Group Invariant Polynomial -- paving the path to operator construction

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@ IPPP Durham; June 19, 2020

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Based on: *arXiv:2004.12830*, arXiv:1905.11047(JHEP 1909 (2019) 035)

How do we proceed ?

Part A: Underlying Mathematics to construct the algorithm.

Part B: Introducing "GrIP" and its utility.

Part C: Impact of the effective operators

Aim: To understand the dynamics and interactions of the subatomic particles

Q. What will be the starting point?

A. Lagrangian !

But Lagrangian is more like a "building". So it's better to start with "bricks"

Q. What are our "bricks"?

A. Quantum fields that represent the particles and the underlying symmetry.

Q. Then, how do we construct a Lagrangian (*up to any mass dimension*)?

A. I hope by the end of this talk we will have the answer.

Initial restriction : space-time is 3+1 dimensional for this talk.

$\boxed{\text{Action:}}$ $$\mathcal{S} = \int d^4x \; \mathcal{L}$$

$\boxed{\text{In natural units:}}$ $$[L] = [T] = [M]^{-1}$$

$\boxed{\text{Mass dimensions:}}$ $$[\mathcal{L}] \equiv 4, \;\; [\mathcal{D}_\mu] \equiv 1 \;\; \Longrightarrow \;\; [\phi] \equiv [A_\mu] \equiv 1, \;\; [\psi] \equiv \frac{3}{2}, \;\; [F_{\mu\nu}] \equiv 2.$$

$$\mathcal{L} = \alpha^{(1)}\mathcal{O}^{(1)} + \alpha^{(2)}\mathcal{O}^{(2)} + \alpha^{(3)}\mathcal{O}^{(3)} + \alpha^{(4)}\mathcal{O}^{(4)} + \alpha^{(5)}\mathcal{O}^{(5)} + \alpha^{(6)}\mathcal{O}^{(6)} + \cdots$$

$$\mathcal{L} = \mathcal{L}_{\text{renorm}} + \sum_{i=5}^{n}\sum_{j=1}^{N_i} \frac{\mathcal{C}_j^{(i)}}{\Lambda^{i-4}}\mathcal{O}_j^{(i)}.$$

$$\boxed{\mathcal{S} = \int d^4x \, \mathcal{L}}$$

Role of Symmetry

"Action" is invariant under the symmetry of the system, so does the "measure".

Thus the Lagrangian(density) must be invariant under the symmetry.

Lagrangian of a real scalar field:

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \mathcal{V}(\phi), \qquad \mathcal{V}(\phi) = \mathcal{M}^3\phi - \frac{1}{2}m^2\phi^2 + \frac{\mu}{3!}\phi^3 + \frac{\lambda}{4!}\phi^4.$$

Lagrangian of a real scalar field with discrete Z_2 symmetry:

$$\mathcal{L}_\rho = \frac{1}{2}(\partial_\mu \rho)(\partial^\mu \rho) - \mathcal{V}(\rho), \qquad \mathcal{V}(\rho) = -\frac{1}{2}m^2\rho^2 + \frac{\lambda}{4!}\rho^4.$$

Lagrangian of a complex scalar field:

$$\phi = \phi_1 + i\phi_2, \quad \phi^* = \phi_1 - i\phi_2; \qquad \phi \to e^{i\theta}\phi, \quad \phi^* \to \phi^* e^{-i\theta}.$$

$$\mathcal{L}_{\phi,\phi^*} = (\partial_\mu \phi^*)(\partial^\mu \phi) - \mathcal{V}(\phi), \qquad \mathcal{V}(\phi) = -m^2(\phi^*\phi) + \lambda(\phi^*\phi)^2.$$

In the same spirit one can construct the SM Lagrangian.

SM Lagrangian:

$$\begin{aligned}\mathcal{L}_{SM} =\ & (\mathcal{D}_\mu\phi)^\dagger(\mathcal{D}^\mu\phi) + \frac{1}{2}m^2(\phi^\dagger\phi) - \frac{\lambda}{4!}(\phi^\dagger\phi)^2 - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} - \frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^{I\mu\nu}W^I_{\mu\nu} \\ & -i\left(\bar{L}^p_L \not{\mathcal{D}} L^p_L + \bar{Q}^p_L \not{\mathcal{D}} Q^p_L + \bar{u}^p_R \not{\mathcal{D}} u^p_R + \bar{d}^p_R \not{\mathcal{D}} d^p_R + \bar{e}^p_R \not{\mathcal{D}} e^p_R\right) \\ & -\left(y_e^{rs}\bar{L}^r_L\phi e^s_R + y_d^{rs}\bar{Q}^r_L\phi d^s_R + y_u^{rs}\bar{Q}^r_L\tilde{\phi} u^s_R\right) + h.c.\end{aligned}$$

Category	Constitution	Operators (for SM)
Scalar Potential	$\phi^n,\ \ n \leq 4$	$(\phi^\dagger\phi),\ (\phi^\dagger\phi)^2$
Scalar Kinetic Term	$\phi^2\mathcal{D}^2$	$(\mathcal{D}_\mu\phi)^\dagger(\mathcal{D}^\mu\phi)$
Fermion Kinetic Term	$\psi^2\mathcal{D}$	$\bar{L}^p_L \not{\mathcal{D}} L^p_L,\ \bar{Q}^p_L \not{\mathcal{D}} Q^p_L,\ \bar{u}^p_R \not{\mathcal{D}} u^p_R,\ \bar{d}^p_R \not{\mathcal{D}} d^p_R,\ \bar{e}^p_R \not{\mathcal{D}} e^p_R$
Gauge Kinetic Term	$(F_{\mu\nu})^2$	$B^{\mu\nu}B_{\mu\nu},\ G^{a\mu\nu}G^a_{\mu\nu},\ W^{I\mu\nu}W^I_{\mu\nu}$
Yukawa Interaction Term	$\psi^2\phi$	$\bar{L}^r_L\phi e^s_R,\ \bar{Q}^r_L\phi d^s_R,\ \bar{Q}^r_L\tilde{\phi} u^s_R$

Table 1: Operator classification for the renormalizable Lagrangian composed of scalars, spinors, gauge field strength tensors, and covariant derivatives.

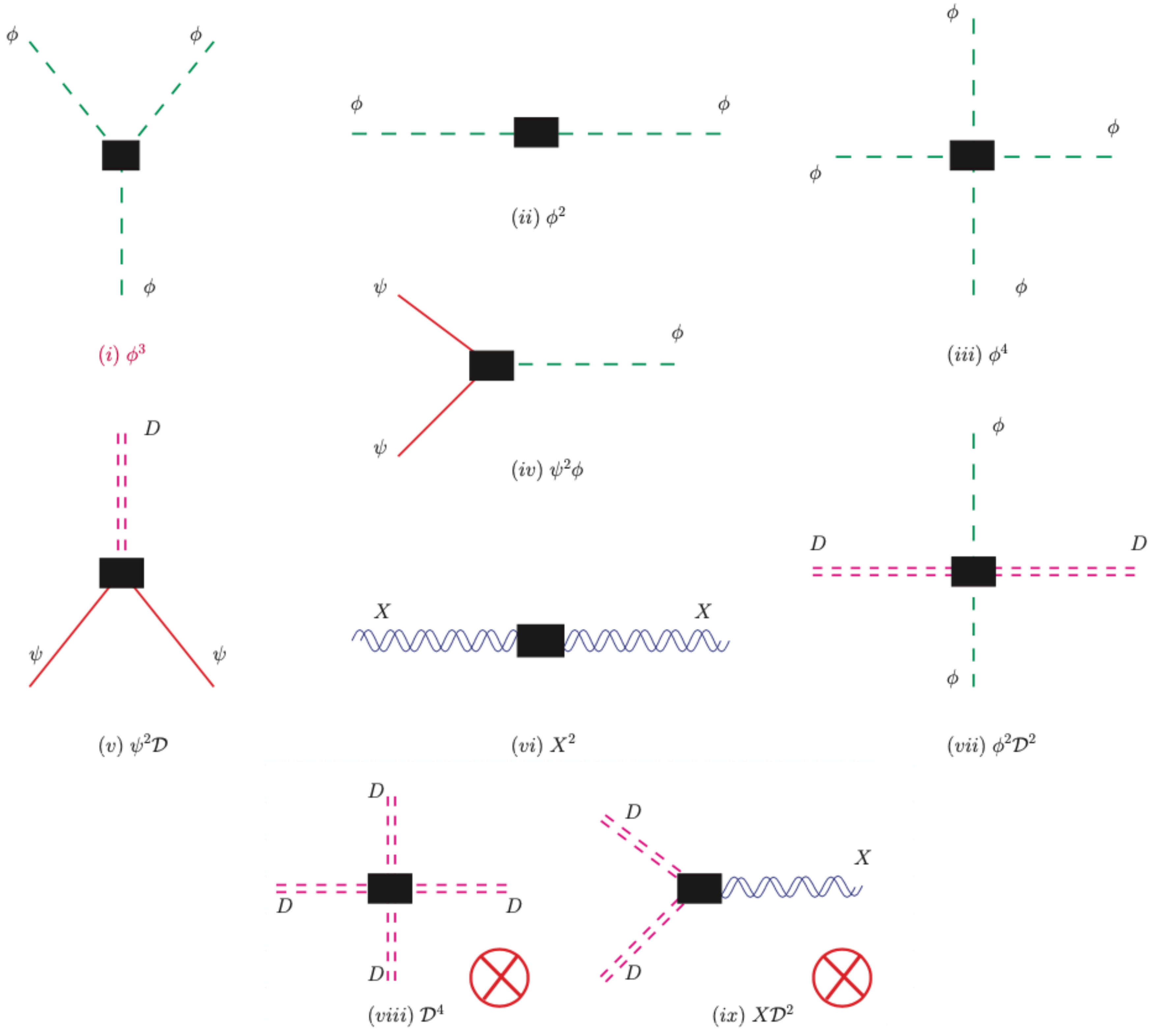

Figure 1: Diagrammatic representation of operator classes at mass dimension-4 (for $3+1$ space-time dimensions) or less constituted by combining spin-0 (ϕ), spin-1/2 (ψ) and Field Strength Tensor (X) of spin-1 fields and the covariant derivative ($\mathcal{D}$). Of these, (i) does not appear in the case of SM hence we have highlighted its caption in colour. Also, the last two structures being total derivative terms are excluded from the Lagrangian.

