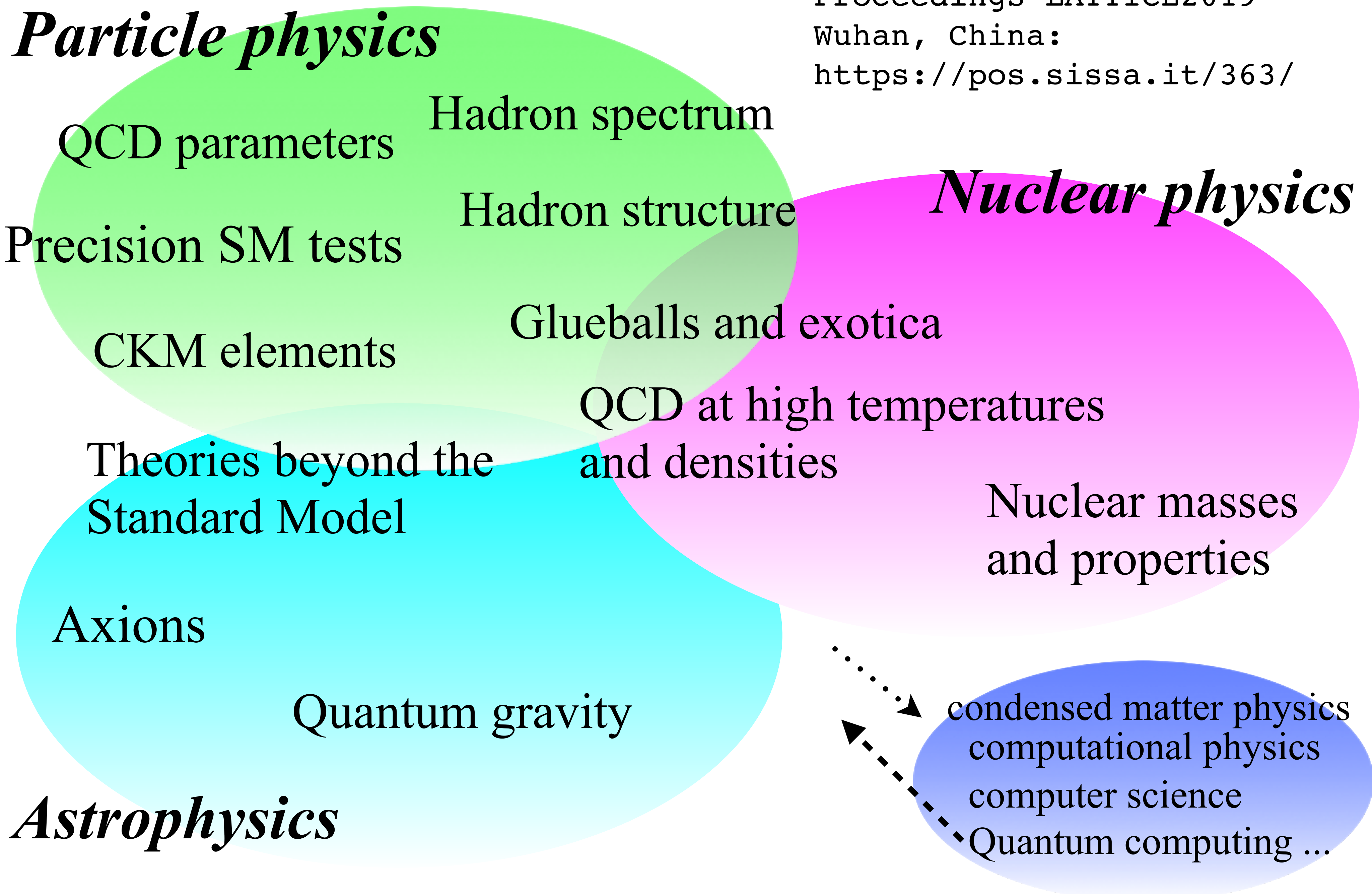
Recent progress in Lattice QCD

Christine Davies
University of Glasgow
HPQCD collaboration

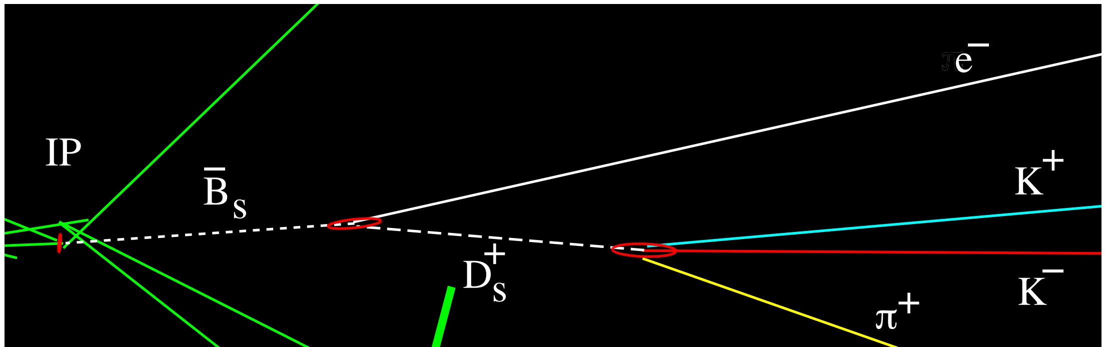
RECONNECT
May 2020

Applications of Lattice QCD/Lattice field theory

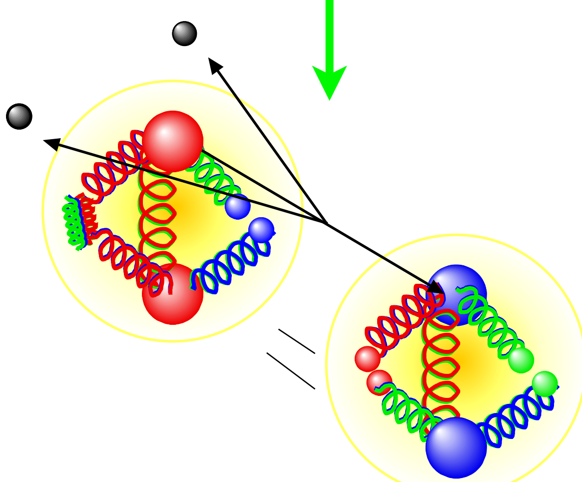
Proceedings LATTICE2019
Wuhan, China:
https://pos.sissa.it/363/

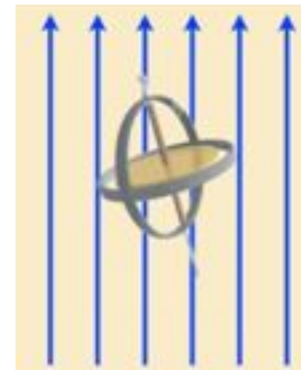


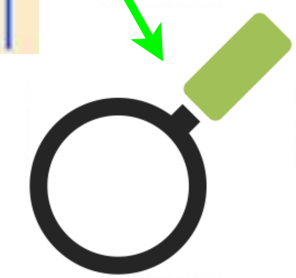
Quark low-energy strong interactions are a major complication in testing the Standard Model

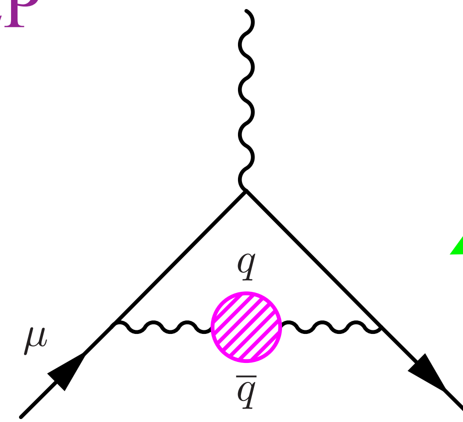


ALEPH@LEP

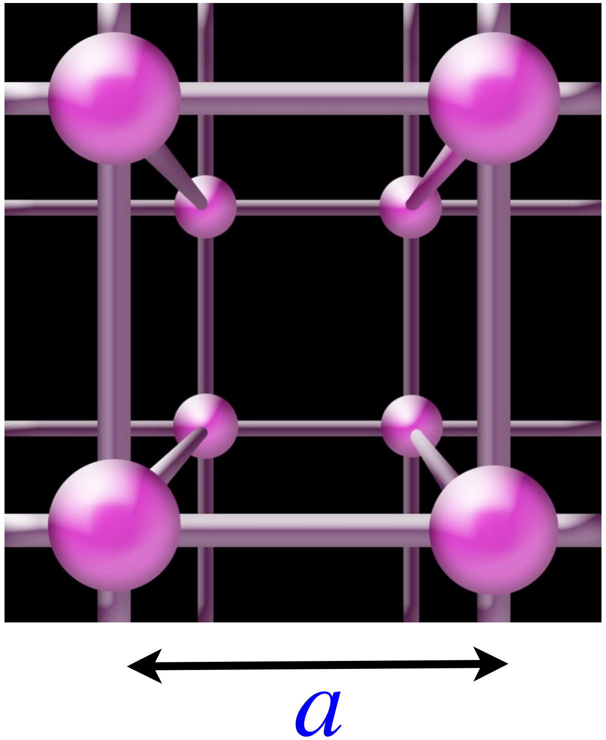


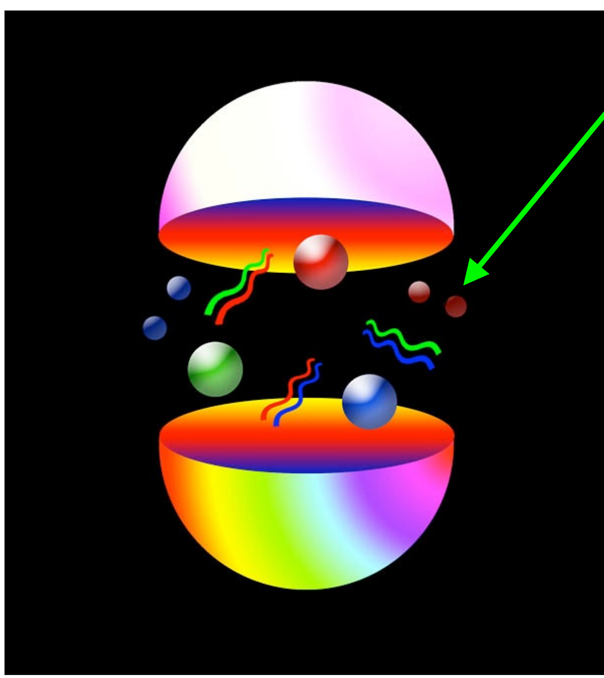






An accurate nonperturbative treatment of QCD - lattice QCD - is key to compare SM and low-energy experimental tests for new physics





Lattice QCD: perform path integral on a discrete space-time lattice

Must be able to determine results in physical continuum limit.

