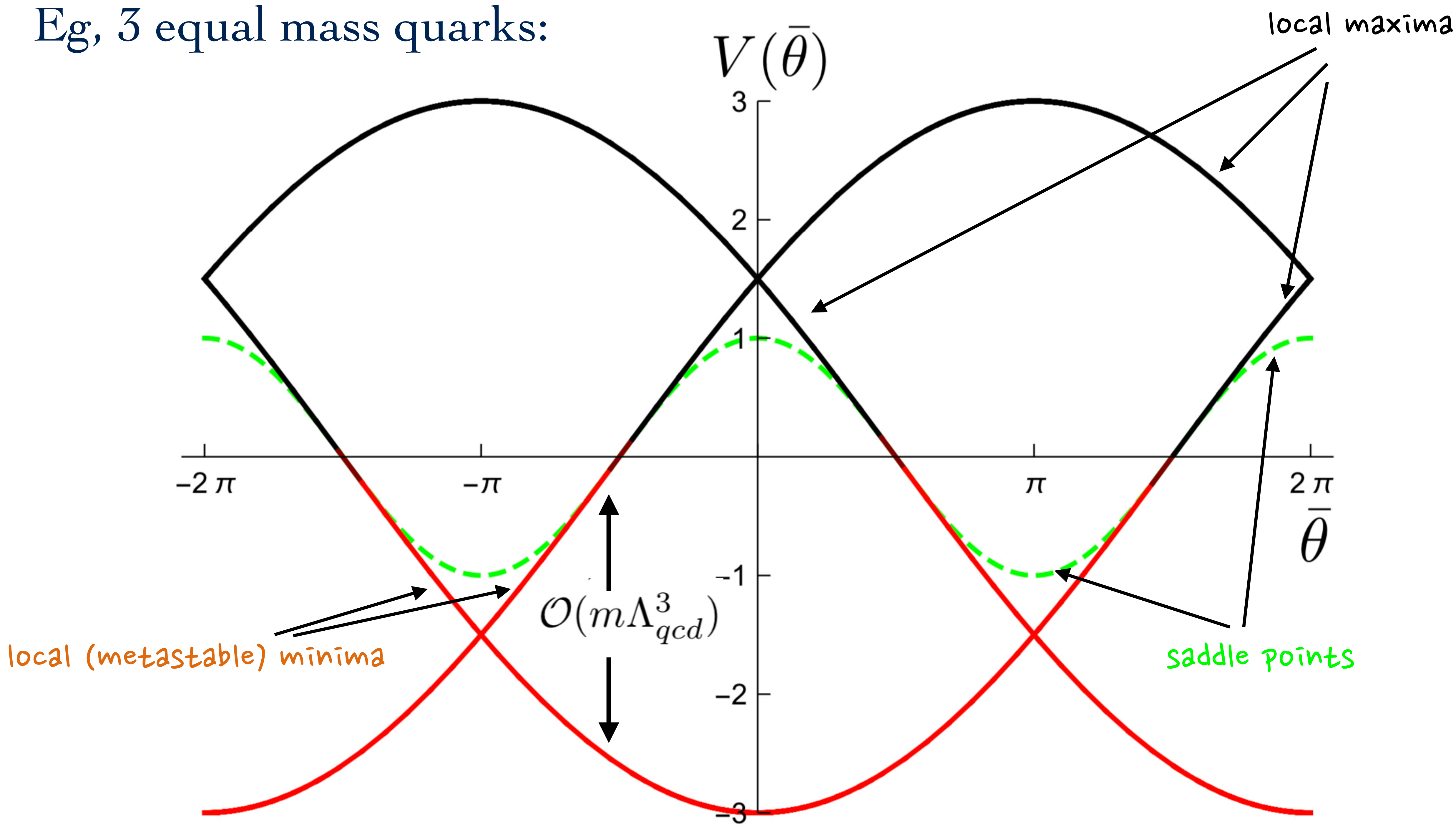

$$V^{(n)} = -NmB_0 \cos\left(\frac{2\pi n - \bar{\theta}}{N}\right)$$