Invariant operator structures:

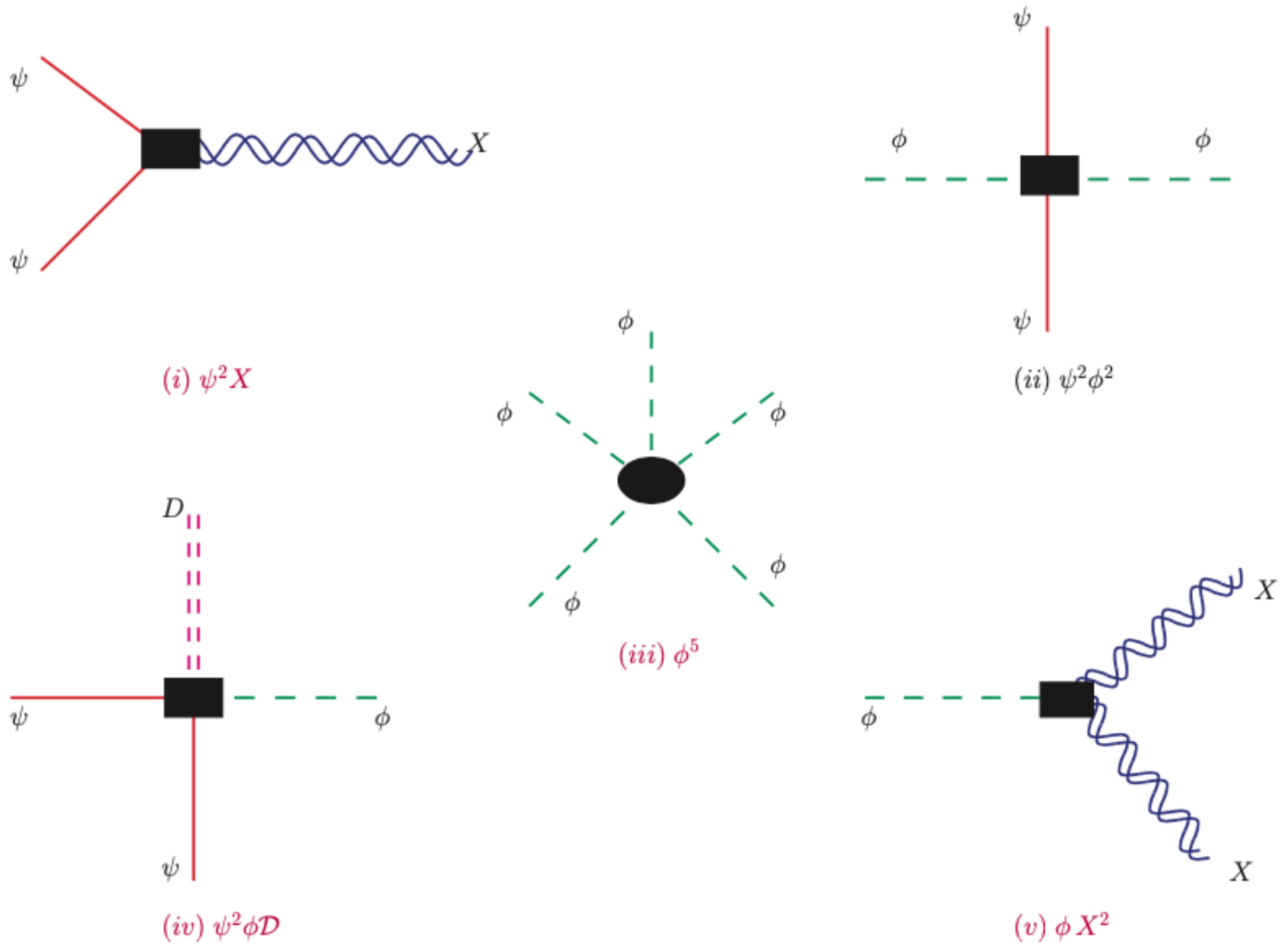

Figure 2: Diagrammatic representation of mass dimension-5 (for 3 + 1 space-time dimensions) operator classes constituted by combining spin-0 (ϕ), spin-1/2 (ψ) and Field Strength Tensor (X) of spin-1 fields and the covariant derivative ($\mathcal{D}$). Only one of these (diagram (ii)) appears in the case of SM. The other structures with red captions (i), (iii), (iv) and (v) may appear for other models with different particle content and symmetry.

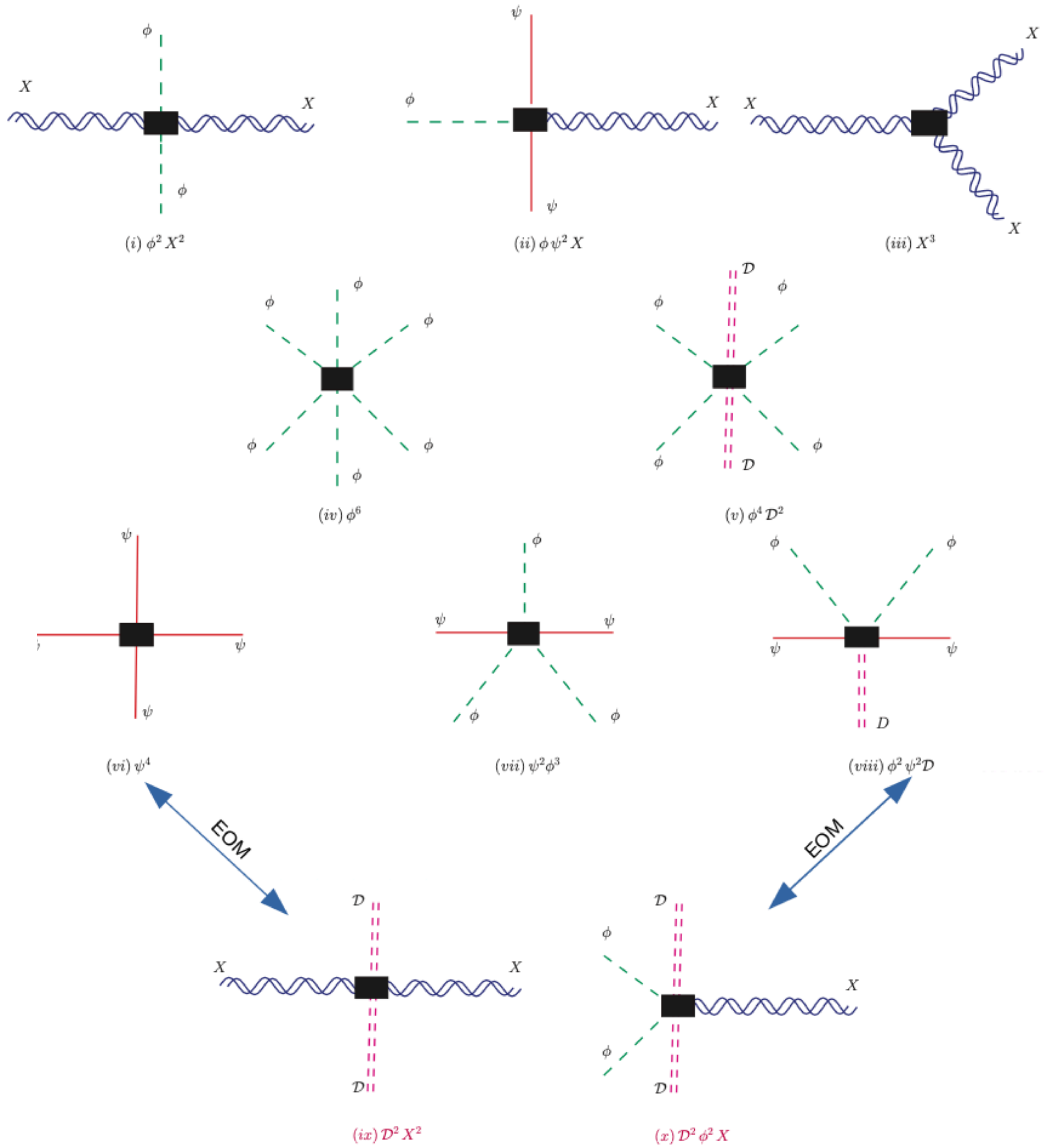

Figure 3: Diagrammatic representation of mass dimension-6 (for 3+1 space-time dimensions) SM operators classes constituted by combining spin-0 (ϕ), spin-1/2 (ψ) and Field Strength Tensor (X) of spin-1 fields and the covariant derivative ($\mathcal{D}$). Operators described by (ix) and (x) are related to (vi) and ($viii$) respectively through the equation of motion of the gauge fields. Based on which terms are included in the operator set we have two popular operator bases. The Warsaw basis [6] includes the operator classes (i) – ($viii$) and forms a complete set. While, the SILH [7] basis trades off (ii), (vi) – ($viii$) (the operators composed of fermionic fields) in favour of (ix) and (x), This forms an under-complete set.

<u>Part A</u>: Underlying Mathematics to construct the algorithm.

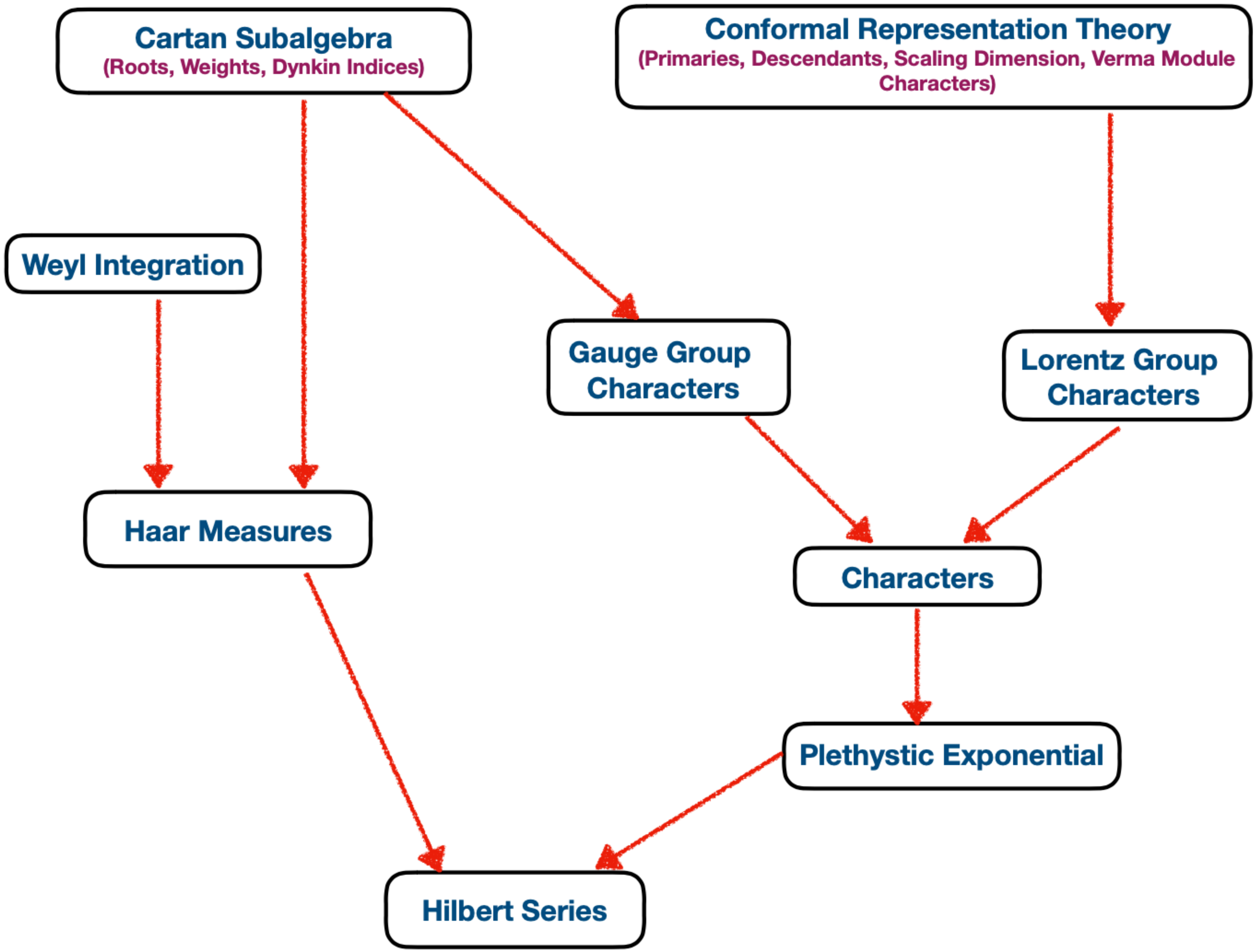

Figure 1: Flow chart depicting the Hilbert Series construction.