Final accuracy depends on :

- control of lattice spacing dependence
- tuning of quark masses
- normalisation of operators in matrix elements

A variety of quark formalisms exist; pros/cons suit them to different physics programmes.

Comparison of results from different formalisms is important

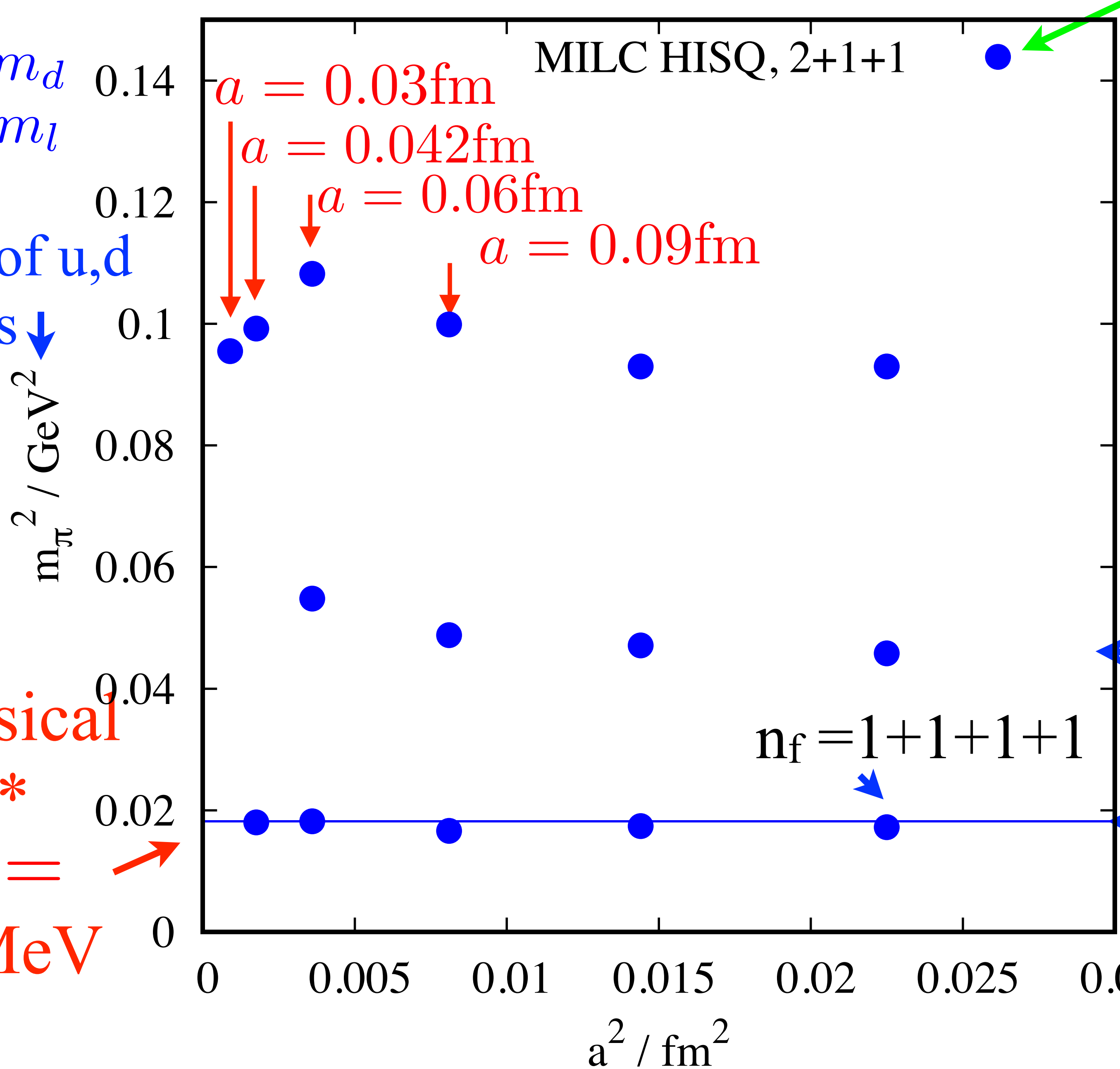
e.g. : Parameters for gluon field configurations generated with Highly Improved Staggered (HISQ) sea quarks

$m_u = m_d$
$= m_l$

mass of u,d quarks

*physical $m_{u/d}$ *

$m_{\pi^0} =$ 135 MeV



"2nd generation" lattices inc. c quarks in sea

HISQ - very accurate discretisation - leading error a^4

HPQCD, hep-lat/0610092.

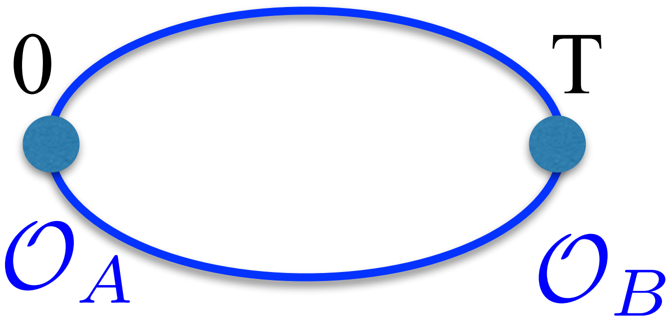
$m_{u,d} \approx m_s/10$

$m_{u,d} \approx m_s/27$

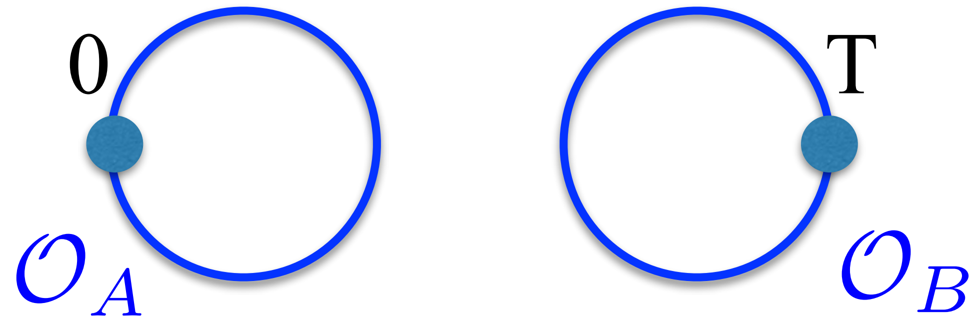
Volume: $m_\pi L > 3$

Physical u/d masses also available in other formalisms

Hadron Correlation functions constructed from valence quark propagators



Connected 2-point function
(for meson)



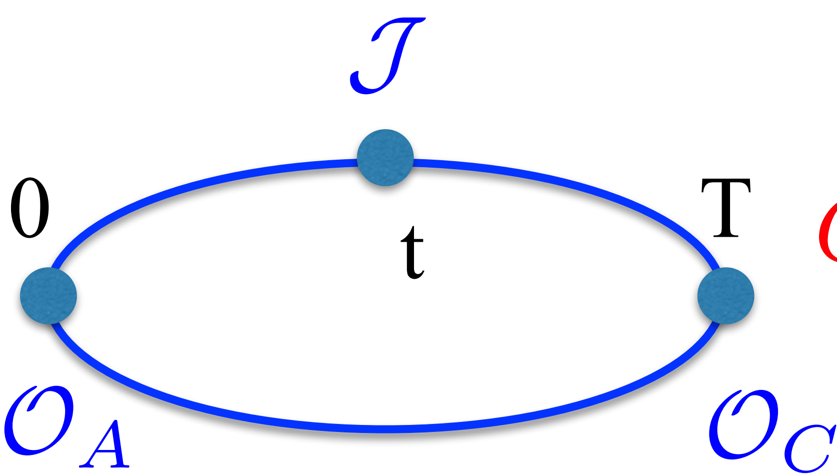
(quark-line) Disconnected 2-point -
difficult and noisy

Multiple states
need to be inc. in fit

$$C_2 = \sum_n A_n B_n e^{-M_n T}$$

$$\langle 0|\mathcal{O}_A|n\rangle$$

decay constant,
if O normalised



$$C_3 = \sum_{m,n} A_n J_{nm} C_m$$

$$e^{-M_n t} e^{-M_m (T-t)}$$

$$\langle n|\mathcal{J}|m\rangle$$

form factor,
if J normalised

Connected 3-point function

Hadron Spectrum

Ground-states very accurate for flavour non-singlet mesons - use to tune quark masses accurately. Baryons also improving.
Issue now is adding in QED (previously often estimated)

Excited and exotic states - experimental interest high (Belle/BES3/JLAB/LHCb/PANDA)

R. Edwards, review LAT2019
S. Prelovsek, review, 2001.01767

Need large basis of operators, O, including multi(=two at present)-hadron operators to determine many energy levels in a given channel as a function of volume. Allows mapping of scattering amplitudes to locate unstable resonance poles.

