

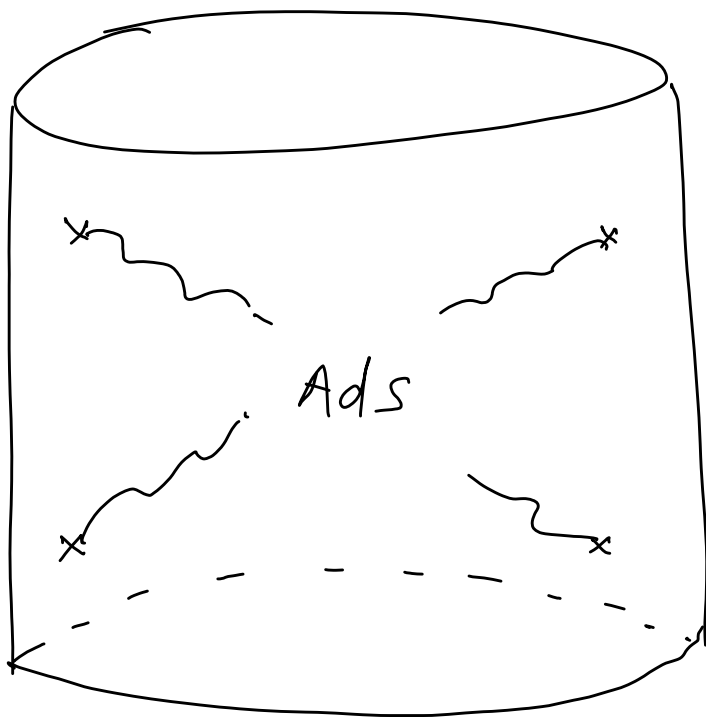
Space-Time , Quantum Mechanics,
and

Positive Geometry at Infinity

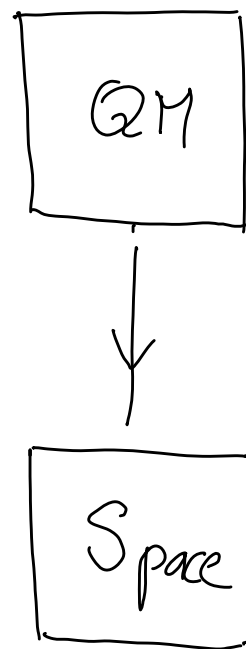
w/ S. He , T. Lam, H. Thomas
M. Spradlin, G. Salvatori
H. Furst, P.-G. Plamondon;
Y. Bai, G. Yan

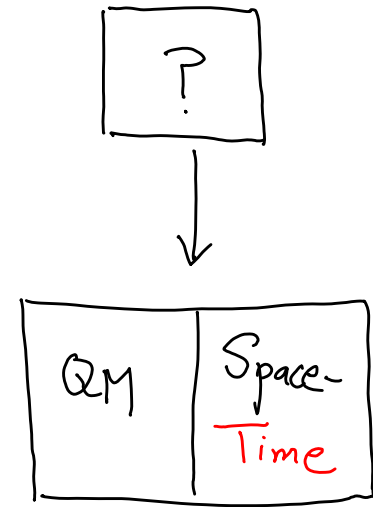
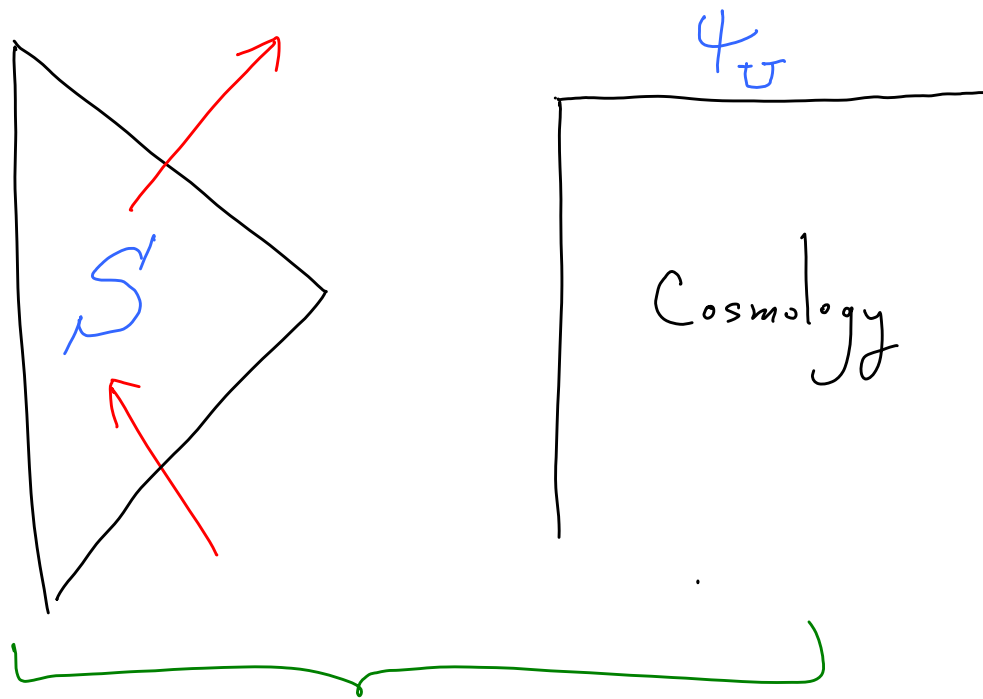
J. Trnka,
J. Bourjaily, ,
P. Benincasa,
S. Caron-Huot
C. Cheung
J. Kaplan

S. Goncharov
A. Postnikov,
F. Cachazo
A. Hodges



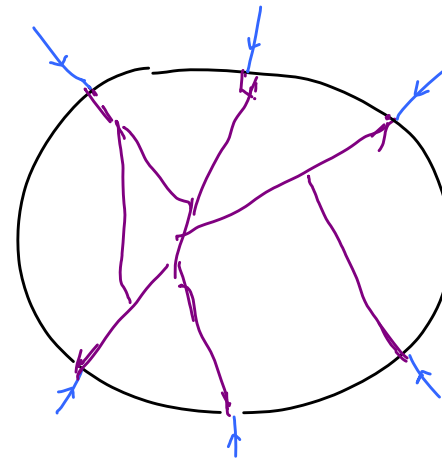
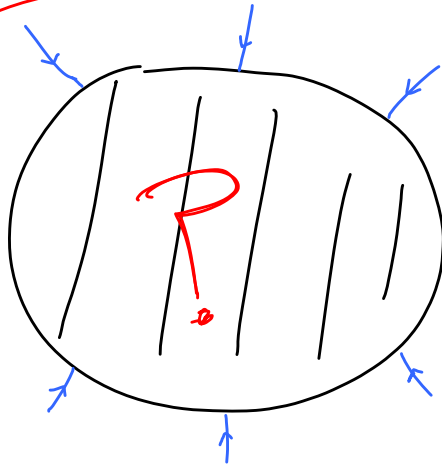
↑ time



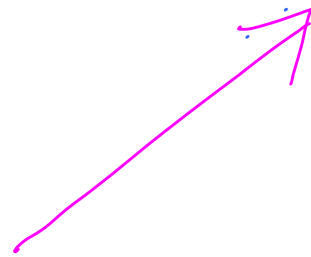


Unlike 2AdS, no
obvious notion of time+locality

What is the Q to which A is the Answer?

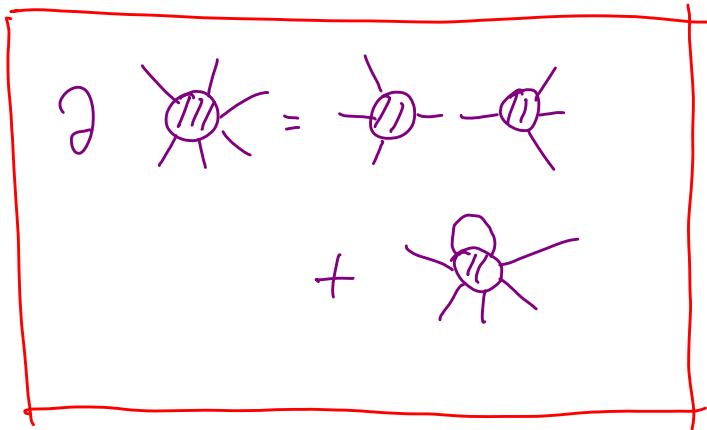
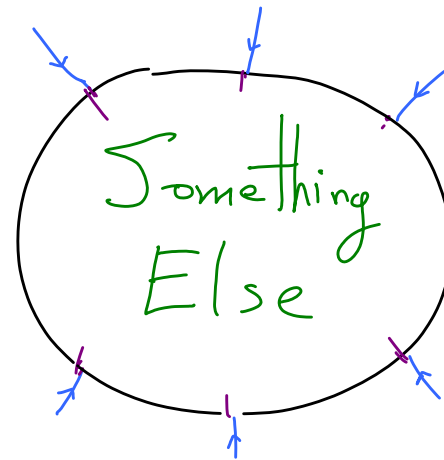
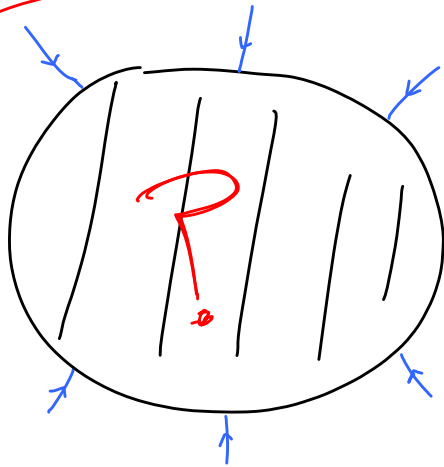


$$\partial \text{ (circle with lines) } = \text{ (circle with lines) } - \text{ (circle with lines) } + \text{ (circle with lines) }$$



Local, Unitary evol. in
Spacetime/Hilbert Space.

What is the Q to which A is the Answer?

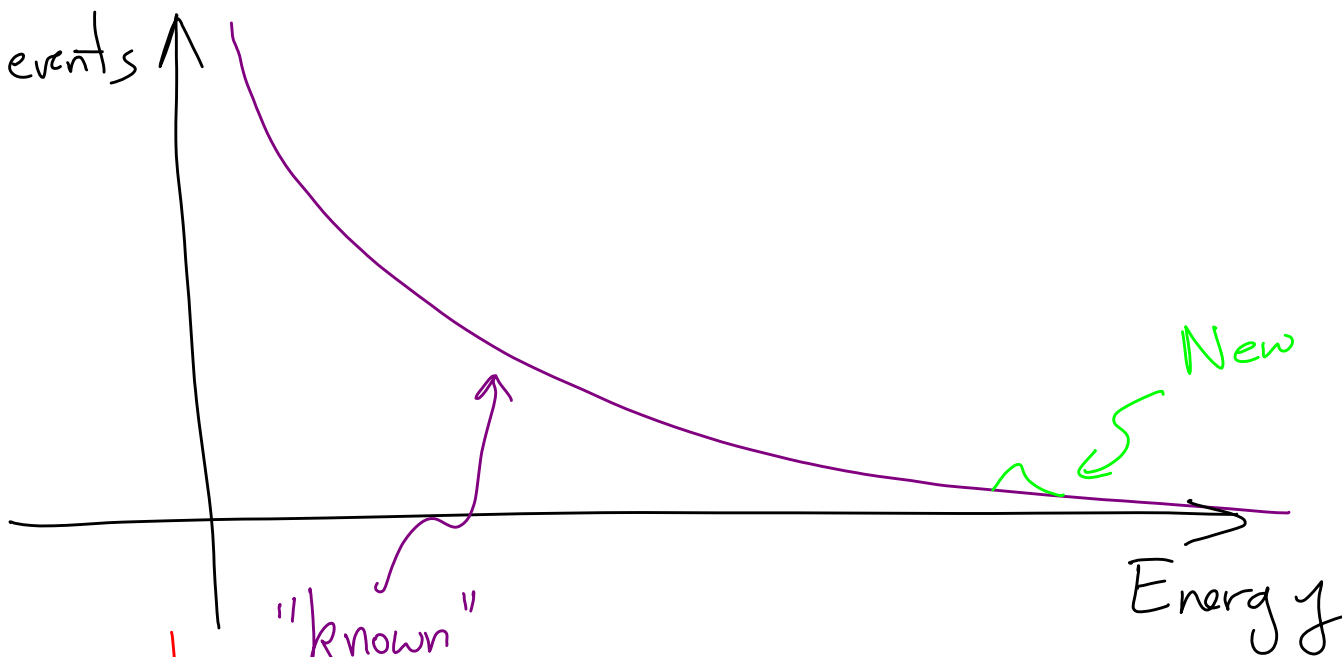


Rules of Spacetime + QM -
Locality + Unitarity -
as derivative outputs

L. H. C.



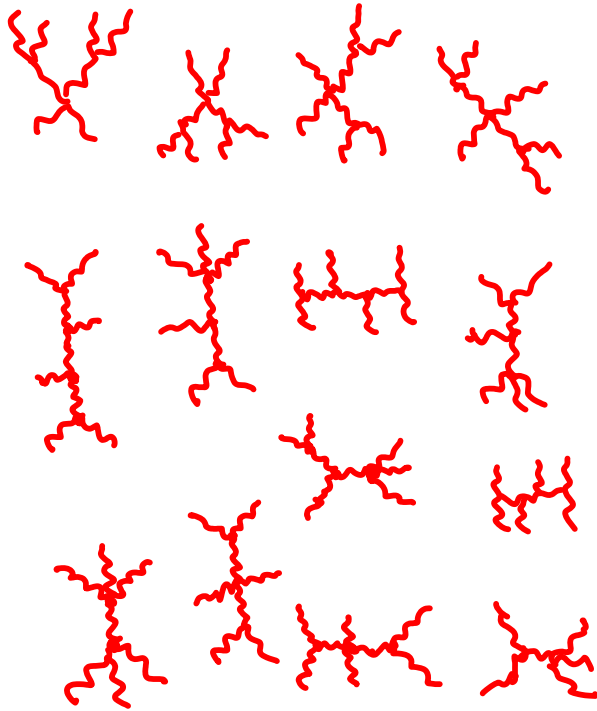
events $\uparrow$



Rates:

~ 1 billion collisions/sec
 ~ 10 top quarks/sec
 ~ 1 squark minute.

Feynman Follies



+ ...

220 Diagrams

10's of thousands
of terms ...