Hilbert Series :

$$\mathcal{H}[\phi] = \prod_{j=1}^{n} \int_{\mathcal{G}_j} \underbrace{d\mu_j}_{\text{Haar Measure}} \underbrace{PE[\varphi, R]}_{\text{Plethystic Exponential}} ,$$

Plethystics :

$$PE[\phi, R] = \exp\left[\sum_{r=1}^{\infty} \frac{\phi^r \chi_R(z_j^r)}{r}\right],$$

$$PE[\psi, R] = \exp\left[\sum_{r=1}^{\infty} (-1)^{r+1} \frac{\psi^r \chi_R(z_j^r)}{r}\right],$$

χ_R is the character of representation R

Orthonormality: $\boxed{\int d\mu_{\mathcal{G}}\, \chi_{R_i}\, \chi_{R_j} = \delta_{ij}}$

ϕ, ψ are spurion variables that tag the fields

Haar Measure :

$$\int_{\mathcal{G}} d\mu_{\mathcal{G}} = \frac{1}{(2\pi i)^k} \oint_{|z_1|=1} \dots \oint_{|z_\sigma|=1} \frac{dz_1}{z_1} \dots \frac{dz_\sigma}{z_\sigma} \prod_{\alpha^+} \left[1 - \prod_{m=1}^{k} (z_m)^{\alpha_m^+}\right],$$

in terms of +ve roots !

$$\int_{SU(N)} d\mu_{SU(N)} = \frac{1}{(2\pi i)^{N-1} N!} \oint_{|z_l|=1} \prod_{l=1}^{N-1} \frac{dz_l}{z_l} \Delta(\epsilon) \Delta(\epsilon^{-1}) .$$

$$\int_{SO(2N+1)} d\mu_{SO(2N+1)} = \frac{1}{(2\pi i)^N 2^N N!} \oint_{|\epsilon_l|=1} \prod_{l=1}^{N} \frac{d\epsilon_l}{\epsilon_l} \Delta(\epsilon) \Delta(\epsilon^{-1}) .$$

$$\int_{SO(2N)} d\mu_{SO(2N)} = \frac{1}{(2\pi i)^N 2^{N-1} N!} \oint_{|\epsilon_l|=1} \prod_{l=1}^{N} \frac{d\epsilon_l}{\epsilon_l} \Delta(\epsilon) \Delta(\epsilon^{-1}) .$$

$$\int_{Sp(2N)} d\mu_{Sp(2N)} = \frac{1}{(2\pi i)^N 2^N N!} \oint_{|\epsilon_l|=1} \prod_{l=1}^{N} \frac{d\epsilon_l}{\epsilon_l} \Delta(\epsilon) \Delta(\epsilon^{-1}) .$$

Character of irreducible representation "R"

$$\boxed{SU(N)}$$

Character of irreducible representation $M(\epsilon)$

$r_1, r_2, \cdots, r_{N-1}$ are integers and $r_1 > r_2 > \cdots > r_{N-1} > 0$

$$\chi^{(M(\epsilon))}_{r_1, r_2, \ldots, r_{N-1}} = \frac{|\epsilon^{r_1}, \epsilon^{r_2}, \ldots, \epsilon^{r_{N-1}}, 1|}{|\epsilon^{N-1}, \epsilon^{N-2}, \ldots., \epsilon, 1|},$$

$$\prod_{a=1}^{N} \epsilon_a = 1, \qquad |\epsilon_a| = 1, \qquad a = 1, 2, \ldots, N.$$

Numerator

$$|\epsilon^{r_1}, \epsilon^{r_2}, \ldots, \epsilon^{r_{N-1}}, 1| = \begin{vmatrix} \epsilon_1^{r_1} & \epsilon_1^{r_2} & \ldots & \epsilon_1^{r_{N-1}} & 1 \\ \epsilon_2^{r_1} & \epsilon_2^{r_2} & \ldots & \epsilon_2^{r_{N-1}} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \epsilon_N^{r_1} & \epsilon_N^{r_2} & \ldots & \epsilon_N^{r_{N-1}} & 1 \end{vmatrix},$$

Vandermonde determinant (denominator)

$$|\epsilon^{N-1}, \epsilon^{N-2}, \ldots., \epsilon, 1| = \begin{vmatrix} \epsilon_1^{N-1} & \epsilon_1^{N-2} & \ldots & \epsilon_1^2 & \epsilon_1 & 1 \\ \epsilon_2^{N-1} & \epsilon_2^{N-2} & \ldots & \epsilon_2^2 & \epsilon_2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \epsilon_N^{N-1} & \epsilon_N^{N-2} & \ldots & \epsilon_N^2 & \epsilon_N & 1 \end{vmatrix} = \prod_{1 \le a < b \le N} (\epsilon_a - \epsilon_b).$$

Realisation of $\vec{r}$ through $\vec{\lambda}$

The r_i's are related to the λ_i's through the following equation:

$$\vec{r} = \vec{\lambda} + \vec{\rho} \qquad \text{where} \qquad \rho_i = N - i, \qquad i = 1, 2, .., N.$$

Since $\lambda_N = 0$ and $\rho_N = N - N = 0$, therefore $r_N = 0$.

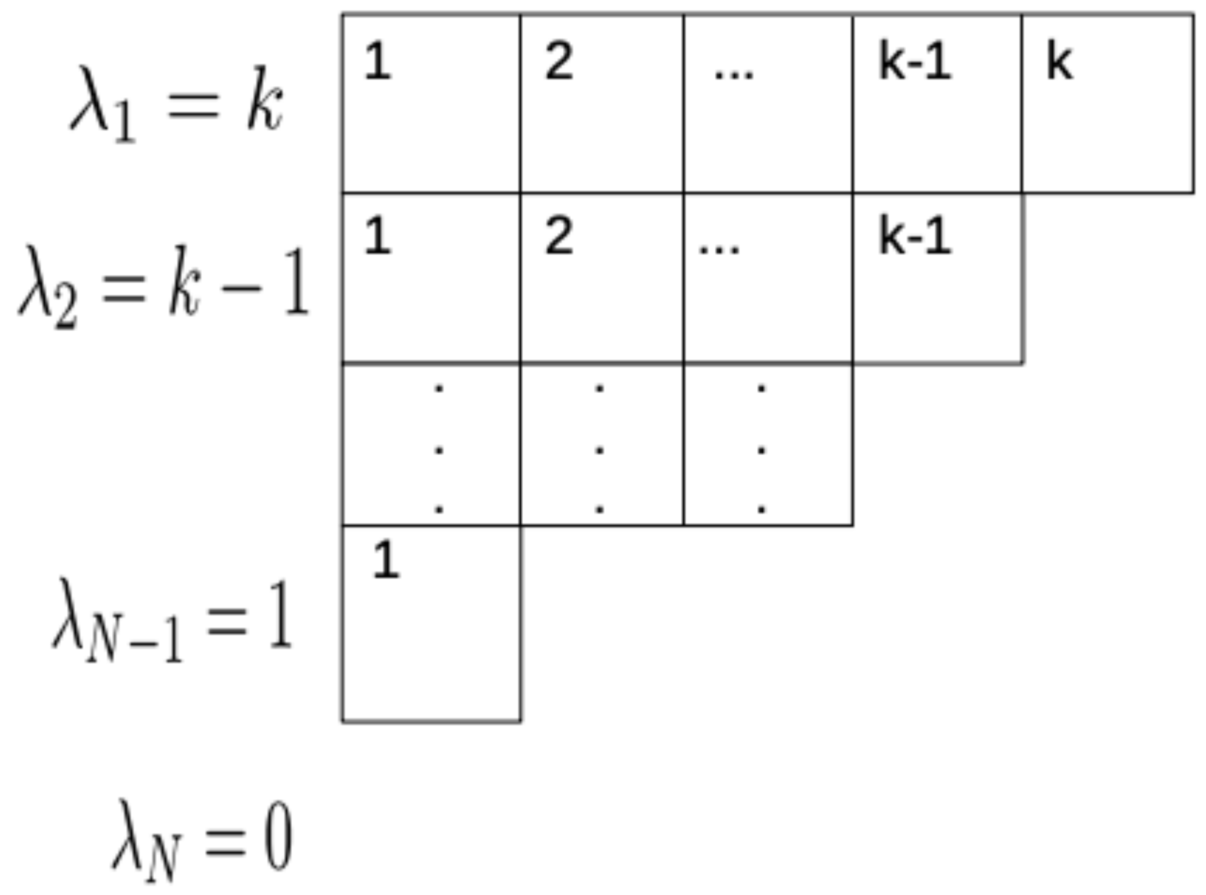

Figure 5: A schematic form of the Young diagram corresponding to an arbitrary representation of $SU(N)$. λ_i is equal to the number of boxes in the i-th row of the diagram. From here one can immediately infer that $\lambda_N = 0$.

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Since $\lambda_N = 0$ and $\rho_N = N - N = 0$, therefore $r_N = 0$.

Weight Tree of LDF

The weight tree corresponding to the LDF representation of $SU(N)$ is

$$\begin{aligned}
L_1 &= \underbrace{(1, 0, 0, ..., 0, 0)}_{(N\text{-}1 \text{ tuple})}, \\
L_2 = L_1 - \alpha_1 &= (-1, 1, 0, ..., 0, 0), \\
&\vdots \\
L_k = L_{k-1} - \alpha_{k-1} &= (0, .., -1, 1, ., 0), \\
&\vdots \\
L_{N-1} = L_{N-2} - \alpha_{N-2} &= (0, 0, ..., -1, 1), \\
L_N = L_{N-1} - \alpha_{N-1} &= (0, 0, ..., 0, -1).
\end{aligned}$$

Cartan Matrix for SU(N)

$$\mathcal{A}_{SU(N)} = \begin{pmatrix} --\alpha_1-- \\ --\alpha_2-- \\ | \\ --\alpha_{N-2}-- \\ --\alpha_{N-1}-- \end{pmatrix} = \begin{pmatrix} 2 & -1 & 0 & \cdots & 0 & 0 & 0 \\ -1 & 2 & -1 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & 2 & -1 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 2 \end{pmatrix}_{(N-1)\times(N-1)}.$$

Dynkin label $(a_1, a_2, ..., a_{N-1})$

$$\lambda_N = 0,$$

$$\lambda_{N-1} = a_{N-1} = \left(\sum_{i=1}^{N-1} a_i\right) - \left(\sum_{j=1}^{N-2} a_j\right),$$

$$\lambda_k = a_k + a_{k+1} + \cdots + a_{N-1} = \left(\sum_{i=1}^{N-1} a_i\right) - \left(\sum_{j=1}^{k-1} a_j\right).$$

$$(a_1, a_2, ..., a_{N-1}) = \lambda_1(1, 0, ..., 0, 0) + \lambda_2(-1, 1, 0, ..., 0) + \cdots + \lambda_{N-1}(0, ..., 0, -1, 1) + \lambda_N(0, 0, ..., 0, -1).$$

Maximal Torus of SU(N)

A torus in a compact Lie group *G* is a compact, connected, abelian Lie subgroup of *G*

A maximal torus is one which is maximal among such subgroups.

$$\mathbb{T}^{N-1} = U(1) \otimes U(1) \cdots \otimes U(1)$$

$$\mathbb{T}^{N-1} : \begin{pmatrix} e^{i\theta_1} & 0 & 0 & \cdots & 0 \\ 0 & e^{i(\theta_2 - \theta_1)} & 0 & \cdots & 0 \\ 0 & 0 & e^{i(\theta_3-\theta_2)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & e^{-i\theta_{N-1}} \end{pmatrix}_{N\times N}.$$

Here, $\boxed{z_i = e^{i\theta_i}}$ and $\boxed{\epsilon_a = \epsilon_a\left(z_1, z_2, ..., z_{N-1}\right)}$

$$\prod_{a=1}^{N} \epsilon_a = 1, \qquad |\epsilon_a| = 1, \qquad a = 1, 2, ..., N.$$