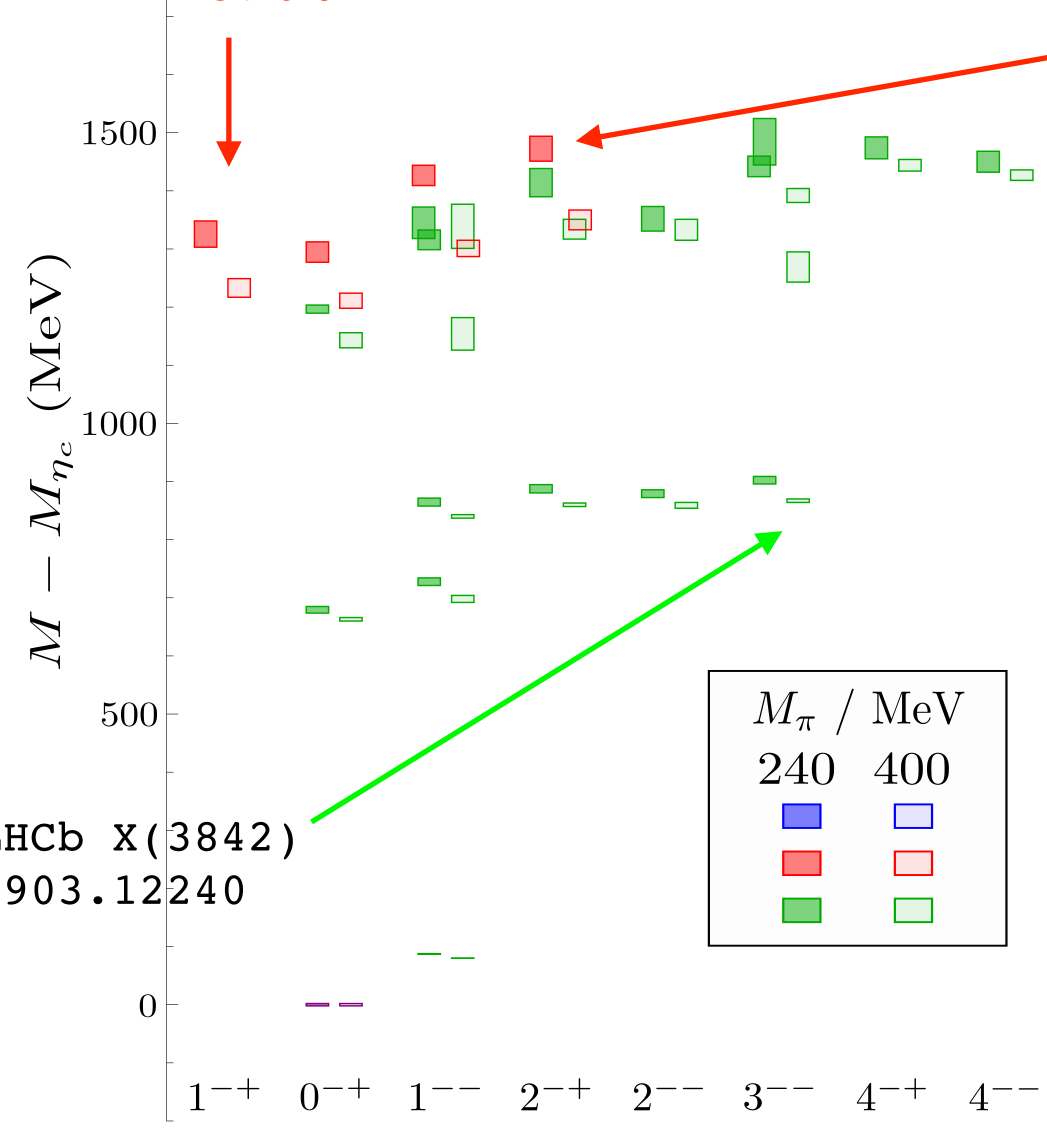
Aim for very complete spectra and search for exotica - hybrids, $\overline{q}qG$
tetraquarks $\overline{qq}qq$ etc

Has not yet been possible to do calculations at physical u/d quark masses or multiple lattice spacings, so precision of results is not high.

Example: charmonium spectrum

Hadspec, 1610.01073, 1709.010147

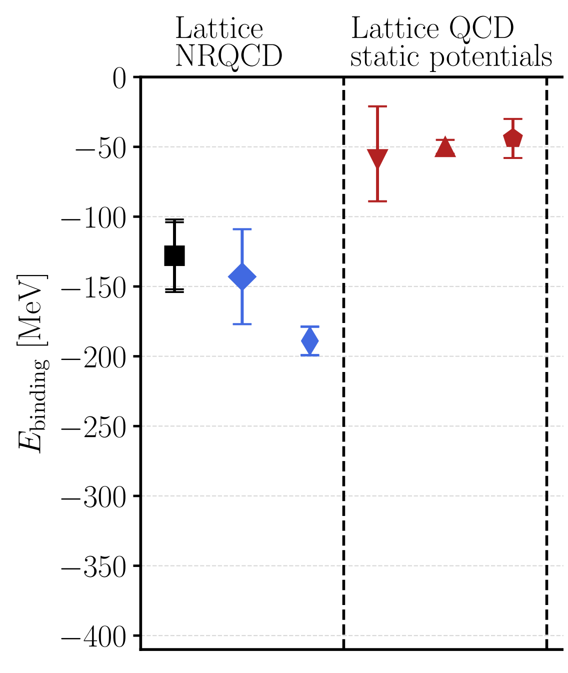
Exotic J^{PC}



'Hybrid' states - lowest multiplet. Arise from coupling to 1^{+-} gluonic excitation which adds ~1.3 GeV. Same picture seen for baryons, light mesons etc.

No sign of tetraquark state that could be Zc+(3900).

LHCb X(3842) 1903.12240



Lattice QCD sees stable tetraquark in

$\overline{bb}ud$

$J^P=1^+, I=0$

Mass below BB* threshold

See also: Prelovsek et al, 1405.7623, 1503.03257

1607.05214,1810.12285,1904.04197

Ground-states : adding QED + isospin-breaking effects

FLAG: 1902.08191

QED and m_u-m_d effects for valence quarks are each ~ few MeV.
Estimated in the past, now being included.
Most calculations ignore effects for sea quarks - 'Quenched QED'

$$S_{\mathrm{Feynman}} \propto k^2 |A_\mu^{\mathrm{QED}}(k)|^2$$

Choose A from Gaussian dist. in mom. space, setting zero-modes to zero: $(\mathrm{QED})_L$

Quark electric charge

$$U_\mu^{\mathrm{QED}} = \exp(-i e_q a A_\mu^{\mathrm{QED}}(x + a\hat{\mu}/2))$$

Feed into Dirac eq. as for gluons

Must allow for finite-volume effects from long-range Coulomb interaction :

BMW:1406.4088 (n-p mass diff.)

$$M_L = M_\infty - Q^2 \alpha \frac{\kappa}{2L} - \ldots$$

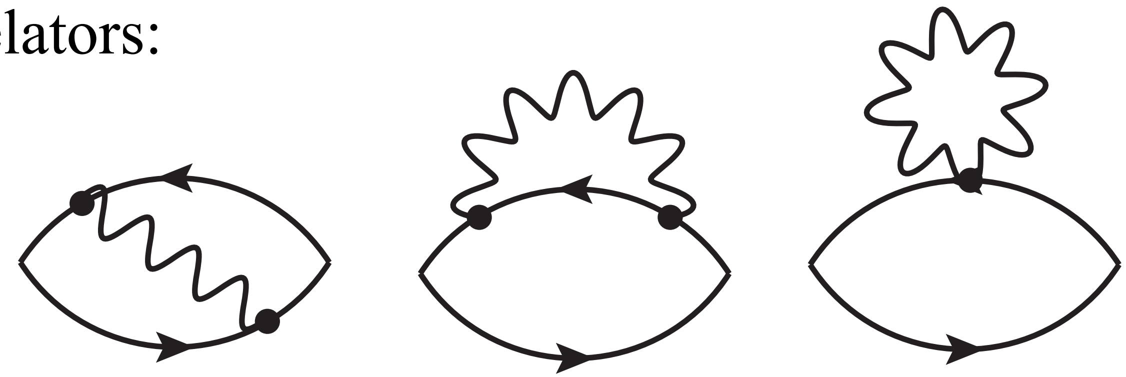
= 0.7 MeV for L=3fm and Q=1

Hadron electric charge

Known higher orders in 1/L

Perturbative approach (ROME123) 1303.4896

Expand path integral to first order in α and d-u mass diff. Calculate slopes from correlators:

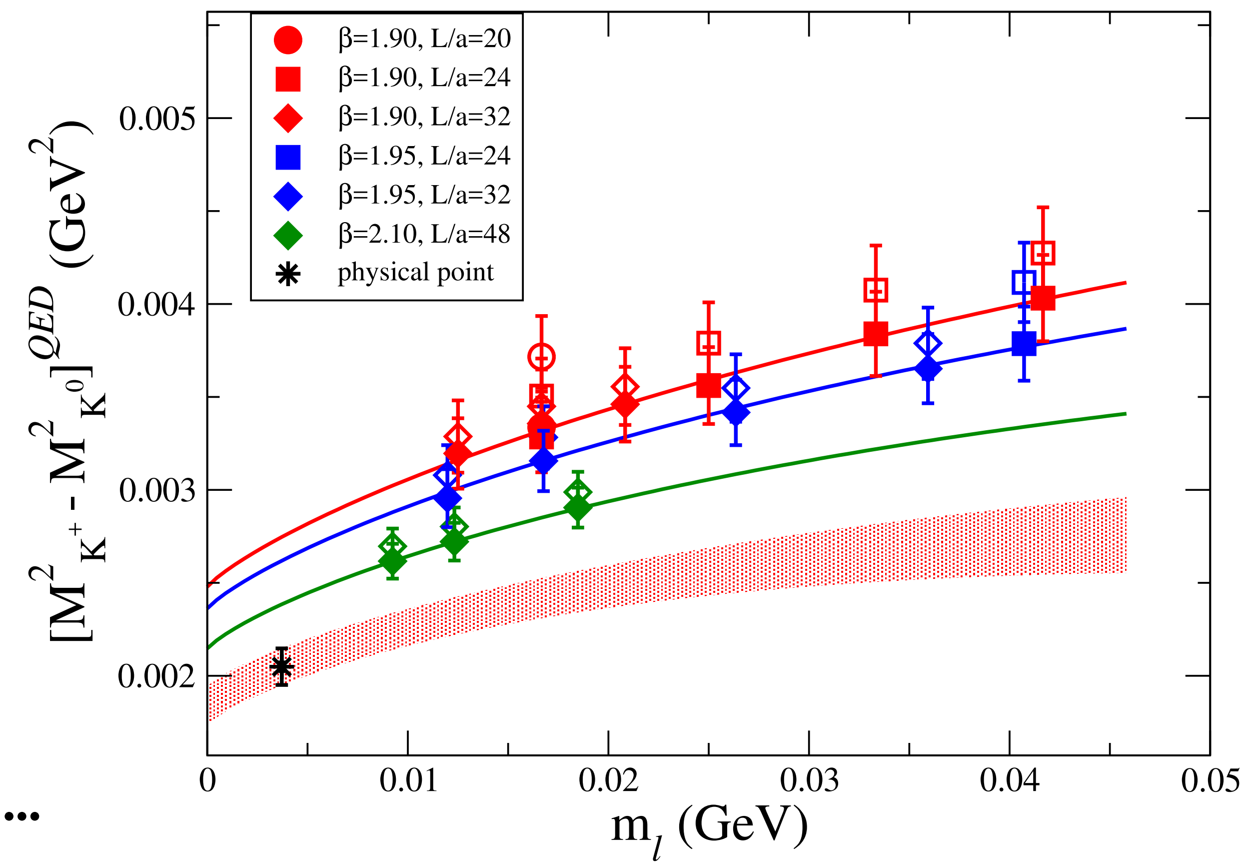


+ mass insertions

e.g. ETMC:1704.06561

Allows separation of strong-isospin-breaking and QED effects, up to scheme dependence. Find e.g.

$$[M_{K^+} - M_{K^0}]^{\mathrm{QED}} = 2.07(15)\mathrm{MeV}$$



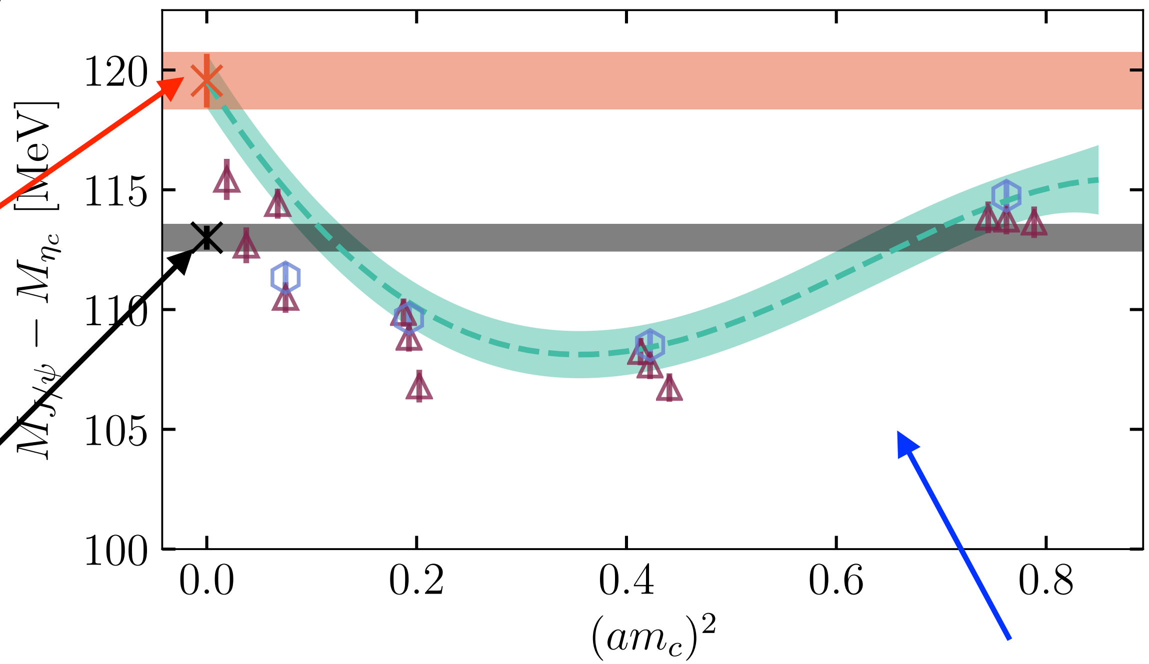
See also RBC/UKQCD:1612.05962 …

Stochastic Approach

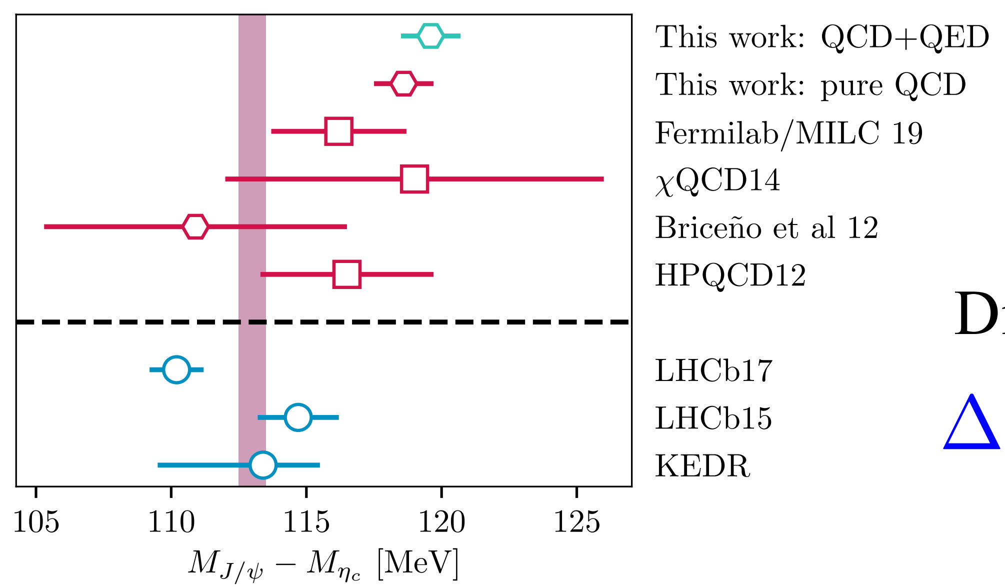
Simply include QED field in calculation; noisy for light quarks, but very accurate for heavy

$$M_{J/\psi} - M_{\eta_c}$$

Lattice 'connected' calculation - ignores $\eta_c \to gg$



Experimental average 113.0(5) MeV



QCD and QCD+QED results - tune m_c to J/ψ. QED effect +1.4% HPQCD:2005.01845

Discrepancy with expt. implies

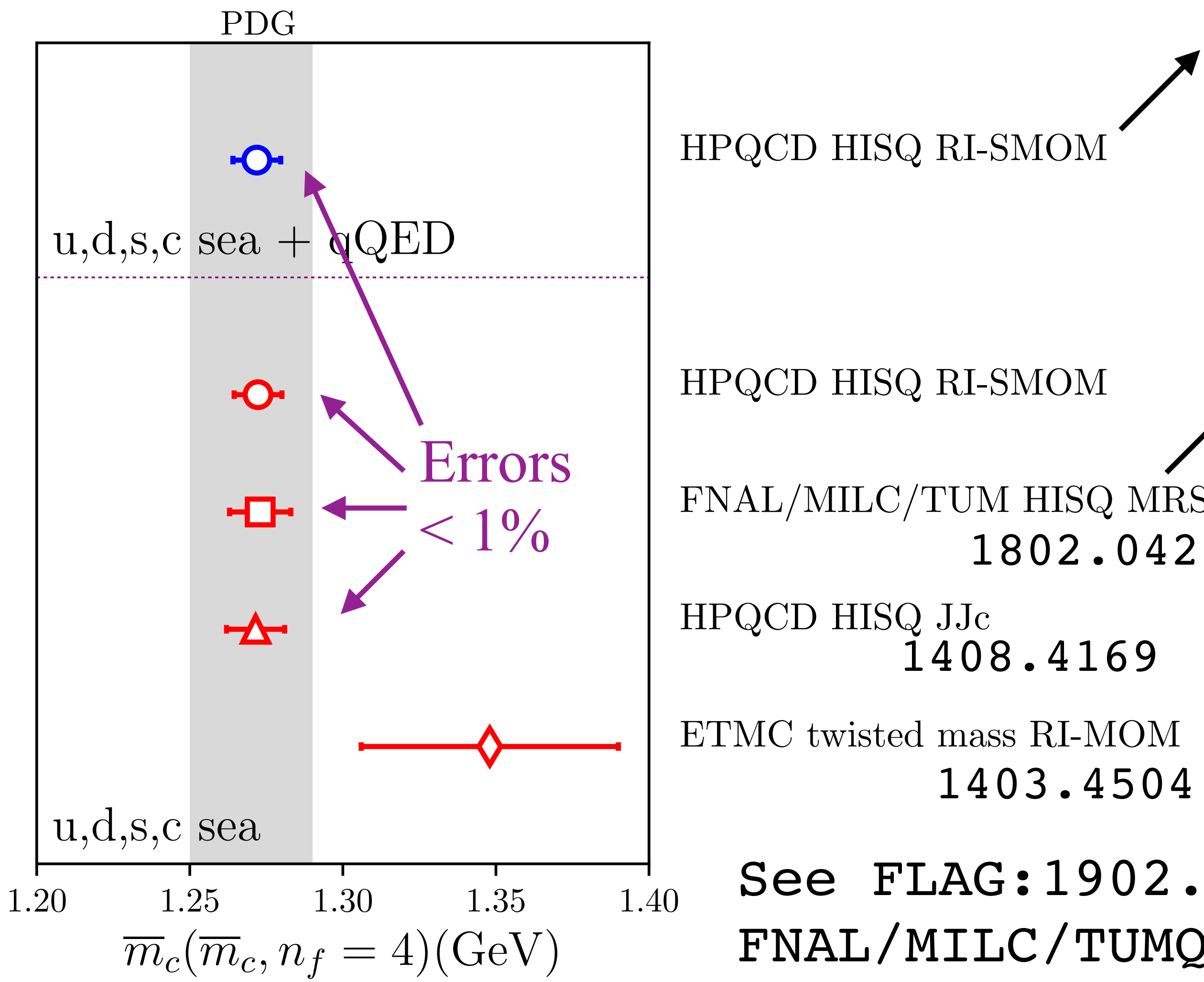
$$\Delta M_{\eta_c}^{\text{annihln.}} = +7.3(1.2)\text{MeV}$$