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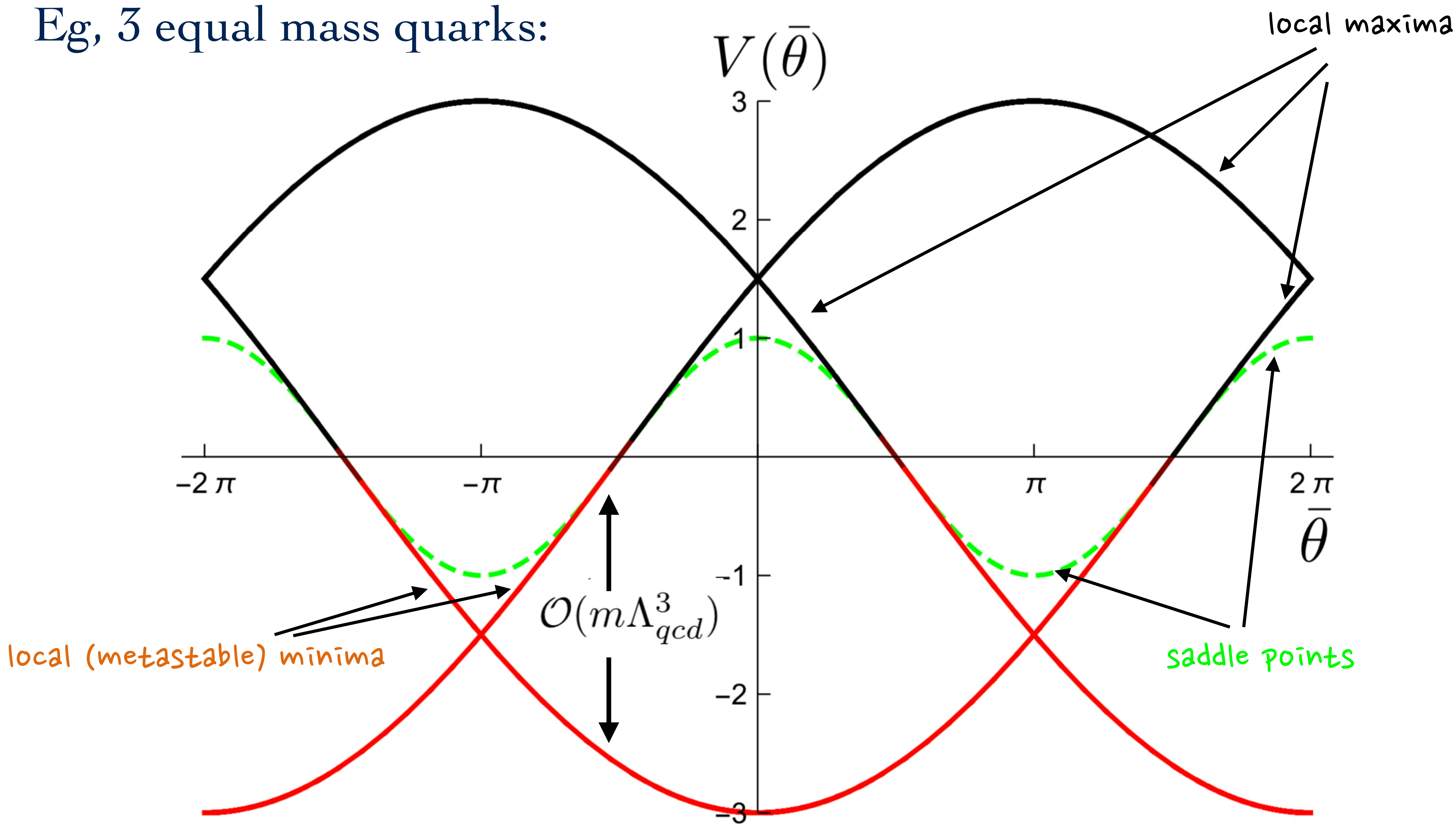
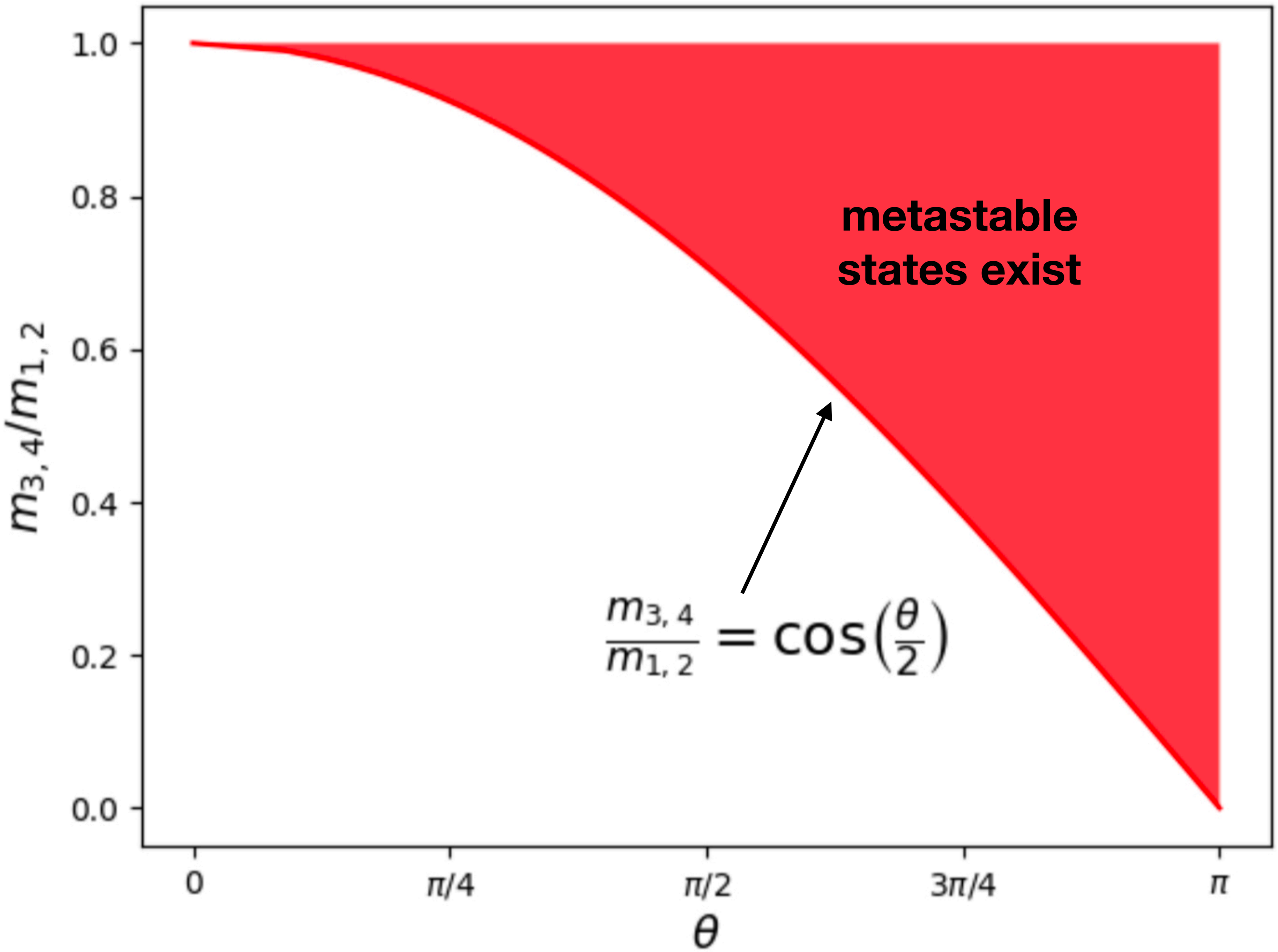