Then, if the $(N-1)$ tuple L_i is denoted as $(l_i^{(1)}, l_i^{(2)}, l_i^{(3)}, ..., l_i^{(N-1)})$, a general formula for ϵ_a can be written in terms of z_i's as:

$$\epsilon_i = z_1^{l_i^{(1)}} \times z_2^{l_i^{(2)}} \times z_3^{l_i^{(3)}} \times ... \times z_{N-1}^{l_i^{(N-1)}}, \tag{2.13}$$

which enables us to write:

$$\begin{aligned} \epsilon_1 &= z_1^1 \times z_2^0 \times z_3^0 \times ... \times z_{N-1}^0 = z_1, & \epsilon_2 &= z_1^{-1} \times z_2^1 \times z_3^0 \times ... \times z_{N-1}^0 = z_1^{-1} z_2, \\ \epsilon_k &= z_1^0 \times ... z_{k-1}^{-1} \times z_k^1 \times ... \times z_{N-1}^0 = z_{k-1}^{-1} z_k, & \epsilon_N &= z_1^0 \times z_2^0 \times z_3^0 \times ... \times z_{N-1}^{-1} = z_{N-1}^{-1}. \end{aligned} \tag{2.14}$$

The weight tree corresponding to the LDF representation of $SU(N)$ is

$$\begin{aligned} L_1 &= \underbrace{(1, 0, 0, ..., 0, 0)}_{(N\text{-}1 \text{ tuple})}, \\ L_2 = L_1 - \alpha_1 &= (-1, 1, 0, ..., 0, 0), \\ &\vdots \\ L_k = L_{k-1} - \alpha_{k-1} &= (0, .., -1, 1, ., 0), \\ &\vdots \\ L_{N-1} = L_{N-2} - \alpha_{N-2} &= (0, 0, ..., -1, 1), \\ L_N = L_{N-1} - \alpha_{N-1} &= (0, 0, ..., 0, -1). \end{aligned}$$

$SU(2)$

The r_i's are related to the λ_i's through the following equation:

$$\vec{r} = \vec{\lambda} + \vec{\rho} \quad \text{where} \quad \rho_i = N - i, \quad i = 1, 2, .., N.$$

Torus coordinates:

$$\epsilon_1 = z_1, \ \epsilon_2 = z_1^{-1}$$

Haar measure:

$$d\mu_{SU(2)} = \frac{1}{2!\,(2\pi i)} \frac{dz_1}{z_1} \Delta(\epsilon)\,\Delta(\epsilon^{-1}) = \frac{1}{2\,(2\pi i)} \frac{dz_1}{z_1} \left(1 - z_1^2\right)\left(1 - \frac{1}{z_1^2}\right)$$

Vandermonde determinant:

$$\Delta(\epsilon) = \begin{vmatrix} \epsilon_1 & 1 \\ \epsilon_2 & 1 \end{vmatrix} = (\epsilon_1 - \epsilon_2) \xrightarrow{\epsilon_i(z_j)} \left(z_1 - \frac{1}{z_1}\right).$$

The (Anti-)Fundamental Representation: $2 \equiv \overline{2} \equiv (1) \equiv \square$

$$\vec{\rho} = (1, 0) \qquad \vec{\lambda} = (1, 0)$$

$$\vec{r} = \vec{\lambda} + \vec{\rho} = (2, 0)$$

$$\chi(\epsilon_1, \epsilon_2) = \frac{|\epsilon^2,\ 1|}{|\epsilon,\ 1|} = \frac{1}{(\epsilon_1 - \epsilon_2)} \begin{vmatrix} \epsilon_1^2 & 1 \\ \epsilon_2^2 & 1 \end{vmatrix} = \epsilon_1 + \epsilon_2, \qquad \Longrightarrow \chi_{(SU(2))_{2(\overline{2})}}(z_1) = z_1 + \frac{1}{z_1}.$$

The Adjoint Representation: $3 \equiv (2) \equiv \square\square$ $\quad \vec{\lambda} = (2, 0)$

$$\chi(\epsilon_1, \epsilon_2) = \frac{|\epsilon^3,\ 1|}{|\epsilon,\ 1|} = \frac{1}{(\epsilon_1 - \epsilon_2)} \begin{vmatrix} \epsilon_3^2 & 1 \\ \epsilon_3^2 & 1 \end{vmatrix} = \frac{\epsilon_1^3 - \epsilon_2^3}{\epsilon_1 - \epsilon_2} = \epsilon_1^2 + \epsilon_1\epsilon_2 + \epsilon_2^2.$$

$$\therefore\ \chi_{(SU(2))_3}(z_1) = z_1^2 + 1 + \frac{1}{z_1^2}.$$

The Quadruplet Representation: $4 \equiv (3) \equiv \square\square\square$

We obtain $\vec{\lambda} = (3, 0)$ and $\vec{r} = \vec{\lambda} + \vec{\rho} = (4, 0)$ and the character is obtained as:

$$\chi(\epsilon_1, \epsilon_2) = \frac{|\epsilon^4,\ 1|}{|\epsilon,\ 1|} = \frac{1}{(\epsilon_1 - \epsilon_2)} \begin{vmatrix} \epsilon_4^2 & 1 \\ \epsilon_4^2 & 1 \end{vmatrix} = \frac{\epsilon_1^4 - \epsilon_2^4}{\epsilon_1 - \epsilon_2} = \epsilon_1^3 + \epsilon_1^2\epsilon_2 + \epsilon_1\epsilon_2^2 + \epsilon_2^3.$$

$$\therefore\ \chi_{(SU(2))_4}(z_1) = z_1^3 + z_1 + \frac{1}{z_1} + \frac{1}{z_1^3}.$$

fundamental $\otimes$ anti-fundamental $=$ adjoint $\oplus$ singlet

$$\Longrightarrow \chi_{\mathrm{f}} \times \chi_{\mathrm{af}} = \chi_{\mathrm{adj}} + \chi_{\mathrm{singl}}. \qquad \therefore\ \chi_{\mathrm{adj}} = \chi_{\mathrm{f}} \times \chi_{\mathrm{af}} - 1.$$

Curious case of space-time symmetry and *derivatives*

1. **We are interested in a complete set of operators, and that includes operators involving *derivatives*.**

2. **Since the *derivatives* transform under space-time symmetry, we need to understand their transformation properties along with the internal symmetries.**

3. **Due to the intrinsic properties of *derivatives*, additional constraints might appear in the form of Equations of Motion (EOMs), and Integration by Parts (IBPs).**

4. **It leads to the over counting of the operators.**

5. **Thus we should carefully identify the redundancy in the operator set to construct only independent operators, so that they can form a basis. Now we will discuss how the HS construction can automatically take care of this issue, which is one of the most significant merits of this program.**

Inclusion of "dynamic variables"

Q. What will happen if the "variables" are dynamic in nature?

Q. What do I mean by the "dynamic variables"?

A. "variables" which respect some Equations of Motion (EOM).

$$\boxed{D\,\Phi_j = 0}$$

Here D is the "differential" operator

EOM behaves more like a constraint in our construction.

Don't forget about IBP.

Q. Why? A. These lead to some sort of constraints.

Thus the "invariants" at a given order of polynomial may not be independent to each other

Curious case of space-time symmetry and *derivatives*

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We are working within **3+1** dimensional framework

$\boxed{\text{conformal group } SO(4,2)}$

1. The extra two dimensions (+2) increase the rank of the conformal group by one unit which appears as a "Scaling" dimension
2. In our analysis, "Scaling" dimension is the mass dimension of the operator/fields, e.g., 1 for boson, 3/2 for fermions,

$\boxed{\Delta \text{ is the scaling dimension}}$

$\boxed{\mathcal{D} \text{ is the covariant derivative operator}}$

Curious case of space-time symmetry

We are working within 3+1 dimension: Lorentz Group

$\mathcal{D}$ is the spurion variable symbolizing the covariant derivative operator.

$SO(4,C)$ character with highest weight $(l_1, l_2) \Rightarrow \chi^{(4)}_{(l_1,l_2)}(x_1, x_2)$

$SU(2)_L$ character with highest weight $(j_1) \Rightarrow \chi_{(j_1)}(y_1)$

$SU(2)_R$ character with highest weight $(j_2) \Rightarrow \chi_{(j_1)}(y_2)$

$l_1 = j_1 + j_2,\ l_2 = j_1 - j_2,\ x_1 = y_1^{1/2} y_2^{1/2}$, and $x_2 = y_1^{1/2} y_2^{-1/2}$

$$\chi^{(4)}_{(l_1,l_2)}(x_1, x_2) = \chi_{j_1}(y_1) * \chi_{j_2}(y_2),$$

$$\begin{aligned}
\chi^{(4)}_{[j_1+j_2+2;j_1,j_2]}(s, y_1, y_2) &= s^{j_1+j_2+2}\left(\chi_{j_1}(y_1)\chi_{j_2}(y_2) - s\chi_{j_1-\frac{1}{2}}(y_1)\chi_{j_2-\frac{1}{2}}(y_2)\right) P^{(4)}(s, y_1, y_2),\\
\chi^{(4)}_{[j_1+1;j_1]}(s, y_1, y_2) &= s^{j_1+1}\left(\chi_{j_1}(y_1) - s\chi_{j_1-\frac{1}{2}}(y_1)\chi_{\frac{1}{2}}(y_2) + s^2\chi_{j_1-1}(y_1)\right) P^{(4)}(s, y_1, y_2),\\
\chi^{(4)}_{[j_2+1;j_2]}(s, y_1, y_2) &= s^{j_2+1}\left(\chi_{j_2}(y_2) - s\chi_{j_2-\frac{1}{2}}(y_2)\chi_{\frac{1}{2}}(y_1) + s^2\chi_{j_2-1}(y_2)\right) P^{(4)}(s, y_1, y_2),
\end{aligned}$$

$$\begin{aligned}
\chi^{(4)}_{[1;(0,0)]}(\mathcal{D},\alpha,\beta) &= \mathcal{D}P^{(4)}(\mathcal{D},\alpha,\beta) \times \left[1-\mathcal{D}^2\right],\\
\chi^{(4)}_{[\frac{3}{2},(\frac{1}{2},0)]}(\mathcal{D},\alpha,\beta) &= \mathcal{D}^{\frac{3}{2}}P^{(4)}(\mathcal{D},\alpha,\beta) \times \left[\alpha+\frac{1}{\alpha}-\mathcal{D}\Big(\beta+\frac{1}{\beta}\Big)\right],\\
\chi^{(4)}_{[\frac{3}{2};(0,\frac{1}{2})]}(\mathcal{D},\alpha,\beta) &= \mathcal{D}^{\frac{3}{2}}P^{(4)}(D,\alpha,\beta) \times \left[\beta+\frac{1}{\beta}-\mathcal{D}\Big(\alpha+\frac{1}{\alpha}\Big)\right],\\
\chi^{(4)}_{[2;(1,0)]}(\mathcal{D},\alpha,\beta) &= \mathcal{D}^2P^{(4)}(\mathcal{D},\alpha,\beta) \times \left[\alpha^2+\frac{1}{\alpha^2}+1-\mathcal{D}\Big(\alpha+\frac{1}{\alpha}\Big)\Big(\beta+\frac{1}{\beta}\Big)+\mathcal{D}^2\right],\\
\chi^{(4)}_{[2;(0,1)]}(\mathcal{D},\alpha,\beta) &= \mathcal{D}^2P^{(4)}(\mathcal{D},\alpha,\beta) \times \left[\beta^2+\frac{1}{\beta^2}+1-\mathcal{D}\Big(\alpha+\frac{1}{\alpha}\Big)\Big(\beta+\frac{1}{\beta}\Big)+\mathcal{D}^2\right],
\end{aligned}$$

$P^{(4)}(s, y_1, y_2)$ is the momentum generating function for $SO(4,\mathbb{C})$

$$P^{(4)}(\mathcal{D},\alpha,\beta) = \left[(1-\mathcal{D}\alpha\beta)\left(1-\frac{\mathcal{D}}{\alpha\beta}\right)\left(1-\frac{\mathcal{D}\alpha}{\beta}\right)\left(1-\frac{\mathcal{D}\beta}{\alpha}\right)\right]^{-1}.$$

Inclusion of "derivative" operators and modifications..