Result of a brute force calculation:

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14. 15. 16. 17. 18. 19. 20. 21. 22. 23. 24. 25. 26. 27. 28. 29. 30. 31. 32. 33. 34. 35. 36. 37. 38. 39. 40. 41. 42. 43. 44. 45. 46. 47. 48. 49. 50. 51. 52. 53. 54. 55. 56. 57. 58. 59. 60. 61. 62. 63. 64. 65. 66. 67. 68. 69. 70. 71. 72. 73. 74. 75. 76. 77. 78. 79. 80. 81. 82. 83. 84. 85. 86. 87. 88. 89. 90. 91. 92. 93. 94. 95. 96. 97. 98. 99. 100. 101. 102. 103. 104. 105. 106. 107. 108. 109. 110. 111. 112. 113. 114. 115. 116. 117. 118. 119. 120. 121. 122. 123. 124. 125. 126. 127. 128. 129. 130. 131. 132. 133. 134. 135. 136. 137. 138. 139. 140. 141. 142. 143. 144. 145. 146. 147. 148. 149. 150. 151. 152. 153. 154. 155. 156. 157. 158. 159. 160. 161. 162. 163. 164. 165. 166. 167. 168. 169. 170. 171. 172. 173. 174. 175. 176. 177. 178. 179. 180. 181. 182. 183. 184. 185. 186. 187. 188. 189. 190. 191. 192. 193. 194. 195. 196. 197. 198. 199. 200. 201. 202. 203. 204. 205. 206. 207. 208. 209. 210. 211. 212. 213. 214. 215. 216. 217. 218. 219. 220. 221. 222. 223. 224. 225. 226. 227. 228. 229. 230. 231. 232. 233. 234. 235. 236. 237. 238. 239. 240. 241. 242. 243. 244. 245. 246. 247. 248. 249. 250. 251. 252. 253. 254. 255. 256. 257. 258. 259. 260. 261. 262. 263. 264. 265. 266. 267. 268. 269. 270. 271. 272. 273. 274. 275. 276. 277. 278. 279. 280. 281. 282. 283. 284. 285. 286. 287. 288. 289. 290. 291. 292. 293. 294. 295. 296. 297. 298. 299. 300. 301. 302. 303. 304. 305. 306. 307. 308. 309. 310. 311. 312. 313. 314. 315. 316. 317. 318. 319. 320. 321. 322. 323. 324. 325. 326. 327. 328. 329. 330. 331. 332. 333. 334. 335. 336. 337. 338. 339. 340. 341. 342. 343. 344. 345. 346. 347. 348. 349. 350. 351. 352. 353. 354. 355. 356. 357. 358. 359. 360. 361. 362. 363. 364. 365. 366. 367. 368. 369. 370. 371. 372. 373. 374. 375. 376. 377. 378. 379. 380. 381. 382. 383. 384. 385. 386. 387. 388. 389. 390. 391. 392. 393. 394. 395. 396. 397. 398. 399. 400. 401. 402. 403. 404. 405. 406. 407. 408. 409. 410. 411. 412. 413. 414. 415. 416. 417. 418. 419. 420. 421. 422. 423. 424. 425. 426. 427. 428. 429. 430. 431. 432. 433. 434. 435. 436. 437. 438. 439. 440. 441. 442. 443. 444. 445. 446. 447. 448. 449. 450. 451. 452. 453. 454. 455. 456. 457. 458. 459. 460. 461. 462. 463. 464. 465. 466. 467. 468. 469. 470. 471. 472. 473. 474. 475. 476. 477. 478. 479. 480. 481. 482. 483. 484. 485. 486. 487. 488. 489. 490. 491. 492. 493. 494. 495. 496. 497. 498. 499. 500. 501. 502. 503. 504. 505. 506. 507. 508. 509. 510. 511. 512. 513. 514. 515. 516. 517. 518. 519. 520. 521. 522. 523. 524. 525. 526. 527. 528. 529. 530. 531. 532. 533. 534. 535. 536. 537. 538. 539. 540. 541. 542. 543. 544. 545. 546. 547. 548. 549. 550. 551. 552. 553. 554. 555. 556. 557. 558. 559. 560. 561. 562. 563. 564. 565. 566. 567. 568. 569. 570. 571. 572. 573. 574. 575. 576. 577. 578. 579. 580. 581. 582. 583. 584. 585. 586. 587. 588. 589. 590. 591. 592. 593. 594. 595. 596. 597. 598. 599. 600. 601. 602. 603. 604. 605. 606. 607. 608. 609. 610. 611. 612. 613. 614. 615. 616. 617. 618. 619. 620. 621. 622. 623. 624. 625. 626. 627. 628. 629. 630. 631. 632. 633. 634. 635. 636. 637. 638. 639. 640. 641. 642. 643. 644. 645. 646. 647. 648. 649. 650. 651. 652. 653. 654. 655. 656. 657. 658. 659. 660. 661. 662. 663. 664. 665. 666. 667. 668. 669. 670. 671. 672. 673. 674. 675. 676. 677. 678. 679. 680. 681. 682. 683. 684. 685. 686. 687. 688. 689. 690. 691. 692. 693. 694. 695. 696. 697. 698. 699. 700. 701. 702. 703. 704. 705. 706. 707. 708. 709. 710. 711. 712. 713. 714. 715. 716. 717. 718. 719. 720. 721. 722. 723. 724. 725. 726. 727. 728. 729. 730. 731. 732. 733. 734. 735. 736. 737. 738. 739. 740. 741. 742. 743. 744. 745. 746. 747. 748. 749. 750. 751. 752. 753. 754. 755. 756. 757. 758. 759. 760. 761. 762. 763. 764. 765. 766. 767. 768. 769. 770. 771. 772. 773. 774. 775. 776. 777. 778. 779. 780. 781. 782. 783. 784. 785. 786. 787. 788. 789. 790. 791. 792. 793. 794. 795. 796. 797. 798. 799. 800. 801. 802. 803. 804. 805. 806. 807. 808. 809. 810. 811. 812. 813. 814. 815. 816. 817. 818. 819. 820. 821. 822. 823. 824. 825. 826. 827. 828. 829. 830. 831. 832. 833. 834. 835. 836. 837. 838. 839. 840.

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 二、经济
 三、文化
 四、教育
 五、科技
 六、军事
 七、外交
 八、内政
 九、法律
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 十三、卫生
 十四、环境
 十五、交通
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 十七、农业
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(The following text is extremely faint and appears to be bleed-through from another page. It contains several lines of illegible text.)

doi:10.1017/S0022292412001909

[The page contains dense, illegible handwritten text, likely bleed-through from the reverse side.]

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a) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$; b) $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$; c) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$; d) $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$; e) $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$; f) $\lim_{x \rightarrow 0} \frac{\sin x - x}{x^3} = -\frac{1}{6}$; g) $\lim_{x \rightarrow 0} \frac{x - \arcsin x}{x^3} = \frac{1}{6}$; h) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} = \frac{1}{2}$; i) $\lim_{x \rightarrow 0} \frac{\ln(1+x) - x}{x^2} = -\frac{1}{2}$; j) $\lim_{x \rightarrow 0} \frac{1 - \cos x - \frac{1}{2}x^2}{x^4} = -\frac{1}{24}$; k) $\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5} = \frac{1}{120}$; l) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x - \frac{1}{2}x^2}{x^3} = -\frac{1}{6}$; m) $\lim_{x \rightarrow 0} \frac{\ln(1+x) - x + \frac{1}{2}x^2}{x^3} = \frac{1}{6}$; n) $\lim_{x \rightarrow 0} \frac{1 - \cos x - \frac{1}{2}x^2 + \frac{1}{24}x^4}{x^6} = \frac{1}{720}$; o) $\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{6}x^3 - \frac{1}{120}x^5}{x^7} = -\frac{1}{5040}$; p) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x - \frac{1}{2}x^2 + \frac{1}{6}x^3}{x^4} = \frac{1}{24}$; q) $\lim_{x \rightarrow 0} \frac{\ln(1+x) - x + \frac{1}{2}x^2 - \frac{1}{3}x^3}{x^4} = -\frac{1}{24}$; r) $\lim_{x \rightarrow 0} \frac{1 - \cos x - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6}{x^8} = \frac{1}{5040}$; s) $\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{6}x^3 - \frac{1}{120}x^5 + \frac{1}{5040}x^7}{x^9} = \frac{1}{362880}$; t) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{24}x^4}{x^5} = -\frac{1}{120}$; u) $\lim_{x \rightarrow 0} \frac{\ln(1+x) - x + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{24}x^4}{x^5} = \frac{1}{120}$; v) $\lim_{x \rightarrow 0} \frac{1 - \cos x - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{4200}x^8}{x^{10}} = -\frac{1}{1680}$; w) $\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{6}x^3 - \frac{1}{120}x^5 + \frac{1}{5040}x^7 - \frac{1}{362880}x^9}{x^{11}} = \frac{1}{645120}$; x) $\lim_{x \rightarrow 0} \frac{e^x - 1 - x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{24}x^4 + \frac{1}{120}x^5}{x^6} = \frac{1}{720}$; y) $\lim_{x \rightarrow 0} \frac{\ln(1+x) - x + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{24}x^4 - \frac{1}{120}x^5}{x^6} = -\frac{1}{720}$; z) $\lim_{x \rightarrow 0} \frac{1 - \cos x - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + \frac{1}{4200}x^8 - \frac{1}{362880}x^{10}}{x^{12}} = \frac{1}{1209600}$.

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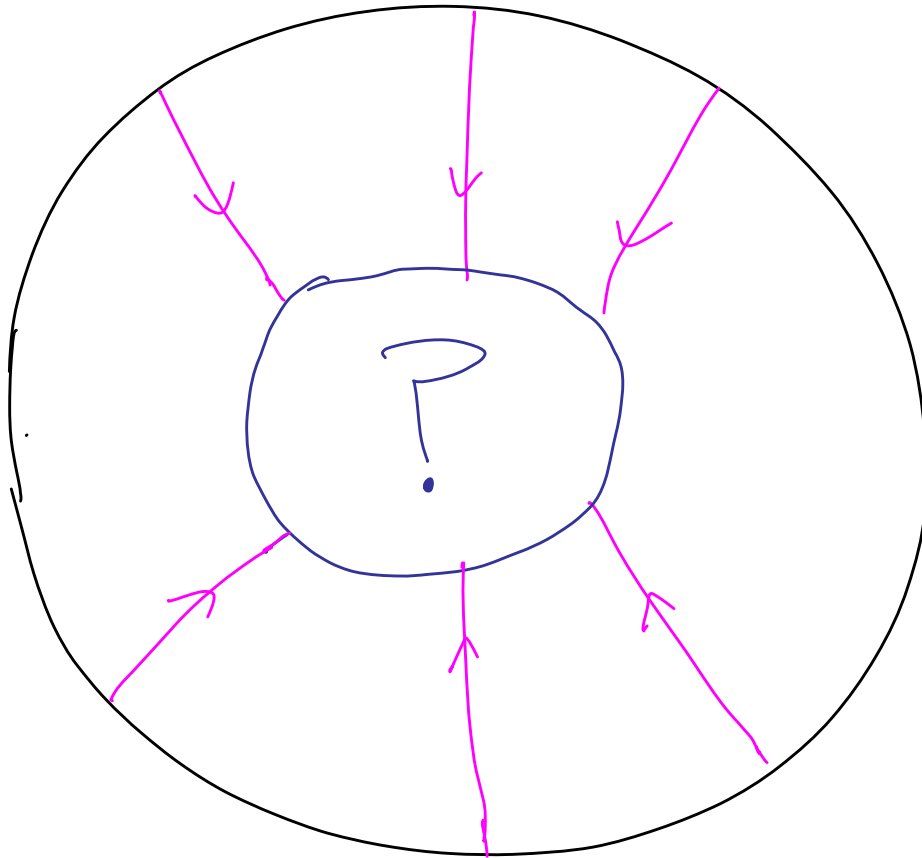
Pages

$$k_1 \cdot k_4 \varepsilon_2 \cdot k_1 \varepsilon_4 \cdot \varepsilon_3 \varepsilon_4 \cdot \varepsilon_5$$

$$(1^- 2^+ 3^- 4^+ 5^+ 6^+)$$

$$\frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 56 \rangle \langle 61 \rangle} (!)$$

Standard way of doing physics makes
usual rules of spacetime + QM
manifest — but is obviously hiding
some extraordinary new structures!



For ~ 10 years
(+ more intensely
in last 2 years)
we are seeing

Combinatorial Positive
Geometries in
Kinematic Space

The Canvas

* Physical momenta

* "Twistor" variables

* "Celestial Sphere"

* Spatial Future

Kin. Space
 $\sim \mathcal{D}$ Minkowski

(Amps)

(ψ_{Univ})

What ideas
breathe
physics-life
into this
space?

The Canvas

| | |
|-----------------------|--|
| * Physical momenta | Kin. Space
$\sim \mathcal{D}$ Minkowski |
| * "Twistor" variables | (Amps) |
| * "Celestial Sphere" | |
| ⋮ | |
| * Spatial Future | (ψ_{Univ}) |

Emerging Picture:

Combinatorics

↕
Positive Geometries

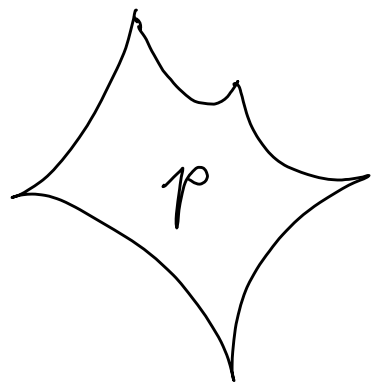
↕
Canonical Forms

* Amplituhedra, Associahedra, Cosmological Polytopes, ...

* Describes REAL WORLD PHYSICS in reasonable approximation, exposes hidden symmetries + new mathematical structures

* Pure magic in physics right under our nose

Positive Geometries



Region $\mathcal{P}$ w/ boundaries
of all Codimension

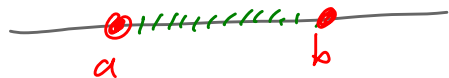
Real Geometry



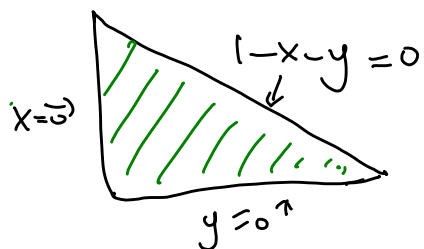
Canonical Forms

$\Omega_{\mathcal{P}}$: unique form with
logarithmic singularities
on (+ only on) boundaries
of $\mathcal{P}$ $\{ \text{locally } \Omega \rightarrow \frac{dx_1}{x_1} \dots \frac{dx_p}{x_p} \}$

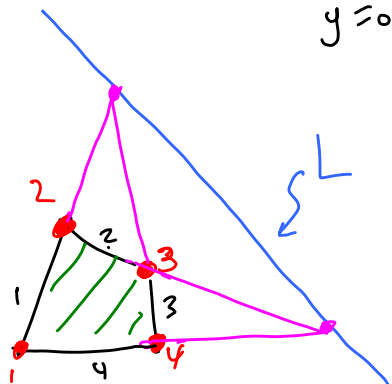
Complex Form



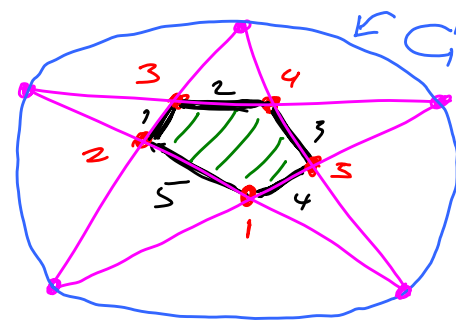
$$\Omega = \frac{dx}{x-a} - \frac{dx}{x-b}$$



$$\Omega = \frac{dx dy}{x y (1-x-y)}$$



$$\Omega = \frac{dx dy}{L_1 L_2 L_3 L_4} \quad \text{L}$$

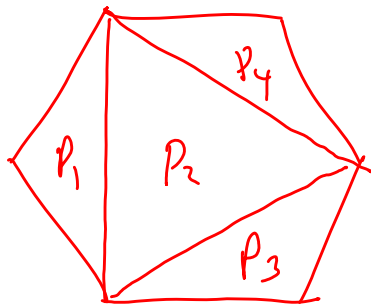


$$\Omega = \frac{dx dy}{L_1 L_2 L_3 L_4 L_5} \quad \text{C}$$

Singularities of $\Omega \longleftrightarrow$

Boundaries/Faces
of P

Triangulation



P tiled by P_i



$$\Omega_P = \sum_i \Omega_{P_i}$$

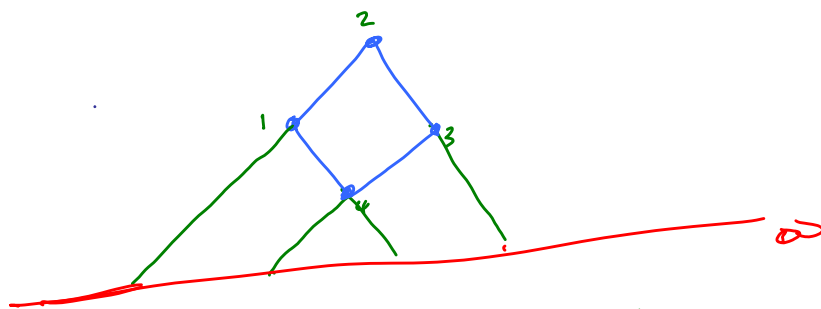
Spurious poles
cancel in sum

Ex: "Simple Polytopes"

$$\Omega_{P_i}$$

$\sum_{\text{vertices}}$

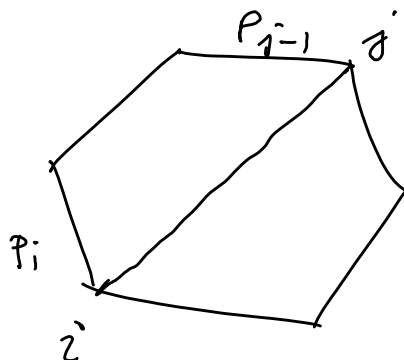
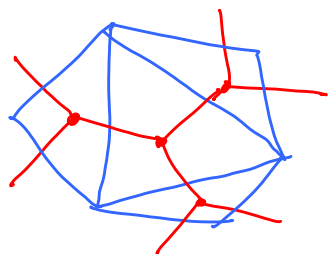
$$\text{sgn}(X_{a_1} \cdots X_{a_n}) d \log X_{a_1} \cdots d \log X_{a_n}$$