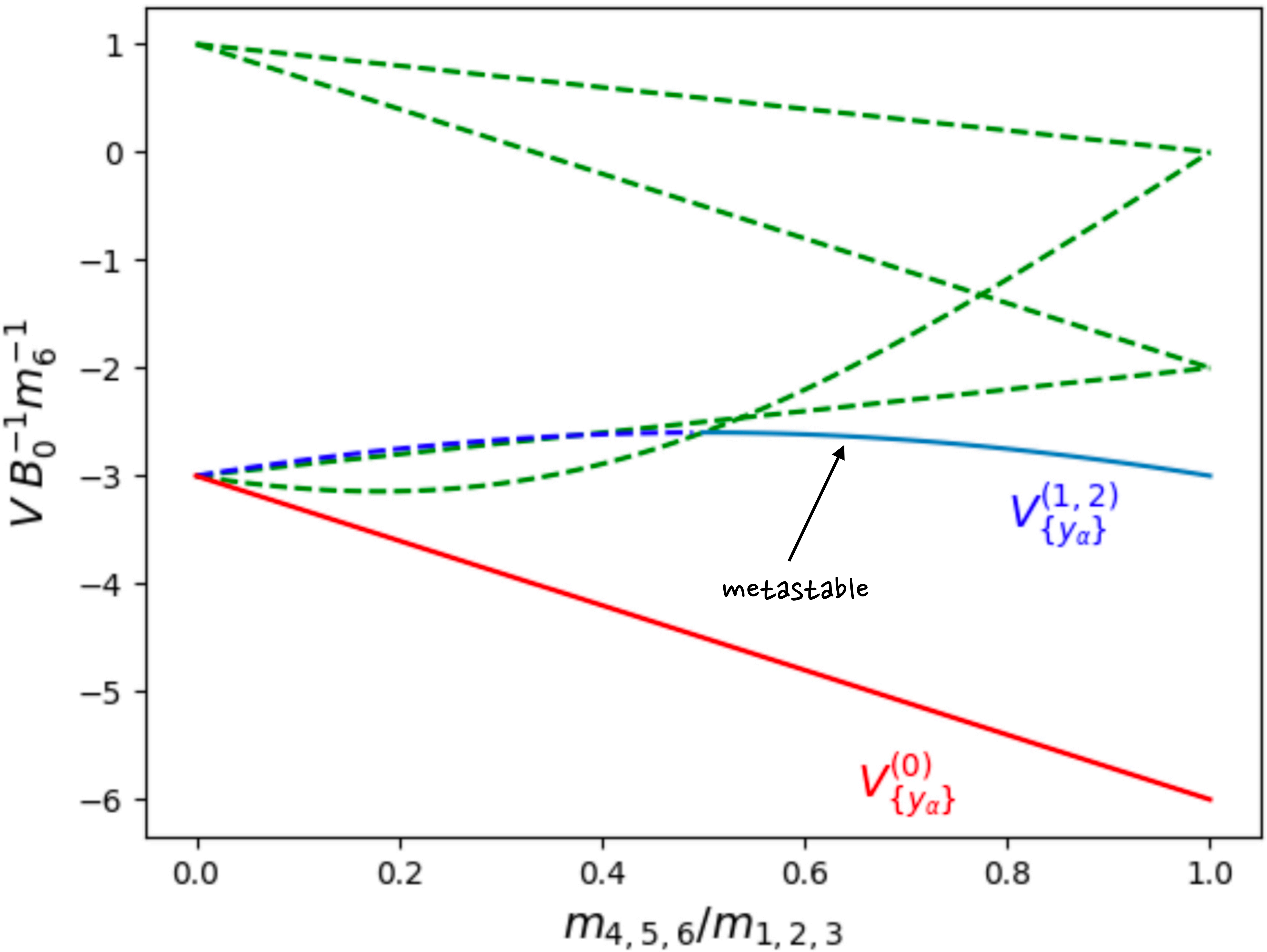
$$V^{(n)} = -NmB_0 \cos\left(\frac{2\pi n - \bar{\theta}}{N}\right)$$

metastable state exists in range $\pi/2 < \bar{\theta} < 3\pi/2$

More complicated case: 4 quarks in 2 equal mass groups



More complicated case: 6 quarks in 2 equal mass groups for $\bar{\theta} = 0$



We argue that all the regions of parameter space with metastable states are plausibly excluded by the SdSC from descending from a theory with gravity