Modified Plethystics:

$$PE[\phi,\mathcal{D},R] = \exp\left[\sum_{r=1}^{\infty}\left(\frac{\phi}{\mathcal{D}^{\Delta_\phi}}\right)^r \frac{\chi_R(z_j^r,\alpha^r,\beta^r)}{r}\right],$$

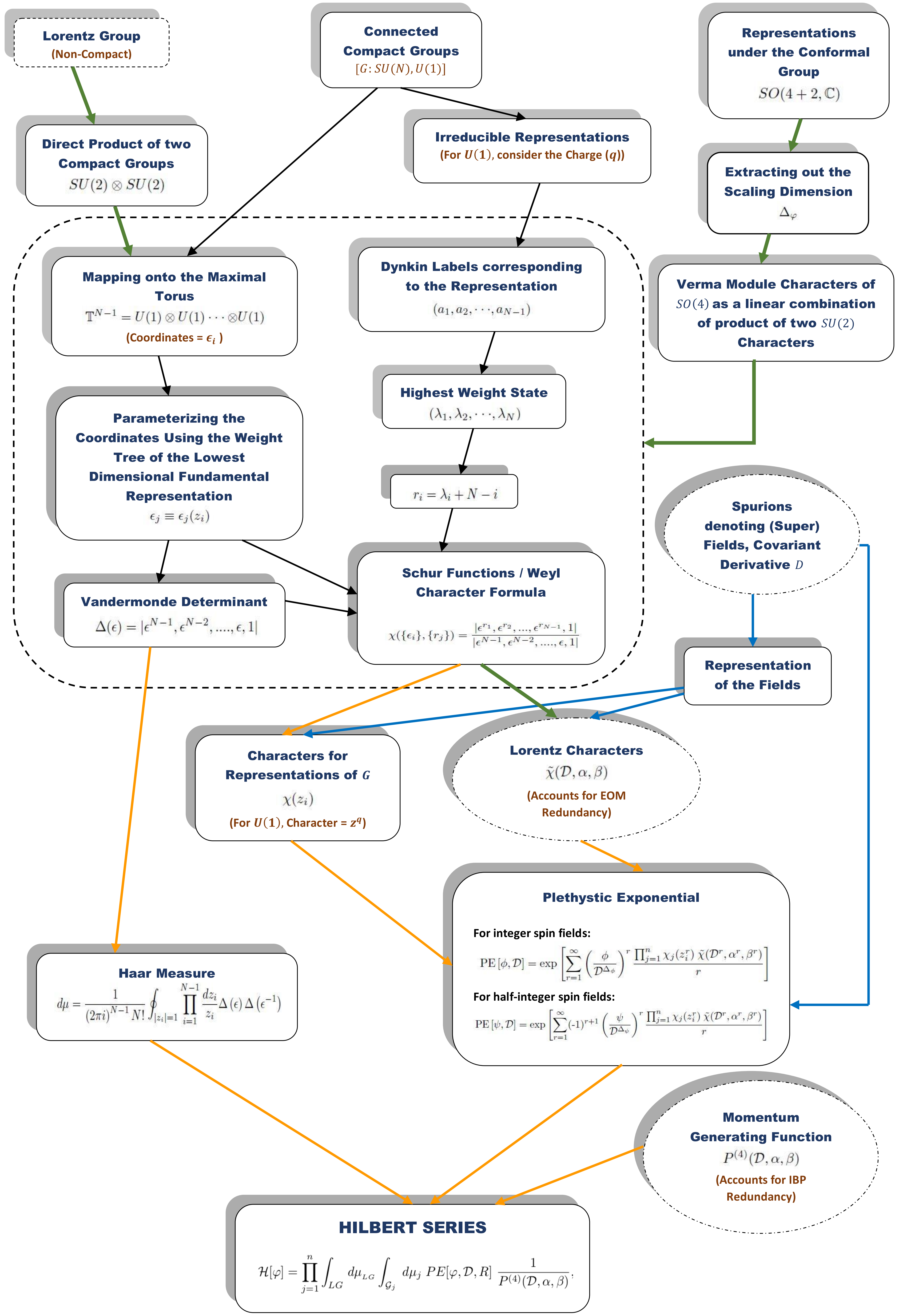
$$PE[\psi,\mathcal{D},R] = \exp\left[\sum_{r=1}^{\infty}(-1)^{r+1}\left(\frac{\psi}{\mathcal{D}^{\Delta_\psi}}\right)^r \frac{\chi_R(z_j^r,\alpha^r,\beta^r)}{r}\right].$$

Modified Haar measure:

$$\begin{aligned}\int d\mu_{Lorentz} &= \int \frac{1}{P^{(4)}(\mathcal{D},\alpha,\beta)} d\mu_{SU(2)} \times d\mu_{SU(2)} \\ &= \frac{1}{P^{(4)}(\mathcal{D},\alpha,\beta)} \times \left[\frac{1}{2\,(2\pi i)}\frac{d\alpha}{\alpha}(1-\alpha^2)\left(1-\frac{1}{\alpha^2}\right)\right] \times \left[\frac{1}{2\,(2\pi i)}\frac{d\beta}{\beta}(1-\beta^2)\left(1-\frac{1}{\beta^2}\right)\right]\end{aligned}$$

Modified Hilbert Series:

$$\mathcal{H}[\varphi] = \int_{LG} d\mu_{SU(2)} \times d\mu_{SU(2)}\ \frac{1}{P^{(4)}(\mathcal{D},\alpha,\beta)} \prod_{j=1}^{n} \int_{\mathcal{G}_j} d\mu_j\ PE[\varphi,\mathcal{D},R],$$



Lorentz Group
(Non-Compact)
Connected Compact Groups
[$G: SU(N), U(1)$]
Representations under the Conformal Group
$SO(4+2,\mathbb{C})$
Direct Product of two Compact Groups
$SU(2)\otimes SU(2)$
Irreducible Representations
(For $U(1)$, consider the Charge (q))
Extracting out the Scaling Dimension
Δ_φ
Mapping onto the Maximal Torus
$\mathbb{T}^{N-1} = U(1)\otimes U(1)\cdots\otimes U(1)$
(Coordinates = ϵ_i)
Dynkin Labels corresponding to the Representation
$(a_1, a_2, \cdots, a_{N-1})$
Verma Module Characters of $SO(4)$ as a linear combination of product of two $SU(2)$ Characters
Highest Weight State
$(\lambda_1, \lambda_2, \cdots, \lambda_N)$
Parameterizing the Coordinates Using the Weight Tree of the Lowest Dimensional Fundamental Representation
$\epsilon_j \equiv \epsilon_j(z_i)$
$r_i = \lambda_i + N - i$
Spurions denoting (Super) Fields, Covariant Derivative D
Schur Functions / Weyl Character Formula
$\chi(\{\epsilon_i\},\{r_j\}) = \frac{|\epsilon^{r_1}, \epsilon^{r_2}, \ldots, \epsilon^{r_{N-1}}, 1|}{|\epsilon^{N-1}, \epsilon^{N-2}, \ldots, \epsilon, 1|}$
Vandermonde Determinant
$\Delta(\epsilon) = |\epsilon^{N-1}, \epsilon^{N-2}, \ldots, \epsilon, 1|$
Representation of the Fields
Characters for Representations of G
$\chi(z_i)$
(For $U(1)$, Character = z^q)
Lorentz Characters
$\tilde{\chi}(\mathcal{D}, \alpha, \beta)$
(Accounts for EOM Redundancy)
Plethystic Exponential
For integer spin fields:
$\mathrm{PE}[\phi, \mathcal{D}] = \exp\left[\sum_{r=1}^{\infty}\left(\frac{\phi}{\mathcal{D}^{\Delta_\phi}}\right)^r \frac{\prod_{j=1}^{n}\chi_j(z_i^r)\,\tilde{\chi}(\mathcal{D}^r, \alpha^r, \beta^r)}{r}\right]$
For half-integer spin fields:
$\mathrm{PE}[\psi, \mathcal{D}] = \exp\left[\sum_{r=1}^{\infty}(-1)^{r+1}\left(\frac{\psi}{\mathcal{D}^{\Delta_\psi}}\right)^r \frac{\prod_{j=1}^{n}\chi_j(z_i^r)\,\tilde{\chi}(\mathcal{D}^r, \alpha^r, \beta^r)}{r}\right]$
Haar Measure
$d\mu = \frac{1}{(2\pi i)^{N-1} N!}\oint_{|z_i|=1}\prod_{i=1}^{N-1}\frac{dz_i}{z_i}\Delta(\epsilon)\,\Delta(\epsilon^{-1})$
Momentum Generating Function
$P^{(4)}(\mathcal{D}, \alpha, \beta)$
(Accounts for IBP Redundancy)
HILBERT SERIES
$\mathcal{H}[\varphi] = \prod_{j=1}^{n}\int_{LG} d\mu_{LG}\int_{\mathcal{G}_j} d\mu_j\; PE[\varphi, \mathcal{D}, R]\;\frac{1}{P^{(4)}(\mathcal{D}, \alpha, \beta)}.$

Two Higgs Doublet Model (2HDM) : an example

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Spin	Lorentz Group
ϕ_1	1	2	1/2	0	Scalar
ϕ_2	1	2	1/2	0	Scalar
Q_L	3	2	1/6	1/2	Spinor
$(u^c)_L$	$\bar{3}$	1	2/3	1/2	Spinor
$(d^c)_L$	$\bar{3}$	1	-1/3	1/2	Spinor
L_L	1	2	-1/2	1/2	Spinor
$(e^c)_L$	1	1	-1	1/2	Spinor
G_μ^A	8	1	0	1	Vector
W_μ^I	1	3	0	1	Vector
B_μ	1	1	0	1	Vector

Table 1: 2HDM: Quantum numbers of the fields.

$$D_\mu \equiv (\partial_\mu + ig_3\frac{T^A}{2}G_\mu^A + ig\frac{\tau^I}{2}W_\mu^I + ig'YB_\mu).$$

$$\begin{aligned} G_{\mu\nu}^A &= \partial_\mu G_\nu^A - \partial_\nu G_\mu^A + g_3 f^{ABC} G_\mu^B G_\nu^C, \\ W_{\mu\nu}^I &= \partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g\epsilon^{IJK} W_\mu^J W_\nu^K, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{2HDM}^{(4)} = &-\frac{1}{4}Tr[G_{\mu\nu}G^{\mu\nu}] - \frac{1}{4}Tr[W_{\mu\nu}W^{\mu\nu}] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ (D_\mu\phi_1)^\dagger(D_\mu\phi_1) + (D_\mu\phi_2)^\dagger(D_\mu\phi_2) - V(\phi_1,\phi_2) \\ &+ i(\bar{L}\gamma^\mu D_\mu L + \bar{Q}\gamma^\mu D_\mu Q + \bar{e}\gamma^\mu D_\mu e + \bar{u}\gamma^\mu D_\mu u + \bar{d}\gamma^\mu D_\mu d) + L_Y + h.c., \end{aligned}$$

$$\mathcal{L}_Y = -y_1^e\bar{L}\phi_1 e - y_2^e\bar{L}\phi_2 e - y_1^d\bar{Q}\phi_1 d - y_2^d\bar{Q}\phi_2 d - y_1^u\bar{Q}\tilde{\phi}_1 u - y_2^u\bar{Q}\tilde{\phi}_2 u + h.c.,$$

$$\begin{aligned} V(\phi_1,\phi_2) = &\, m_{11}^2(\phi_1^\dagger\phi_1) + m_{22}^2(\phi_2^\dagger\phi_2) - m_{12}^2(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1) + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 \\ &+\lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{1}{2}\lambda_5[(\phi_1^\dagger\phi_2)^2 + (\phi_2^\dagger\phi_1)^2] \\ &+\left(\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right)(\phi_1^\dagger\phi_2 + \phi_2^\dagger\phi_1). \end{aligned}$$

2HDM Fields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	**Spin**
ϕ_1	1	2	1/2	0
ϕ_2	1	2	1/2	0
Q_L^p	3	2	1/6	1/2
u_R^p	3	1	2/3	1/2
d_R^p	3	1	-1/3	1/2
L_L^p	1	2	-1/2	1/2
e_R^p	1	1	-1	1/2
$B_{\mu\nu}$	1	1	0	1
$W_{\mu\nu}^I$	1	3	0	1
$G_{\mu\nu}^a$	8	1	0	1
$\mathcal{D}_\mu$	**Covariant Derivative**			

Table 2: 2HDM: Quantum numbers of fields under the gauge groups and their spins under the Lorentz group. I = 1,2,3 and a = 1,2,.....,8 correspond to $SU(2)$ and $SU(3)$ gauge indices respectively, and p = 1,...,N_f denotes the flavor index. The color and isospin indices have been suppressed. L and R denote the chirality, i.e., the left or right handedness of the field.