Spurious Poles @ ∞ cancel in sum.

$$\underline{\phi^3_{B-A}}$$

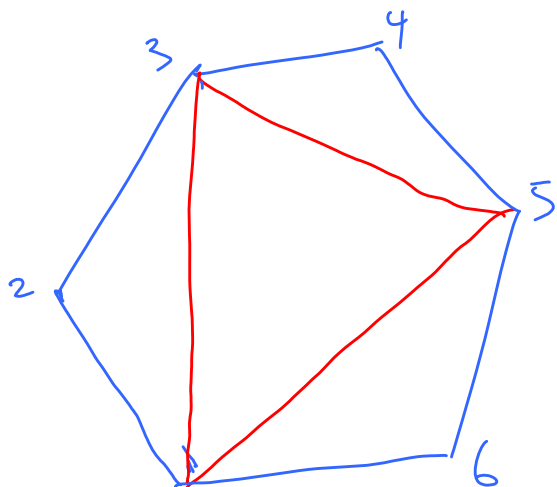
* Tree-level



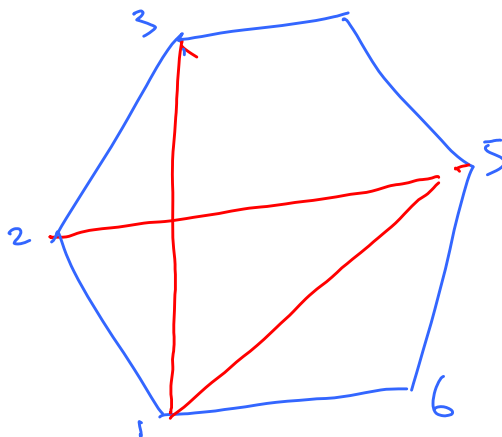
$$X_{ij} = (p_i + \dots + p_{j-1})^2 = X_{ji}$$

$$[X_{ii+1} = 0]$$

$$A^{\text{tree}} = \sum_{\text{Triang. } T} \prod_{ij \in T} \frac{1}{X_{ij}}$$

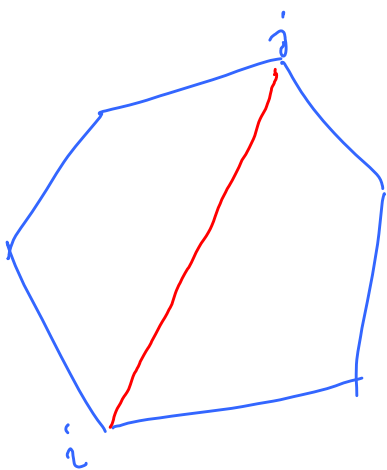


Allowed

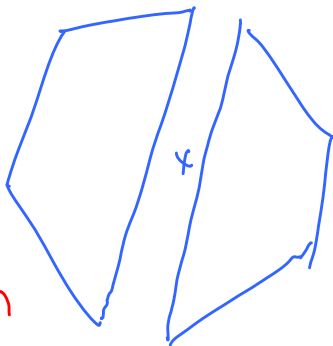


Crossing Chords not allowed.

Locality



$x_{ij} \rightarrow 0$
Factorization



Unitarity

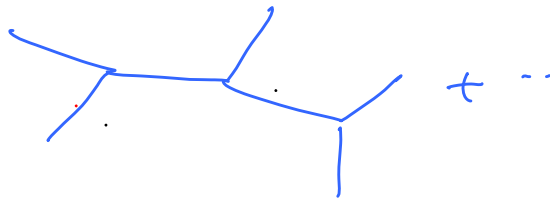
Big Clue Seen By $\mathcal{N}=4$ Theorist Experimentalist

Amplitude has poles when $(\sum p)^2 \rightarrow 0$, + Factorizes!



F

"It's



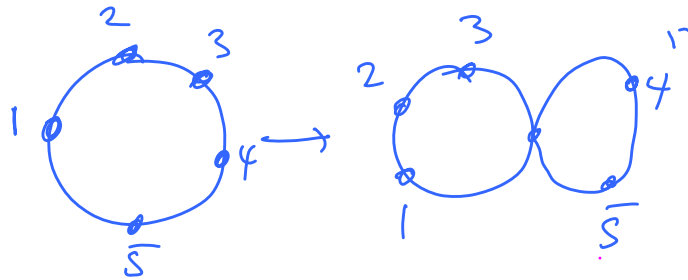
"

Particles in
Spacetime

D

"

"It's



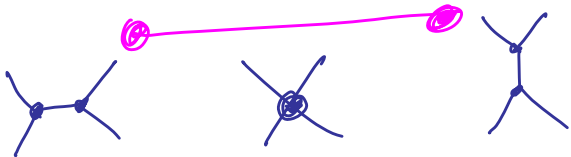
String Worksheet

Bigger Clue Seen By Lazier but Smarter Experm.

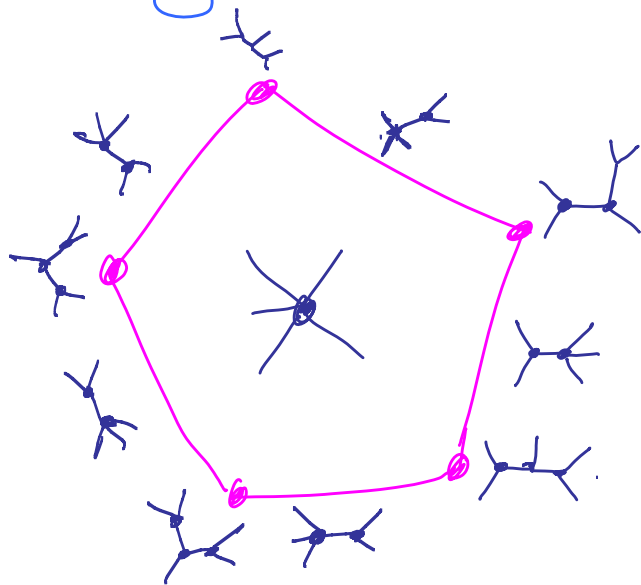
"Only certain patterns of poles come together!"

$n=4$ s, t but not s, t

$n=5$ s_{12}, s_{123} + cyclic but not other comb.



$n=4$

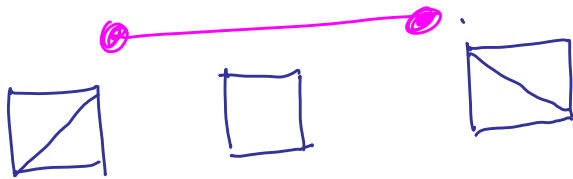


Bigger Clue Seen By Lazier but Smarter Experm.

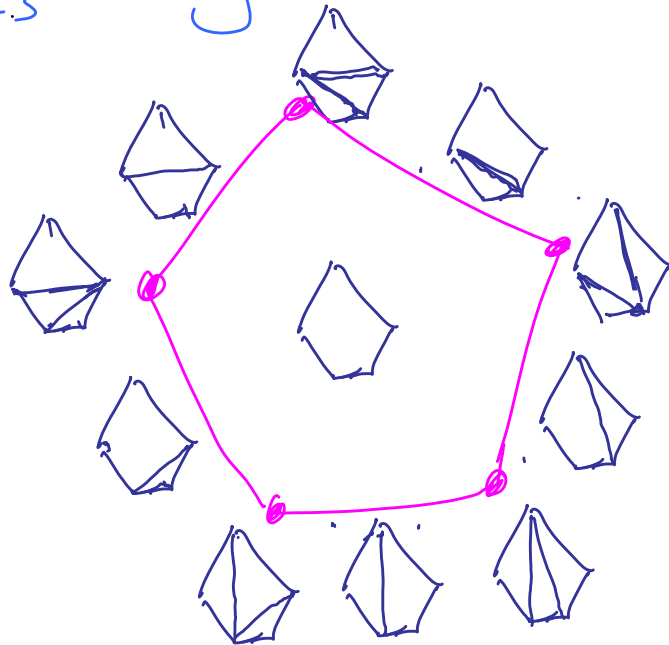
"Only certain patterns of poles come together!"

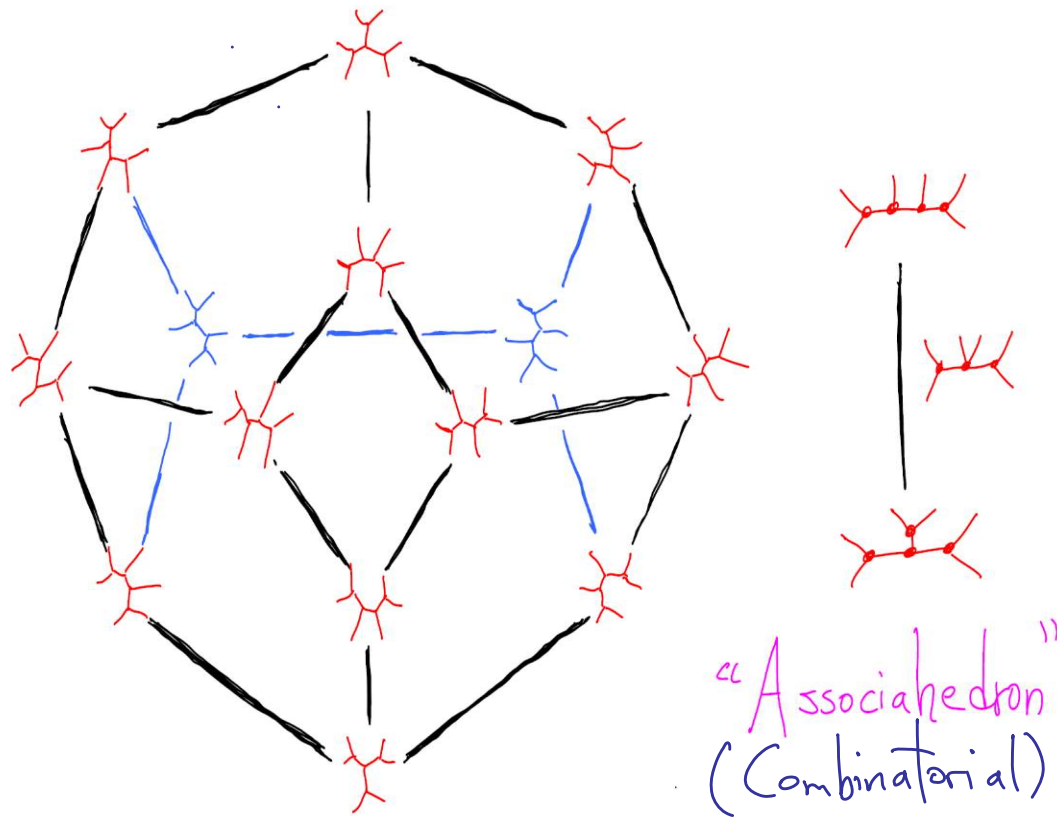
$n=4$ s, t but not s, t

$n=5$ s_{12}, s_{123} + cyclic but not other comb.



$n=4$





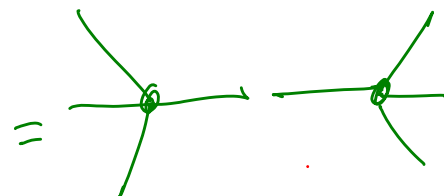
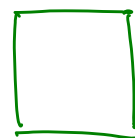
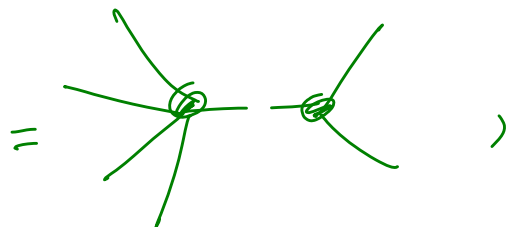
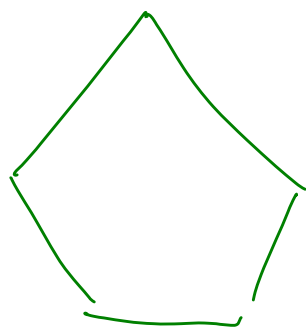
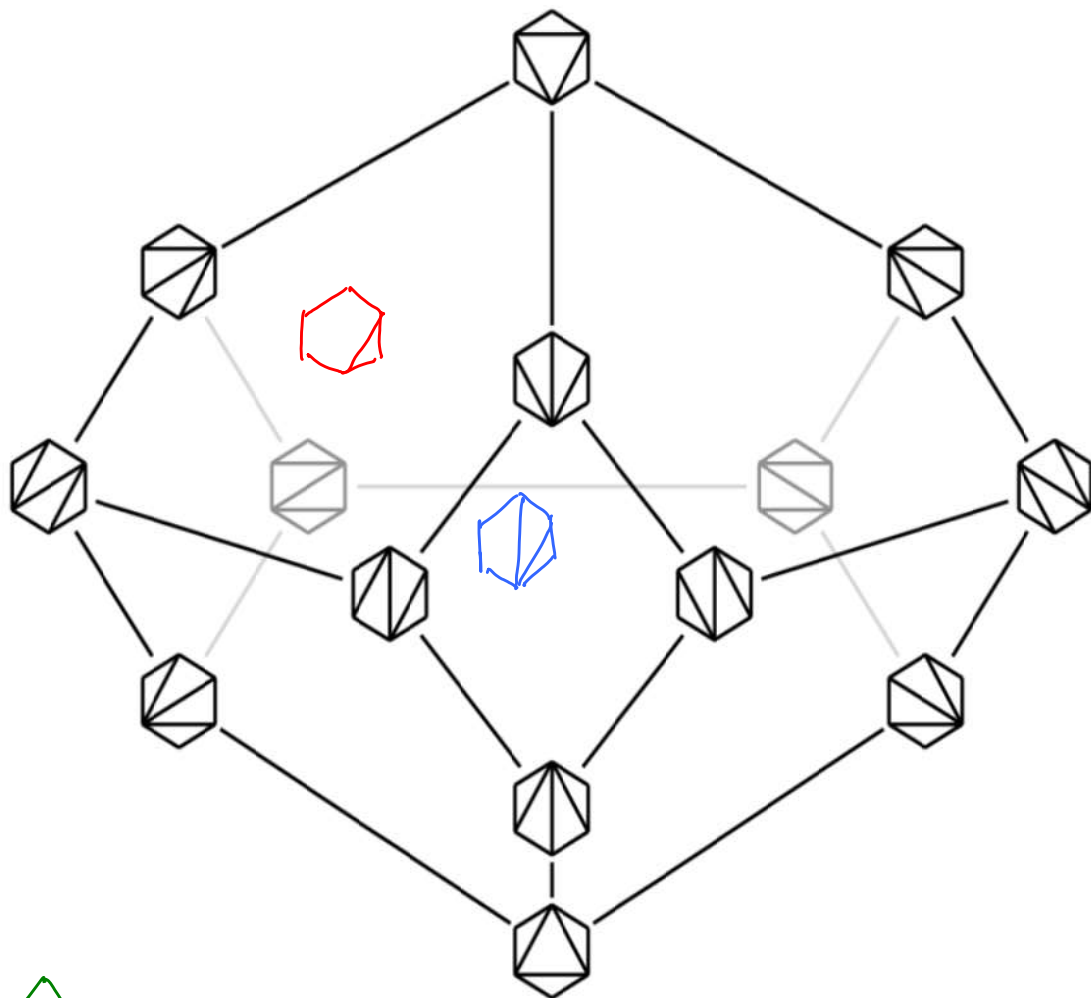
Particle + String
picture don't
make this
obvious!

But Assoc.
does see
everything
since

$$\partial A_n = A_{n_1} \times A_{n_2}$$

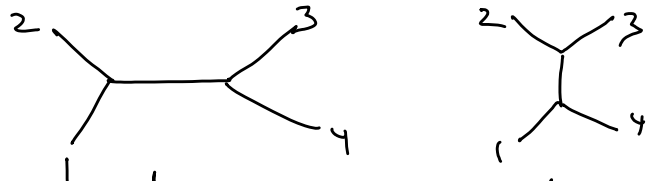
HOW CAN WE MAKE THIS OBVIOUS?

