Utrecht University

EMMEΦ

Deterministic Building Blocks

for theories of particles and forces

Work under construction

arxiv:2005.06374 (updated version)

RECONNECT
Remote Conference on New Concepts in Particle Theory

Gerard 't Hooft

May 28, 2020

Utrecht University

EMMEΦ

Progress on

Deterministic Building Blocks

for theories of particles and forces

Work under construction

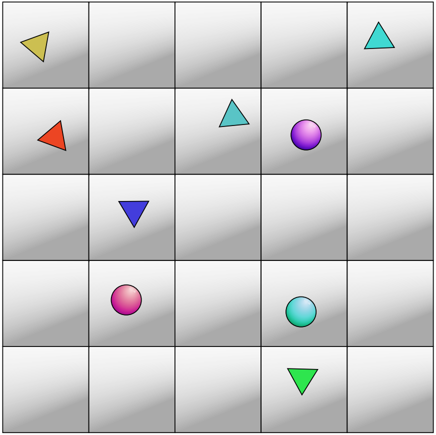
arxiv:2005.06374 (updated version)

RECONNECT
Remote Conference on New Concepts in Particle Theory

Gerard 't Hooft

May 28, 2020

The Cellular Automaton: Only *classical* evolution equations.



Quantum field lattice: same with *quantum* evolution equations.

Conjecture:

- Every cellular automaton is mathematically equivalent to a genuine quantum field theory on a lattice.
- Every lattice quantum field theory can be accurately approximated by a *classical* cellular automaton.

At *ultra-high energies*, the departure may be big, but the quantum theory will stay truly quantum $\rightarrow$ there may be predictions!

But we have the objection raised by Bell's theorem, and the puzzles posed by "paradoxes" such as Greenberger - Horne - Zeilinger (GHZ), etc.

To be discussed after the lecture:

These objections are philosophical. The arguments are known not to be infinitely accurate (just a "tiny loop hole"). And that's why they fail. The arguments in favor of this 'hidden variable' conjecture are much stronger.

I'll show you how they go.

Operators are arranged in the following classes:

- Beables,

  refer to things that are 'truly there'.

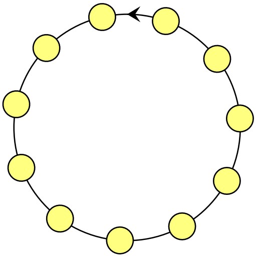
  All beable operators commute with one another, at all times.

- Changeables,

  transform beables into other beables,

- Superimposables,

  all other operators.



1. The periodic chain.

Ontological states:

$$|0\rangle, \ |1\rangle, \ \dots \ |N-1\rangle$$

Evolution law:

$$|k\rangle_{t+\delta t} = U(\delta t)\,|k\rangle_t$$

$$U(\delta t)|k\rangle = |k+1 \mod N\rangle$$

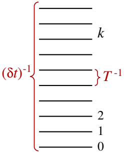
$$U(\delta t) = e^{-iH\,\delta t} \ , \quad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle$$

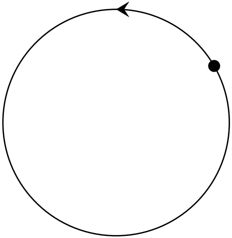
$$|n\rangle^E \stackrel{\text{def}}{=} \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi ikn/N} |k\rangle^{\text{ont}} \ , \qquad k = 0, \cdots, \ N-1 \ ;$$

$$n = 0, \cdots, \ N-1 \ .$$

$$|k\rangle^{\text{ont}} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} e^{-2\pi ikn/N} |n\rangle^E \ .$$

$$H = \frac{2\pi}{N\,\delta t}\, n = \omega n$$





2. The continuum limit.

Ontological states: $|\phi\rangle$

Evolution law:

$\frac{\mathrm{d}}{\mathrm{d}t}|\phi\rangle_t = \omega$

$$U(\delta t)|\phi\rangle = |\phi + \omega t\rangle$$

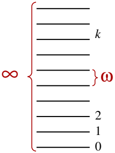
$$U(\delta t) = e^{-iH\,\delta t}\ , \qquad \frac{\mathrm{d}|\psi\rangle}{\mathrm{d}t} = -i\,H|\psi\rangle$$

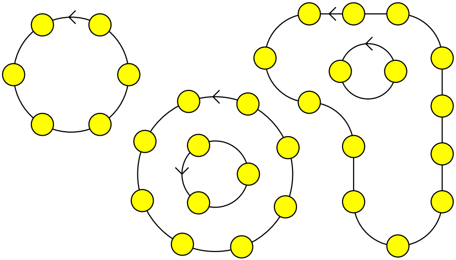
$$|n\rangle^E \stackrel{\mathrm{def}}{=} \frac{1}{\sqrt{2\pi}}\oint e^{i\phi n/N}|\phi\rangle^{\mathrm{ont}}\ , \qquad 0 \le \phi < 2\pi\,;$$

$$n = 0, \cdots,\ \infty\ .$$

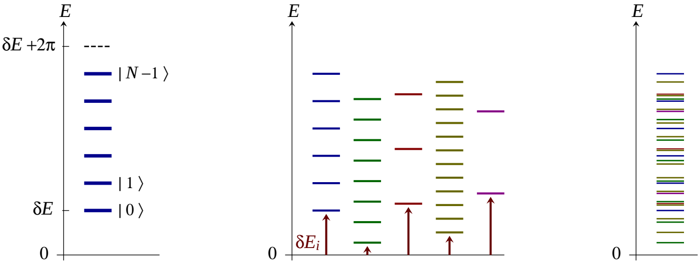
$$|\phi\rangle^{\mathrm{ont}} = \frac{1}{\sqrt{2\pi}}\sum_{n=0}^{\infty} e^{-i\phi n/N}|n\rangle^E\ .$$

We generate exactly the spectrum of the harmonic oscillator: $H = \omega\, n$





Finite,
deterministic,
time reversible
models



Since the time steps δt are discrete, . . .

the Hamiltonian is periodic in energy,

There are only N energy levels.