Arguments of Plethystics

$$
\begin{aligned}
Bl &\to \frac{1}{r} Bl^r\, \chi^{(4)}_{(1,0)}(\mathcal{D}^r,\alpha^r,\beta^r)\,, \\
Wl &\to \frac{1}{r} Wl^r\, \chi^{(4)}_{(1,0)}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(2))_3}(z_1^r), \\
Gl &\to \frac{1}{r} Gl^r\, \chi^{(4)}_{(1,0)}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(3))_8}(z_1^r,z_2^r), \\
\phi_1 &\to \frac{1}{r} \phi_1^r\, \chi^{(4)}_{(0,0)}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(2))_2}(z_1^r)\ z^{1/2}, \\
\phi_2 &\to \frac{1}{r} \phi_2^r\, \chi^{(4)}_{(0,0)}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(2))_2}(z_1^r)\ z^{r/2}, \\
e &\to \frac{(-1)^{r+1}}{r} e^r\, \chi^{(4)}_{(0,\frac{1}{2})}(\mathcal{D}^r,\alpha^r,\beta^r)\ z^{-r},
\end{aligned}
$$

$$
\begin{aligned}
Br &\to \frac{1}{r} Br^r\, \chi^{(4)}_{(0,1)}(\mathcal{D}^r,\alpha^r,\beta^r)\,, \\
Wr &\to \frac{1}{r} Wr^r\, \chi^{(4)}_{(0,1)}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(2))_3}(z_1^r), \\
Gr &\to \frac{1}{r} Gr^r\, \chi^{(4)}_{(0,1)}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(3))_8}(z_1^r,z_2^r), \\
\phi_1^\dagger &\to \frac{1}{r} (\phi_1^\dagger)^r\, \chi^{(4)}_{(0,0)}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(2))_{\overline{2}}}(z_1^r)\ z^{-r/2}, \\
\phi_2^\dagger &\to \frac{1}{r} (\phi_2^\dagger)^r\, \chi^{(4)}_{(0,0)}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(2))_{\overline{2}}}(z_1^r)\ z^{-r/2}, \\
e^\dagger &\to \frac{(-1)^{r+1}}{r} (e^\dagger)^r\, \chi_{(\frac{1}{2},0)}(\mathcal{D}^r,\alpha^r,\beta^r)\ z^{r},
\end{aligned}
$$

$$
\begin{aligned}
u &\to \frac{(-1)^{r+1}}{r} u^r\, \chi^{(4)}_{(0,\frac{1}{2})}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi^{(4)}_{(SU(3))_3}(z_2^r,z_3^r)\ z^{2r/3}, \\
u^\dagger &\to \frac{(-1)^{r+1}}{r} (u^\dagger)^r\, \chi^{(4)}_{(\frac{1}{2},0)}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(3))_{\overline{3}}}(z_2^r,z_3^r)\ z^{-2r/3}, \\
d &\to \frac{(-1)^{r+1}}{r} d^r\, \chi^{(4)}_{(0,\frac{1}{2})}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(3))_3}(z_2^r,z_3^r)\ z^{-r/3}, \\
d^\dagger &\to \frac{(-1)^{r+1}}{r} (d^\dagger)^r\, \chi^{(4)}_{(\frac{1}{2},0)}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(3))_{\overline{3}}}(z_2^r,z_3^r)\ z^{r/3}, \\
L &\to \frac{(-1)^{r+1}}{r} L^r\, \chi^{(4)}_{(\frac{1}{2},0)}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(2))_2}(z_1^r)\ z^{-r/2}, \\
L^\dagger &\to \frac{(-1)^{r+1}}{r} (L^\dagger)^r\, \chi^{(4)}_{(0,\frac{1}{2})}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(2))_{\overline{2}}}(z_1^r)\ z^{r/2}, \\
Q &\to \frac{(-1)^{r+1}}{r} Q^r\, \chi^{(4)}_{(\frac{1}{2},0)}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(3))_3}(z_2^r,z_3^r)\ \chi_{(SU(2))_2}(z_1^r)\ z^{r/6}, \\
Q^\dagger &\to \frac{(-1)^{r+1}}{r} (Q^\dagger)^r\, \chi^{(4)}_{(0,\frac{1}{2})}(\mathcal{D}^r,\alpha^r,\beta^r)\ \chi_{(SU(3))_{\overline{3}}}(z_2^r,z_3^r)\ \chi_{(SU(2))_{\overline{2}}}(z_1^r)\ z^{-r/6}.
\end{aligned}
$$

Dictionary for translating HS output to covariant forms of operators

Q	$\rightarrow$	Q_L^p	u	$\rightarrow$	u_R^p	d	$\rightarrow$	d_R^p
$Q^\dagger$	$\rightarrow$	$\overline{Q}_L^p$	$u^\dagger$	$\rightarrow$	$\overline{u}_R^p$	$d^\dagger$	$\rightarrow$	$\overline{d}_R^p$
L	$\rightarrow$	L_L^p	e	$\rightarrow$	e_R^p	$\phi_{1,2}, \phi_{1,2}^\dagger$	$\rightarrow$	$\phi_{1,2}, \phi_{1,2}^\dagger$
$L^\dagger$	$\rightarrow$	$\overline{L}_L^p$	$e^\dagger$	$\rightarrow$	$\overline{e}_R^p$	$\mathcal{D}$	$\rightarrow$	$\mathcal{D}_\mu$
(Wl, Wr)	$\rightarrow$	$(W_{\mu\nu}^I, \tilde{W}_{\mu\nu}^I)$	(Gl, Gr)	$\rightarrow$	$(G_{\mu\nu}^a, \tilde{G}_{\mu\nu}^a)$	(Bl, Br)	$\rightarrow$	$(B_{\mu\nu}, \tilde{B}_{\mu\nu})$

2HDM: example

Mass Dimension-2			
Operator Type	**HS Output**	**Covariant Form**	**No. of Operators (including h.c.)**
Pure SM	$\phi_1^\dagger\phi_1$	$\phi_1^\dagger\phi_1$	1
Pure BSM	$\phi_2^\dagger\phi_2$	$\phi_2^\dagger\phi_2$	1
Mixed	$\phi_1^\dagger\phi_2$	$\phi_1^\dagger\phi_2$	2

Mass Dimension-4			
Operator Type	**HS Output**	**Covariant Form**	**No. of Operators (including h.c.)**
Pure SM	$Bl^2 + Br^2$, $Gl^2 + Gr^2$, $Wl^2 + Wr^2$, $N_f^2 L^\dagger\phi_1 e,\ N_f^2 Q^\dagger\phi_1 d,\ N_f^2 Q^\dagger\phi_1^\dagger u$, $^\clubsuit N_f^2 Q^\dagger Q\mathcal{D},\ N_f^2 u^\dagger u\mathcal{D},\ N_f^2 d^\dagger d\mathcal{D}$, $N_f^2 L^\dagger L\mathcal{D},\ N_f^2 e^\dagger e\mathcal{D},\ \phi_1^\dagger\phi_1\mathcal{D}^2,\ (\phi_1^\dagger\phi_1)^2$	$B^{\mu\nu}B_{\mu\nu},\quad B^{\mu\nu}\tilde{B}_{\mu\nu}$, $G^{a\mu\nu}G_{\mu\nu}^a,\quad G^{a\mu\nu}\tilde{G}_{\mu\nu}^a$, $W^{I\mu\nu}W_{\mu\nu}^I,\quad W^{I\mu\nu}\tilde{W}_{\mu\nu}^I$, $\overline{L}_L^r\phi_1 e_R^s,\ \overline{Q}_L^r\phi_1 d_R^s,\ \overline{Q}_L^r\tilde{\phi}_1 u_R^s$, $^\clubsuit\overline{Q}_L^r \not{\mathcal{D}} Q_L^r,\ \overline{u}_R^r \not{\mathcal{D}} u_R^r,\ \overline{d}_R^r \not{\mathcal{D}} d_R^r$, $\overline{L}_L^r \not{\mathcal{D}} L_L^r,\ \overline{e}_R^r \not{\mathcal{D}} e_R^r,\ (\mathcal{D}_\mu\phi_1)^\dagger(\mathcal{D}^\mu\phi_1),\ (\phi_1^\dagger\phi_1)^2$	$11N_f^2 + 8$
Pure BSM	$\phi_2^\dagger\phi_2\mathcal{D}^2,\ (\phi_2^\dagger\phi_2)^2$	$(\mathcal{D}_\mu\phi_2)^\dagger(\mathcal{D}^\mu\phi_2),\ (\phi_2^\dagger\phi_2)^2$	2
Mixed	$N_f^2 L^\dagger\phi_2 e,\ N_f^2 Q^\dagger\phi_2 d,\ N_f^2 Q^\dagger\phi_2^\dagger u$, $(\phi_1^\dagger)^2(\phi_2)^2,\ (\phi_1^\dagger)^2\phi_1\phi_2,\ (\phi_2^\dagger)^2\phi_1\phi_2$ $2\phi_1^\dagger\phi_1\phi_2^\dagger\phi_2,\quad \phi_1^\dagger\phi_2\mathcal{D}^2$	$\overline{L}_L^r\phi_2 e_R^s,\ \overline{Q}_L^r\phi_2 d_R^s,\ \overline{Q}_L^r\tilde{\phi}_2 u_R^s$, $(\phi_1^\dagger\phi_2)^2,\ (\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2),\ (\phi_2^\dagger\phi_1)(\phi_2^\dagger\phi_2)$ $(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2), (\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1),\ (\mathcal{D}_\mu\phi_1)^\dagger(\mathcal{D}^\mu\phi_2)$	$6N_f^2 + 10$

2HDM: example

HS output

Mass Dimension-5	
Operator Class	**Operators (in non-covariant form)**
$\Psi^2\Phi^2$	$\frac{1}{2}(N_f^2+N_f)\phi_1^2L^2$, $\frac{1}{2}(N_f^2+N_f)\phi_2^2L^2$, $N_f^2\phi_1\phi_2L^2$

