(Re)Connecting Dark Matter, Inflation, CMB

Gravitational Particle Production in the Early Universe

RECONNECT 2020

My collaborators in aspects of this work:

Ivone Albuquerque, Daniel Chung, Patrick Crotty, Michael Fedderke, Gian Giudice, Lam Hui, Andrew Long, Toni Riotto, Leo Senatore, Alexi Starobinski, Igor Tkachev, Mark Wyman

Review paper in progress with Andrew Long

Rocky Kolb

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Gravitational Particle Production in the Early Universe

Edward W. Kolb[1] and Andrew J. Long[2]

[1]Kavli Institute for Cosmological Physics and Enrico Fermi Institute
The University of Chicago, 5640 S. Ellis Ave., Chicago, IL 60637
[2]Department of Physics and Astronomy, Rice University, Houston, Texas 77251

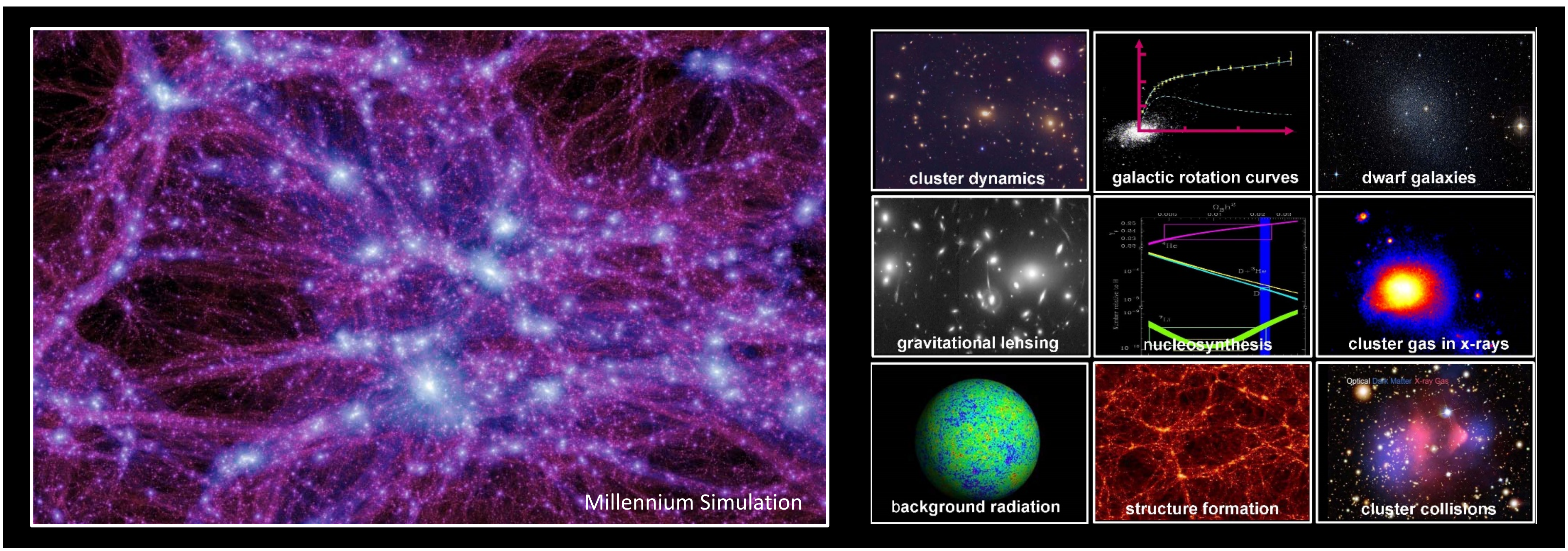
Abstract

We review early-universe production of particles by gravitational interactions. The primary emphasis of this review is production of particles solely by the expansion of the universe, with particular attention to the possibility that the particles could be the dark matter. For the evolution of the background spacetime we assume an initial inflationary phase, followed by a transition to a matter-dominated phase, then followed by the transition to a hot radiation-dominated phase. The two basic requirements for a particle to be produced by the expansion of the universe are 1) the contribution to the matter action from the particle must violate conformal invariance (the trace of the matter stress-energy tensor involving the new field must be nonzero), and 2) the mass of the particle must not be too much in excess of the expansion rate of the universe during inflation. We will treat particle production by expansion for particles of spin 0, 1/2, 1, 3/2, and 2. For each spin, we start with the Minkowski-spacetime action, promote it to a curved spacetime, specialize to a Robertson-Walker metric, and then calculate the spectrum of particles resulting from the expansion of the universe. We then turn to other mechanisms for cosmological particle production through gravity: particle production from the standard-model plasma through graviton exchange, particle production through black-hole evaporation, and particle production through a misalignment mechanism. Following the review of production mechanisms we summarize the criteria for the resulting density of particles to be sufficient to account for the dark matter, as well as several other cosmological implications. We also briefly discuss the implications of further endowing the particles with suppressed interactions with standard-model fields.

Gravitational Particle Production in the Early Universe

My astronomer friends tell me two important things!

1) There is dark matter in the universe today
   - It has gravitational interactions
   - Mass somewhere in the range $10^{-22}\,\mathrm{eV}$ to $30\,\mathrm{M}_\odot$
   - If associated with a particle, mass in range $10^{-22}\,\mathrm{eV}$ to $10^{19}\,\mathrm{GeV}$

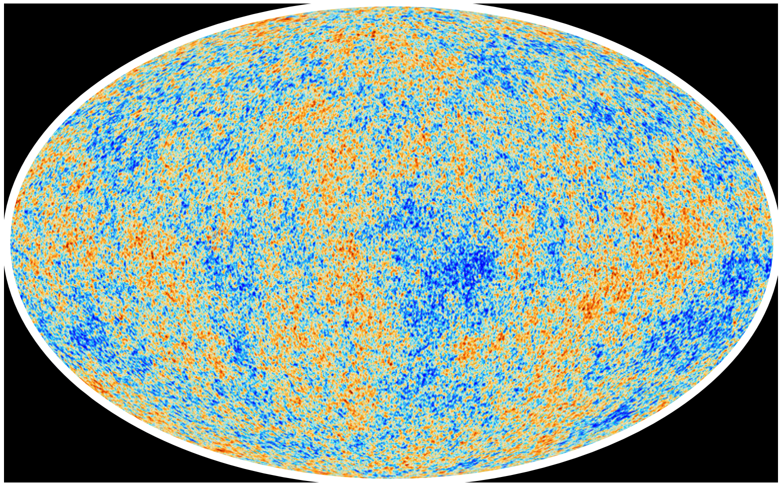


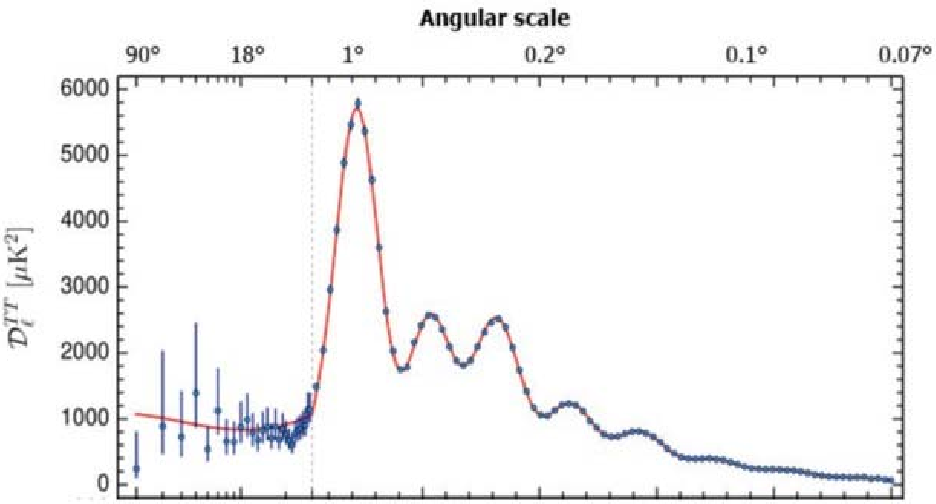
Gravitational Particle Production in the Early Universe

My astronomer friends tell me two important things!

2) Something like inflation occurred in the early universe

- Temperature anisotropies are small and as expected in inflation
- Perturbation spectrum is not Harrison-Zel'dovich ($n_S = 1$), but close to it
- No indications (yet) of an isocurvature component
- No indications (yet) of nongaussianities
- No indications (yet) of tensor mode, implies $H \lesssim 10^{14}\ \mathrm{GeV}$ during inflation





Motivations for Gravitational Particle Production

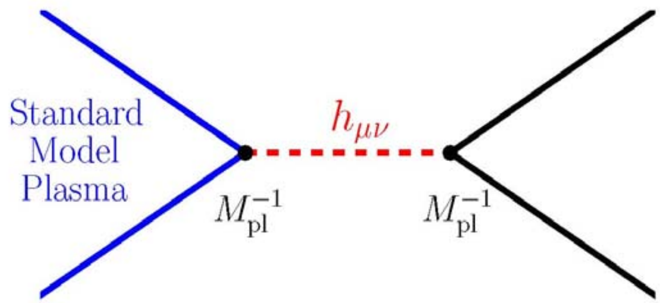
1. Dark Matter: What do we "know" about dark matter? **<u>If</u>** it is a particle it
   a. Is stable, or lifetime $\gg$ age of universe.
   b. Is massive.
   c. Has gravitational interactions.
   d. No compeling reason to go beyond CDM
      - Interacts feebly (if at all) through standard model processes.
      - (IMO) no compelling evidence for self-interactions.
   e. So far has cleverly evaded detection.

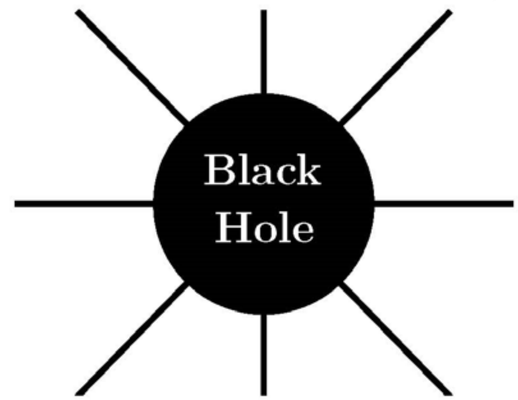
Which leads to the question: What if it has **<u>only</u>** gravitational interactions?
If so, gravity must be involved in production.