(Not calculable in lattice QCD so far)

Determining quark masses from lattice QCD

Lattice quark masses tuned very accurately from ground-state meson masses, now inc. QED. Issue is accurate conversion to $\overline{\text{MS}}$

Update on m_c : HPQCD:2005.01845



Momentum-subtraction scheme, inc. analysis of nonpert. condensate effects

$\overline{m}_c(\overline{m}_c) = 1.2719(78)\text{GeV}$

Heavy-light meson masses + minimal renormalon-sub. scheme

1802.04248

Time-moments of pseudoscalar correlators

1408.4169

1403.4504

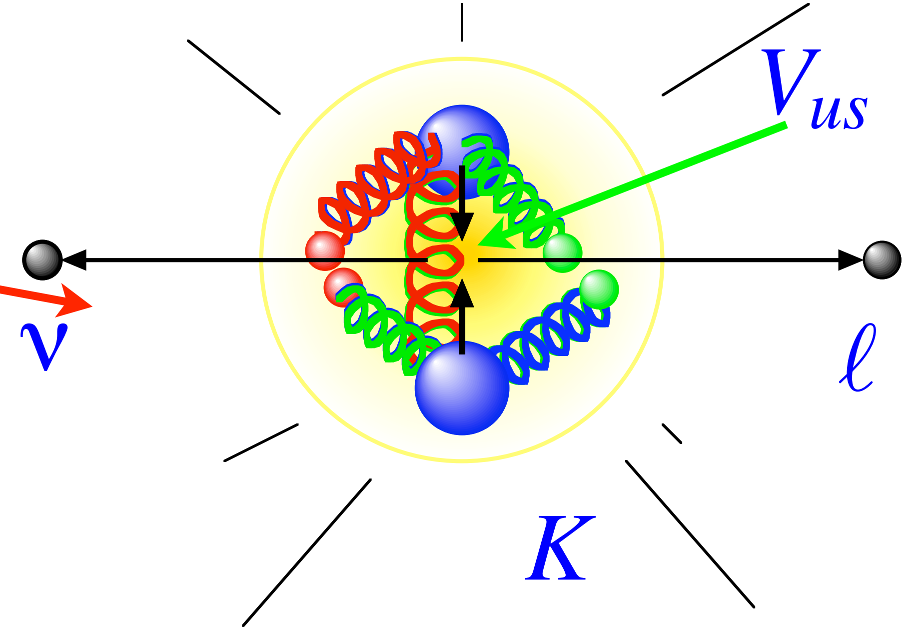
PDG: take note

See FLAG:1902.08191+ FNAL/MILC/TUMQCD:1802.04248 for other masses + ratios

Weak decays probe hadron structure and quark couplings.
(Semi)-leptonic decays and mixing calculable in lattice QCD

S. Gottlieb, review LAT2019

$$\left(\begin{array}{ccc} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to l\nu & K \to l\nu & B \to l\nu \\ & K \to \pi l\nu & B \to \pi l\nu \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \pi l\nu & D \to K l\nu & B \to D^{(*)} l\nu \\ D \to l\nu & D_s \to l\nu & \Lambda_b \to \Lambda_c l\nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ \langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle & \end{array}\right)$$



See HPQCD, 1907.01025

CKM matrix

$$Br(M \to \mu\nu) \propto V_{ab}^2 f_M^2$$

Expt = CKM x theory(QCD)

Need precision lattice QCD to get accurate CKM elements to test Standard Model.

Aim : sub-1% errors

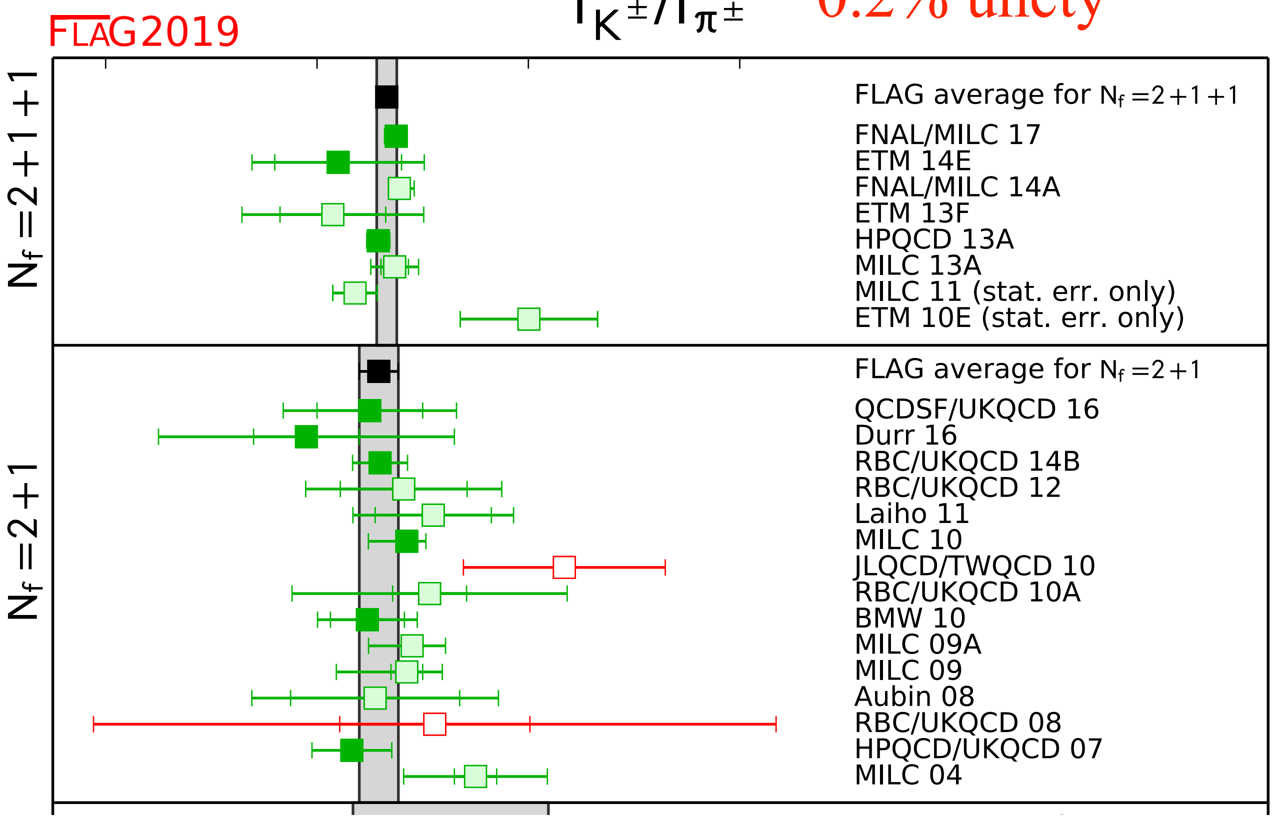
If V_{ab} known, compare lattice to experiment to test QCD

ALSO - test corresponding electromagnetic processes

Lattice QCD results for K/π decay constants

FLAG:1902.08191

$f_{K^\pm}/f_{\pi^\pm}$ 0.2% uncty



Combine w. ratio of exptl leptonic decay rates:

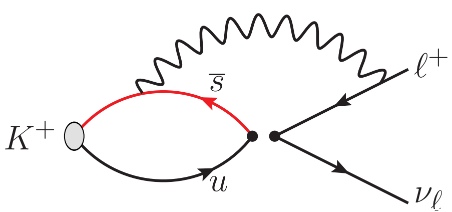
$$\frac{|V_{us}|}{|V_{ud}|} = 0.2313(5)$$

Tensions at ~2 σ with Vus from semilept. K decay and first-row unitarity.

FNAL/MILC:1809.02827

Exptl leptonic rate is

$$\Gamma(\pi/K \to \mu\overline{\nu}[\gamma])$$



Di Carlo et al:1904.08731 + Martinelli LAT2019

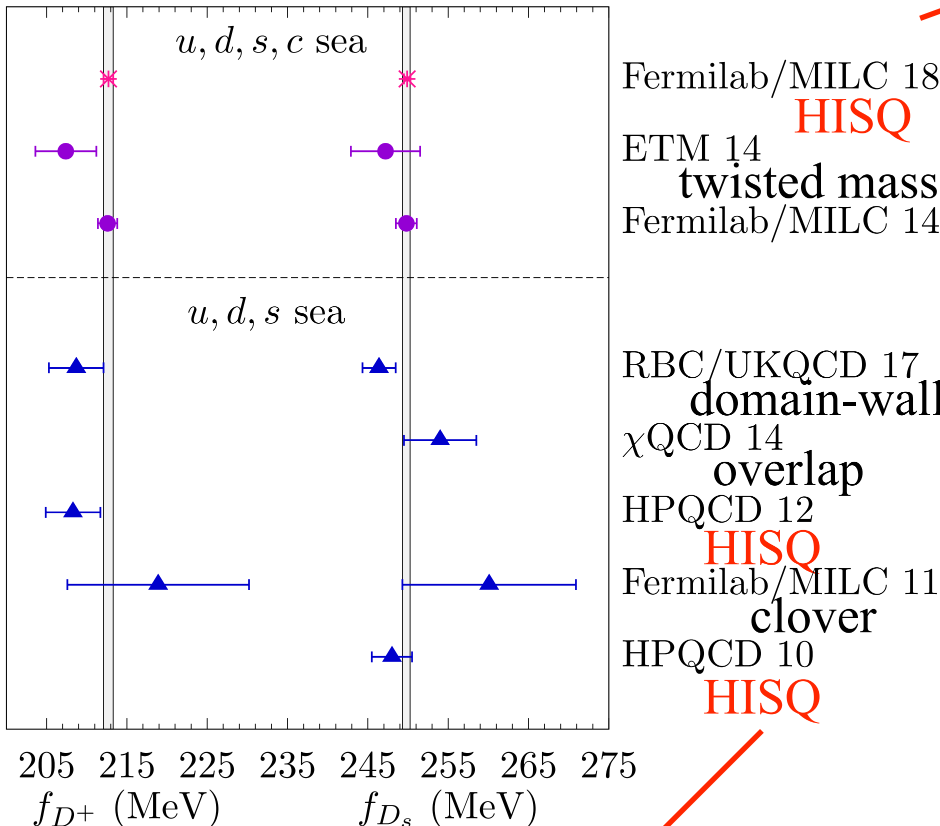
Now calculate QED radiative corrns in lattice QCD. So far agree with previous estimates. Extend to heavy mesons, semilept. decays