A Qualitative Clue to the "theory"
in Kinematic Space



Hidden Symmetry: Projective Invariance

$n=4$



$$\frac{dX_{13}}{X_{13}} - \frac{dX_{24}}{X_{24}} = d \log X_{13}/X_{24}.$$

General n : invariance of $\Omega_{(n-3)}$ under $X_{ij} \rightarrow \alpha(x) X_{ij}$

$\Rightarrow d \log X_{ij} \rightarrow d \log X_{ij} + d \log \alpha \rightarrow$ for each set

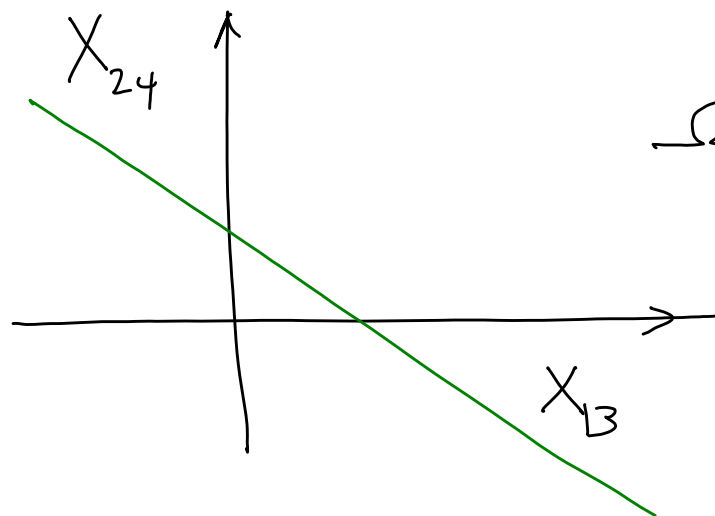
of $(n-4)$ comp. propagators, coeff. of $d \log \alpha$ vanishes.

Many more eqns than unknown signs.

Remarkably: signs can be chosen so $\Omega_{(n-3)}$ is projective!

[Analog of Dual Conf Inv. in $N=4$ SYM].

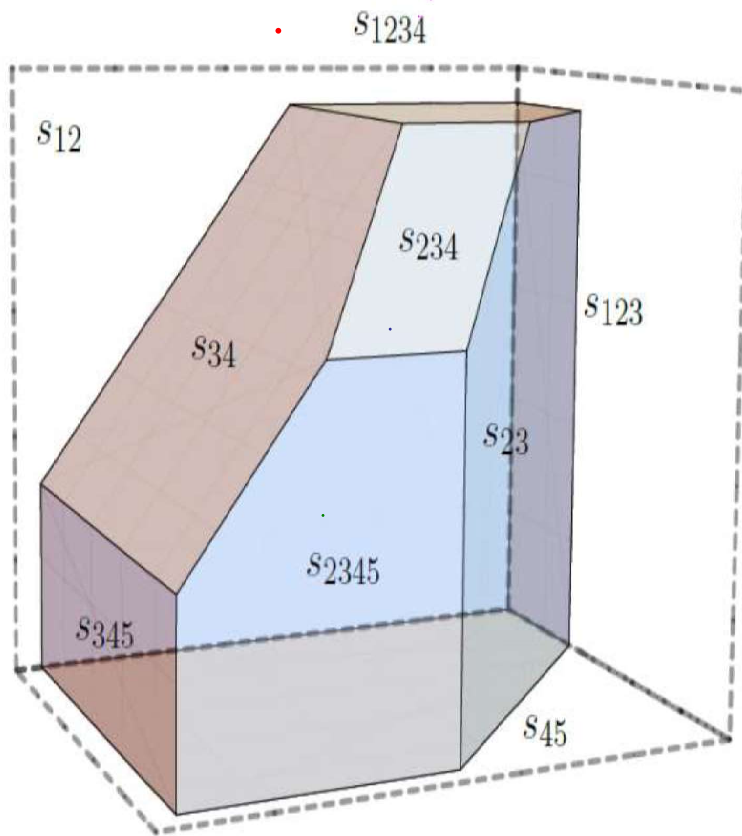
Pull back to Amplitude



$$X_{13} + X_{24} = c$$

$$|\Omega| = dX_{13} \left(\frac{1}{X_{13}} + \frac{1}{X_{24}} \right)$$

General n : $|\Omega|_{ABHY} = d^{(n-3)} X \cdot (\text{Amplitude})$

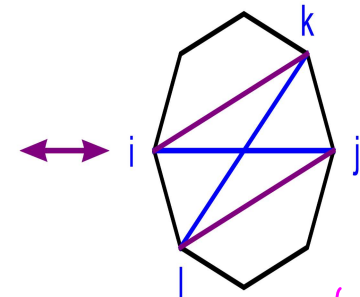
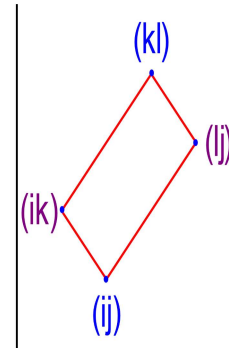
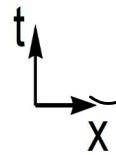
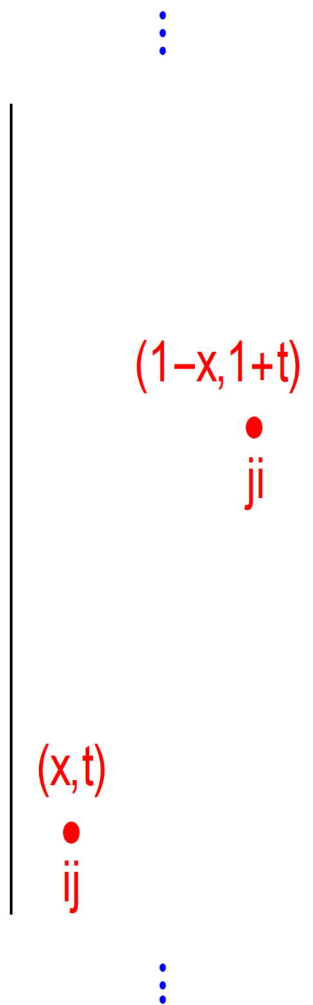
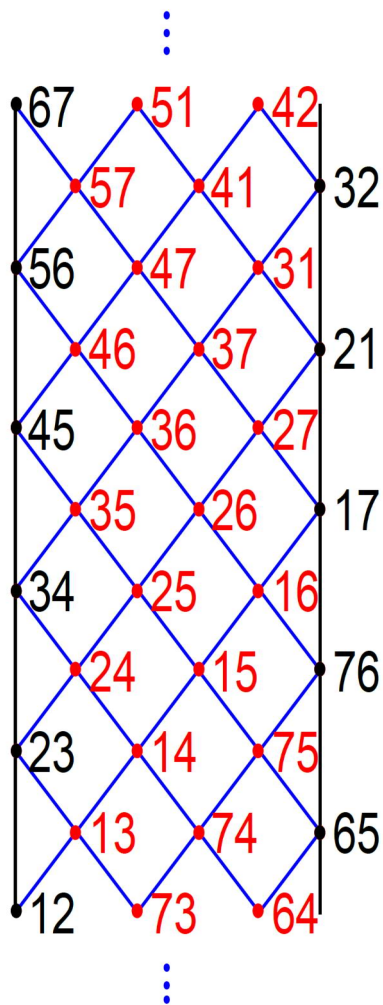


"ABHY"
 Associahedron
 in Kinematic
 Space

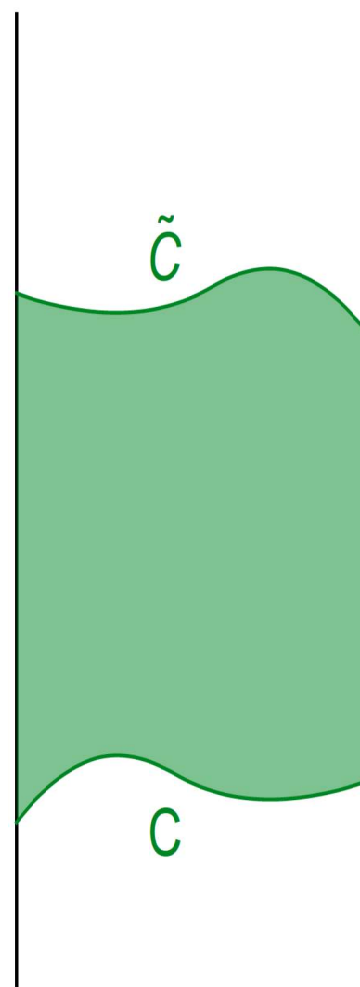
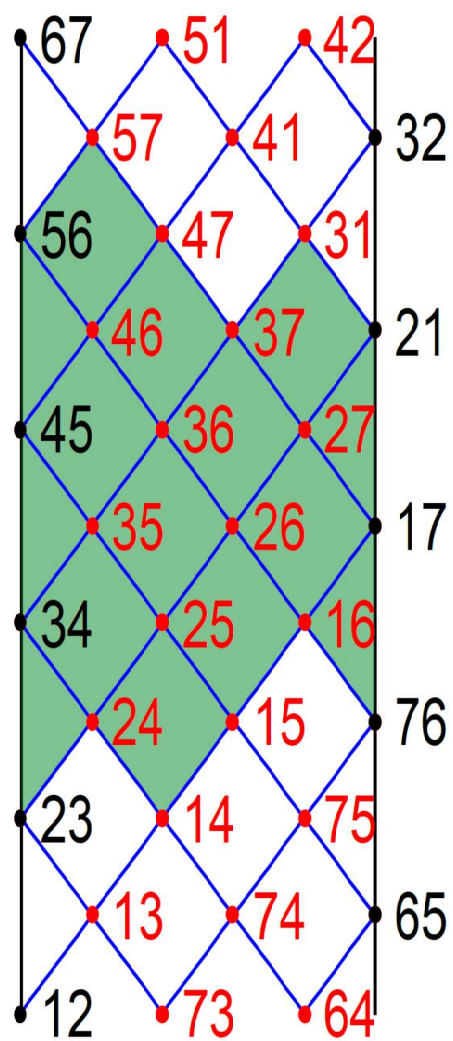
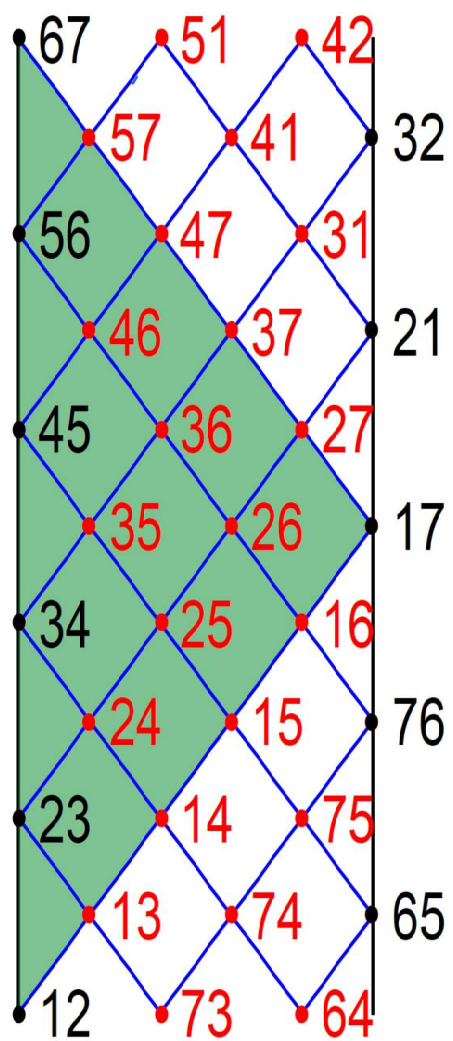
↓
 "Cluster Algebra"
 Polytopes
 Associated with
 Dynkin Diagrams

HOW CAN WE MAKE THIS OBVIOUS?
 A Qualitative Clue to the "theory"
 in Kinematic Space

Kinematic "Spacetime"



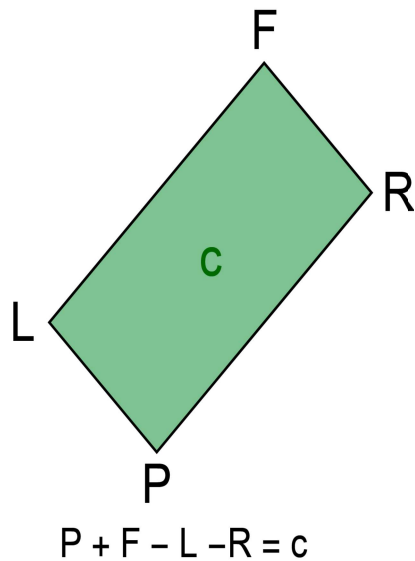
P, F corners $\longleftrightarrow$ incompatible chords
of Causal Diamond



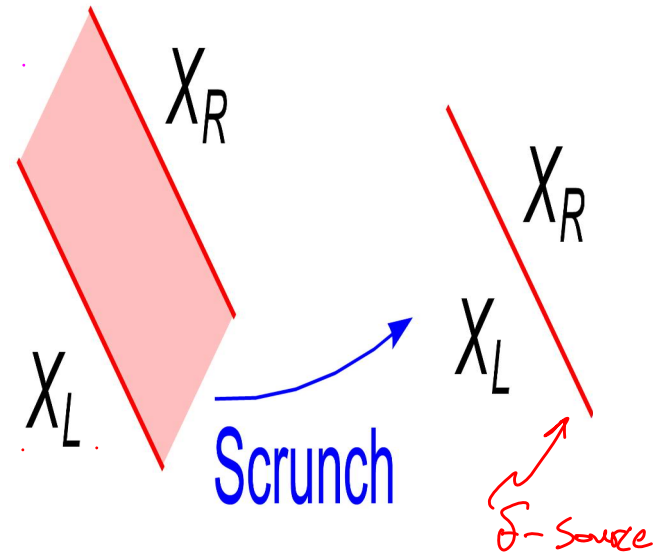
Wave Equation in Kin. "Spacetime"

$$\partial_u \partial_v X = c$$

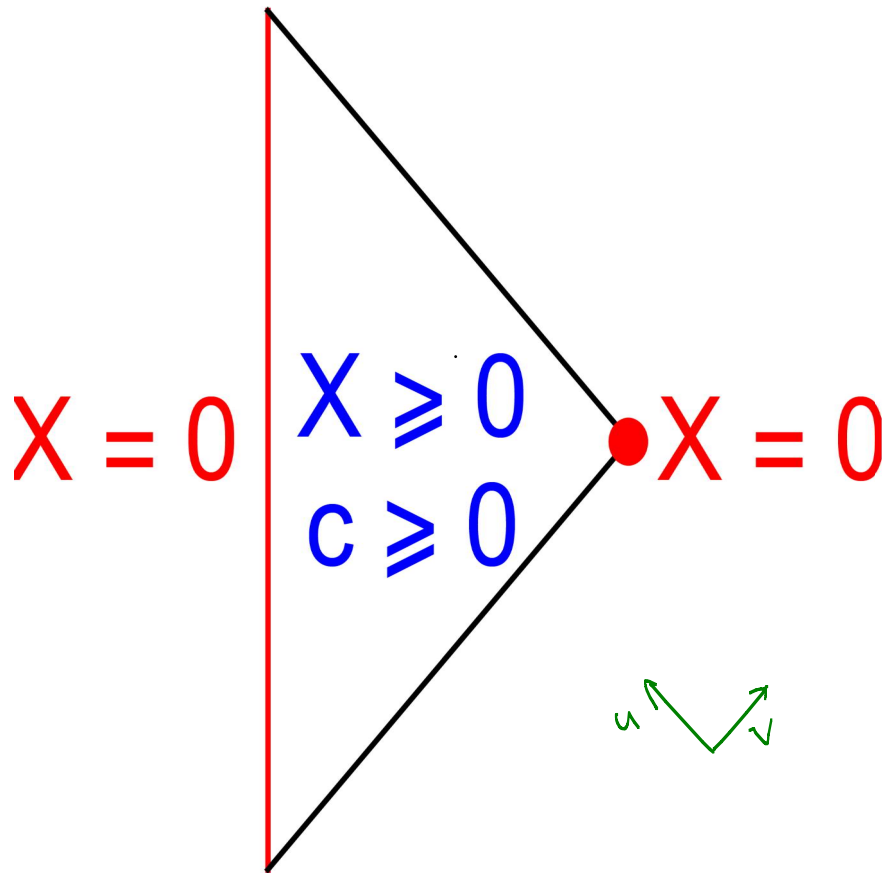
"Gauss Law"