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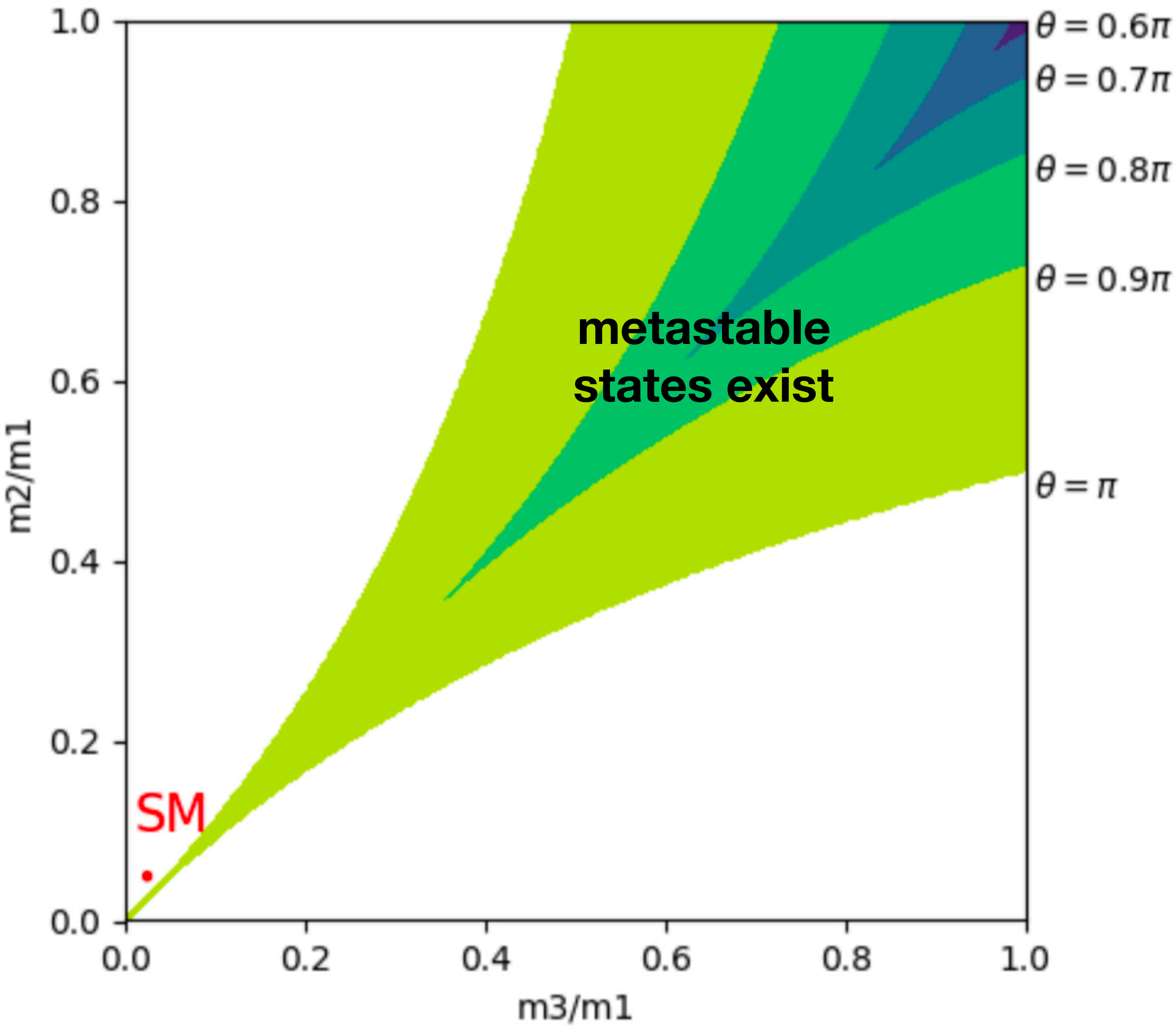
What about the *observed* SM parameter values:
3 light quark masses, $m_{u,d,s}$ and $\bar{\theta} = 0$?

For SM:



1.0
0.8
0.6
0.4
0.2
0.0
m2/m1
0.0
0.2
0.4
0.6
0.8
1.0
m3/m1
SM
metastable
states exist
θ = 0.6π
θ = 0.7π
θ = 0.8π
θ = 0.9π
θ = π

Amusingly the condition that theory is safe from metastable states for any value of $\bar{\theta}$ in an N-light-quark theory is

$$\frac{1}{m_N} > \frac{1}{m_1} + \cdots + \frac{1}{m_{N-1}}$$

This is satisfied by the SM light quarks

$$\frac{1}{m_u} > \frac{1}{m_d} + \frac{1}{m_s}$$

But if had, say, 5 or 6 light quarks with not large mass ratios then would have metastable states at $\bar{\theta} = 0$

Robustness Against Quintessence

So far haven't addressed obvious question of how SdSC is consistent with our presently observed cosmic acceleration (CA)