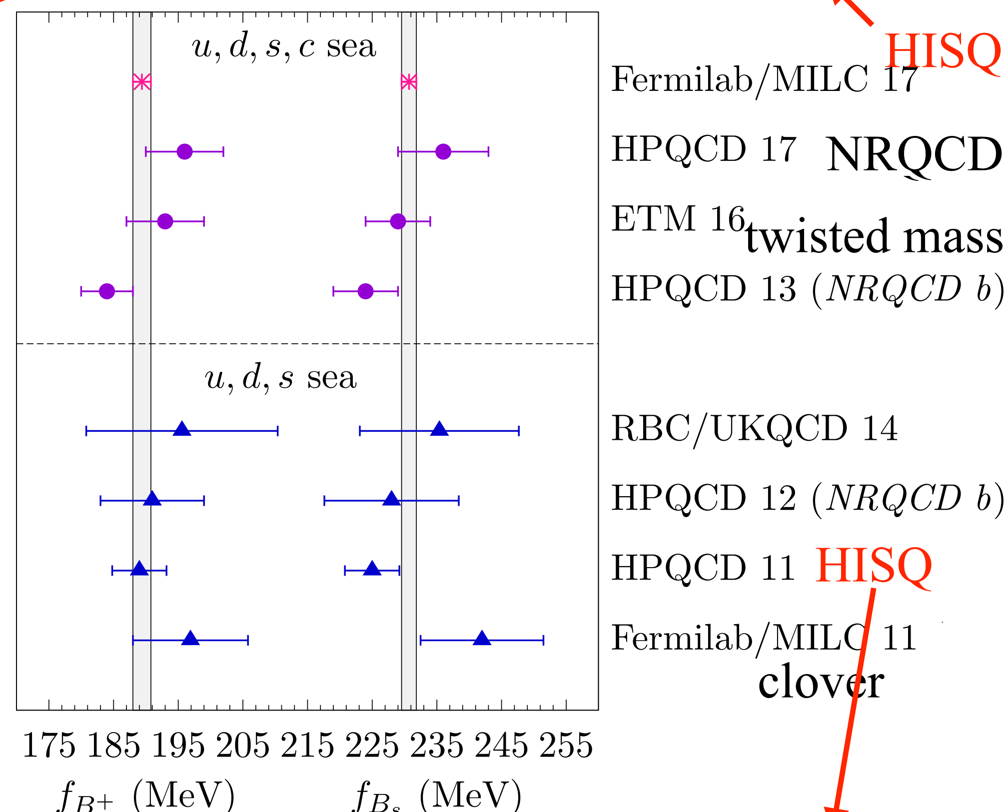
Electroweak corrections to super-allowed nuclear β decay affect Vud and must be pinned down.

B and D meson decay constants

uncty. now 0.2-0.7%
<< exptl. error on lept.rates

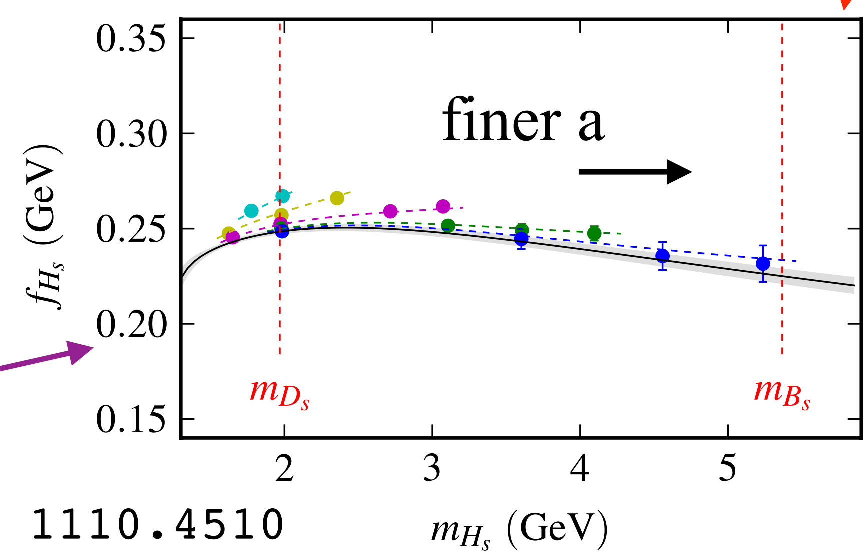
FNAL/MILC:1712.09262





Key to accuracy for c and b quarks is a highly improved relativistic action, such as HISQ, so disc. effects and operator norm. are controlled.

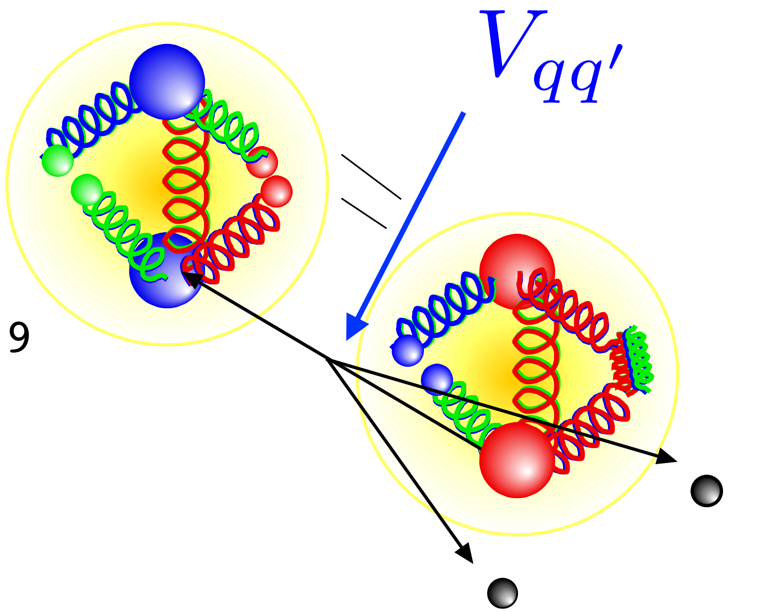
Multiple am_h and multiple a allow determination of value at b - 'heavy-HISQ' approach



HPQCD 1110.4510

Semileptonic form factors

Encode QCD meson structure info. from which differential rate calculated, up to CKM factor.



e.g. $B \to D^* \ell \overline{\nu}$ A. Lytle, review LAT2019

(exclusive) expt. + lattice QCD gives Vcb.

$$\langle D^*(\vec{p}) | V^\mu - A^\mu | B(\vec{p} = 0) \rangle$$

$$\longrightarrow A_0(q^2), A_1(q^2), A_2(q^2), V(q^2) \longleftarrow q^2 = (p_i - p_f)^2$$

Issues for form factor calculations are :

$$0 < q^2 < (M_i - M_f)^2$$

• Discretisation errors, operator normalisation (as for decay constants)

$$|\vec{p}_f|_{q^2=0} = \frac{M_i}{2} - \frac{M_f^2}{2M_i}$$

• Dependence on q^2 . Early calculations done for A_1 at zero recoil (q^2_{max}), compared to expt. extrapolated to that point.
MUCH better to compare direct to expt. across FULL q^2 range.

Large B mass and q^2 range make this hard - much work going on ….