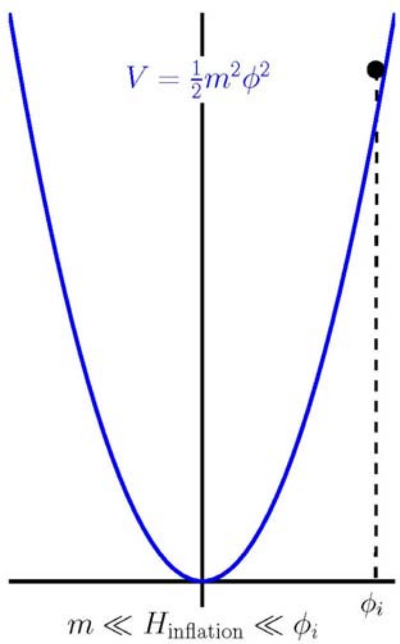
2. Even if not dark matter
   a. Could have late decays
   b. Decays could lead to interesting phenomenology.

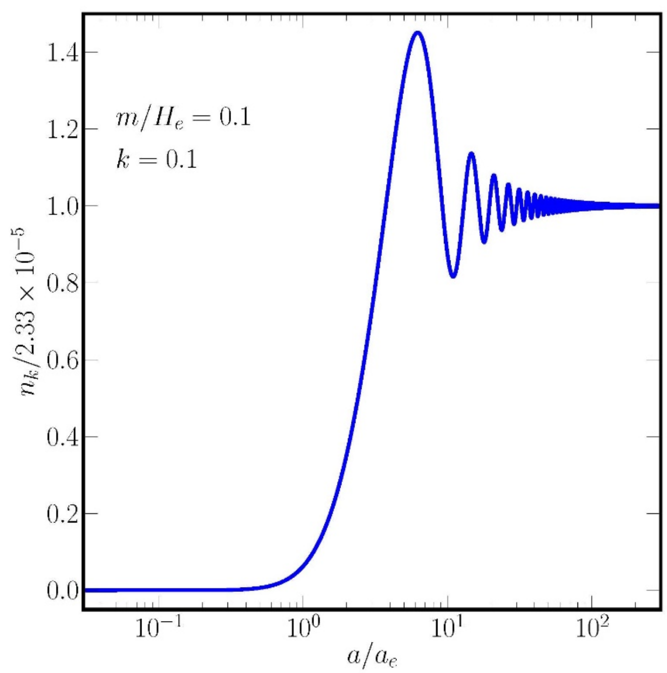
3. Can't hide from Gravity.
   a. Gravitational production is not optional.
   b. Might as well understand it.

Gravitational Particle Production in the Early Universe









Gravitational Particle Production in the Early Universe

The expansion of the universe creates particles*

$$H_{\text{crit}} = m \qquad\qquad \Gamma = \exp\left(-\pi\, H_{\text{crit}} / H\right)$$

Analogy: Schwinger effect (1951) (Sauter 1931; Heisenberg & Euler 1935; Weiskopf 1936)

$$\left|\vec{E}_{\text{crit}}\right| = \frac{m_e^2 c^3}{e\hbar} \approx 10^{16}\ \text{V cm}^{-1} \qquad\qquad \Gamma = \exp\left(-\pi \left|\vec{E}\right|_{\text{crit}} \Big/ \left|\vec{E}\right|\right)$$

Explore possibility that dark matter is produced during inflation through its gravitational interaction.

Assume standard QFT

Assume standard GR

Assume chaotic ϕ^2 inflation model

* If Lagrangian breaks Weyl conformal symmetry

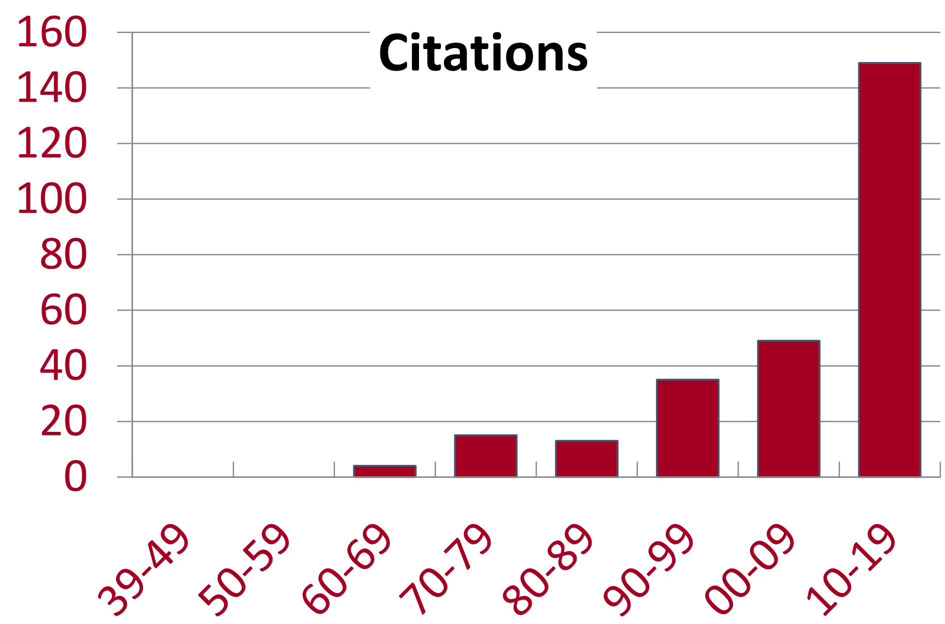
Gravitational Particle Production in the Early Universe

- 1939: Schrödinger, *The Proper Vibrations of the Early Universe*

"The decomposition of an arbitrary wave function into proper vibrations is rigorous. The two proper vibrations cannot be rigorously separated in the expanding universe. …. If in a certain moment only one of them is present, the other can turn up in the course of time.

Generally speaking this is a phenomenon of outstanding importance. With particles it would mean the production or annihilation of matter merely by expansion.

There will be a mutual adulteration of positive and negative frequency terms in the course of time, giving rise to what in the introduction I called the "alarming phenomenon."



Gravitational Particle Production in the Early Universe

- 1965: Leonard Parker's Thesis and papers following

  … for the early stages of a Friedmann expansion it [particle creation] may well be of great cosmological significance, especially since it seems inescapable if one accepts quantum field theory and general relativity.   (1968)

- 1968: Ya. B. Zel'dovich

  Looked for "great cosmological significance" in the isotropization of an anisotropic universe.

- 1983-84: Mukhanov; Sasaki; Kodama, ….

  Inflaton and graviton perturbations from inflation is "great cosmological significance."

- 1998-present: Chung, Kolb & Riotto; Kuzmin & Tkachev … many people

  Gravitational production may be origin of dark matter.

- 2005-present: Chung, Kolb, Riotto & Senatore … a few people

  Gravitational production may lead to curvature fluctuations detectable in CMB.

- 2013-present: Chung & Yoo … a couple of people

  Gravitational production may lead to nongaussianities detectable in CMB.

Scalars as Exemplars

Couple scalar field ϕ to gravity: $S = \int d^4x \sqrt{-g}\left[\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}m^2\phi^2 - \xi\mathcal{R}\phi^2\right]$

$\xi\mathcal{R}\phi^2$ is lowest-dimension non-minimal gravitational coupling

Flat FRW metric: $ds^2 = a^2(\eta)\left[d\eta^2 - d\vec{x}^2\right]$ conformally equivalent to Minkowski

Mode functions in conformal time satisfy ($a\phi \to \chi$): $\chi_k''(\eta) + \omega_k^2(\eta)\chi_k(\eta) = 0$

$$\omega_k^2(\eta) = \left|\vec{k}\right|^2 + a^2(\eta)m^2 + \tfrac{1}{6}(1-6\xi)a^2R$$

$\xi = 0$ (minimally coupled)
$\xi = 1/6$ (conformally coupled)

Wave equation with <u>time-dependent</u> mass, depending on the evolution in time of the scale factor $a(\eta)$.

Weyl conformal transformation of metric $g_{\mu\nu} \to e^{2\Omega(x)} g_{\mu\nu}$ leads to

$\delta S_M = \int d^4x\sqrt{-g}\, T^{\mu\nu}\,\delta g_{\mu\nu} = \int d^4x\sqrt{-g}\;\Omega(x)\, T^\mu{}_\mu$ Have to break Weyl invariance: $T^\mu{}_\mu \neq 0$

For scalar field: $T^\mu{}_\mu \supset m$ and $(\xi - 1/6)$

Noninteracting (Spectator) Fields

- Assume a particle with no SM interactions and no direct coupling to inflaton.
- Assume it is not dynamically important in determining expansion rate of the universe.
- It is a "spectator" field.
- Specify coupling to gravity

  Minimal $S=\int d^4x\sqrt{-g}\ \mathcal{L}\,(\eta_{\mu\nu}\rightarrow g_{\mu\nu}\,;\,\partial_\mu\ \rightarrow D_\mu$; fermions: tetrads, spin connection, etc.)

  Non-minimal $\mathcal{L}\ \supset$ relevant operators $R\,\phi^2$; $R\,g^{\mu\nu}\,A_\mu A_\nu$; $R^{\mu\nu}\,A_\mu A_\nu$

  (ignore irrelevant operators like $R\ \psi\,\psi$)

- Conformal invariance: flat FRW $g_{\mu\nu}=\ a^2(\eta)\ \eta_{\mu\nu}$, is conformally equivalent to Minkowski.

  If $\mathcal{L}_{\text{matter}}$ unchanged under Weyl rescaling $g_{\mu\nu}\rightarrow e^{\,\Lambda(x)}\,g_{\mu\nu}$, action for spectator field doesn't feel expansion.

  Weyl invariance $\rightarrow$ stress-tensor traceless $\rightarrow$ no particle production; e.g., Maxwell action.