Assuming the SdSC is correct I am aware of at least three possible responses to CA

i) CA is not due to a cosmological constant but an evolving quintessence field, a possibility which is (marginally) consistent with data and the SdSC

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This is potentially deadly as adding new states, eg, quintessence, in the far IR can possibly destabilise local metastable minima and nullify our results

Thus we add an ultra-light quintessence field and consider the two general possibilities

a) Sequestered: $V \approx V^{(n)}_{\{y_\alpha\},\mathrm{QCD}}(\pi^a) + \tilde{V}(\varphi)$

b) Un-sequestered wrt SM: $V = V^{(n)}_{\{y_\alpha\},\mathrm{QCD}}(\pi^a, \varphi)$

branches of potential

the SM parameters - quark masses, theta-angle

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branches of potential

the SM parameters - quark masses, theta-angle

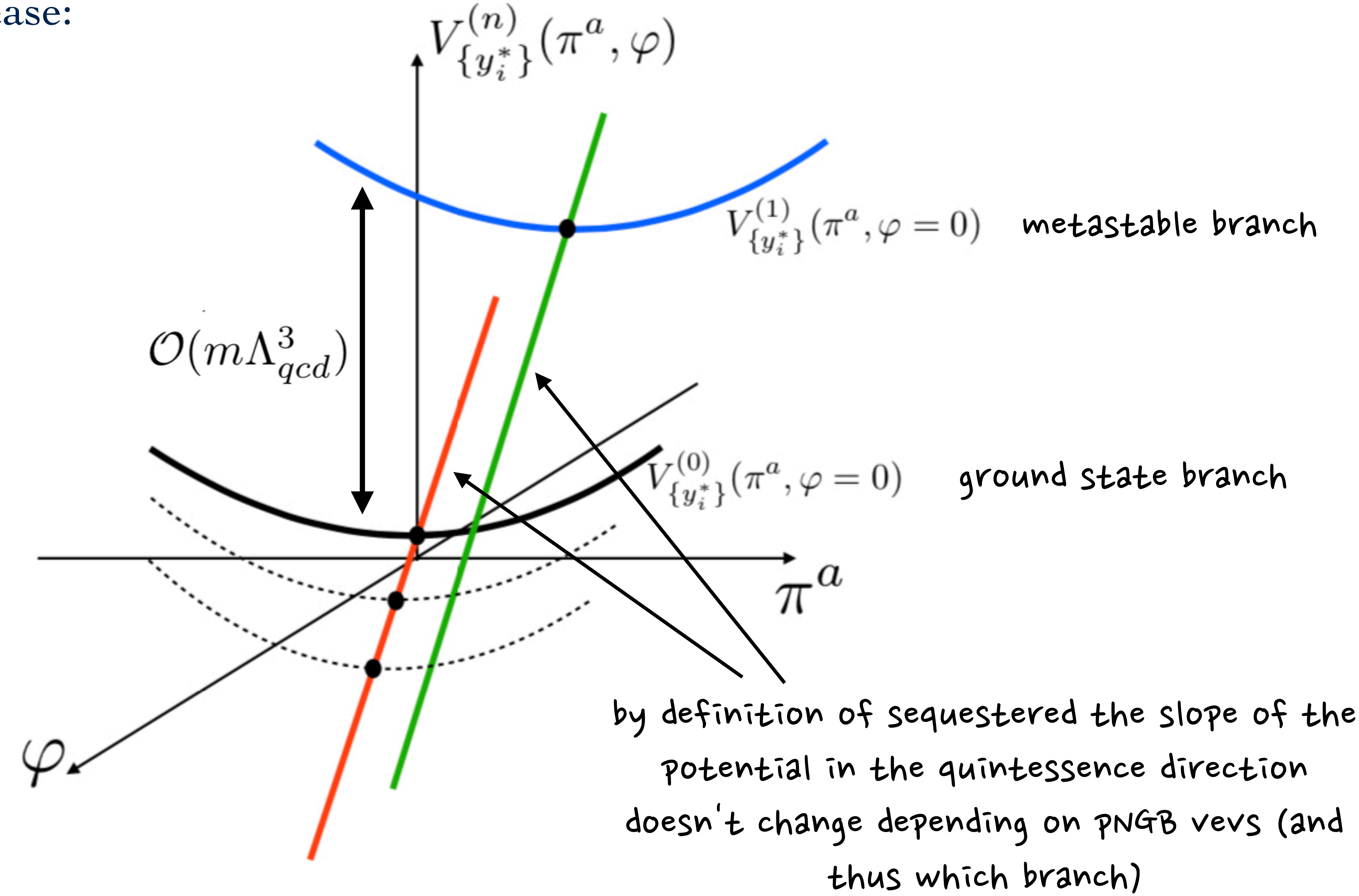
We must now enforce the SdSC conditions

$$|\nabla V| \geq c \frac{V}{M_{\mathrm{pl}}}$$

$$\text{or} \qquad \min(\nabla_i \nabla_j V) \leq -c' \frac{V}{M^2_{\mathrm{pl}}}$$

and see if a successful quintessence potential is consistent

Sequestered case:



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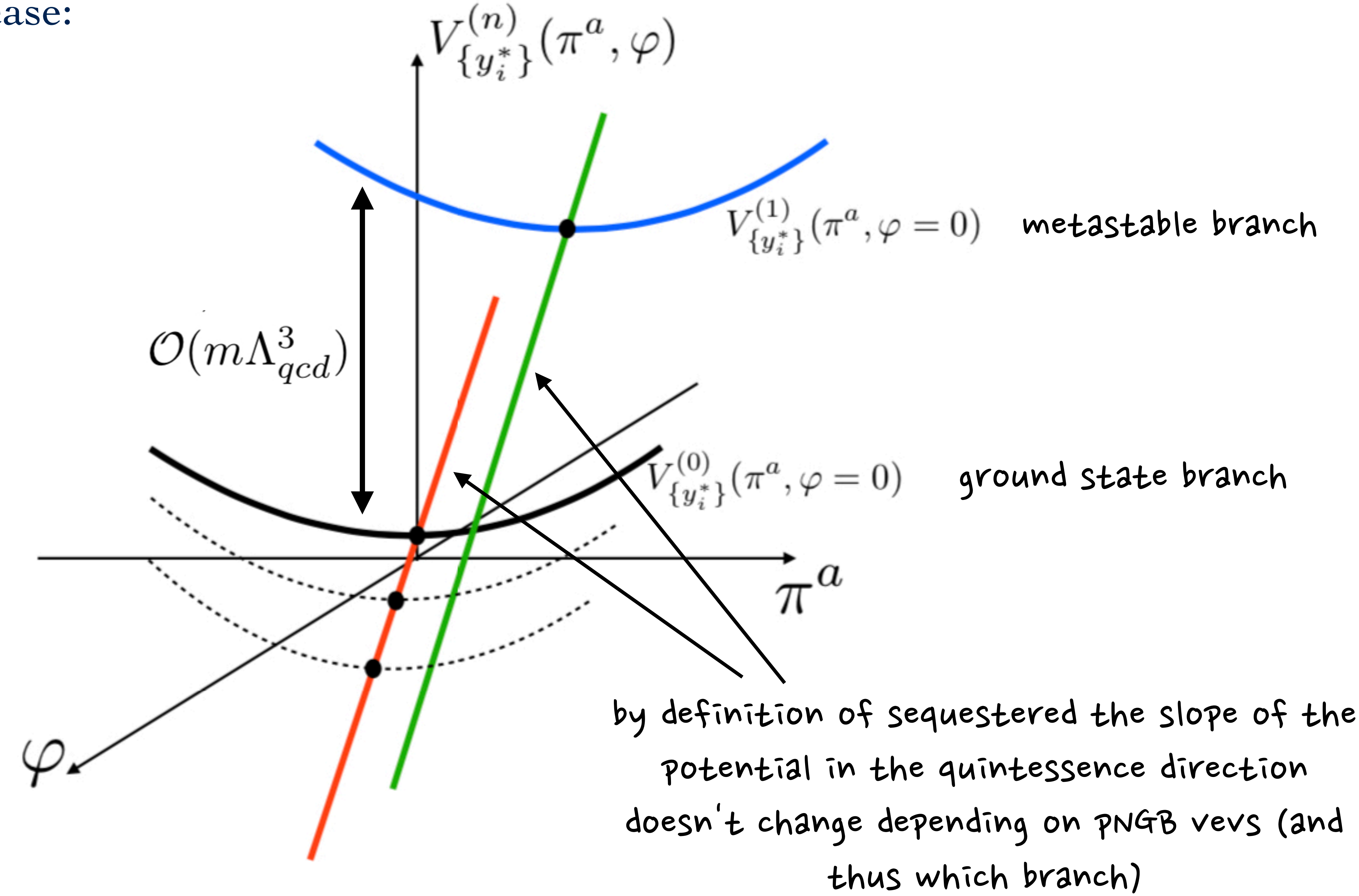

Applying the SdSC conditions in φ direction to the metastable branch

$$|\nabla_\varphi \tilde{V}(\varphi)| \geq c\frac{m_q\Lambda^3_{qcd}}{M_{\rm pl}} \quad \text{or} \quad \nabla^2_\varphi \tilde{V}(\varphi) \leq -c'\frac{m_q\Lambda^3_{qcd}}{M^2_{\rm pl}}$$

In either situation the slope in the quintessence direction is then forced to be huge and φ almost immediately evolves to deep AdS and a big crunch

Eg, in case of 1st condition being satisfied, in both branches the field evolves as

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Then vacuum energy in ground-state branch evolves to AdS in a extremely small fraction of a Hubble time