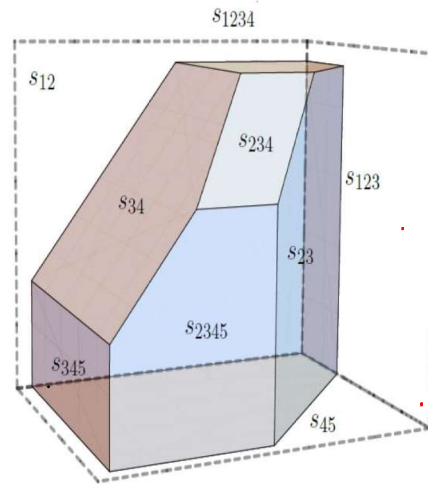
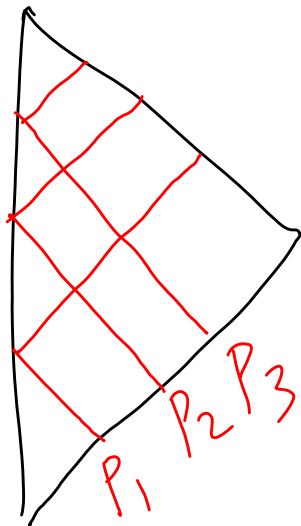
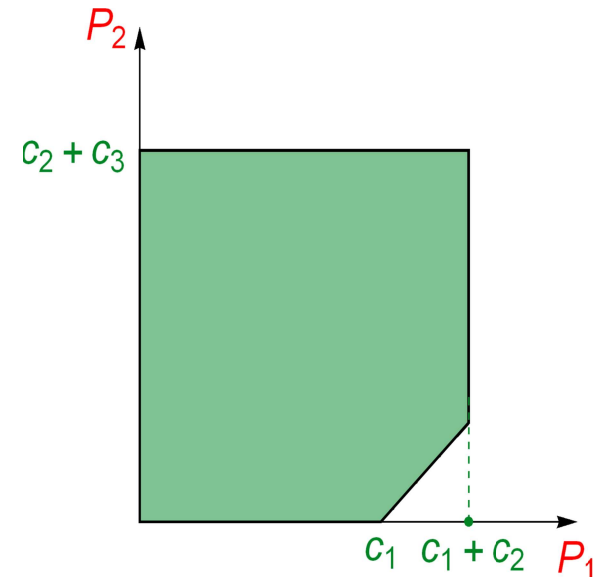
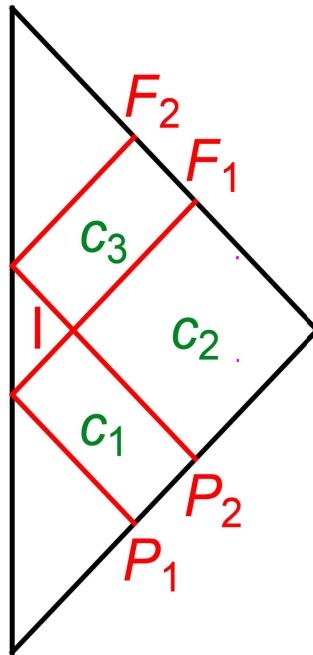
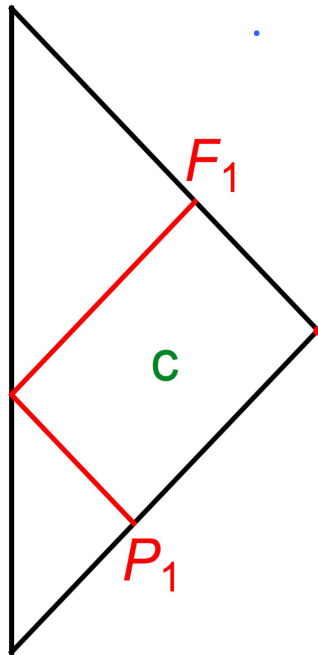
"Scrunch rule"



Positivity

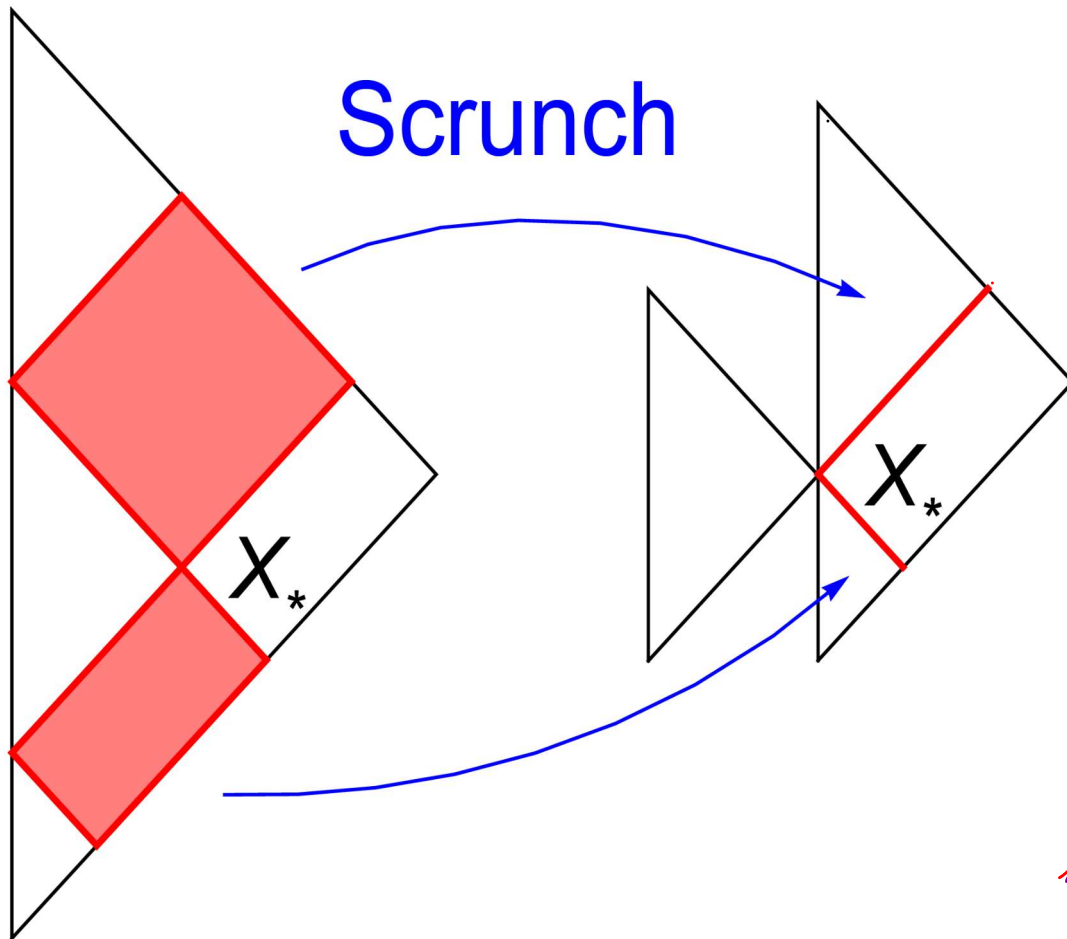


$$\partial_u \partial_v X = c$$

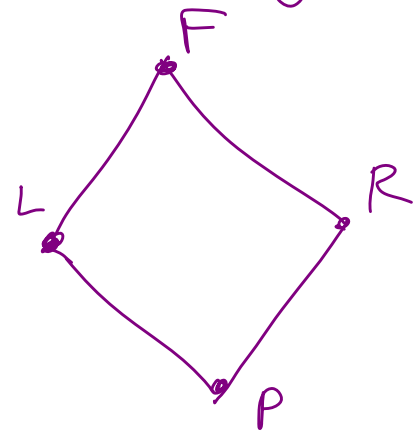


ABHT
Associations

Factorization!



Crucial point:

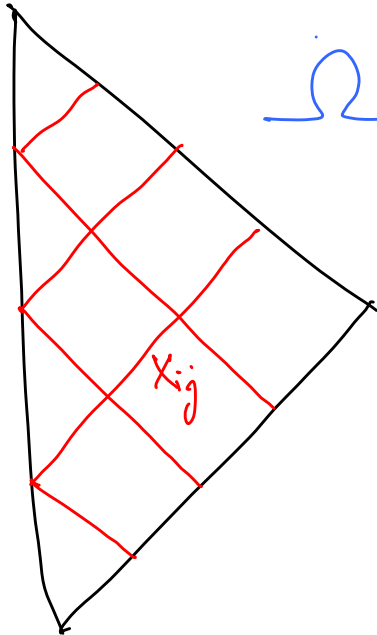


can't put $P=0, F=0$.

$$\cancel{P+F} - L - R = c$$

neg = pos X

Scattering Form as Canonical Form



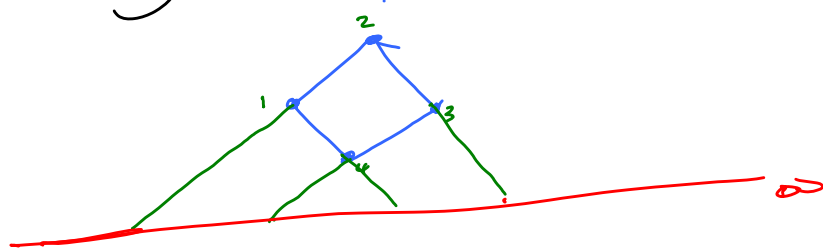
Ω fixed by : $\Omega \Big|_{\substack{\text{Wave} \\ \text{Eqn}}} = \omega^{\text{can}} [\text{Assoc}]$

* Trivially Projective Invariant

* Since $\det[g_1, \dots, g_{n-3}]$ of g -vectors
in a cluster cone $= \pm 1$,

$$\Omega \Big|_{WE} = \omega^{\text{can}} [\text{Assoc}] = (dVol) \times \text{Amplitude!}$$

* Feynman diagrams are one particular triangulation :

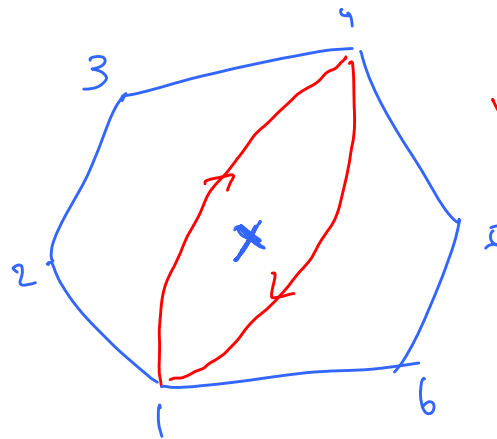
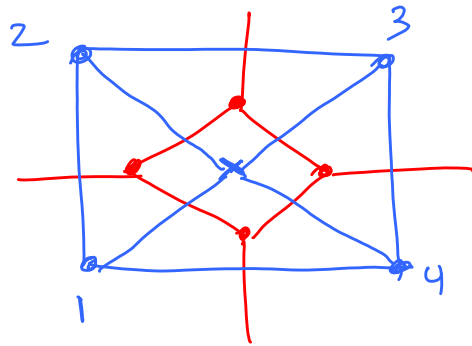


Spurious poles @
 ∞ break proj.
invariance term-
by-term

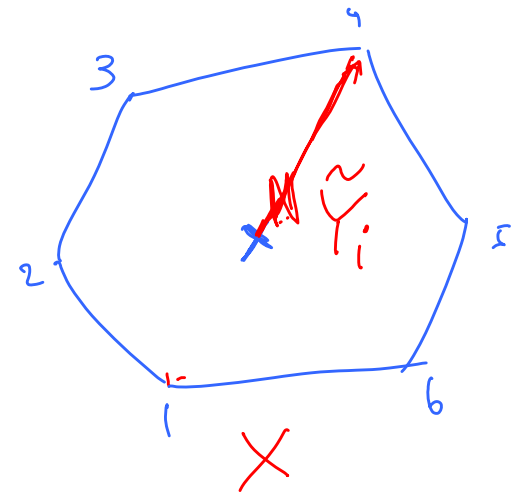
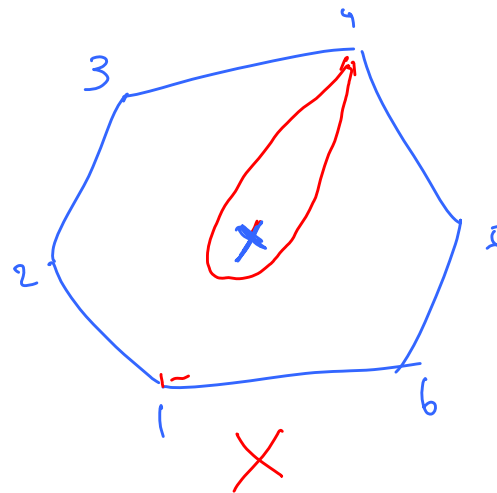
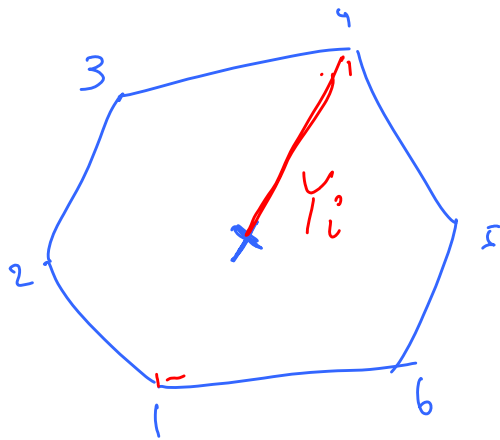
* Many other triangulations! Manifest proj.
invariance term-by-term, vastly more efficient
than Feynman Diagrams

* (Another general method for computing can.
form, in this case, directly gives string amplitudes!)

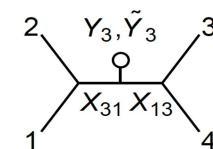
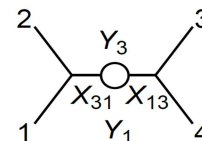
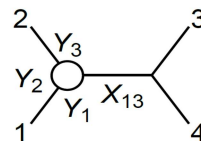
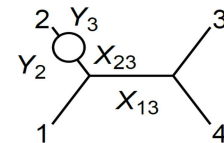
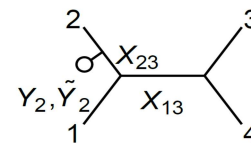
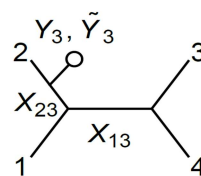
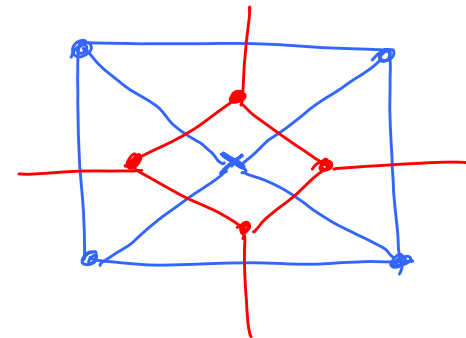
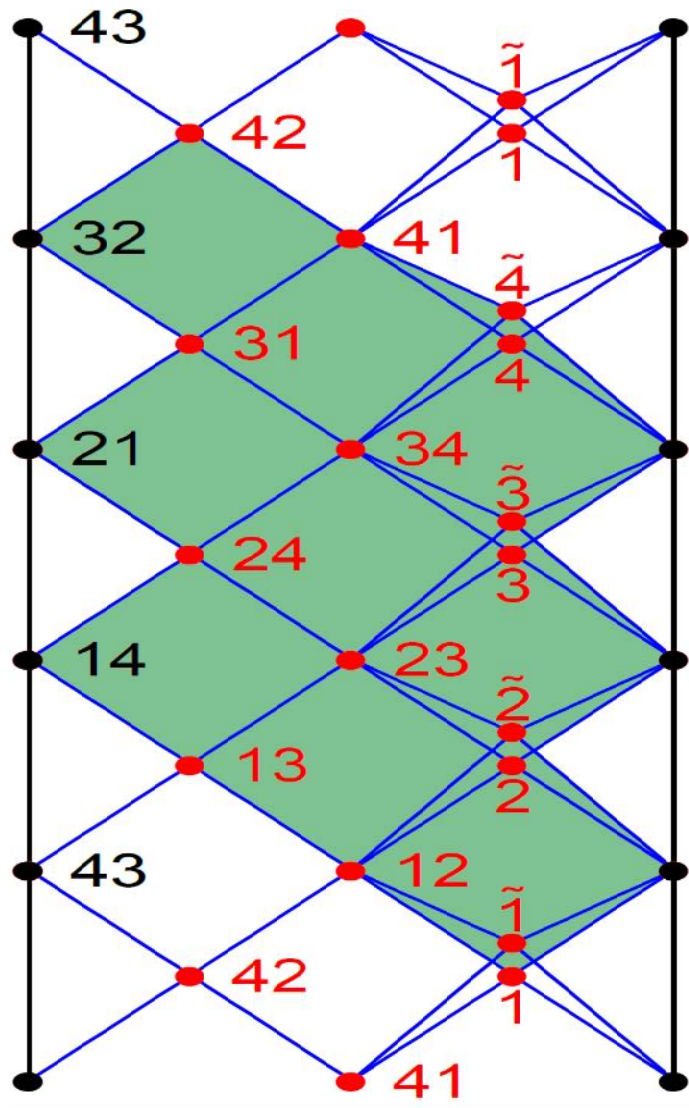
Variables for 1-loop Integrand