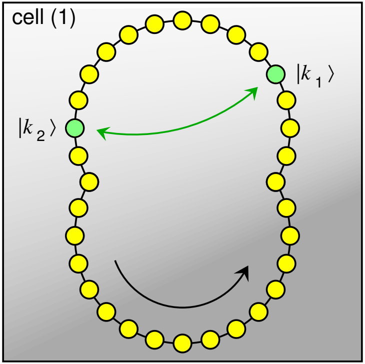
Therefore,

There is a lowest energy state ("vacuum state"), and there is *a highest energy state* ("anti-vacuum")

possibly important in black hole physics,
where the horizon flips over the time coordidate

Interactions

The ‘exchange interaction’



H contains $|k_1\rangle\langle k_2|$

Calculation:

$$H = p + \pi|\psi\rangle\langle\psi| \ ,$$

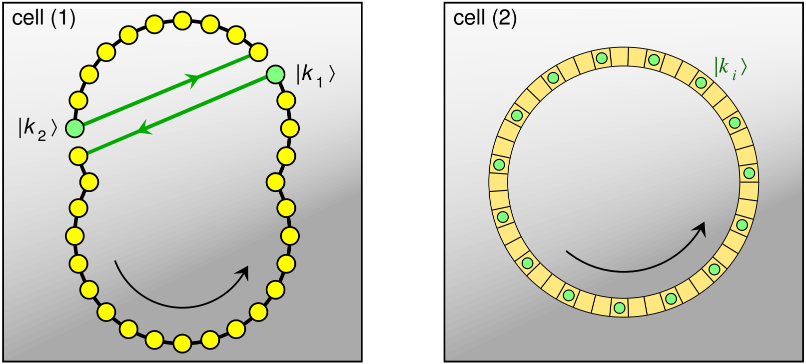
$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|k_1\rangle^{(1)} - |k_2\rangle^{(1)}\right)$$

footnotes:

1. easier in continuum limit.
2. Uses $e^{\pi i} = -1$.
3. Separates states odd and even in $k_1 \leftrightarrow k_2$.

But this does not (yet) give quantum mechanics . . .

Now let cell (2) act as a *sieve*



$$H = p^{(1)} + p^{(2)} + \pi|\psi\rangle^{(1)}\,\langle\psi|^{(1)}\,\left(\sum_i |k_i\rangle^{(2)}\,\langle k_i|^{(2)}\right)$$

We have $\quad H = p^{(1)} + p^{(2)} + \pi|\psi\rangle^{(1)}\,\langle\psi|^{(1)}\left(\sum_i |k_i\rangle^{(2)}\,\langle k_i|^{(2)}\right)$

$\sum_i |k_i\rangle^{(2)}\,\langle k_i|^{(2)}$ is a projection operator. Its vacuum expectation value is small; this operator only becomes sizeable if we include all very high energy states:

$$\left\langle\ \sum_i |k_i\rangle^{(2)}\,\langle k_i|^{(2)}\ \right\rangle_{\mathrm{vac}} = \alpha\ , \qquad |\alpha| \ll 1$$

So, at low energies, we get as effective interaction Hamiltonian:

$H^{\mathrm{int}} = \varepsilon|\psi\rangle\,\langle\psi|$ with

$$\varepsilon = \pi\alpha, \quad |\varepsilon| \ll 1\ ; \qquad |\psi\rangle = \tfrac{1}{\sqrt{2}}\big(|k_1^{(1)}\rangle - |k_2^{(1)}\rangle\big)\ .$$

And this is real quantum mechanics!

Easy questions:

Locality: $H = \sum_{\vec{x}} \mathcal{H}(\vec{x}\,)$

At fixed time t, when $\vec{x} \neq \vec{x}\,'$ (outside the light cone):

$$[\mathcal{H}(\vec{x}\,),\ \mathcal{H}(\vec{x}\,'\,)] = 0\ .$$

This means that the interchanges of beables must commute outside the light cone, which is easy to guarantee in classical models (classical locality)

Difficult questions, yet to be answered:

- Construct fundamental models
- Continuous symmetries. In particular
  Lorentz invariance (special relativity)
- Free particles (non-interacting quantum fields)
  Scalar fields, Dirac equation, . . .
- More general, more efficient procedures
- General relativity

On our program:

- *Construct the simplest cellular automaton that may represent free fermions or bosons* (This has not yet been done - very difficult)
- Find the simplest way to introduce interactions. (since we use the quantum mechanical language, *some* Hamiltonian will come out)
- Find the various continuum limits

Physical laws should in principle not depend on statistical regularities:

If you believe in determinism, you have to believe it all the way

$$=\approx= \infty =\approx=$$