Schrödinger's Alarming Phenomenon

Solutions to wave equation include both + and – frequency terms

$$\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}} e^{-i\int \omega_k(\eta) d\eta} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}} e^{+i\int \omega_k(\eta) d\eta}$$

Pure outgoing (+ frequency) is a good solution if

$$A_k = \left|\partial_\eta \omega_k(\eta) / \omega_k^2(\eta)\right| \ll 1$$

Adiabaticity parameter

Abrupt changes in *$a(\eta)$* leads to nonadiabatic changes in $\omega_k(\eta)$, which *adulterates* positive and negative frequency modes, leading to *Schrödinger's Alarming Phenomenon* of particle creation in the expanding universe.

Comoving number density of particles at late time is

$$n\,a^3 = \int \frac{dk}{k} \boxed{\frac{k^3}{2\pi^2}\left|\beta_k(\eta)\right|^2} = \int \frac{dk}{k} \boxed{n_k}$$

Program:

- Assume adiabatic initial and final conditions (true for inflation and matter dom.)
- Solve wave equation with initial conditions $\quad \left|\alpha_k\right|^2 = 1, \quad \left|\beta_k\right|^2 = 0.$
- At late-times (adiabatic conditions) $\quad \left|\beta_k\right|^2 = \frac{\omega_k}{2}\left|\chi_k\right|^2 + \frac{1}{2}\omega_k\left|\partial_\eta \chi_k\right|^2 - \frac{1}{2}$

Inflaton Mass May Represent New Mass Scale

Mass

Perhaps there are other fields with mass $\sim m_{\text{inflaton}} \sim H$

10^{12} ? GeV Inflaton: Schrödinger's alarming phenomenon $\rightarrow$ curvature fluctuations

They would also participate in Schrödinger's alarming phenomenon

(Twelve-Step) Program to Calculate Present Number Density

For massive fields of spin 0, $1/2$, 1, $3/2$, and 2

1. Promote classical Minkowski action to curved space. Include relevant operators coupling field to curvature scalar and tensor.
2. Derive classical field equation(s) of motion in FRW background.
3. Derive classical (Belinfante-Rosenfeld) stress-energy tensor and calculate $\rho = T^0{}_0$.
4. Express action in conformal time and define a commoving field to have canonical kinetic term.
5. Derive field equations (with time-dependent mass) for comoving field.
6. Promote classical theory to quantum theory and expand field in positive and negative frequency mode functions.
7. Derive evolution equations (in conformal time) for mode functions.
8. Express stress tensor in terms of mode functions and find $\rho = \left\langle 0 \right| : \hat{T}^0{}_0 : \left| 0 \right\rangle$.
9. Set initial conditions of pure positive-frequency modes (Bunch—Davies).
10. Solve mode equations Particle creation corresponds to evolution mixing positive and negative frequency terms.
11. Calculate late-time value of $\left\langle 0 \right| : \hat{T}^0{}_0 : \left| 0 \right\rangle$ for each mode, yielding $n_k\, a^3$.
12. Integrate over all modes and determine late-time number density of particles created by expansion, yielding $n\, a^3$.

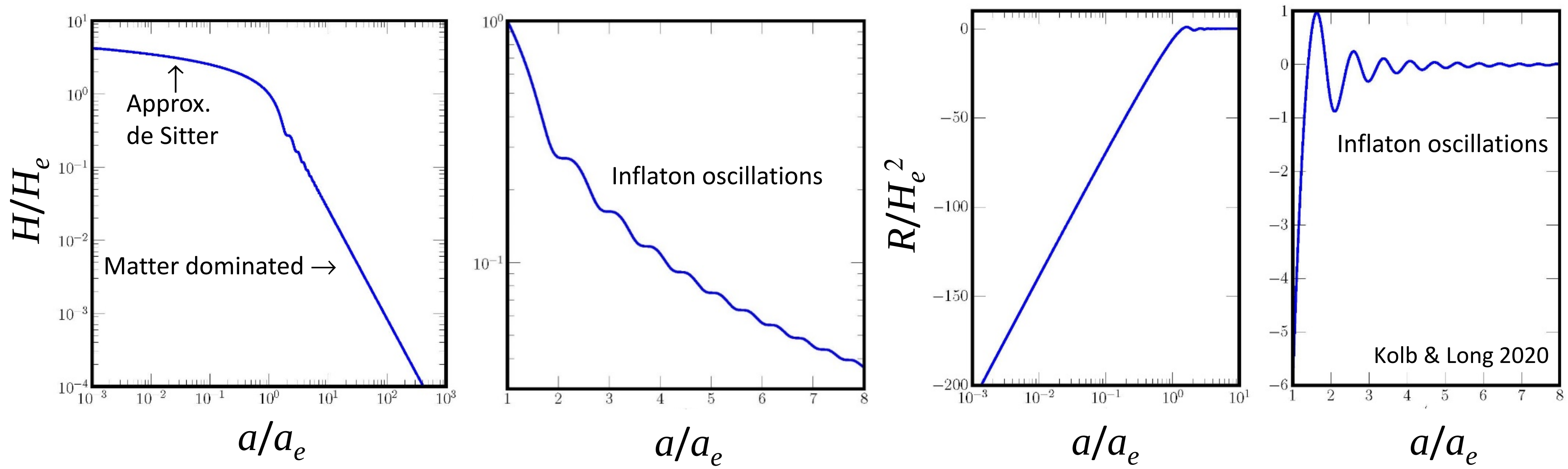
Background Geometry: Inflation $\rightarrow$ Matter $\rightarrow$ Radiation

- Exact de Sitter $\rightarrow$ Matter dominated (inflation ends at $a = a_e$ with expansion rate H_e):

$$H = H_e \qquad\qquad H = H_e\left(\frac{a_e}{a}\right)^{3/2}$$

$$\frac{R}{H_e^2} = -12 \qquad\qquad \frac{R}{H_e^2} = -3\left(\frac{a_e}{a}\right)^3$$

- Assume chaotic inflation: $V(\varphi) = ½\, \mu^2\varphi^2$



- Other inflation models have been studied (Chung, Crotty, Kolb, Riotto Phys. Rev. D64). Basic idea robust.
- Many more inflation models out there.

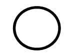
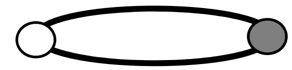
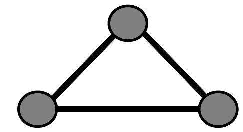
One- Two- and Three-Point Functions

One-point function $\rightarrow$ number density

Two-point function $\rightarrow$ isocurvature

Three-point function $\rightarrow$ nongaussianities

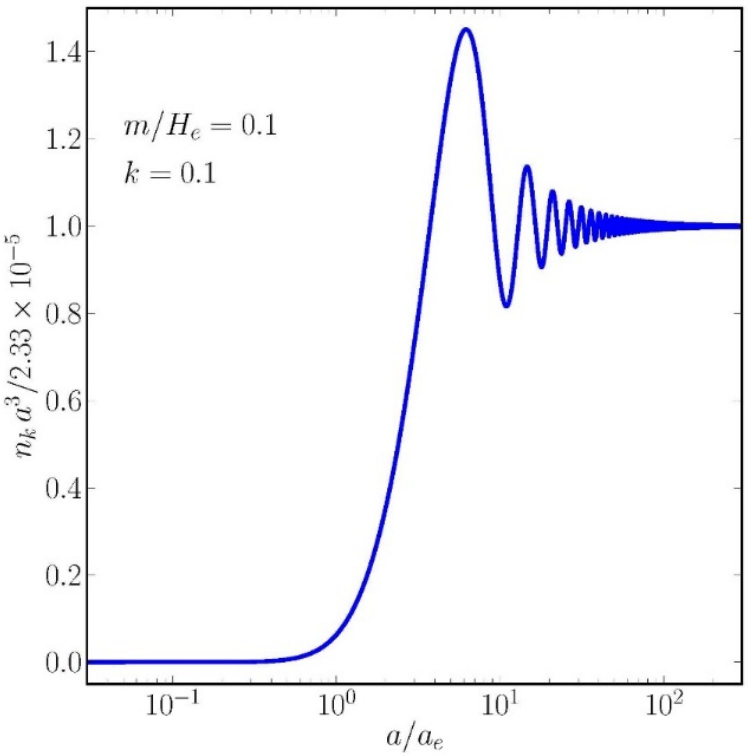
Complexity ↓

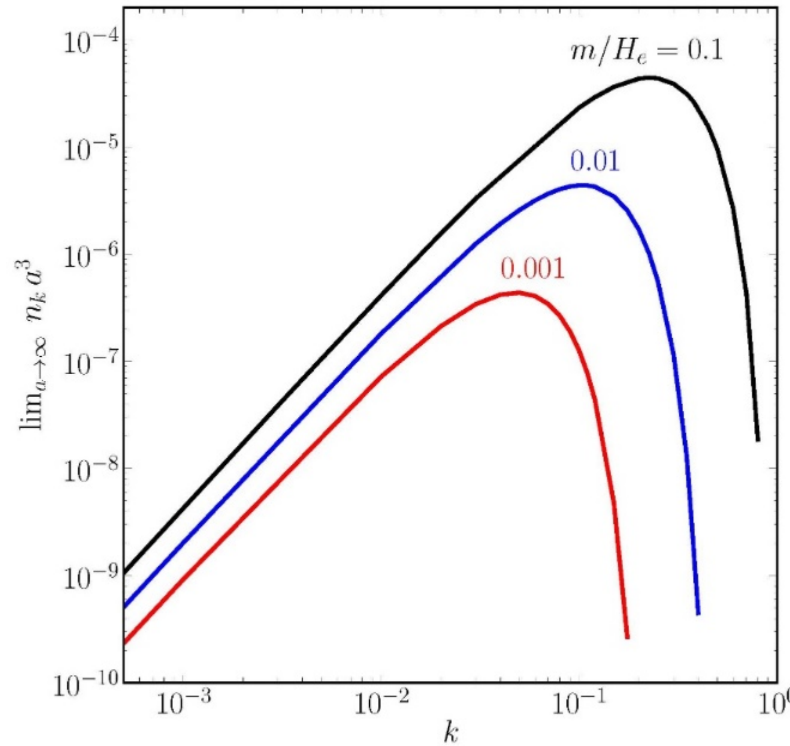
*** related to but not cosmological collider program**	**1-pt function** 	**2-pt function** 	**3-pt function*** 
Observable	**Dark Matter**	**Isocurvature Fluctuations**	**CMB Non-Gaussianities**
Massive scalar field (conformal)	Kuzmin & Tkachev (99)	Expected to be very small	Chung & Yoo
Massive scalar field (minimal)	Kuzmin & Tkachev (99)	Chung, Kolb, Riotto & Senatore	
Massive Dirac field	Chung, Kolb & Riotto (98, 99)	Similar to conformal scalar	Similar to conformal scalar?
Proca-de Broglie field	Massive: Kolb & Long (in progress) Light: Graham, Mardon & Rajendran		
Massive Rarita-Schwinger field	Several people gravitini, e.g., Giudice, Riotto & Tkachev; Kallosh, Kofman, Linde & Van Proeyn		
Massive Fierz-Pauli field	Kolb & Long (in progress)		