$$\Delta t \equiv \tau / H$$

$$\tau_{AdS} \sim \frac{V_0}{m_q \Lambda_{qcd}^3} \ll 1$$

$$H^{-1} \sim M_{\rm pl}/\sqrt{V_0}$$

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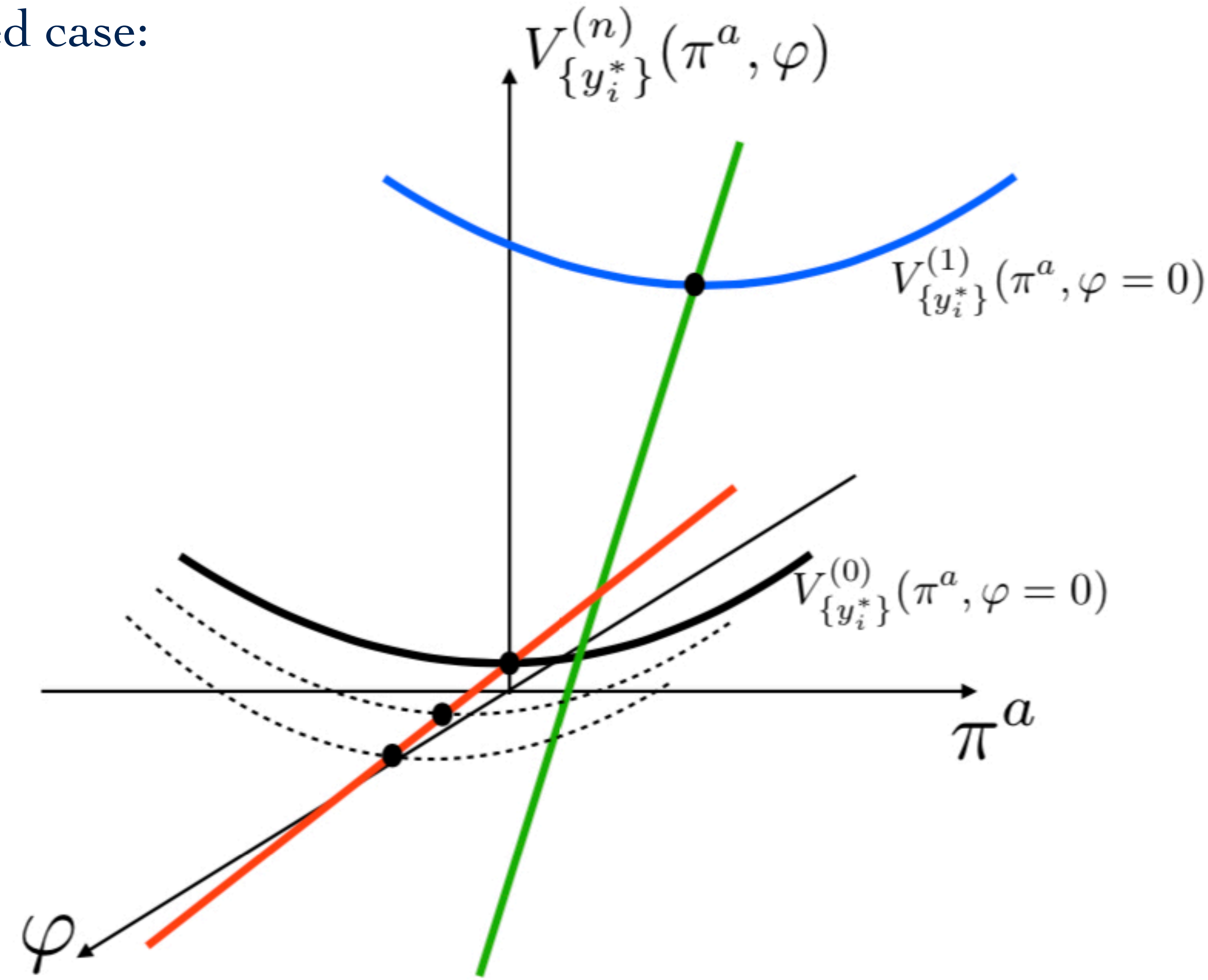
Similar conclusion if 2nd condition is satisfied...

Fundamental reason for failure is huge disparity between our vacuum energy and $m_q\Lambda_{QCD}^3$

Un-sequestered case:



Now, in principle slope in quintessence direction could change greatly between metastable branch (where pNGB vevs are $\mathcal{O}(f_\pi)$) and ground state branch of QCD simultaneously satisfying SdSC in both cases and not implying immediate big crunch

Problem is that this needs relatively huge couplings of φ to the pion fields

destabilises required flatness of quintessence potential unless extreme tuning of multiple terms

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Particularly interesting as we can achieve this limit in two ways

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In case ii) our thinking might lead to a new perspective on the hierarchy problem

In the limit $v_{EW} \gg 50$ TeV the IR theory is pure $SU(3) \times U(1)_{EM}$

Because there is no matter the two gauge groups are totally decoupled

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What is the basic argument?

multiple branches of the potential energy are required to reconcile

i) Must be a periodic function $V(\bar{\theta}) = V(\bar{\theta} + 2\pi)$

ii) Has the form $V(\bar{\theta}) = N_c^2 f(\bar{\theta}/N_c)$ with $f(x)$ 2π-periodic

These two properties compatible if a multi-branched function

$$V(\bar{\theta}) = \min_n V^{(n)}(\bar{\theta}) \quad \text{with} \quad V^{(n)}(\bar{\theta}) = N_c^2 f\left(\frac{\bar{\theta} + 2\pi n}{N_c}\right)$$