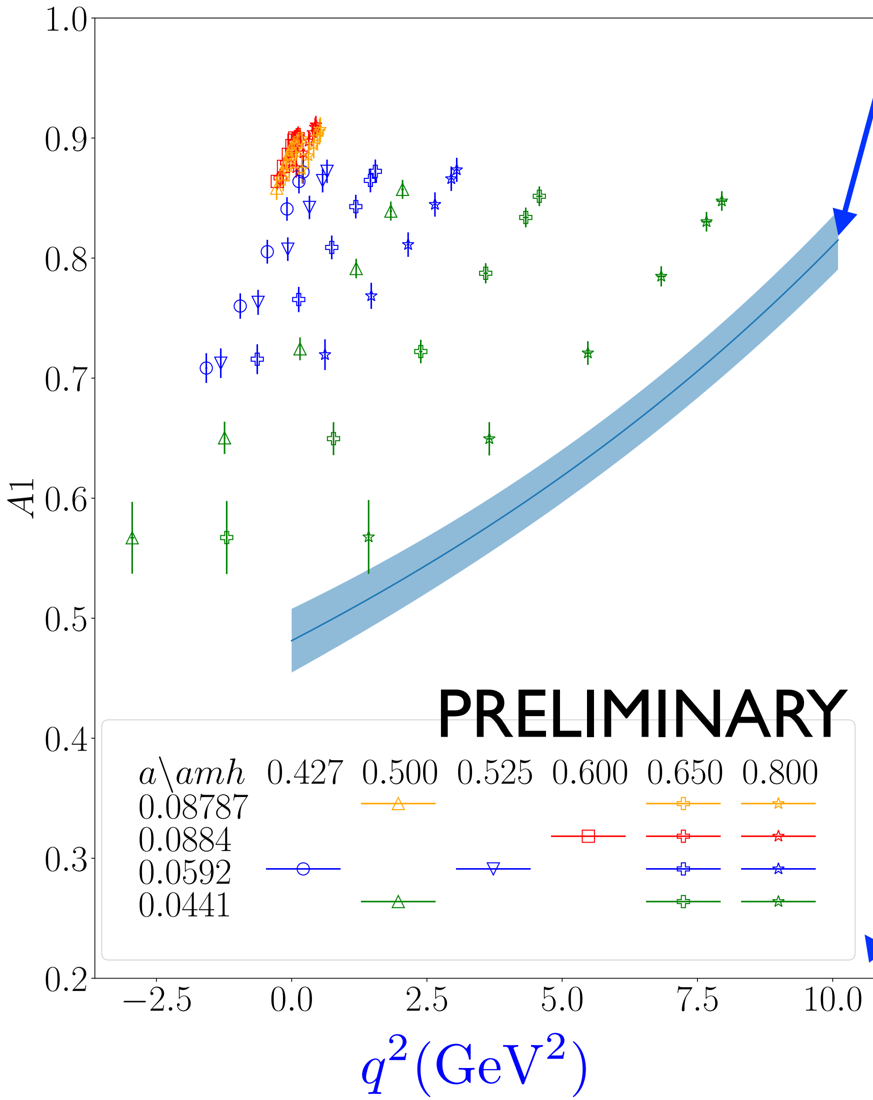
For B/D* see FNAL/MILC,1906.01019

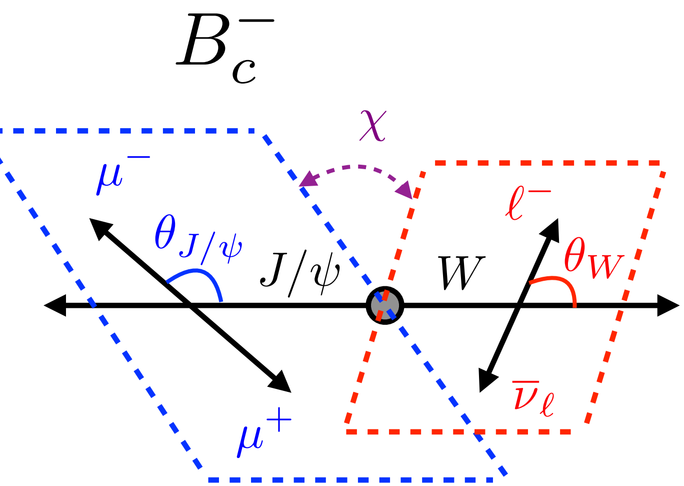
$B_c \to J/\psi \ell \overline{\nu}$

FIRST lattice QCD calculation

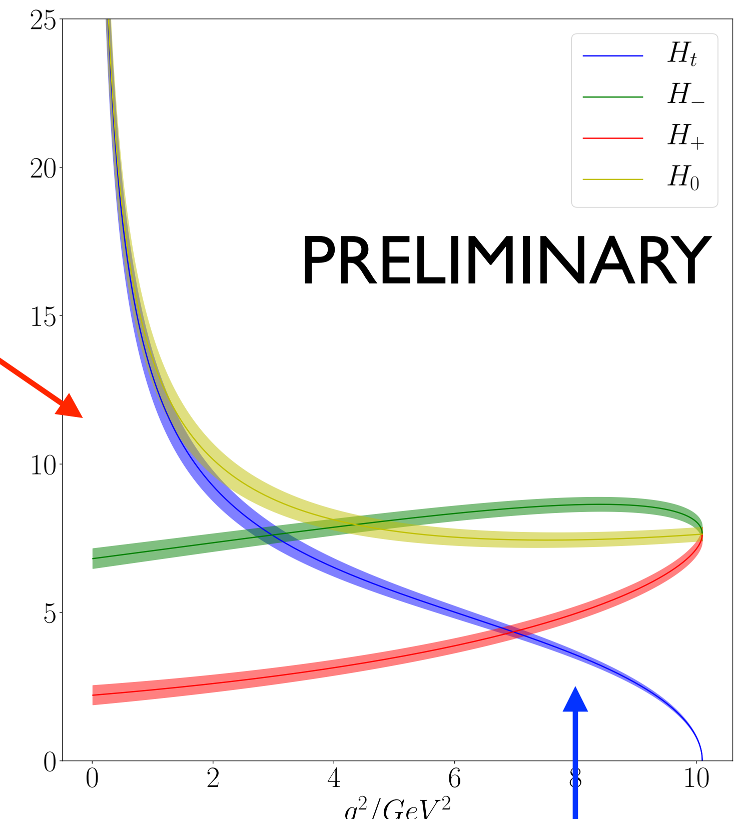
`Harrison et al,`
`HPQCD, in prep.`

Continuum result at b mass





Resulting helicity amplitudes combining all form factors



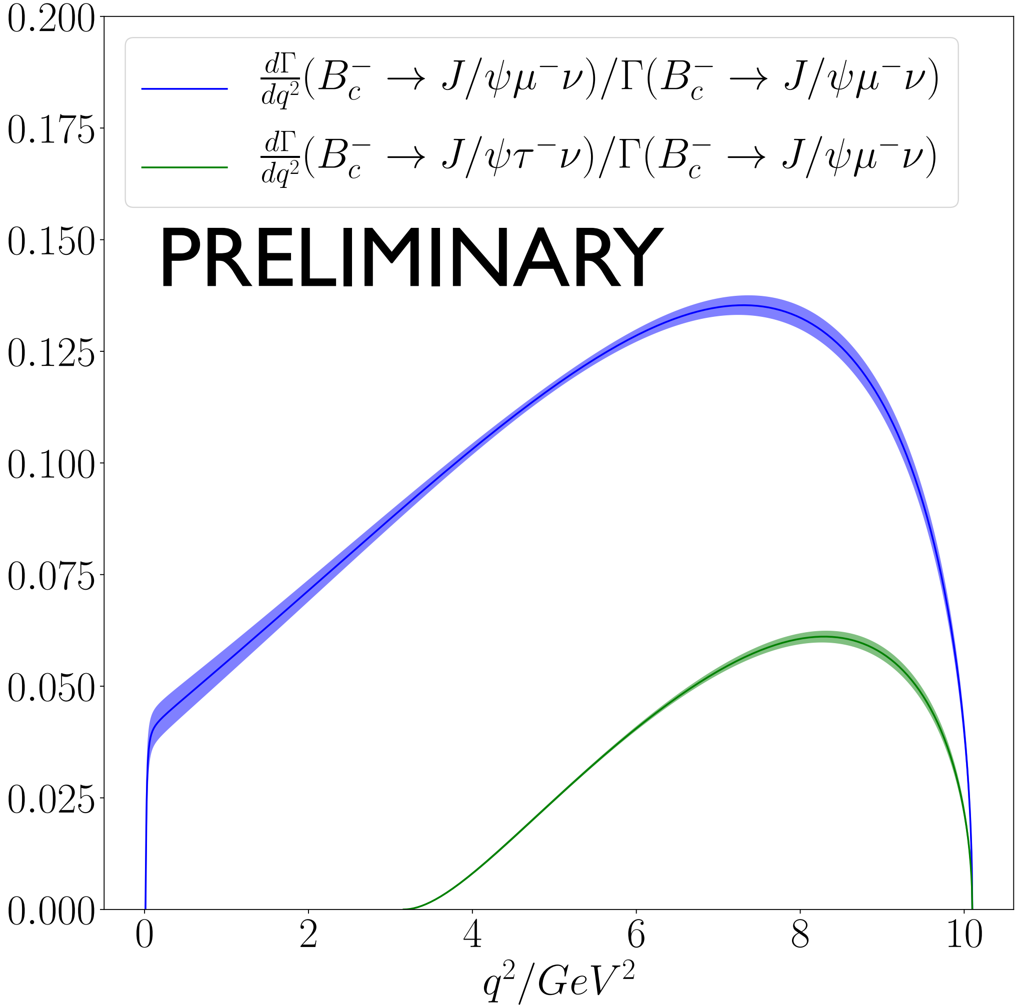
Heavy-HISQ approach covers full q² range - expands with mass range on finer lattices

Contributes for τ

`HPQCD, 1906.00701`

differential distributions for W decay to (massless) μ or (massive) τ

J. Harrison et al, HPQCD, in prep.



lepton universality test

$$R(J/\psi) = \frac{\mathrm{Br}(B_c \to J/\psi\, \tau\overline{\nu}_\tau)}{\mathrm{Br}(B_c \to J/\psi\, \mu\overline{\nu}_\mu)}$$

We find, in SM:

$$R(J/\psi) = 0.2636(37)$$

PRELIMINARY

close to R(D*)