Complexity →

Spin 0: ϕ with $\xi = 1/6$

$$\omega_k^2(\eta) = k^2 + a^2(\eta) m^2$$





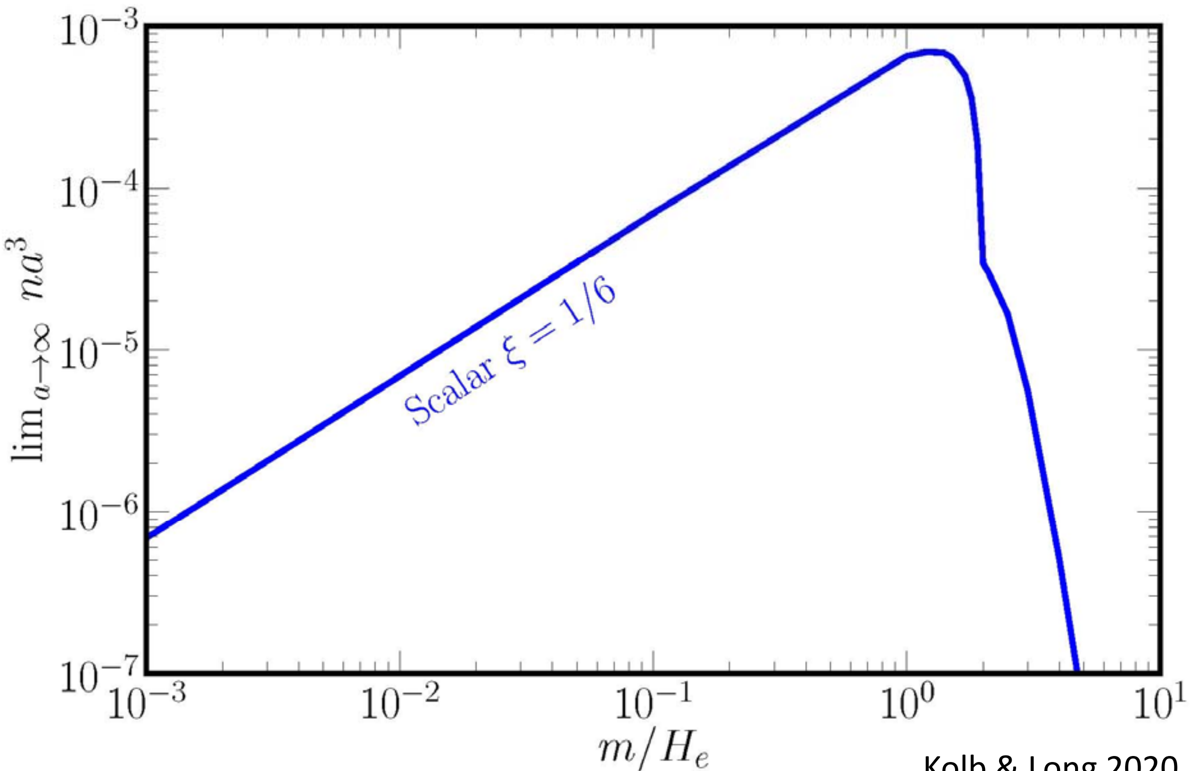
Images from
Kolb & Long 2020

- Adiabaticity parameter: $A_k = \dfrac{a^3 H m^2}{\left(k^2 + a^2 m^2\right)^{3/2}}$
- Most nonadiabatic at end of inflation ($m/H_e > 1$), or just into MD era ($m/H_e < 1$)
- Suppression in spectrum at $k > 1$
- Suppression in spectrum for $m/H_e > 1$

Final Number Density Spin 0: ϕ with $\xi = 1/6$

- Comoving abundance $n\,a^3$ calculable in terms of m/H_e.
- Translation of comoving abundance to present mass density has additional dependence on H_e & T_{RH}.

$$H_e \leq 3\times 10^{14}\ \mathrm{GeV}$$
$$T_{\mathrm{RH}} \geq 10^{2}\ \mathrm{GeV}$$



Kolb & Long 2020

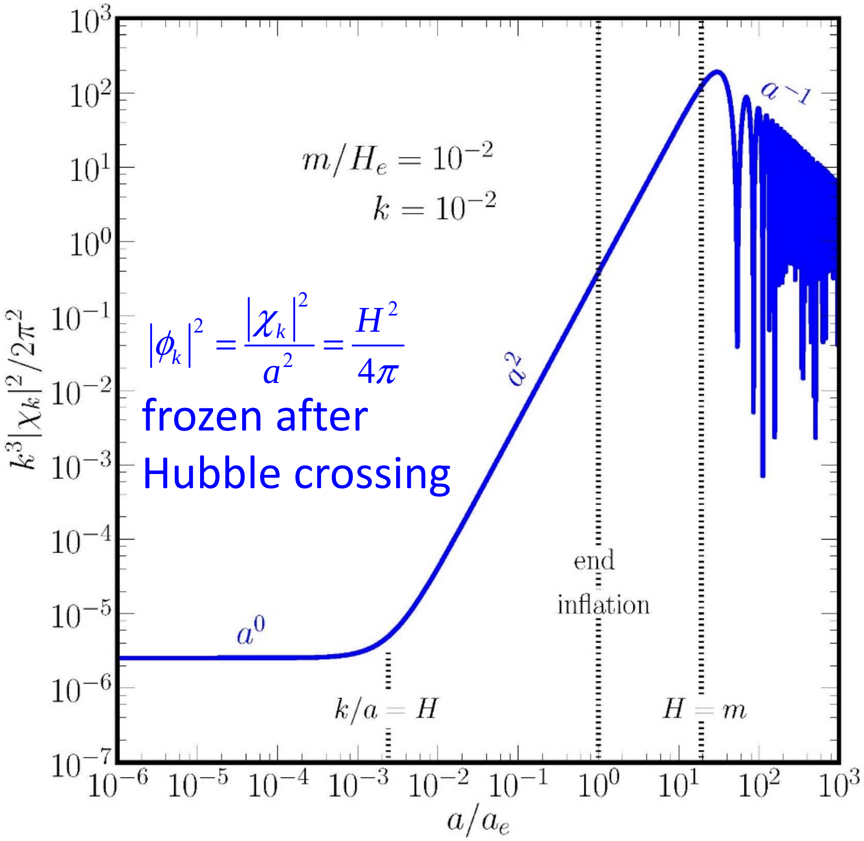
- For $m > H_e$, $n\,a^3 \propto e^{-c\,m/He}$
- For $m < H_e$, $n\,a^3 \propto m^{+1}$ conformally coupled

$$\frac{\Omega h^2}{0.12} = \frac{m}{H_e}\left(\frac{H_e}{10^{12}\,\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^{9}\,\mathrm{GeV}}\right)\left[\frac{n\,a^3}{10^{-5}}\right]$$

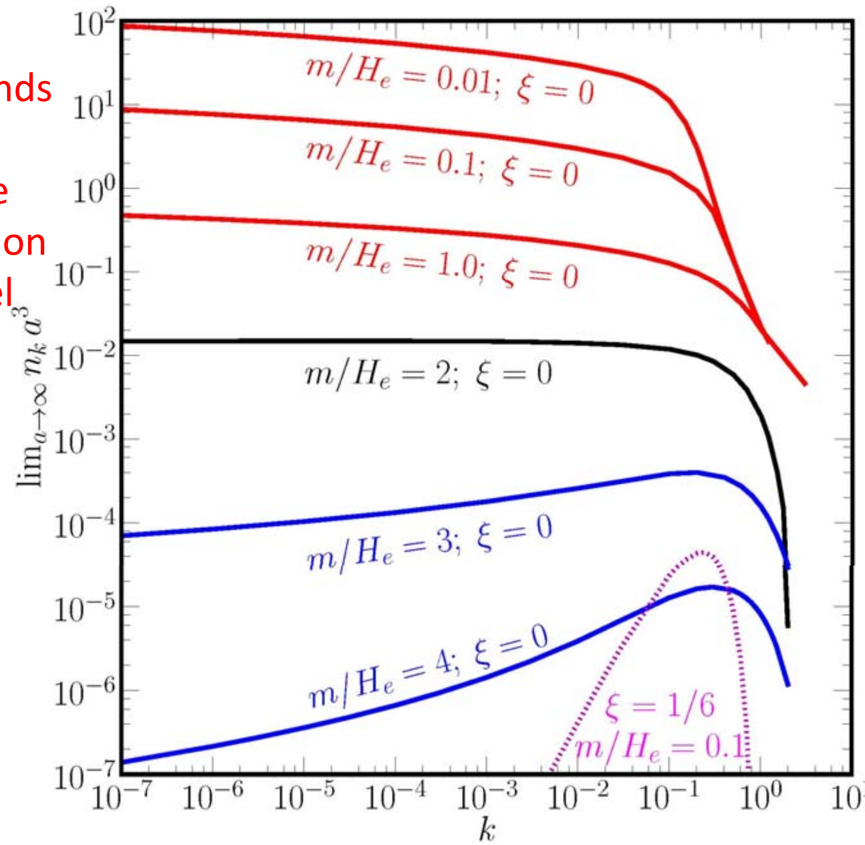
- If DM candidate, require large H_e and large T_{RH}

Spin 0: ϕ with $\xi = 0$

$\omega_k^2(\eta) = k^2 + a^2(\eta)m^2 + \frac{1}{6}a^2(\eta)R(\eta) \approx k^2 + a^2(\eta)m^2 - 2a^2(\eta)H_e^2$ (in de Sitter, $R \sim -H^2$)