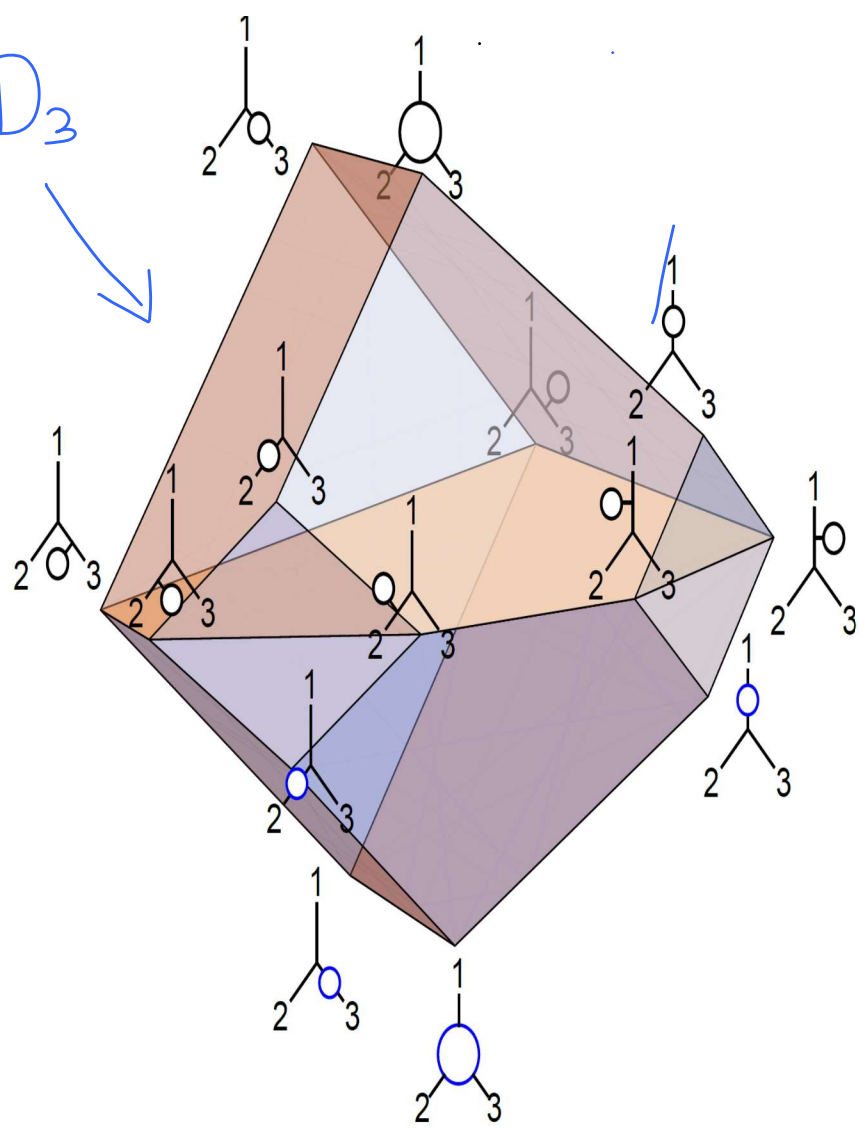
$$X_{ij} \neq X_{ji}$$



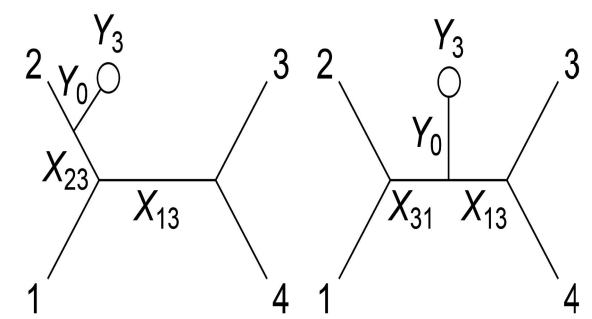
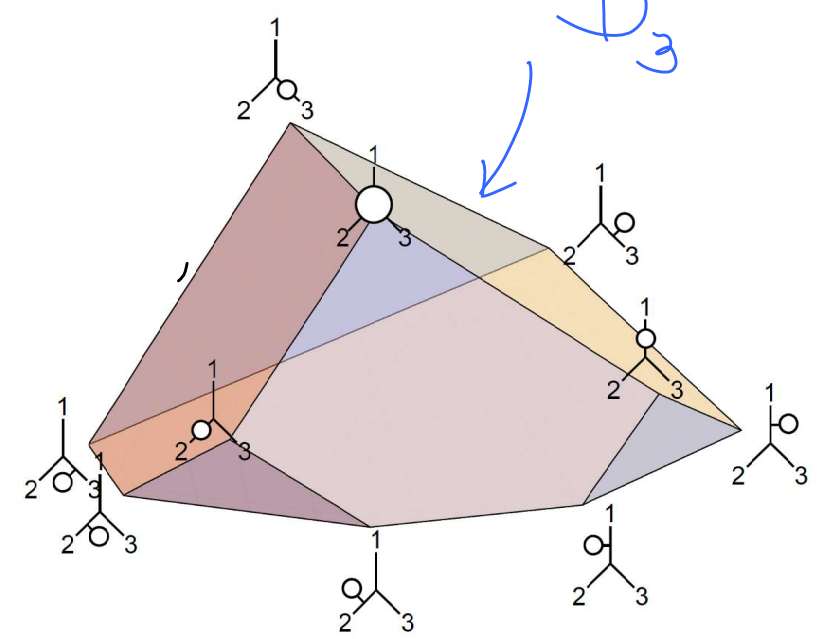
Kinematic "Spacetime" @ $l-\text{loop}$



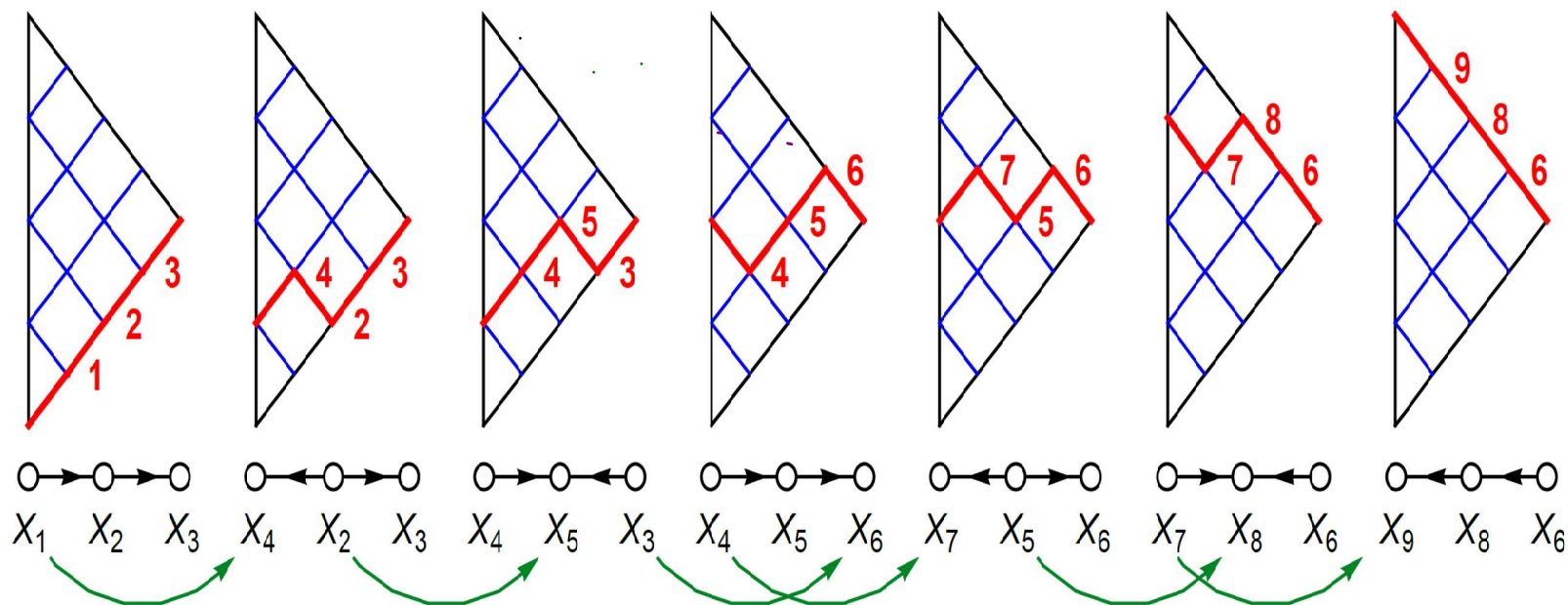
D_3



D_3



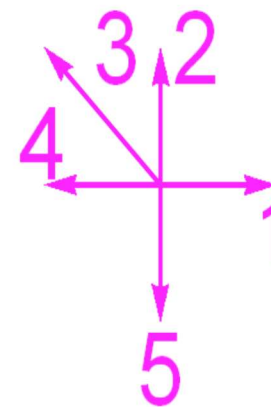
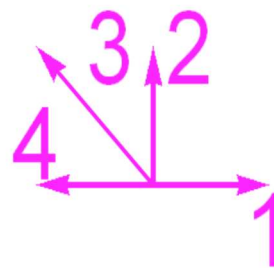
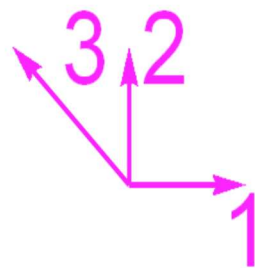
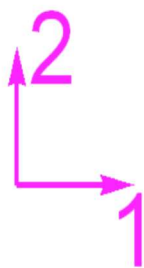
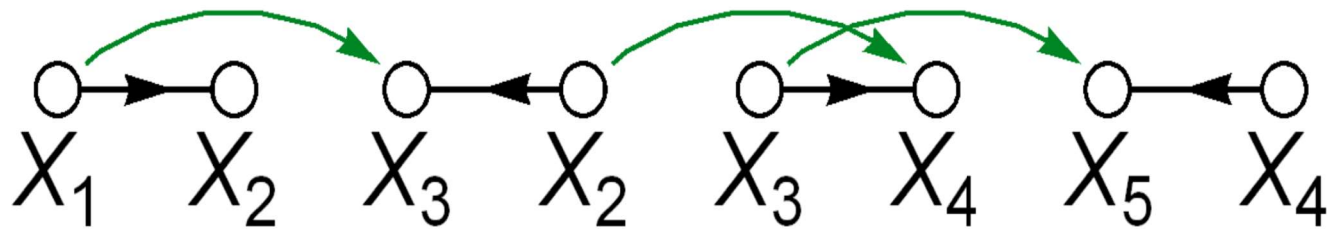
"Time Evolution" in Kinematic Space



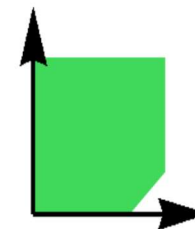
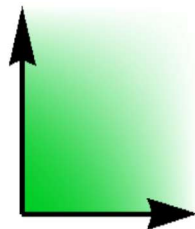
"Walk" by mutating only on sources

$$X_v + X'_v - \sum_{v \rightarrow w} X_w = c_v$$

(~ "tropicalized mutation")

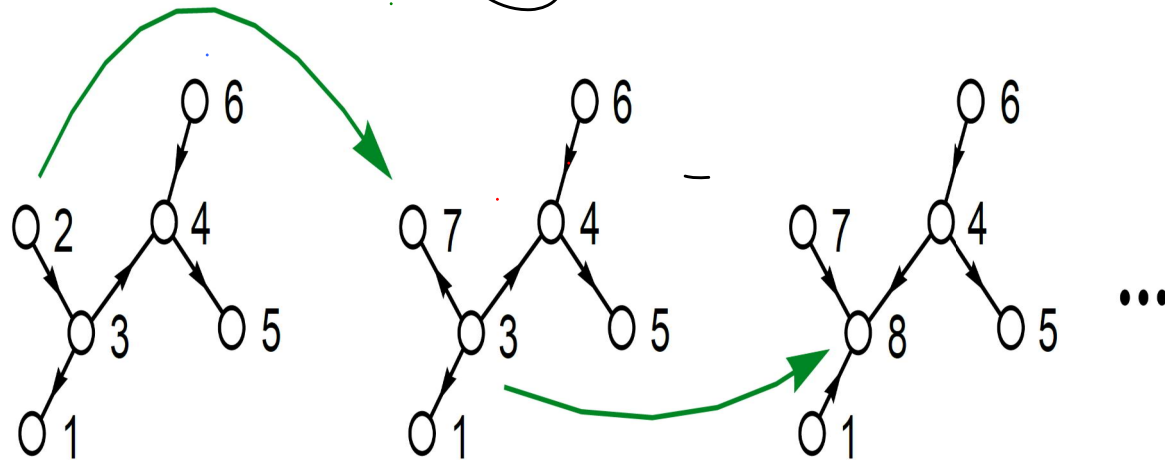


STOP



STOP

Abstract Away "Time Evolution"



$$X_v + X'_v - \sum_{v \rightarrow w} X_w = C_v$$

(Auslander-Reiten Walk)

Can Define For Any Tree Quiver

$$Q \xrightarrow[\text{@ } i]{\text{mutate}} Q' \implies X' = A_i X$$

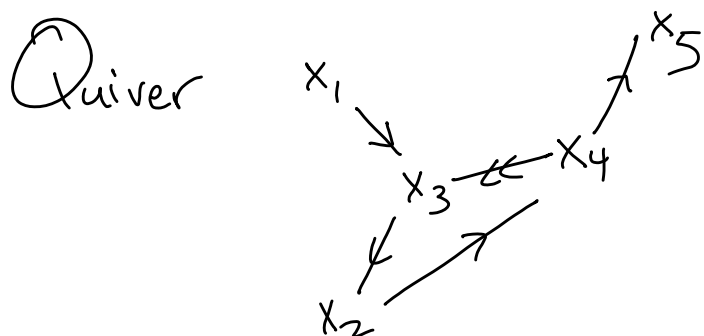
A_i generators of Coxeter Group of Q

ABHY Polytope Finite $\longrightarrow$ Coxeter Group Finite

$\longrightarrow$ Dynkin Classification $\longrightarrow$ ABHY Real.

of all Finite Cluster Algebra Polytopes

Some Cluster Algebra Basics



• "Mutation" @ v

1) Reverse all arrow

2) If $a \rightarrow v \rightarrow b$ add $a \rightarrow b$.