First LHCb result :

$$R_{J/\psi} = 0.71(17)(18)$$

LHCb, 1711.05623

$$R(D^*)|_{\mathrm{SM}} = 0.258(5)$$ HFLAV

$$R(D^*)|_{\mathrm{Belle}} = 0.283(23)$$ 1910.05864

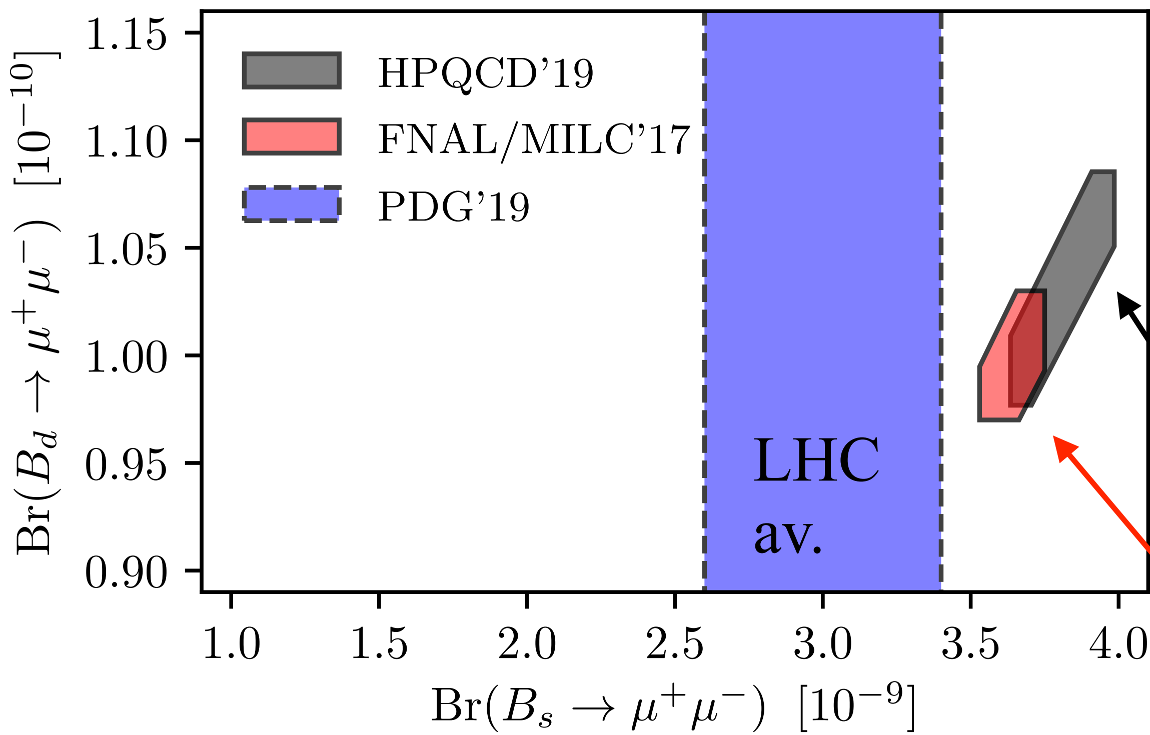
LHCb aim for 2% in R with 300fb^{-1}

LHCb, 1808.08865

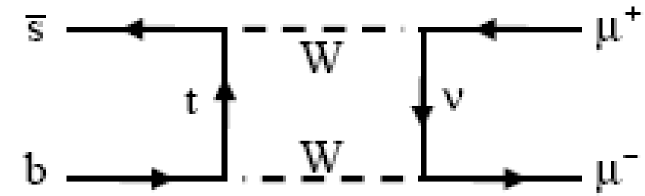
Precision tests of the Standard Model

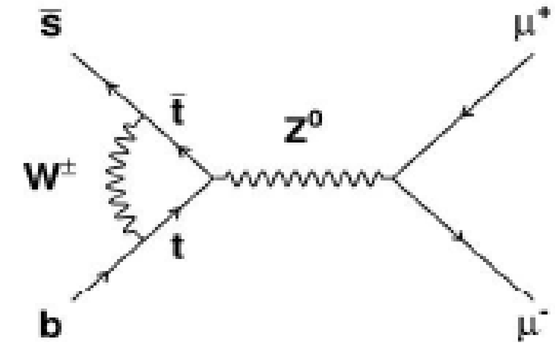
$B_{s/d} \to \mu^+\mu^-$

rare process that may give access to BSM physics



HPQCD, 1907.01025





Rate requires lattice QCD input: Either

$$\text{Rate} \propto \frac{\Delta M_q}{\hat{B}_{B_q}}$$

4q-op Bag param.

OR

$$\text{Rate} \propto f_{B_q}^2 |V_{tb}^* V_{tq}|^2$$

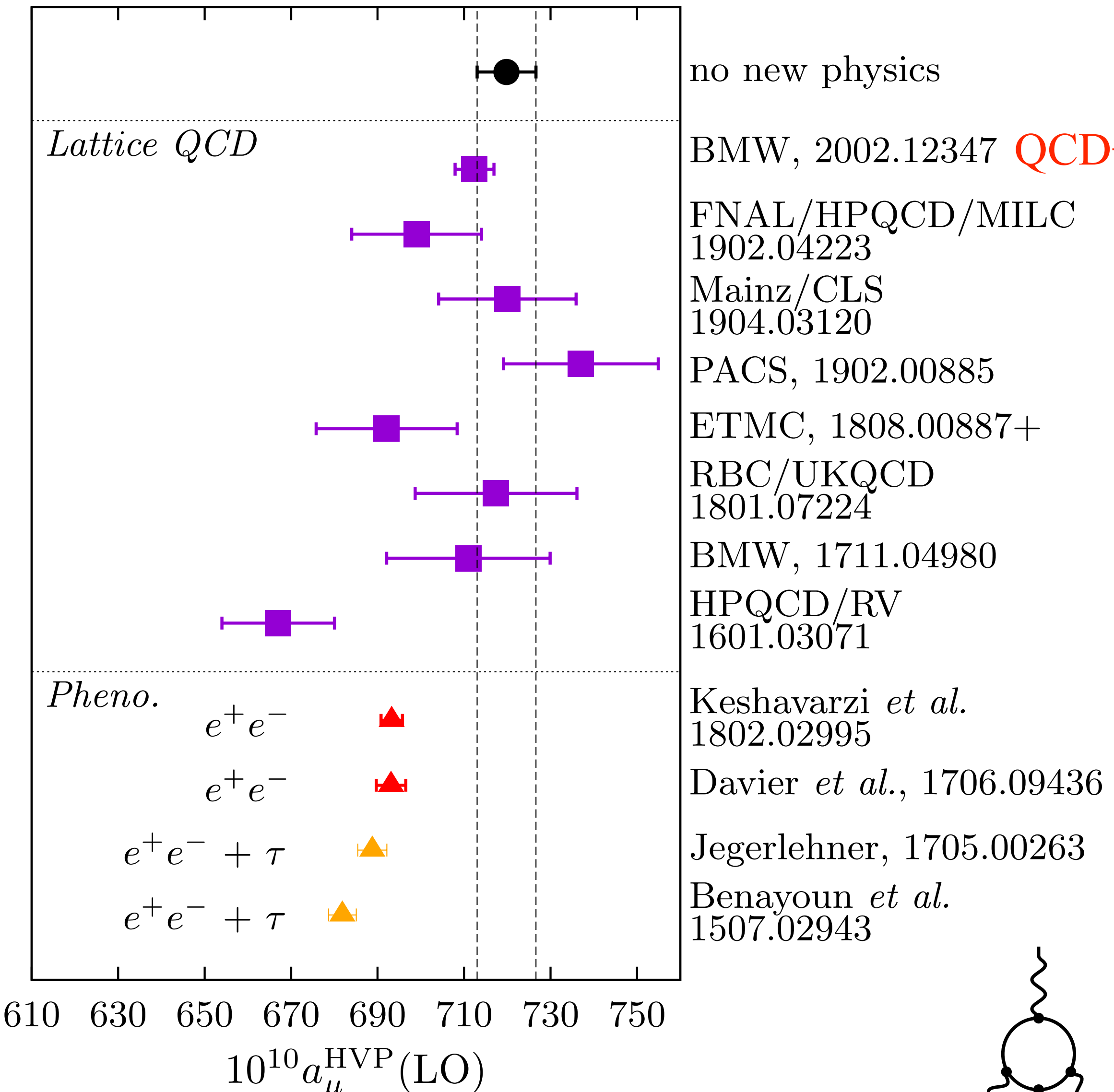
FNAL/MILC:1712.09262

agree: CKM consistent with unitarity

See Kane et al, 1907.00279 for first steps towards QED radiative corrections from lattice QCD

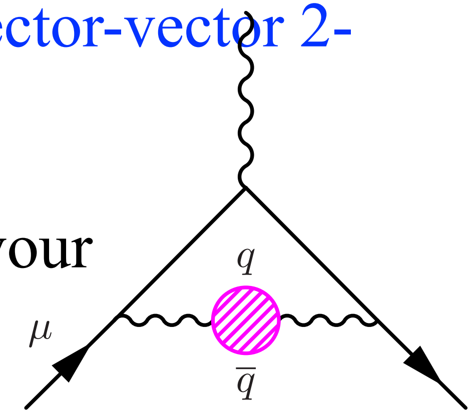
QCD effects in the anomalous magnetic moment of the muon

V.Gulpers, review LAT2019

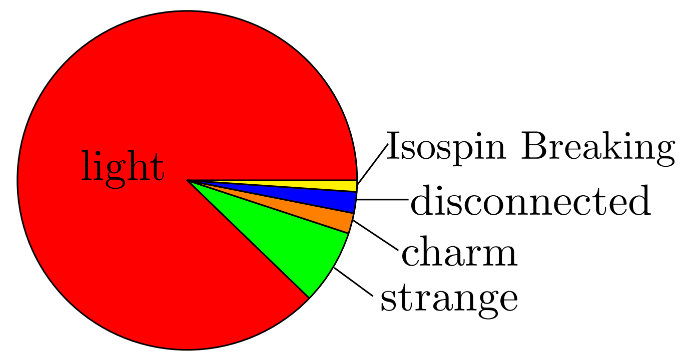


QCD+QED

Hadronic vacuum polarisation contribution
Needs vector-vector 2-point

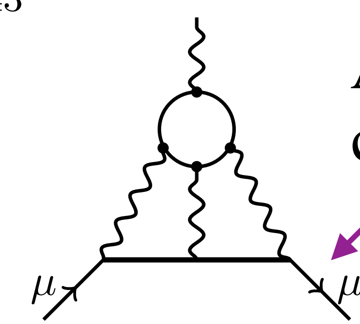


Separate by flavour + disc.



Lots of lattice results - not yet at accuracy of pheno. from R(e+e- to hadrons).

New exptl. results awaited ..



Also hadronic light-by-light contribution in QCD+QED

RBC:1911.08123

$$a_\mu^{\rm HLbL} = 7.9(3.5) \times 10^{-10}$$

Not large ...

See also Mainz:1911.05573

Conclusions

Lattice QCD continues slow but steady progress

Precision quantities, such as hadron ground-rates masses and decay constants continue to improve. Progress now is through inclusion of QED and d-u mass difference effects.

Harder calculations, such as those of exotic spectroscopy, semileptonic form factors, QCD effects on $\boldsymbol{a}_\mu$, are improving through competition between different groups.

The range of calculations is growing e.g. in baryon physics, not discussed here.