$$n = 0, \ldots, N_c - 1$$

ground state potential

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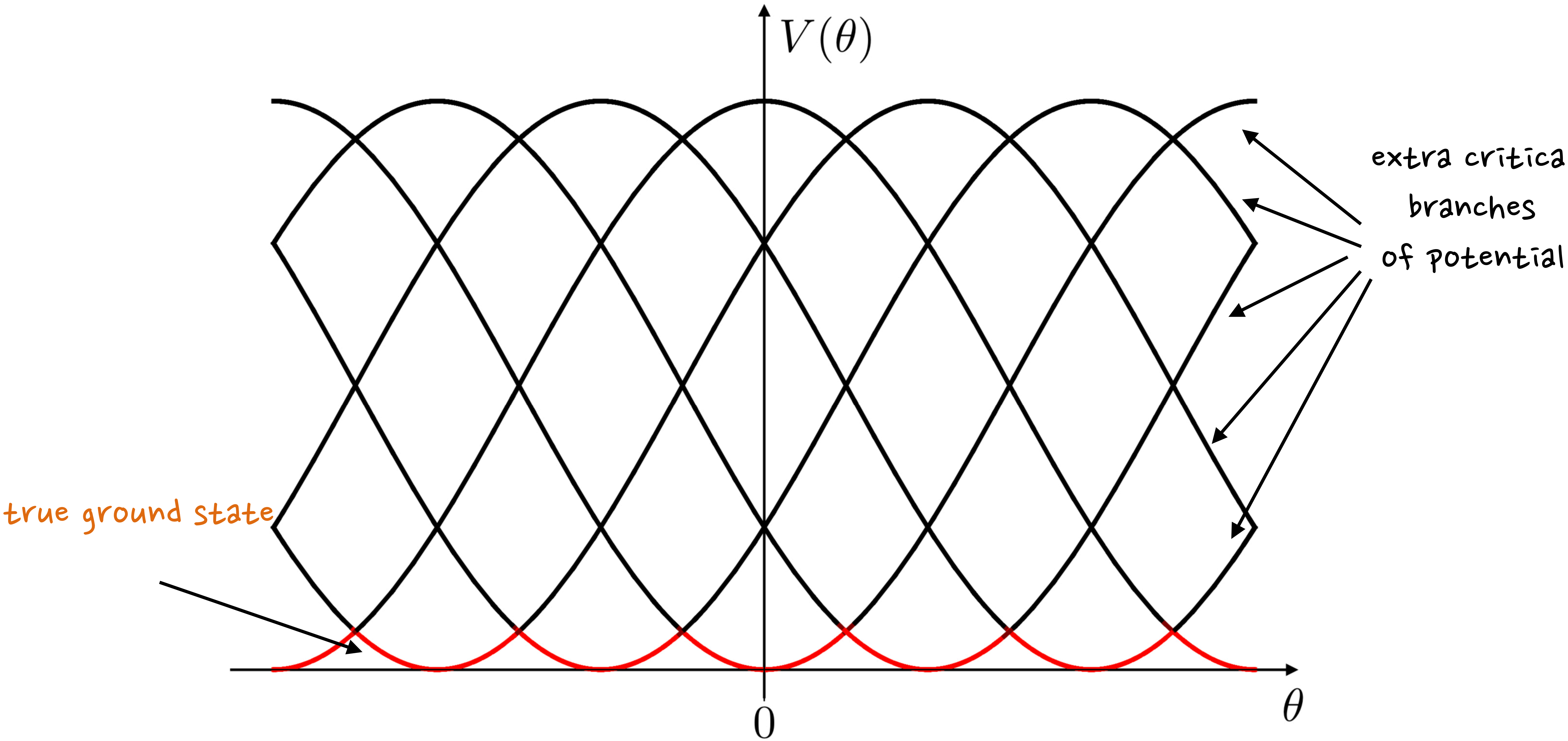

(Witten argued that in $N_c \to \infty$ limit all the extra critical points were *local minima* and even at $\theta = 0$ there would be $(N_c - 1)$ metastable states)

Properties of these states?

(Gabadaze & Shifman, 2002)

Define the topological susceptibility

$$\chi = \int d^4x \langle Q(x) Q(0) \rangle \qquad Q = \frac{1}{8\pi^2} \mathrm{Tr}(G^{\mu\nu} \tilde{G}_{\mu\nu})$$

Then one order parameter distinguishing the various states is

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Thus the metastable states are not CP-invariant even if $\theta = 0, \pi$

Moreover, for $n \ll N_c$ the difference in energy densities is

$$V^{(n)}(0) - V^{(0)}(0) \simeq \frac{(2\pi n)^2}{2}\chi$$

$$\chi_{SM} \simeq (190\ \mathrm{MeV})^4$$

What happens at *finite* N_c ?

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$$N_s = \begin{cases} 2\left[\frac{N_c}{4}\right] & \theta = 0 \\ \left[\frac{N_c}{4}\right] + \left[\frac{N_c+3}{4}\right] - 1 & 0 < \theta < \pi/2 \\ \left[\frac{N_c+1}{2}\right] - 1 & \theta = \pi/2 \\ \left[\frac{N_c+1}{4}\right] + \left[\frac{N_c+2}{4}\right] - 1 & \pi/2 < \theta < \pi \\ 2\left[\frac{N_c+2}{4}\right] - 1 & \theta = \pi \end{cases}$$

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Different semiclassical arguments do find metastable state at $\theta = 0$ for N_c=3 QCD

(Halperin & Zhitnitsky, 1998)

There have been *no* sufficiently good lattice studies of this question, so we don't know truth!

Thus it is just possible that N_f=0, N_c=3 QCD has metastable states and therefore the SdSC could forbid the $v_{EW} \gg 50$ TeV limit!

Will be very interesting to have lattice studies of this question...

Thanks for your attention!