3) Delete 

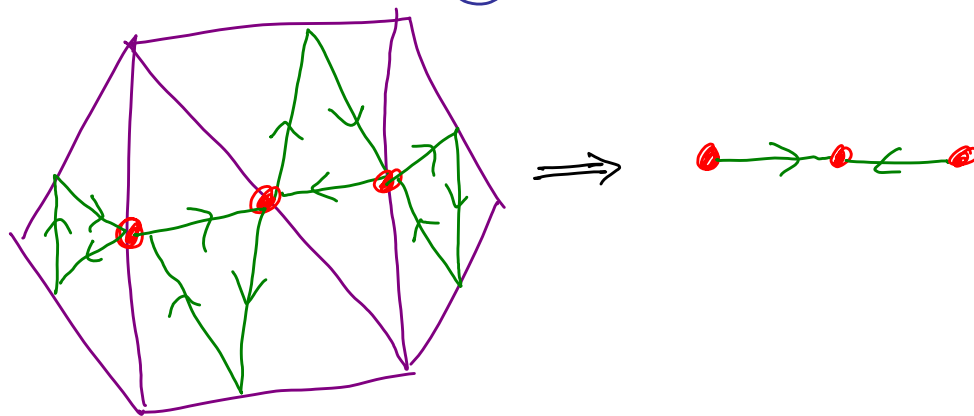
$$\bullet x_v x_{v'} = \prod_{w \rightarrow v} x_w + \prod_{w \leftarrow v} x_w$$

• Also "tropicalized", pair:

$$\left\{ \begin{aligned} &X_v + X_{v'} - \sum_{w \leftarrow v} X_w, \\ &X_v + X_{v'} - \sum_{w \rightarrow v} X_w \end{aligned} \right\}$$

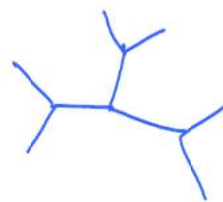
Surface Cluster Algebras

* Quivers from triangulations of surface



* Mutation $\longleftrightarrow$ Skein relations: $X = \left(\begin{array}{c} \text{arc} \\ \text{arc} \end{array} \right) + \left(\begin{array}{c} \text{arc} \\ \text{arc} \end{array} \right)$

A_n

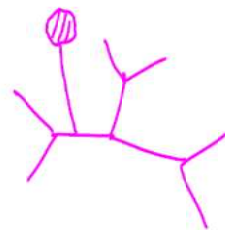


"tree"

B_n



}

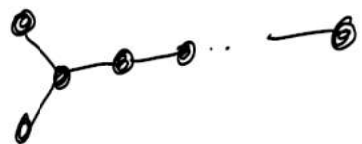


"tadpole"

C_n



D_n

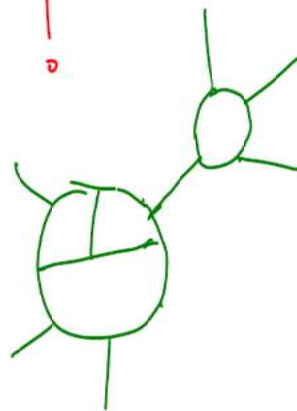


"loop"

Exceptional Finite



?

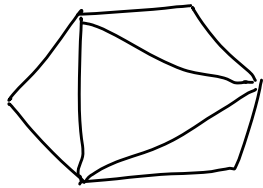


Infinite type



(Cluster Algebra of Punctured Surfaces)

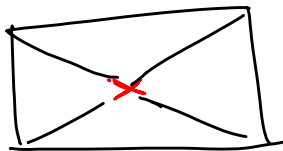
All-loop, all orders in $1/N$ Φ_{BA}^3



trees $\longrightarrow$



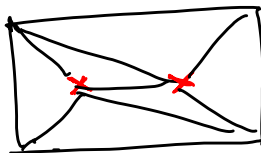
A_{n-3}



1-loop $\longrightarrow$



D_n

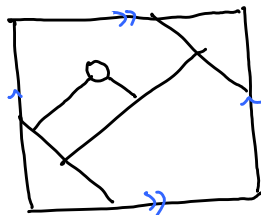


2-loop $\longrightarrow$



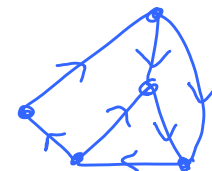
$\tilde{D}_{n+2}$

"Infinite Type"



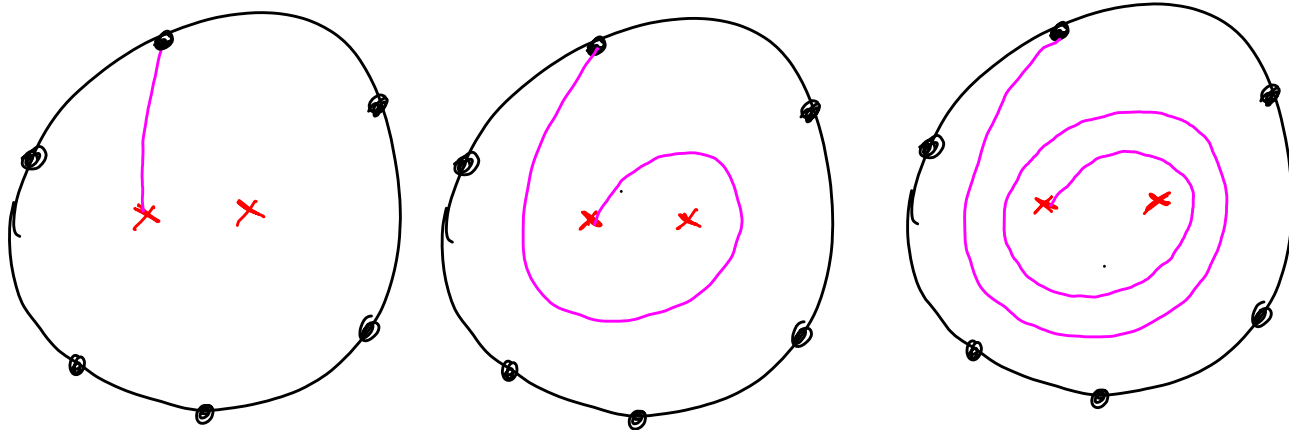
2-loop, leading $1/N$ [contours]

$\longrightarrow$



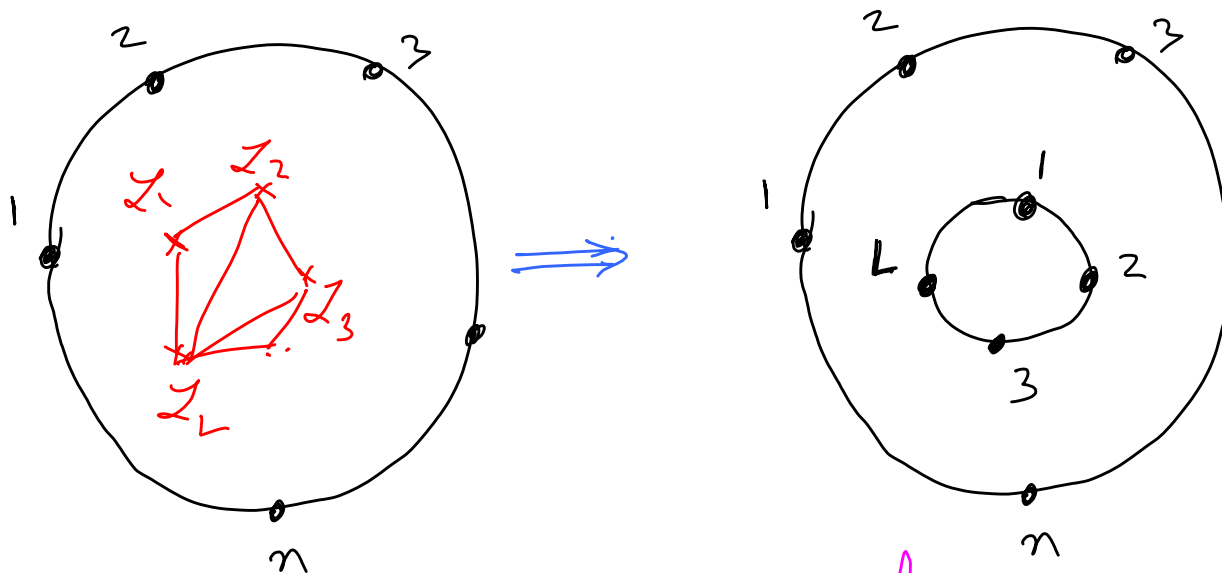
Seeking "Clusterhedra"

Infinite Winding



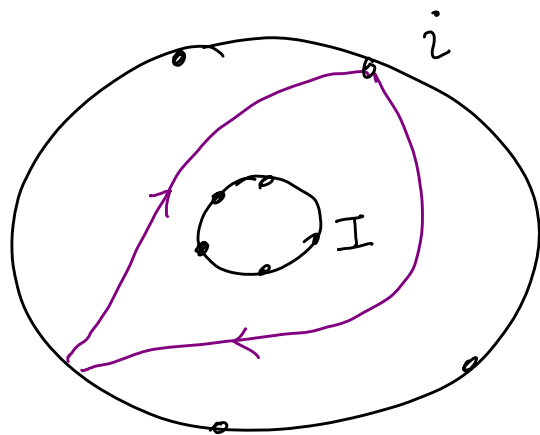
* Infinite action of Mapping class groups,
(Braid group on L strands)

Class of All-loop Cuts

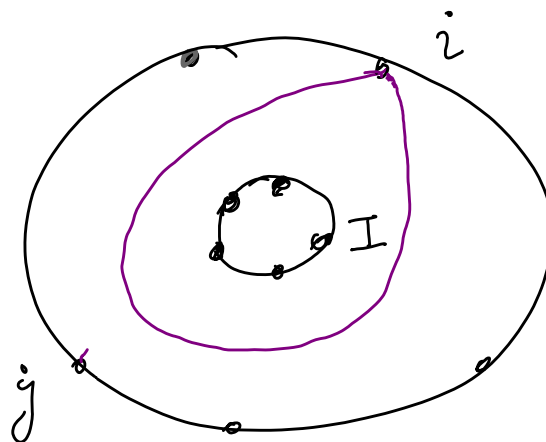


$A_{n,L}$

Variables for $A_{n,L}$



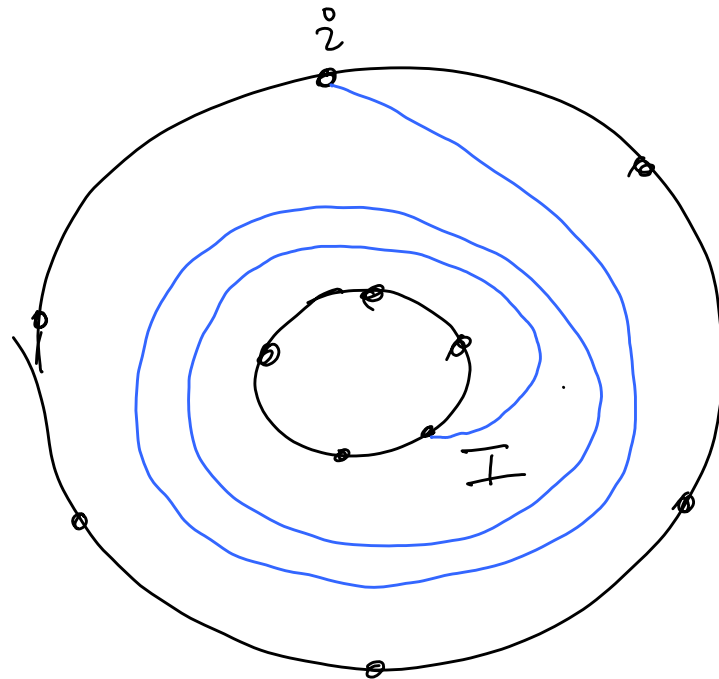
$$X_{ij} \neq X_{ji}$$



$$X_{i \pm n}$$

"Out/Out" or " I_n/I_n "

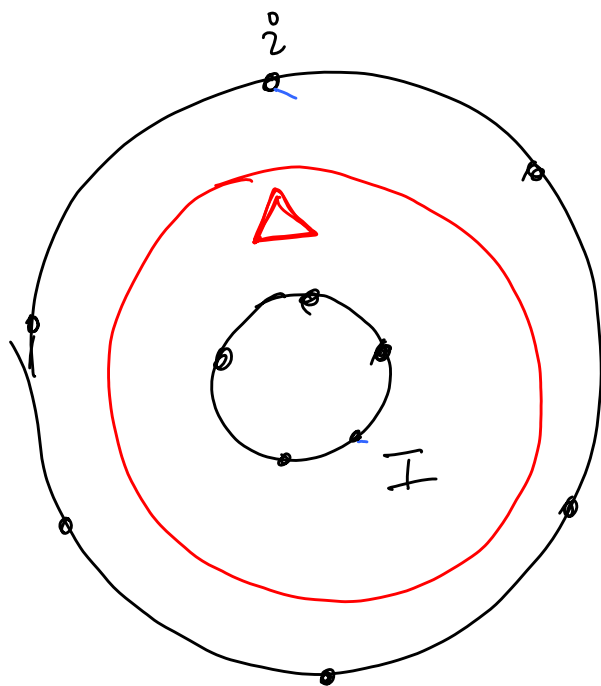
Variables for $A_{n,L}$



$\times^{\omega}$
 iI

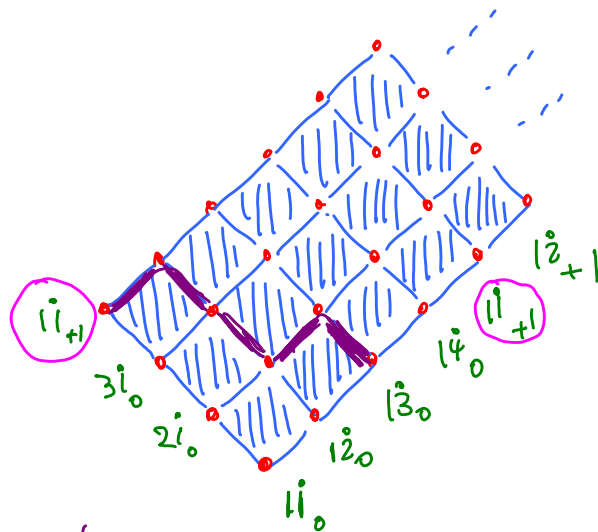
" $I_n - Out$ "

Variables for $A_{n,L}$



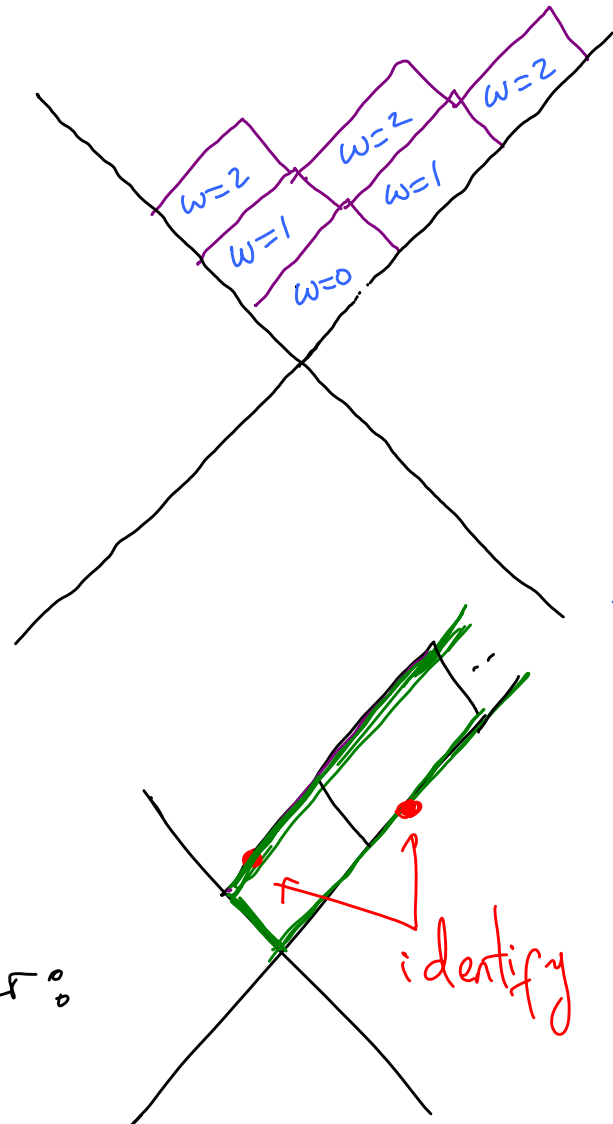
Not a variable
we would see
in triangulation/
dual of Feyn. Diag.