Slope depends on H, hence inflation model



- Nonadiabatic deep in inflation as mode becomes tachyonic
- *Irruption* when tachyonic: $k = aH$
- Suppression in spectrum at $k > 1$ and $m/H_e > 1$
- Spectrum diverges in IR for $m/H_e < 2 \rightarrow$ isocurvature issues

Spin 0: ϕ with $\xi = 0$

$$\chi_k''(\eta) + \omega_k^2(\eta)\chi_k(\eta) = 0 \qquad \omega_k^2(\eta) = k^2 + a^2(\eta)m^2 + \tfrac{1}{6}a^2(\eta)R(\eta)$$

de Sitter $(-\infty < \eta < 0)$

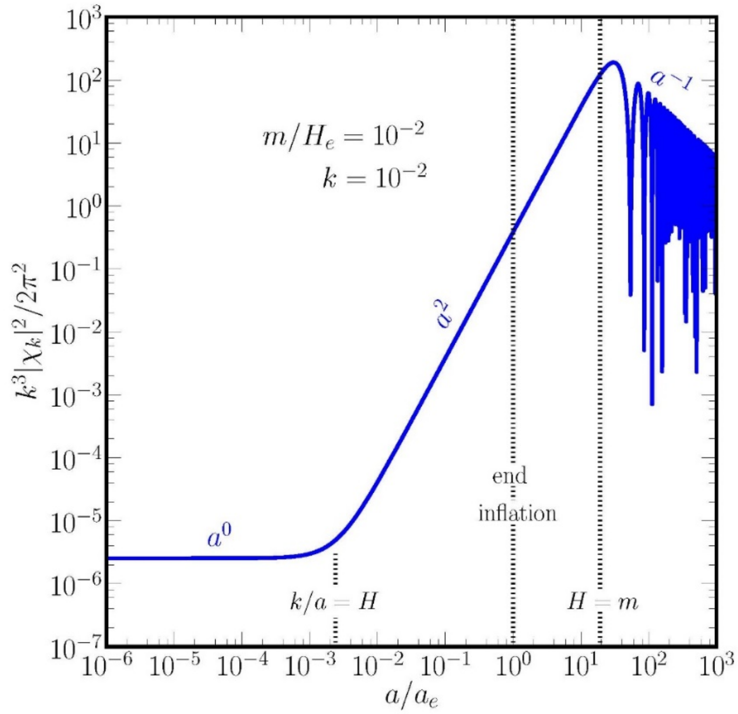
$$a = \frac{a_e}{1-\eta} \qquad R = -12H_e^2$$

k^2 dominates: $|\chi_k|^2 \propto \eta^0 \propto a^0$
m^2 dominates: $|\chi_k|^2 \propto \eta^1 \propto a^{-1}$
R dominates: $|\chi_k|^2 \propto \eta^{-2} \propto a^2$

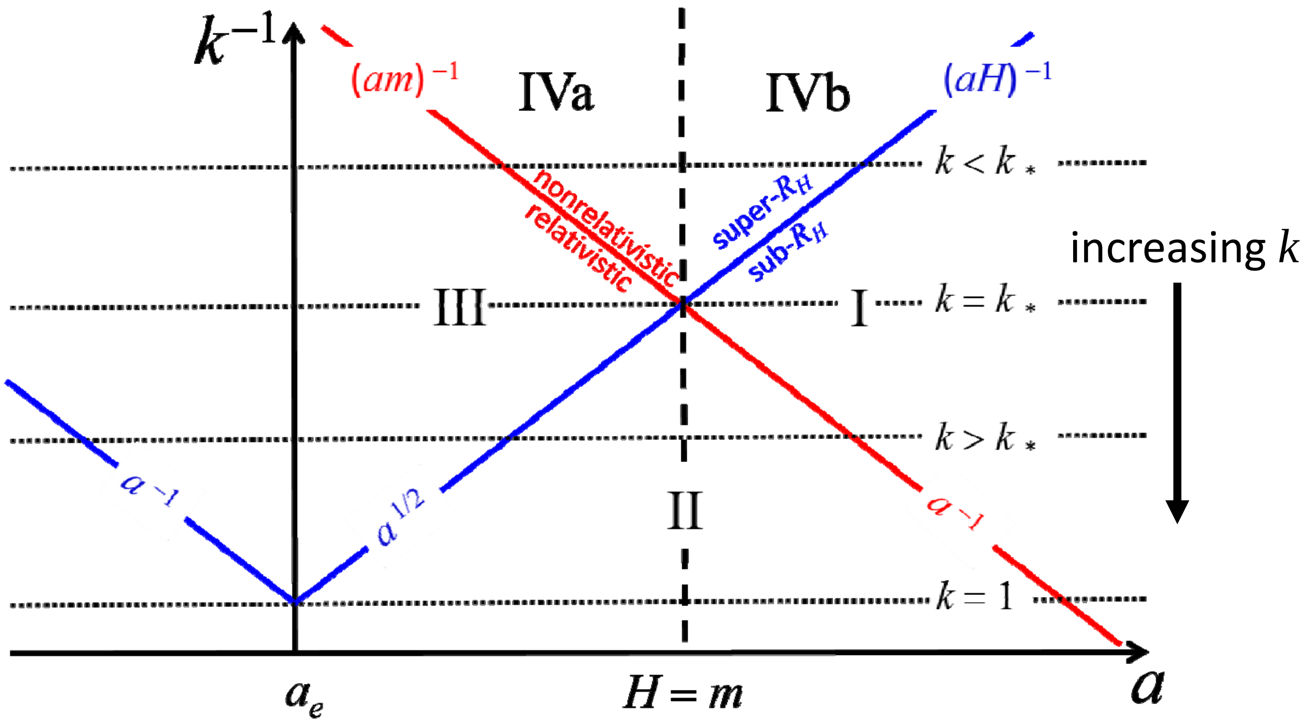
Matter Dominated $(0 < \eta < \infty)$

$$a = \left(1 + \tfrac{1}{2}\eta\right)^2 \qquad R = -\frac{3H_e^2}{\left(1+\frac{1}{2}\eta\right)^6} = -\frac{3H_e^2}{\left(a/a_e\right)^3} = -3H^2$$

k^2 dominates: $|\chi_k|^2 \propto \eta^0 \propto a^0$
m^2 dominates: $|\chi_k|^2 \propto \eta^{-2} \propto a^{-1}$
R dominates: $|\chi_k|^2 \propto \eta^4 \propto a^2$



$$\rho_k \sim m^2\phi_k^2 \propto \frac{|\chi_k|^2}{a^2}$$



Complexities:

- Evolution not exactly de Sitter or matter-dominated, esp. at end of inflation.
- R oscillates around end of inflation (inflation model dependent).
- Transition to radiation-dominated, $(aH)^{-1} \propto a$.
- Energy density also contains a term $|\partial_\eta \chi_k|^2$.
- R and H grow as $\eta \to -\infty$.

$$\omega_k^2(\eta) = k^2 + a^2(\eta) m^2 - \tfrac{1}{2} a^2(\eta) H^2(\eta)$$

Region	**Relativistic/ Nonrelativistic**	**Super/Sub Hubble Radius**	$H > m$ $H < m$	**Dominant term in** ω_k^2	**Evolution of** $\lvert\chi_k\rvert^2$	$\dfrac{\lvert\chi_k\rvert^2}{a^2}$
I	NR $(k/a < m)$	Sub $(k/a > H)$		$a^2 m^2$	a^{-1}	a^{-3}
II	R $(k/a > m)$	Sub $(k/a > H)$		k^2	a^0	a^{-2}
III	R $(k/a > m)$	Super $(k/a < H)$		$a^2 R$	a^2	a^0
IVa	NR $(k/a < m)$	Super $(k/a < H)$	$>$	$a^2 R$	a^2	a^0
IVb	NR $(k/a < m)$	Super $(k/a < H)$	$<$	$a^2 m^2$	a^{-1}	a^{-3}

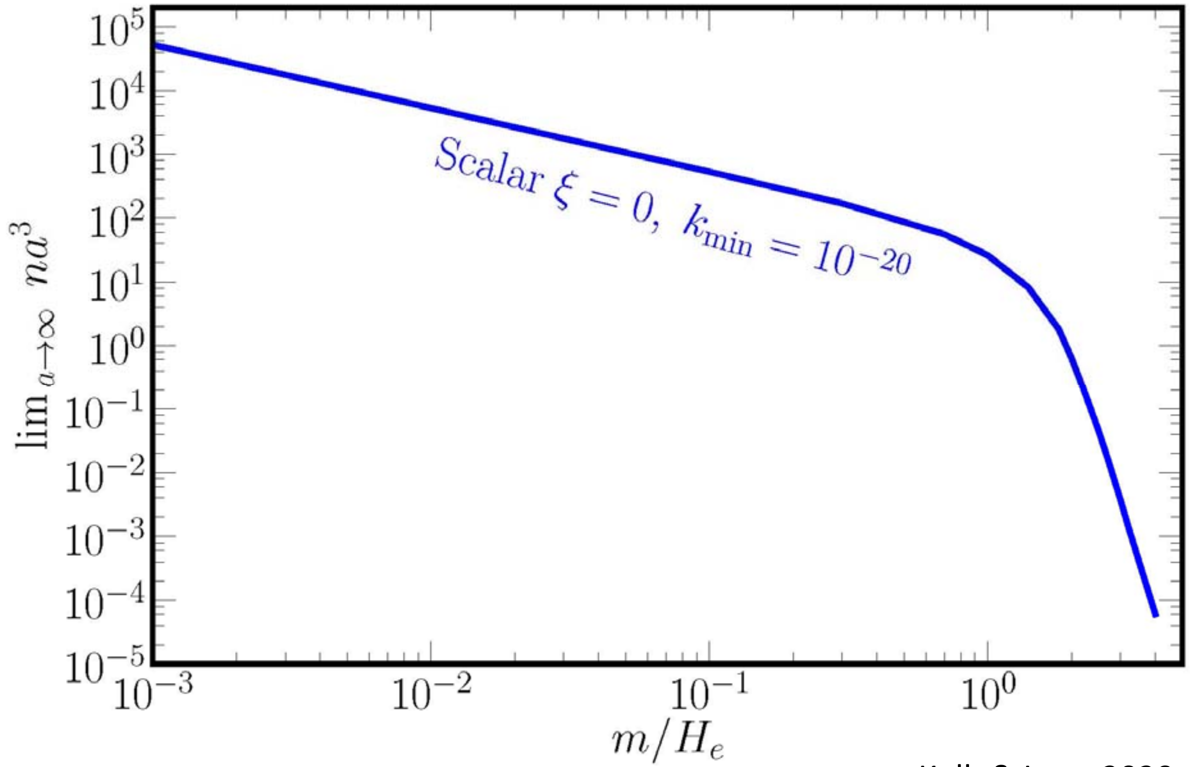
Final Number Density Spin 0: ϕ with $\xi = 0$

- Comoving abundance $n\,a^3$ calculable in terms of m/H_e.