Mass Dimension-6	
Operator Class	**Operators (in non-covariant form)**
X^3	Wl^3, Wr^3, Gl^3, Gr^3
Φ^6	$(\phi_1^\dagger\phi_1)^3$, $(\phi_2^\dagger\phi_2)^3$, $2\phi_1^2(\phi_1^\dagger)^2\phi_2\phi_2^\dagger$, $2\phi_1\phi_1^\dagger\phi_2^2(\phi_2^\dagger)^2$, $(\phi_1^\dagger)^3\phi_2^3$, $\phi_1^2(\phi_1^\dagger)^3\phi_2$, $\phi_1(\phi_1^\dagger)^3\phi_2^2$, $(\phi_1^\dagger)^2\phi_2^3\phi_2^\dagger$, $\phi_1^\dagger\phi_2^3(\phi_2^\dagger)^2$, $2\phi_1(\phi_1^\dagger)^2\phi_2^2\phi_2^\dagger$
Φ^2X^2	$Gl^2\phi_1^\dagger\phi_1$, $Gl^2\phi_2^\dagger\phi_2$, $Gl^2\phi_1^\dagger\phi_2$, $Gl^2\phi_1\phi_2^\dagger$, $Wl^2\phi_1^\dagger\phi_1$, $Wl^2\phi_2^\dagger\phi_2$, $Wl^2\phi_1^\dagger\phi_2$, $Wl^2\phi_1\phi_2^\dagger$, $Bl^2\phi_1^\dagger\phi_1$, $Bl^2\phi_2^\dagger\phi_2$, $Bl^2\phi_1^\dagger\phi_2$, $Bl^2\phi_1\phi_2^\dagger$, $BlWl\phi_1^\dagger\phi_1$, $BlWl\phi_2^\dagger\phi_2$, $BlWl\phi_1^\dagger\phi_2$, $BlWl\phi_1\phi_2^\dagger$
$\Psi^2\Phi X$	$(N_f^2)Gl\phi_1^\dagger d^\dagger Q$, $(N_f^2)Gl\phi_2^\dagger d^\dagger Q$, $(N_f^2)Gl\phi_1u^\dagger Q$, $(N_f^2)Gl\phi_2u^\dagger Q$, $(N_f^2)Wl\phi_1^\dagger d^\dagger Q$, $(N_f^2)Wl\phi_2^\dagger d^\dagger Q$, $(N_f^2)Wl\phi_1u^\dagger Q$, $(N_f^2)Wl\phi_2u^\dagger Q$, $(N_f^2)Wl\phi_1^\dagger e^\dagger L$, $(N_f^2)Wl\phi_2^\dagger e^\dagger L$, $(N_f^2)Bl\phi_1^\dagger d^\dagger Q$, $(N_f^2)Bl\phi_2^\dagger d^\dagger Q$, $(N_f^2)Bl\phi_1u^\dagger Q$, $(N_f^2)Bl\phi_2u^\dagger Q$, $(N_f^2)Bl\phi_1^\dagger e^\dagger L$, $(N_f^2)Bl\phi_2^\dagger e^\dagger L$
$\Psi^2\Phi^2\mathcal{D}$	$(N_f^2)dd^\dagger\phi_1\phi_1^\dagger\mathcal{D}$, $(N_f^2)dd^\dagger\phi_2\phi_2^\dagger\mathcal{D}$, $(N_f^2)ee^\dagger\phi_1\phi_1^\dagger\mathcal{D}$, $(N_f^2)ee^\dagger\phi_2\phi_2^\dagger\mathcal{D}$, $(2N_f^2)LL^\dagger\phi_1\phi_1^\dagger\mathcal{D}$, $(2N_f^2)LL^\dagger\phi_2\phi_2^\dagger\mathcal{D}$, $(2N_f^2)QQ^\dagger\phi_1\phi_1^\dagger\mathcal{D}$, $(2N_f^2)QQ^\dagger\phi_2\phi_2^\dagger\mathcal{D}$, $(N_f^2)uu^\dagger\phi_1\phi_1^\dagger\mathcal{D}$, $(N_f^2)uu^\dagger\phi_2\phi_2^\dagger\mathcal{D}$, $(N_f^2)dd^\dagger\phi_1^\dagger\phi_2\mathcal{D}$, $(N_f^2)ee^\dagger\phi_1^\dagger\phi_2\mathcal{D}$, $(N_f^2)uu^\dagger\phi_1^\dagger\phi_2\mathcal{D}$, $(2N_f^2)LL^\dagger\phi_1^\dagger\phi_2\mathcal{D}$, $(2N_f^2)QQ^\dagger\phi_1^\dagger\phi_2\mathcal{D}$, $(N_f^2)ud^\dagger(\phi_1^\dagger)^2\mathcal{D}$, $(N_f^2)ud^\dagger(\phi_2^\dagger)^2\mathcal{D}$, $(N_f^2)ud^\dagger(\phi_1^\dagger)(\phi_2^\dagger)\mathcal{D}$
$\Psi^2\Phi^3$	$(N_f^2)\phi_1(\phi_1^\dagger)^2e^\dagger L$, $(N_f^2)\phi_2(\phi_2^\dagger)^2e^\dagger L$, $(N_f^2)\phi_1(\phi_2^\dagger)^2e^\dagger L$, $(N_f^2)(\phi_1^\dagger)^2\phi_2e^\dagger L$, $(2N_f^2)\phi_1\phi_1^\dagger\phi_2^\dagger e^\dagger L$, $(2N_f^2)\phi_1^\dagger\phi_2\phi_2^\dagger e^\dagger L$, $(N_f^2)\phi_1(\phi_1^\dagger)^2d^\dagger Q$, $(N_f^2)\phi_2(\phi_2^\dagger)^2d^\dagger Q$, $(N_f^2)\phi_1(\phi_2^\dagger)^2d^\dagger Q$, $(N_f^2)(\phi_1^\dagger)^2\phi_2d^\dagger Q$, $(2N_f^2)\phi_1\phi_1^\dagger\phi_2^\dagger d^\dagger Q$, $(2N_f^2)\phi_1^\dagger\phi_2\phi_2^\dagger d^\dagger Q$, $(N_f^2)\phi_1^2\phi_1^\dagger u^\dagger Q$, $(N_f^2)\phi_2^2\phi_2^\dagger u^\dagger Q$, $(N_f^2)\phi_1^2\phi_2^\dagger u^\dagger Q$, $(N_f^2)\phi_1^\dagger\phi_2^2u^\dagger Q$, $(2N_f^2)\phi_1\phi_1^\dagger\phi_2u^\dagger Q$, $(2N_f^2)\phi_1\phi_2\phi_2^\dagger u^\dagger Q$
$\Phi^4\mathcal{D}^2$	$2\phi_1^2(\phi_1^\dagger)^2\mathcal{D}^2$, $2\phi_2^2(\phi_2^\dagger)^2\mathcal{D}^2$, $4\phi_1\phi_2\phi_1^\dagger\phi_2^\dagger\mathcal{D}^2$, $2\phi_1^2(\phi_2^\dagger)^2\mathcal{D}^2$, $2\phi_1\phi_2(\phi_2^\dagger)^2\mathcal{D}^2$, $2\phi_1(\phi_1^\dagger)^2\phi_2\mathcal{D}^2$
Ψ^4	$(N_f^4)ee^\dagger uu^\dagger$, $(\frac{1}{2}N_f^2+\frac{1}{2}N_f^4)L^2(L^\dagger)^2$, $(N_f^2+N_f^4)Q^2(Q^\dagger)^2$, $(2N_f^4)LL^\dagger QQ^\dagger$, $(N_f^4)ee^\dagger dd^\dagger$, $(\frac{1}{4}N_f^2+\frac{1}{2}N_f^3+\frac{1}{4}N_f^4)e^2(e^\dagger)^2$, $(\frac{1}{2}N_f^2+\frac{1}{2}N_f^4)d^2(d^\dagger)^2$, $(\frac{1}{2}N_f^2+\frac{1}{2}N_f^4)u^2(u^\dagger)^2$, $(2N_f^4)dd^\dagger uu^\dagger$, $(N_f^4)ee^\dagger LL^\dagger$, $(N_f^4)uu^\dagger LL^\dagger$, $(N_f^4)dd^\dagger LL^\dagger$, $(N_f^4)ee^\dagger QQ^\dagger$, $(2N_f^4)uu^\dagger QQ^\dagger$, $(2N_f^4)dd^\dagger QQ^\dagger$, $(N_f^4)eL^\dagger d^\dagger Q$, $(2N_f^4)ud(Q^\dagger)^2$, $(2N_f^4)eL^\dagger uQ^\dagger$, $(\frac{1}{3}N_f^2+\frac{2}{3}N_f^4)LQ^3$, $(\frac{1}{2}N_f^3+\frac{1}{2}N_f^4)e\,u\,Q^2$, $(N_f^4)eu^2d$, $(N_f^4)L\,u\,d\,Q$

EFT Operator classification

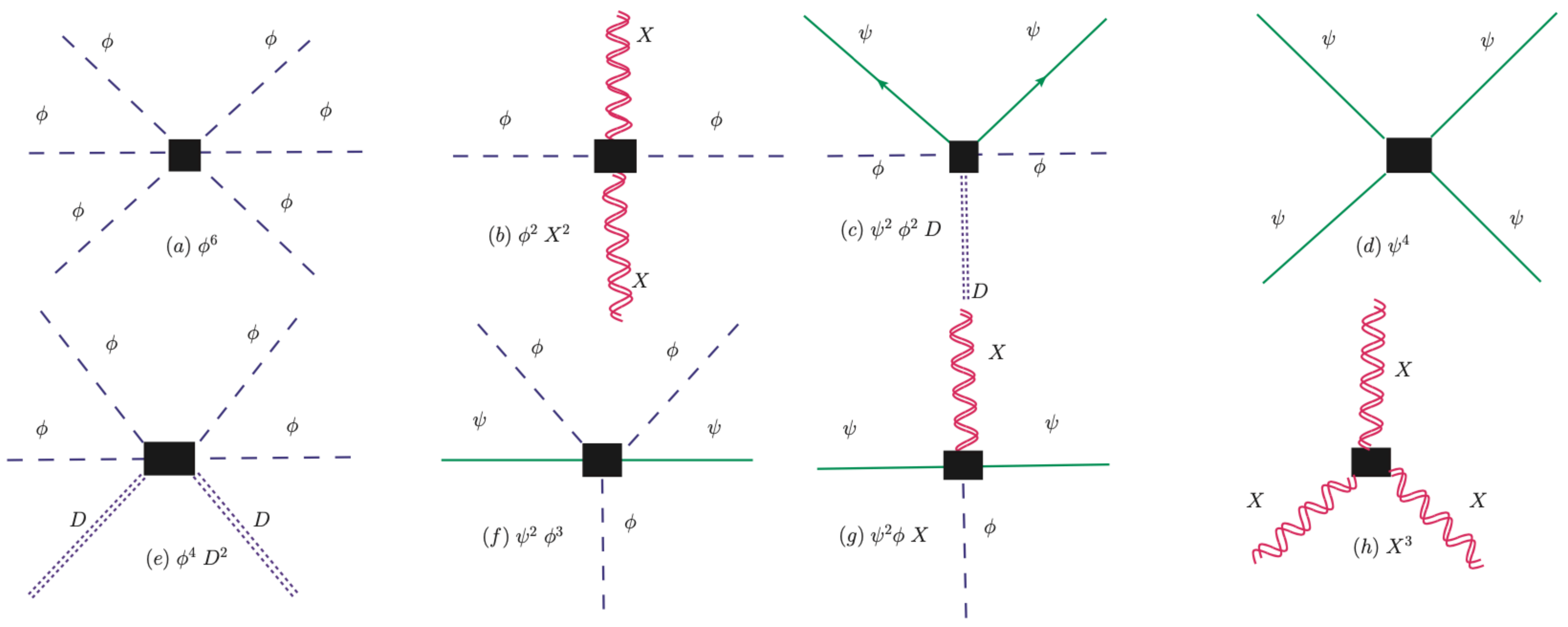


Figure 2: Effective operators in "Warsaw"-like basis representing the following class of operators: (a) ϕ^6 [6 + 7×2 = 20], (b) ϕ^2X^2 [32], (c) $\psi^2\phi^2D$ [14 + 10×2 = 34], (d) ψ^4 [20 + 5×2 = 30] (Baryon Number Conserving) + [4×2 = 8] (Baryon Number Violating), (e) ϕ^4D^2 [8 + 6×2 = 20], (f) $\psi^2\phi^3$ [24×2 = 48], (g) $\psi^2\phi X$ [16×2 = 32], (h) X^3 [4].

2HDM: dim-6 effective operators in covariant form

$\mathcal{O}_{\phi L}^{11}$	$(L^T C i\tau_2\phi_1)(\tilde{\phi_1}^{\dagger}L) + h.c.$	$\mathcal{O}_{\phi L}^{22}$	$(L^T C i\tau_2\phi_2)(\tilde{\phi_2}^{\dagger}L) + h.c.$	$\mathcal{O}_{\phi L}^{12}$	$(L^T C i\tau_2\phi_1)(\tilde{\phi_2}^{\dagger}L) + h.c.$

Table 3: 2HDM: $\phi^2 L^2$ [$3\times2 = 6$] class of operators.