In-Out "Kinematical Spacetime"

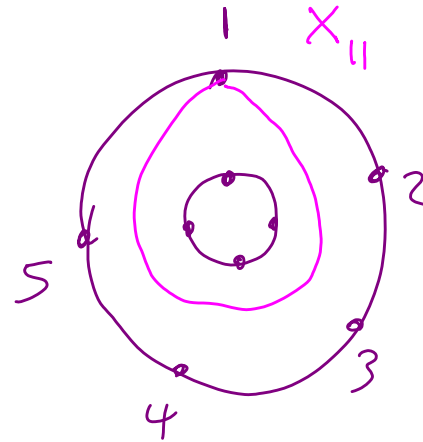
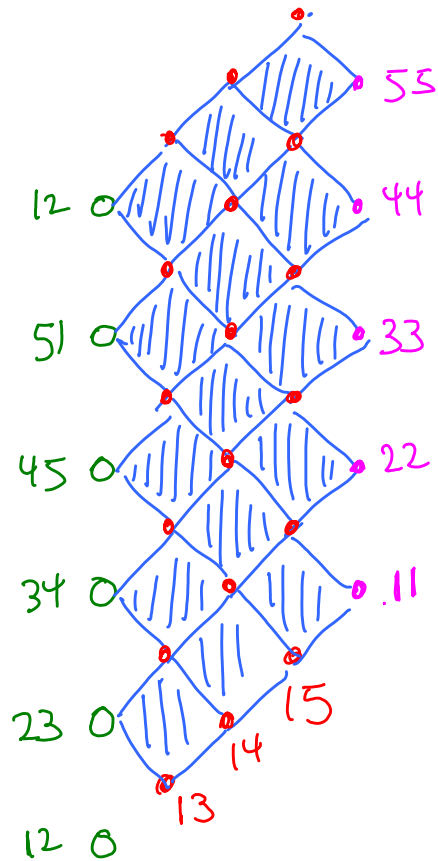


Initial "Cauchy Surface"
set by initial cluster

* So, we have an infinite cylinder:

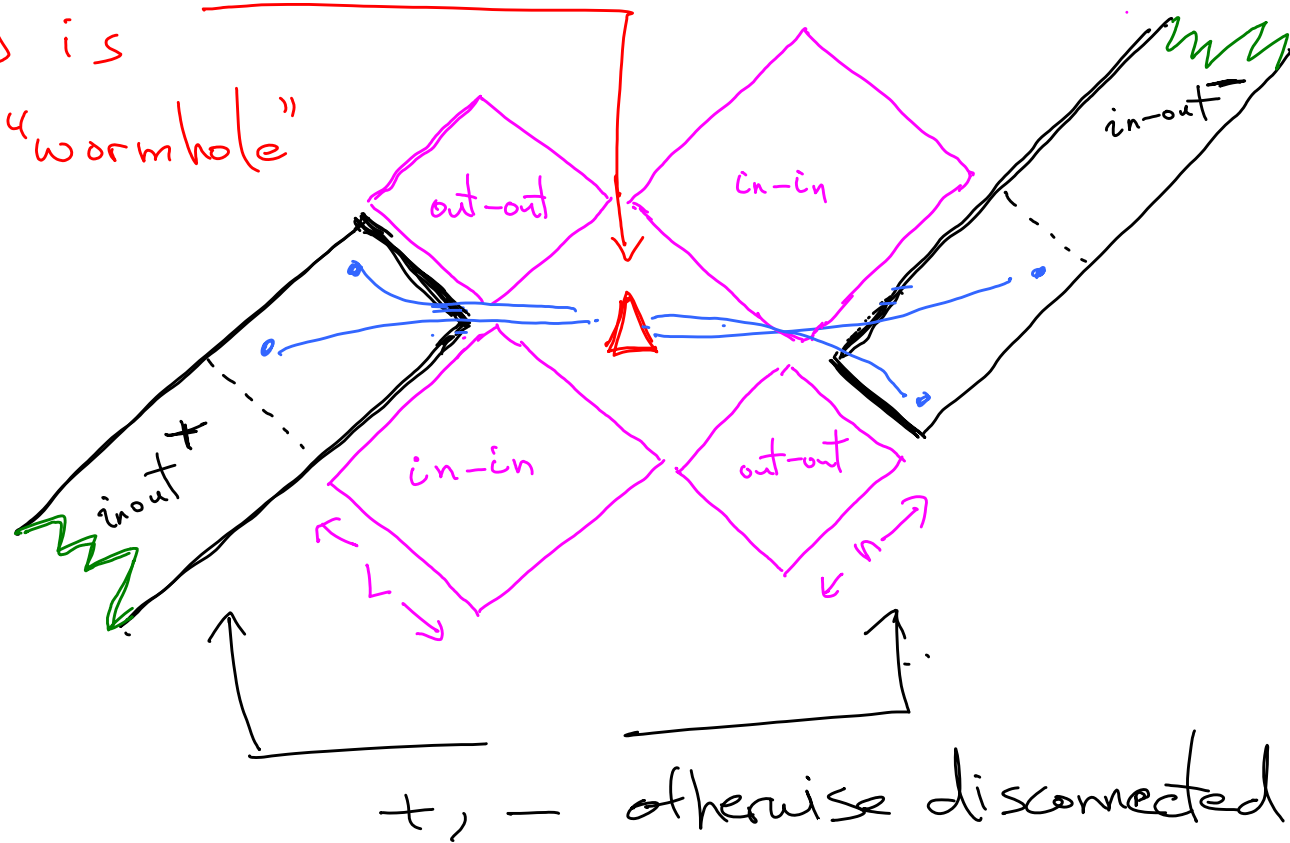


$I_n - I_n / \text{Out-Out}$ "Kinematic Spacetime"



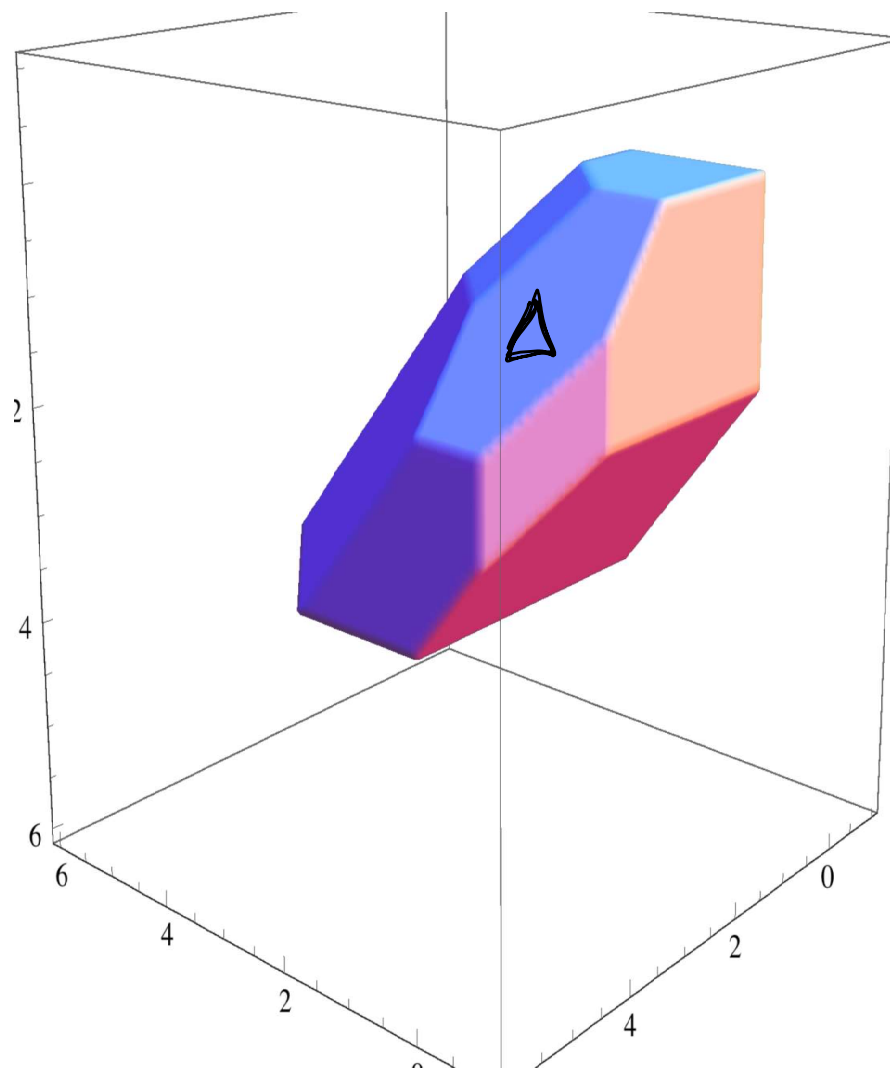
The Final "Kinematical Spacetime" for A_n, L

Δ is
a "wormhole"

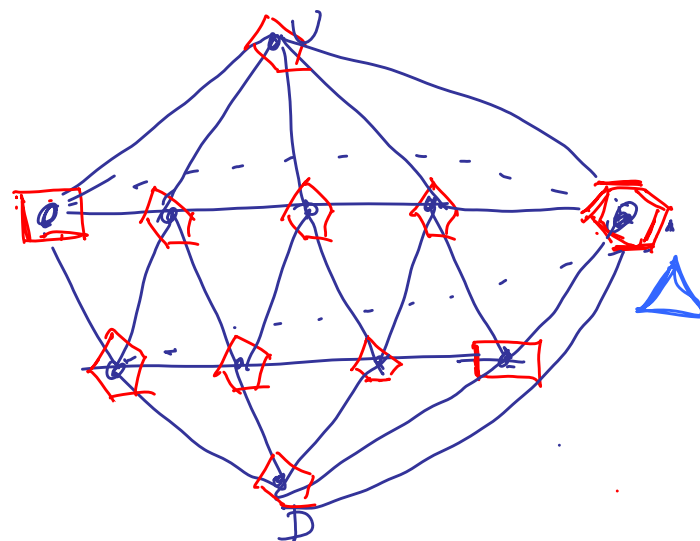


Affine Clustered

- * Exactly the same ideas work for $\tilde{D}_{n+2}$
= 2-loop. So, we have clustered through
2-loops and all L cut of $A_{n,L}$.
- * Δ is "mutation partner" for every missing var;
all vertices $\implies$ all vertices unamb. assoc. w/ diagonals.
- * All non- Δ vertices counted same # of
times
- * All normal facet det's involving Δ are
 ± 1 , get all $+1$'s for amplitude



$A_{2,1}$ Clusterhedron



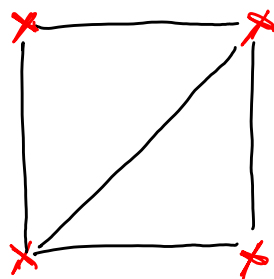
So we are done through 2-loops +
some all-loop cuts. On our way
to all loops. Simple Strategy:

Understand decoupled "Topological Sectors,"

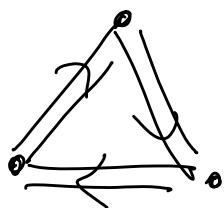
Find " Δ 's" $\longleftrightarrow$ "Wormholes" to
glue them together

Novelty for $L \geq 3$ / non-planar: "Cosmological" Kinematic Space

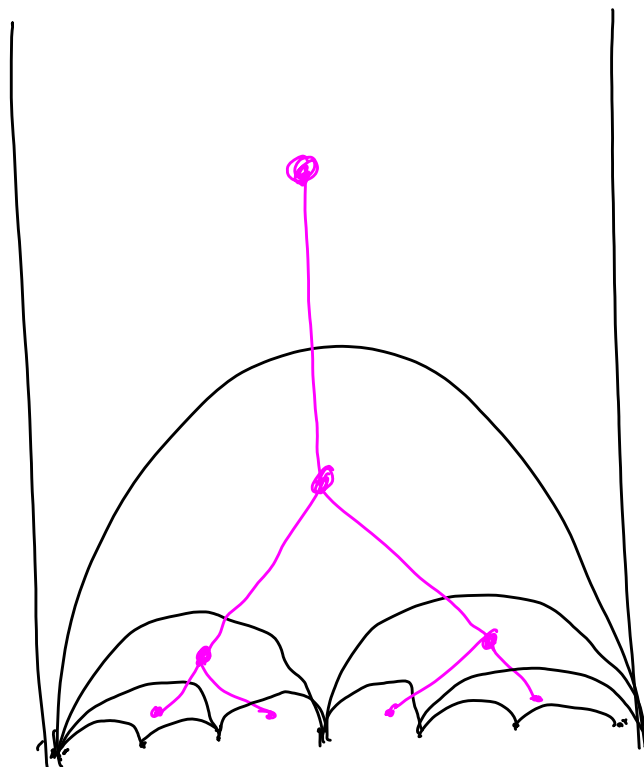
e.g. once-punctured
torus



$SL(2, \mathbb{Z})$
→
for gen.
triang.

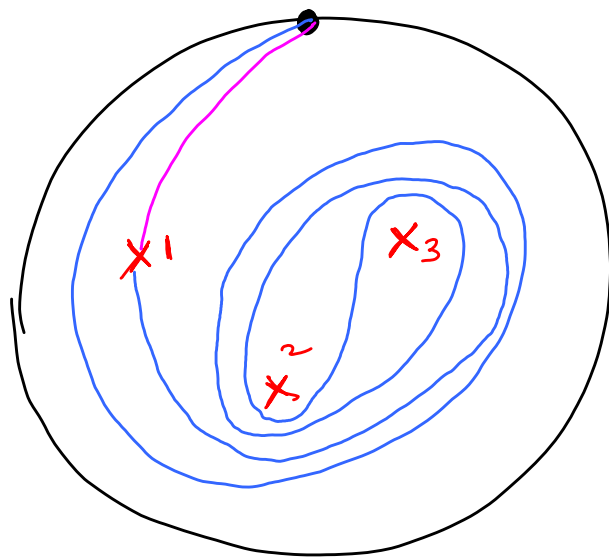


Markov



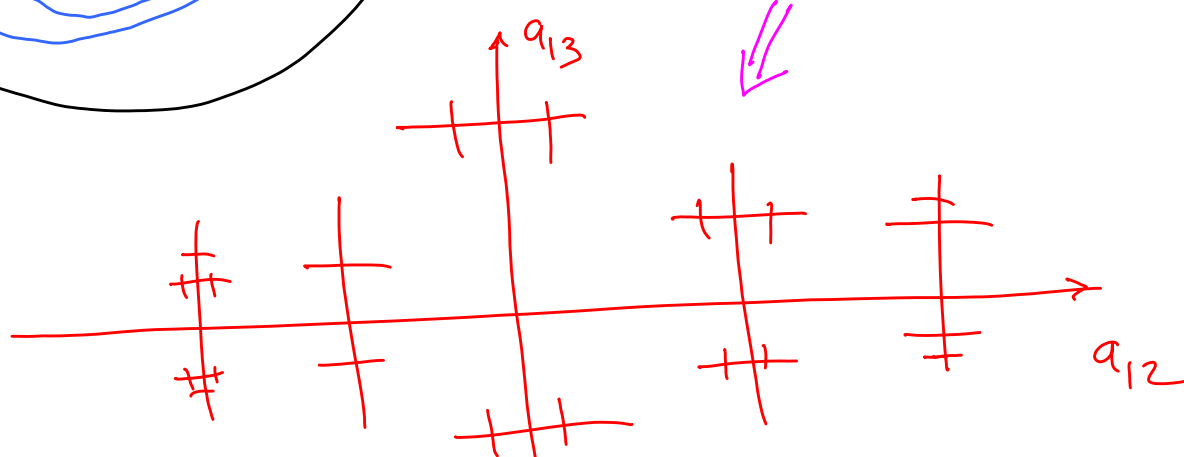
" dS " in KinSpace.

Novelty for $L \geq 3$ / non-planar: "Cosmological" Kinematic Space



Arcs labelled by
Braid₃ / Braid₂

Free₂ (a_{12}, a_{13})



"False vacuum eternal inflation"

Outlook

- * Over the past few years, we have been seeing concrete examples where rules of spacetime + quantum mechanics — locality + unitarity — are seen to emerge from simple new structures in kinematical space @ infinity
- * All-loop planar $\mathcal{N}=4$ SYM $\rightarrow$ Amplituhedra
- * Non-SUSY ϕ_{BA}^3 , heading towards all-loop, all orders in $1/N \rightarrow$ Clusterhedra
- * These ideas are directly related to real-world physics in reasonable approximations, expose hidden symmetries, and describe universal features of wide class of theories

Seeking Principles Underlying Magic

Grand Synthesis Yet to Come!