- Translation of commoving abundance to present mass density has additional dependence on H_e & T_{RH}.

$$H_e \leq 3\times 10^{14}\ \mathrm{GeV}$$

$$T_{\mathrm{RH}} \geq 10^2\ \mathrm{GeV}$$



Kolb & Long 2020

- For $m > H_e$, $n\,a^3 \propto e^{-c\,m/He}$

- For $m < H_e$, $n\,a^3 \propto m^{-1}$ minimally coupled

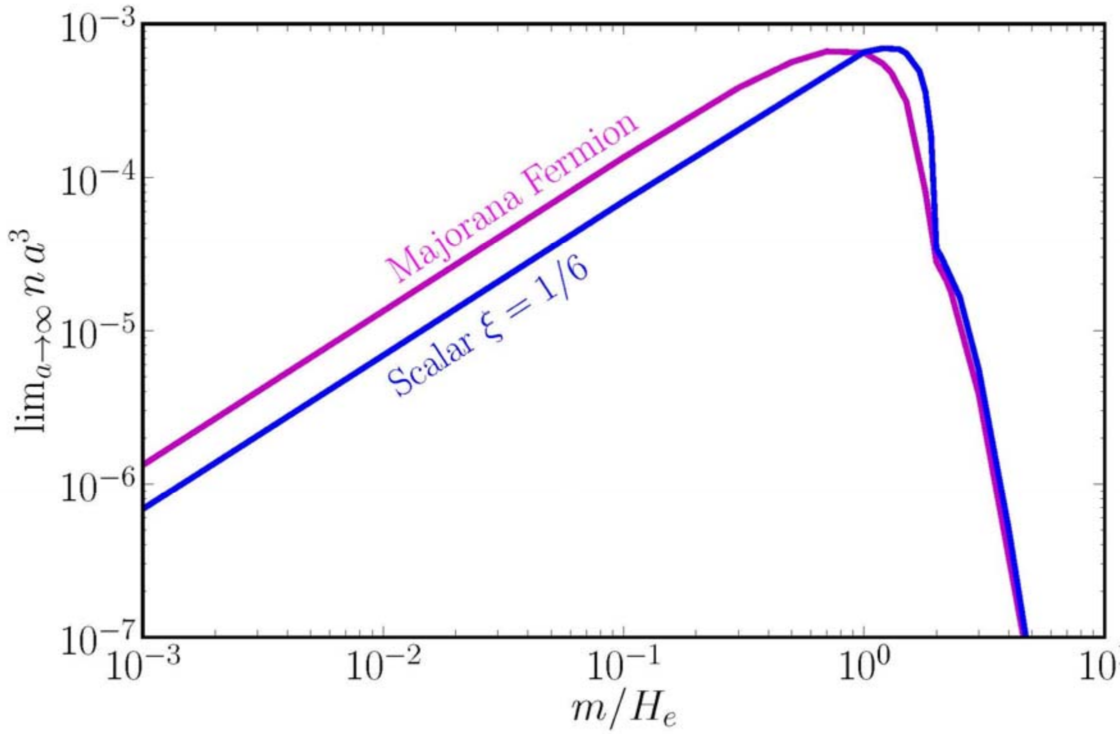
$$\frac{\Omega h^2}{0.12} = \frac{m}{H_e}\left(\frac{H_e}{10^{12}\,\mathrm{GeV}}\right)^2\left(\frac{T_{\mathrm{RH}}}{10^9\,\mathrm{GeV}}\right)\left[\frac{n\,a^3}{10^{-5}}\right]$$

- If DM candidate, require large H_e and large T_{RH}

Spin ½: ψ

- Weyl invariant in massless limit ($T^{\mu}{}_{\mu} = m\bar{\psi}\psi$)
- No relevant nonminimal operators
- Resembles conformally-coupled scalars
- Adiabaticity parameter:

$$A_k = \frac{a^2 Hmk}{\left(k^2 + a^2 m^2\right)^{3/2}}$$
$$= (k/m \times \text{conformal scalar})$$



$$\frac{\Omega h^2}{0.12} = \frac{m}{H_e}\left(\frac{H_e}{10^{12}\,\text{GeV}}\right)^2 \left(\frac{T_{\text{RH}}}{10^{9}\,\text{GeV}}\right) \frac{\lim_{a\to\infty} n\, a^3}{10^{-5}}$$

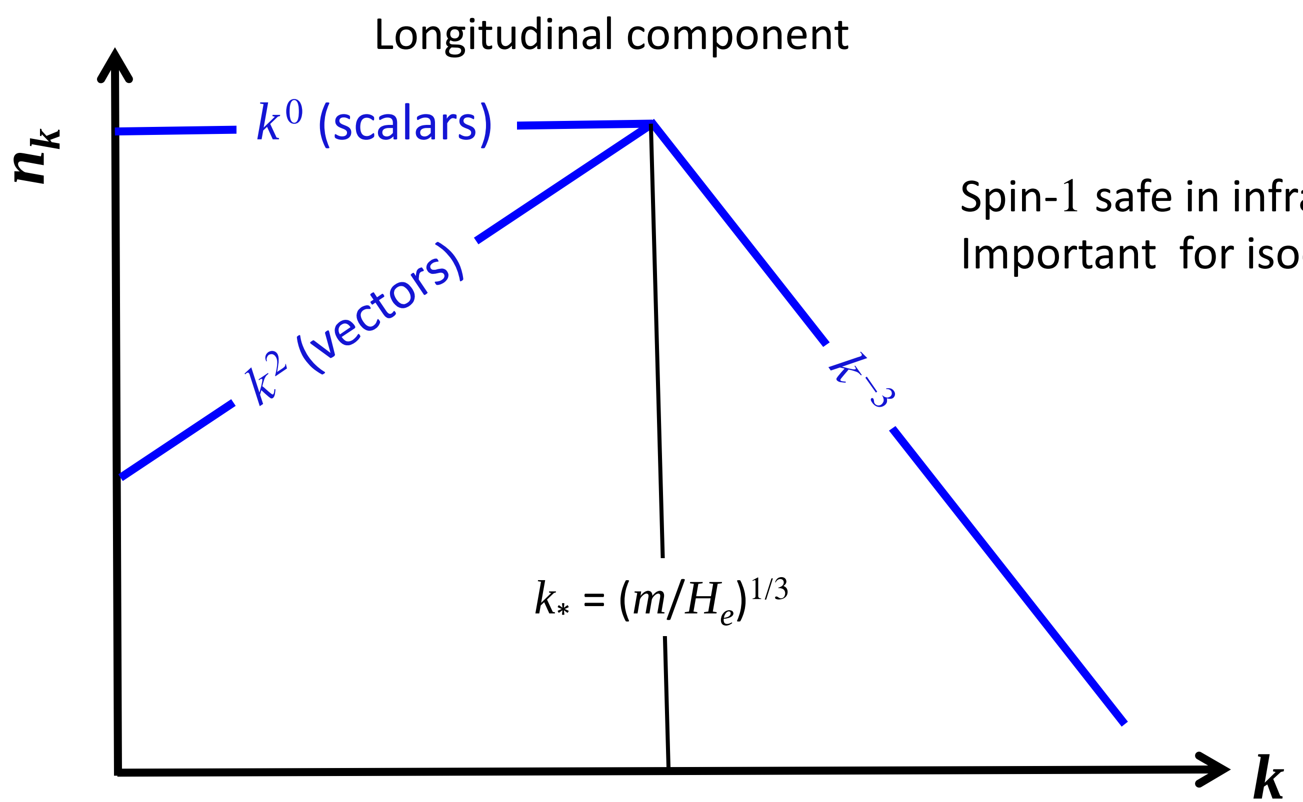
Spin 1: A^μ

- (Optional) start with Steuckelberg Lagrangian (Abelian Higgs model). Integrate out scalar. In $\lim g \to 0,\ v \to \infty;\ g\,v \to$ const. Steuckelberg leads to …
- … de Broglie-Proca Lagrangian for massive spin-1 field.
- Mass term: $m^2 g^{\mu\,\nu} A_\mu A_\nu \to m^2\, a^{-2} A_i^2$ (cf. $m^2 \phi^2$ for scalars) .
- A^0 is auxiliary field.
- Two possible non-minimal couplings: $\xi_1\, R\, g^{\mu\nu}\, A_\mu A_\nu\,;\ \ \xi_2\, R^{\mu\nu}\, A_\mu A_\nu\,.$
- Decompose field into transverse and longitudinal modes.
- Transverse modes have exact action of conformally-coupled scalar.
- Longitudinal mode more complicated, but resembles minimally-coupled scalars (light-mass interesting: Graham, Mardon, Rajendran).
- In relativistic limit longitudinal mode decouples (Goldstone boson equivalence theorem).

Spin 1: A^μ

3 DOF:
- 2 Transverse A^T $\qquad \omega_T^2 = k^2 + a^2 m^2$
- 1 Longitudinal $A^L = \sqrt{\frac{k^2 + a^2 m^2}{a^2 m^2}}\, \phi^L$ $\qquad \omega_L^2 = k^2 + a^2 m^2 + 3\frac{k^2 a^4 H^2 m^2}{\left(k^2 + a^2 m^2\right)^2} + \frac{1}{6}\frac{k^2 a^2 R}{k^2 + a^2 m^2}$

(kinetic term of longitudinal mode can be ghostly)

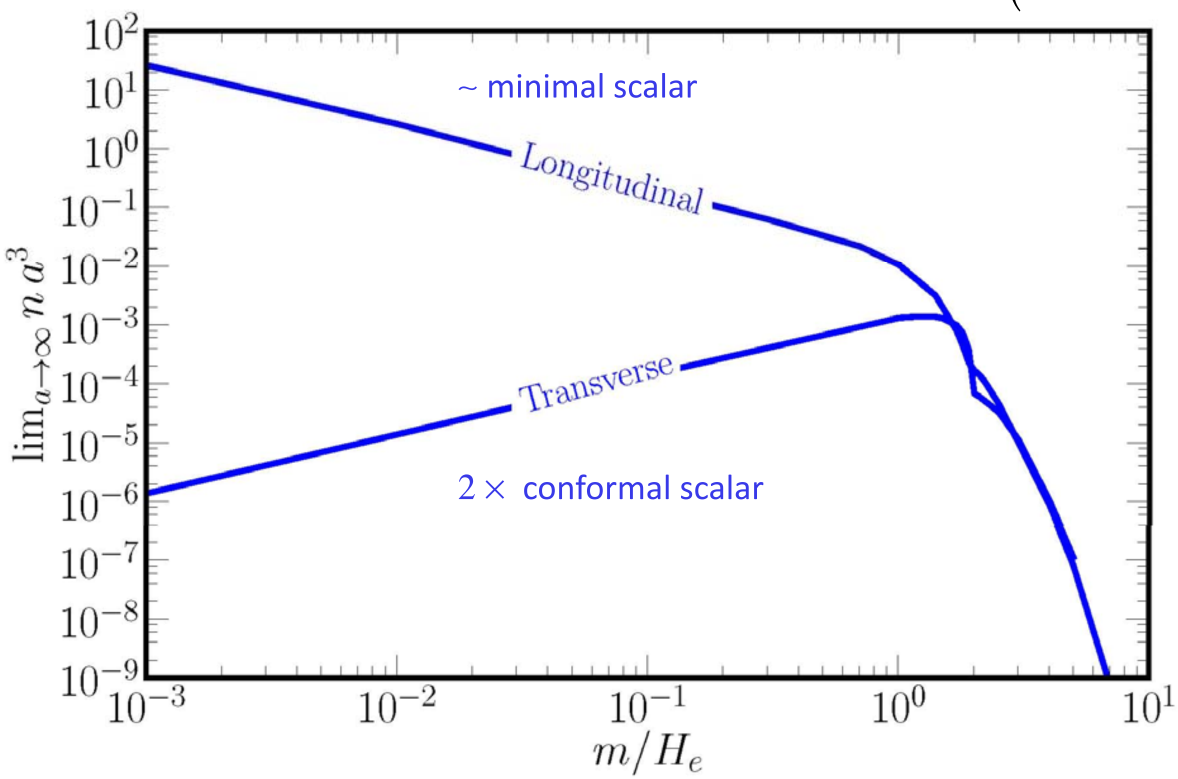


Spin-1 safe in infrared for low mass.
Important for isocurvature constraints.

Graham, Mardon, Rajendran

Final Number Density Spin 1: A^μ

3 DOF:
- 2 Transverse $\quad A^T \qquad \omega_T^2 = k^2 + a^2 m^2$
- 1 Longitudinal $\quad A^L = \sqrt{\dfrac{k^2 + a^2 m^2}{a^2 m^2}}\,\phi^L \qquad \omega_L^2 = k^2 + a^2 m^2 + 3\dfrac{k^2 a^4 H^2 m^2}{\left(k^2 + a^2 m^2\right)^2} + \dfrac{1}{6}\dfrac{k^2 a^2 R}{k^2 + a^2 m^2}$



$$\frac{\Omega h^2}{0.12} = \begin{cases} 10^3 \left(\dfrac{H_e}{10^{12}\ \mathrm{GeV}}\right)^2 \left(\dfrac{T_{\mathrm{RH}}}{10^9\ \mathrm{GeV}}\right) & m > 3\ \mathrm{GeV}\dfrac{T_{\mathrm{RH}}}{10^9\ \mathrm{GeV}} \\ 10^{-1} \left(\dfrac{H_e}{10^{12}\ \mathrm{GeV}}\right)^2 \left(\dfrac{m}{\mathrm{eV}}\right)^{1/2} & m < 3\ \mathrm{GeV}\dfrac{T_{\mathrm{RH}}}{10^9\ \mathrm{GeV}} \end{cases}$$

Spin 3/2: Ψ_μ

- Useful references: Giudice, Riotto & Tkachev and Kallosh, Kofman, Linde, Van Proeyen
- Start with Rarita-Schwinger action.
- Constraint equations ($\gamma^\mu \psi_\mu = 0$ and $\eta^{\mu\nu} \partial_\mu \psi_\nu = 0$) $\rightarrow$ constraint equation for ψ_0.
- Two propagating degrees of freedom with polarizations 3/2 and 1/2
- Mass term: $\frac{1}{2} m \bar{\psi}_\mu \left(\underline{\gamma}^\mu \underline{\gamma}^\sigma - \underline{\gamma}^\sigma \underline{\gamma}^\mu \right) \psi_\sigma$ with $\underline{\gamma}^\mu = e_a{}^\mu \gamma^a$
- Needn't consider non-minimal couplings.
- Trace of stress tensor has term proportional to m and Hubble-induced background term.
- Define $\chi_\mu = a^{1/2} \psi_\mu$
- Field equations for Fourier modes:
  C_A & C_B complicated functions of m, H, R

$$\left[i\gamma^0 \partial_\eta - am - k^3 \gamma^3 \right] \chi_{3/2} = 0$$ Like spin 1/2

$$\left[i\gamma^0 \partial_\eta - am - k^3 \left(C_A + iC_B \gamma^0 \right) \gamma^3 \right] \chi_{1/2} = 0$$ Not …

Spin 2: $\boldsymbol{h_{\mu\nu}}$

- Start with Fierz-Pauli field $h_{\mu\nu}(x)$ in flat space

- Five constraint equations ($\partial^{\mu} h_{\mu\nu} = 0$ and $h = 0$) reduce 10 dof to expected 5 dof.

- If naively promote to curved space, run into potential problems:
  - Discontinuity with GR in limit $m \to 0$
  - Only four constraint equations $\to$ left with too many dof
  - Unwanted degree of freedom is a ghost (Boulware-Deser ghost).

- de Rahm-Gabadadze-Tolley showed how to construct ghost-free massive gravity in 2010.

- Hassan and Rosen followed lead to construct ghost-free bimetric theory.

- ✓ It’s complicated

Isocurvature Considerations

- If dark matter arises through gravitational production, there may be a signature as isocurvature perturbations in CMB

- Isocurvature perturbation between gravitationally-produced DM field X and radiation R

$$\mathcal{S}(\eta,\vec{x}) = \frac{1}{1+w_X}\frac{\delta\rho_X}{\bar{\rho}_X} - \frac{1}{1+w_R}\frac{\delta\rho_R}{\bar{\rho}_R} = \frac{\delta\rho_X}{\bar{\rho}_X} - \frac{3}{4}\frac{\delta\rho_R}{\bar{\rho}_R}$$

$\mathcal{S}$ is gauge invariant, but most easily calculated in commoving gauge where inflaton perturbations (hence radiation perturbations) vanish.

$$(\eta,\vec{x}) = \frac{\delta\rho_X}{\bar{\rho}_X} \equiv \delta_X \quad \text{(comoving gauge)}$$

- Power spectrum for 2-point function:

$$\Delta^2_{\mathcal{S}}(q) = \frac{q^3}{2\pi^2}\int d^3r\, e^{-i\vec{q}\cdot\vec{r}} \left\langle 0 \middle| :\hat{\delta}_X(\vec{x})\,\hat{\delta}_X(\vec{x}+\vec{r}): \middle| 0 \right\rangle$$

- After a few pages of mode expansions, normal ordering, canonical commutation relations, …

$$\Delta^2_{\mathcal{S}}(q) \propto q^3 \int d^3\vec{k}\, \left|\beta_{\vec{k}}\right|^2 \left|\beta_{|\vec{k}-\vec{q}|}\right|^2$$

Isocurvature Considerations

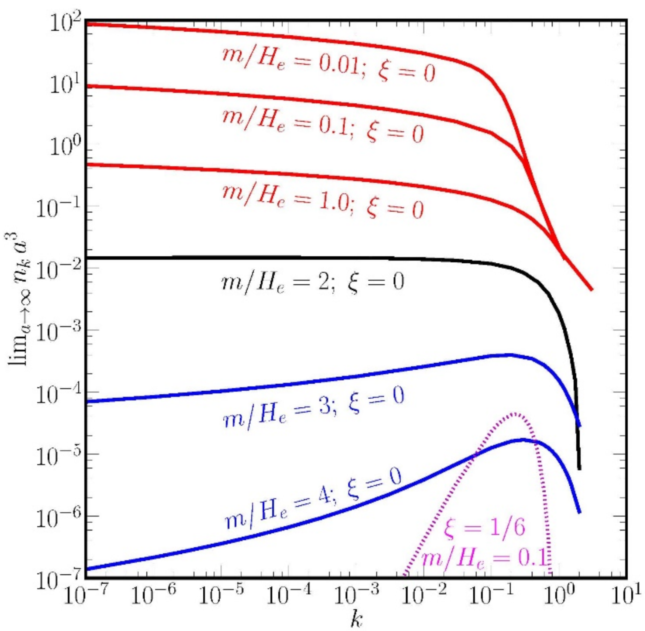
- CMB temperature anisotropies result from both curvature and isocurvature perturbations.
- At last scattering $\frac{\Delta T}{T}(\vec{x}) = -\frac{1}{5}\mathcal{R}(\eta_{\rm rec},\vec{x}) - \frac{2}{5}\mathcal{S}(\eta_{\rm rec},\vec{x})$

curvature perturbation — isocurvature perturbation

- CMB measurements place limits on

$$\beta_{\rm iso}(q_0) \equiv \frac{\Delta_{\mathcal{S}}^2}{\Delta_{\mathcal{R}}^2 + \Delta_{\mathcal{S}}^2} \cong \frac{\Delta_{\mathcal{S}}^2}{\Delta_{\mathcal{R}}^2}$$

$$\beta_{\rm iso}\left(q_0 = 0.05\ \mathrm{Mpc}^{-1}\right) < 0.038\ \ (95\%\ \mathrm{CL})$$



red spectrum is problematic $\rightarrow$ $m/H_e >$ few for minimal coupling
(Chung, Kolb, Riotto, Senatore)

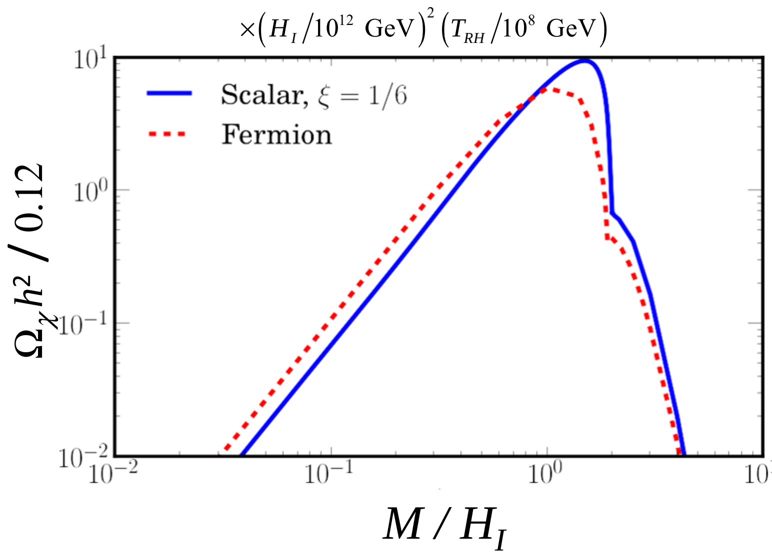
Only complete study for minimal coupling scalars.

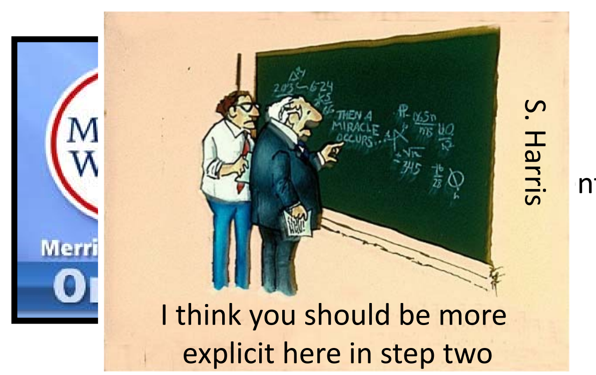
Nongaussianities

- Studied for scalars with minimal coupling (Chung & Yoo 2013)
- “Nonthermal dark matter particles can produce local nongaussianities large enough to be observed by ongoing and near future experiments without being in conflict with the existing isocurvature bounds.”
- “… can be observable through local nongaussianities even when they form a very small fraction of the total dark matter content”

WIMPzillas

- Inflation signifies a new mass scale.
- H_e, expansion rate at end of inflation, comparable to inflaton mass.
- Expect other particles with mass comparable to inflaton mass.
- If one stable, natural candidate for dark matter (don't say WIMPzilla miracle).
- If not stable but long-lived, decays can produce entropy, baryon number, …





Variations on a Theme

- Irruption. Many models have non-canonical kinetic term. Irruption. (Fedderke, Kolb, Wyman).

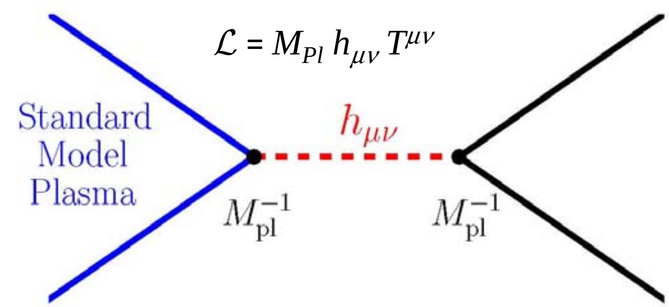
- Hilltop Inflation: $H_{\mathrm{INFLATION}} > H_e$. Softens $e^{-m/H}$ suppression to power law (Ema, Nakayama, Tang; Chung, Kolb, Long)

- Can visualize WIMPzilla production as scattering process (Ema, Nakayama, Tang)

- Same process can produce particles in reheating, assumes $m < T_{\mathrm{RH}}$ (Garny, Sandora, & Sloth)

- Can endow WIMPzilla with SM interactions

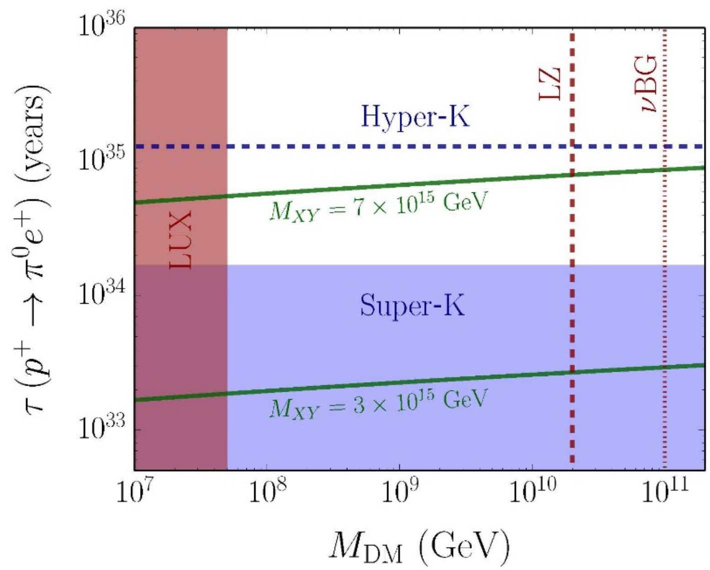
$$\frac{\Omega h^2}{0.12} \simeq \left(\frac{\langle \sigma \mathrm{v} \rangle}{T^2/M_{Pl}{}^4}\right)\left(\frac{m}{10^{13}\,\mathrm{GeV}}\right)\left(\frac{T_{\mathrm{RH}}}{10^{14}\,\mathrm{GeV}}\right)^3$$



Variations on a Theme

Can endow WIMPzillas with other Couplings

Gutzillas:
Hiragaya, Lin, Lou, JHEP 1609 (2016) 014
WIMPzillas interact with SM through Z exchange

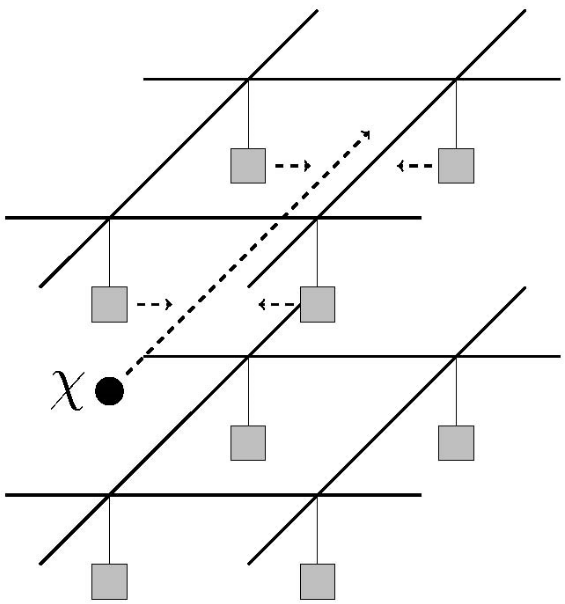


Higgs Portal:
Kolb & Long JHEP 1609 (2016) 014
$\chi\,\chi\;\Phi\,\Phi$: WIMPzillas interact through Higgs exchange

Detect WIMPzillas with only Gravitational Coupling?

"Gravitational Direct Detection of Dark Matter"
Carney, Ghosh, Krnjaic, Taylor arXiv: 1903.00492

$$\mathrm{SNR}^2 = 10^4 \left(\frac{M_\chi}{1\ \mathrm{mg}}\right)^2 \left(\frac{M_D}{1\ \mathrm{mg}}\right)^2 \left(\frac{1\ \mathrm{mm}}{d}\right)^4$$



Meter-scale detector

Billion microgram to milligram sensors

Lattice spacing millimeter to centimeter

Detect DM of mass greater than Planck mass

How about 10^{-6} Planck mass?

Goal:
Fill in the empty boxes and summarize everything in a review (Kolb & Long)

Complexity

* related to but not cosmological collider program	1-pt function	2-pt function	3-pt function*
Observable	**Dark Matter**	**Isocurvature Fluctuations**	**CMB Non-Gaussianities**
Massive scalar field (conformal)	Kuzmin & Tkachev (99)	Expected to be very small	Chung & Yoo
Massive scalar field (minimal)	Kuzmin & Tkachev (99)	Chung, Kolb, Riotto & Senatore	
Massive Dirac field	Chung, Kolb & Riotto (98, 99)	Similar to conformal scalar	Similar to conformal scalar?
Proca-de Broglie field	Massive: Kolb & Long (in progress) Light: Graham, Mardon & Rajendran		
Massive Rarita-Schwinger field	Several people gravitini, e.g., Giudice, Riotto & Tkachev; Kallosh, Kofman, Linde & Van Proeyn		
Massive Fierz-Pauli field	Kolb & Long (in progress)		

Complexity

Review assumes single-field chaotic inflation, ξ either 0 or $1/6$, & standard reheating. Template for further investigations.

Dream:
Dark Matter is a WIMPzilla, see effects in CMB (isocurvature & nongaussianities), & somehow have laboratory or astronomical signature of WIMPzillas

(Re)Connecting Dark Matter, Inflation, CMB

Gravitational Particle Production in the Early Universe

RECONNECT 2020

My collaborators in aspects of this work:
Ivone Albuquerque, Daniel Chung, Patrick Crotty, Michael Fedderke, Gian Giudice, Lam Hui,
Andrew Long, Toni Riotto, Leo Senatore, Alexi Starobinski, Igor Tkachev, Mark Wyman

Review paper in progress with Andrew Long

Rocky Kolb

KICP THE UNIVERSITY OF CHICAGO