$\mathcal{O}_6^{111111}$	$(\phi_1^\dagger\phi_1)^3$	$\mathcal{O}_6^{222222}$	$(\phi_2^\dagger\phi_2)^3$
$\mathcal{O}_6^{111122}$	$(\phi_1^\dagger\phi_1)^2(\phi_2^\dagger\phi_2)$	$\mathcal{O}_6^{112222}$	$(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)^2$
$\mathcal{O}_6^{122111}$	$(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)(\phi_1^\dagger\phi_1)$	$\mathcal{O}_6^{122122}$	$(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)(\phi_2^\dagger\phi_2)$
$\mathcal{O}_6^{121211}$	$(\phi_1^\dagger\phi_2)^2(\phi_1^\dagger\phi_1)$ + h.c.	$\mathcal{O}_6^{121222}$	$(\phi_1^\dagger\phi_2)^2(\phi_2^\dagger\phi_2)$ + h.c.
$\mathcal{O}_6^{111112}$	$(\phi_1^\dagger\phi_1)^2(\phi_1^\dagger\phi_2)$ + h.c.	$\mathcal{O}_6^{122222}$	$(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_2)^2$ + h.c.
$\mathcal{O}_6^{121221}$	$(\phi_1^\dagger\phi_2)^2(\phi_2^\dagger\phi_1)$ + h.c.	$\mathcal{O}_6^{121212}$	$(\phi_1^\dagger\phi_2)^3$ + h.c.
$\mathcal{O}_6^{112212}$	$(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2)$ + h.c.		

Table 4: 2HDM: ϕ^6 [$6 + 7\times2 = 20$] class of operators.

$\mathcal{O}_{dG}^{1}$	$(\bar{Q}\sigma^{\mu\nu}T^A d)\phi_1 G_{\mu\nu}^A$	$\mathcal{O}_{dG}^{2}$	$(\bar{Q}\sigma^{\mu\nu}T^A d)\phi_2 G_{\mu\nu}^A$
$\mathcal{O}_{uG}^{\tilde{1}}$	$(\bar{Q}\sigma^{\mu\nu}T^A u)\tilde{\phi}_1 G_{\mu\nu}^A$	$\mathcal{O}_{uG}^{\tilde{2}}$	$(\bar{Q}\sigma^{\mu\nu}T^A u)\tilde{\phi}_2 G_{\mu\nu}^A$
$\mathcal{O}_{dW}^{1}$	$(\bar{Q}\sigma^{\mu\nu}d)\tau^I\phi_1 W_{\mu\nu}^I$	$\mathcal{O}_{dW}^{2}$	$(\bar{Q}\sigma^{\mu\nu}d)\tau^I\phi_2 W_{\mu\nu}^I$
$\mathcal{O}_{uW}^{\tilde{1}}$	$(\bar{Q}\sigma^{\mu\nu}u)\tau^I\tilde{\phi}_1 W_{\mu\nu}^I$	$\mathcal{O}_{uW}^{\tilde{2}}$	$(\bar{Q}\sigma^{\mu\nu}u)\tau^I\tilde{\phi}_2 W_{\mu\nu}^I$
$\mathcal{O}_{dB}^{1}$	$(\bar{Q}\sigma^{\mu\nu}d)\phi_1 B_{\mu\nu}$	$\mathcal{O}_{dB}^{2}$	$(\bar{Q}\sigma^{\mu\nu}d)\phi_2 B_{\mu\nu}$
$\mathcal{O}_{uB}^{\tilde{1}}$	$(\bar{Q}\sigma^{\mu\nu}u)\tilde{\phi}_1 B_{\mu\nu}$	$\mathcal{O}_{uB}^{\tilde{2}}$	$(\bar{Q}\sigma^{\mu\nu}u)\tilde{\phi}_2 B_{\mu\nu}$
$\mathcal{O}_{eW}^{1}$	$(\bar{L}\sigma^{\mu\nu}e)\tau^I\phi_1 W_{\mu\nu}^I$	$\mathcal{O}_{eW}^{2}$	$(\bar{L}\sigma^{\mu\nu}e)\tau^I\phi_2 W_{\mu\nu}^I$
$\mathcal{O}_{eB}^{1}$	$(\bar{L}\sigma^{\mu\nu}e)\phi_1 B_{\mu\nu}$	$\mathcal{O}_{eB}^{2}$	$(\bar{L}\sigma^{\mu\nu}e)\phi_2 B_{\mu\nu}$

Table 7: 2HDM: $\psi^2\phi X$ [$16\times2 = 32$] class of operators. Each of these operators also has a distinct Hermitian Conjugate.

$\mathcal{O}_G$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$\mathcal{O}_{\tilde{G}}$	$f^{ABC}\tilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$\mathcal{O}_W$	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$
$\mathcal{O}_{\tilde{W}}$	$\epsilon^{IJK}\tilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$

Table 6: 2HDM: X^3 [4] class of operators.

$\mathcal{O}_{\phi G}^{11}$	$(\phi_1^\dagger\phi_1)G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{\phi\tilde{G}}^{11}$	$(\phi_1^\dagger\phi_1)G_{\mu\nu}^A\tilde{G}^{A\mu\nu}$	$\mathcal{O}_{\phi G}^{22}$	$(\phi_2^\dagger\phi_2)G_{\mu\nu}^A G^{A\mu\nu}$
$\mathcal{O}_{\phi W}^{11}$	$(\phi_1^\dagger\phi_1)W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{\phi\tilde{W}}^{11}$	$(\phi_1^\dagger\phi_1)W_{\mu\nu}^I\tilde{W}^{I\mu\nu}$	$\mathcal{O}_{\phi\tilde{G}}^{22}$	$(\phi_2^\dagger\phi_2)G_{\mu\nu}^A\tilde{G}^{A\mu\nu}$
$\mathcal{O}_{\phi B}^{11}$	$(\phi_1^\dagger\phi_1)B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{\phi\tilde{B}}^{11}$	$(\phi_1^\dagger\phi_1)B_{\mu\nu}\tilde{B}^{\mu\nu}$	$\mathcal{O}_{\phi W}^{22}$	$(\phi_2^\dagger\phi_2)W_{\mu\nu}^I W^{I\mu\nu}$
$\mathcal{O}_{\phi B}^{22}$	$(\phi_2^\dagger\phi_2)B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{\phi\tilde{B}}^{22}$	$(\phi_2^\dagger\phi_2)B_{\mu\nu}\tilde{B}^{\mu\nu}$	$\mathcal{O}_{\phi\tilde{W}}^{22}$	$(\phi_2^\dagger\phi_2)W_{\mu\nu}^I\tilde{W}^{I\mu\nu}$
$\mathcal{O}_{\phi WB}^{11}$	$(\phi_1^\dagger\tau^I\phi_1)W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{\phi\tilde{W}B}^{11}$	$(\phi_1^\dagger\tau^I\phi_1)\tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{\phi WB}^{22}$	$(\phi_2^\dagger\tau^I\phi_2)W_{\mu\nu}^I B^{\mu\nu}$
$\mathcal{O}_{\phi\tilde{W}B}^{22}$	$(\phi_2^\dagger\tau^I\phi_2)\tilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{\phi G}^{12}$	$(\phi_1^\dagger\phi_2)G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{\phi\tilde{G}}^{12}$	$(\phi_1^\dagger\phi_2)G_{\mu\nu}^A\tilde{G}^{A\mu\nu}$
$\mathcal{O}_{\phi G}^{21}$	$(\phi_2^\dagger\phi_1)G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{\phi\tilde{G}}^{21}$	$(\phi_2^\dagger\phi_1)G_{\mu\nu}^A\tilde{G}^{A\mu\nu}$	$\mathcal{O}_{\phi W}^{12}$	$(\phi_1^\dagger\phi_2)W_{\mu\nu}^I W^{I\mu\nu}$
$\mathcal{O}_{\phi\tilde{W}}^{12}$	$(\phi_1^\dagger\phi_2)W_{\mu\nu}^I\tilde{W}^{I\mu\nu}$	$\mathcal{O}_{\phi W}^{21}$	$(\phi_2^\dagger\phi_1)W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{\phi\tilde{W}}^{21}$	$(\phi_2^\dagger\phi_1)W_{\mu\nu}^I\tilde{W}^{I\mu\nu}$
$\mathcal{O}_{\phi B}^{12}$	$(\phi_1^\dagger\phi_2)B_{\mu\nu}B^{\mu\nu}$	$\mathcal{O}_{\phi\tilde{B}}^{12}$	$(\phi_1^\dagger\phi_2)B_{\mu\nu}\tilde{B}^{\mu\nu}$	$\mathcal{O}_{\phi B}^{21}$	$(\phi_2^\dagger\phi_1)B_{\mu\nu}B^{\mu\nu}$
$\mathcal{O}_{\phi\tilde{B}}^{21}$	$(\phi_2^\dagger\phi_1)B_{\mu\nu}\tilde{B}^{\mu\nu}$	$\mathcal{O}_{\phi WB}^{12}$	$(\phi_1^\dagger\tau^I\phi_2)W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{\phi\tilde{W}B}^{12}$	$(\phi_1^\dagger\tau^I\phi_2)\tilde{W}_{\mu\nu}^I B^{\mu\nu}$
$\mathcal{O}_{\phi WB}^{21}$	$(\phi_2^\dagger\tau^I\phi_1)W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{\phi\tilde{W}B}^{21}$	$(\phi_2^\dagger\tau^I\phi_1)\tilde{W}_{\mu\nu}^I B^{\mu\nu}$		

Table 5: 2HDM: $\phi^2 X^2$ [32] class of operators.

$\mathcal{O}_{Le}^{1(21)}$	$(\bar{L}e\phi_1)(\phi_2^\dagger\phi_1)$	$\mathcal{O}_{Le}^{2(22)}$	$(\bar{L}e\phi_2)(\phi_2^\dagger\phi_2)$	$\mathcal{O}_{Le}^{2(11)}$	$(\bar{L}e\phi_2)(\phi_1^\dagger\phi_1)$
$\mathcal{O}_{Le}^{1(12)}$	$(\bar{L}e\phi_1)(\phi_1^\dagger\phi_2)$	$\mathcal{O}_{Qd}^{1(21)}$	$(\bar{Q}d\phi_1)(\phi_2^\dagger\phi_1)$	$\mathcal{O}_{Qd}^{2(22)}$	$(\bar{Q}d\phi_2)(\phi_2^\dagger\phi_2)$
$\mathcal{O}_{Qd}^{2(11)}$	$(\bar{Q}d\phi_2)(\phi_1^\dagger\phi_1)$	$\mathcal{O}_{Qd}^{1(12)}$	$(\bar{Q}d\phi_1)(\phi_1^\dagger\phi_2)$	$\mathcal{O}_{Qu}^{\tilde{2}(22)}$	$(\bar{Q}u\tilde{\phi}_2)(\phi_2^\dagger\phi_2)$
$\mathcal{O}_{Qu}^{\tilde{1}(12)}$	$(\bar{Q}u\tilde{\phi}_1)(\phi_1^\dagger\phi_2)$	$\mathcal{O}_{Qu}^{\tilde{2}(11)}$	$(\bar{Q}u\tilde{\phi}_2)(\phi_1^\dagger\phi_1)$	$\mathcal{O}_{Qu}^{\tilde{1}(21)}$	$(\bar{Q}u\tilde{\phi}_1)(\phi_2^\dagger\phi_1)$
$\mathcal{O}_{Le}^{1(11)}$	$(\bar{L}e\phi_1)(\phi_1^\dagger\phi_1)$	$\mathcal{O}_{Le}^{2(12)}$	$(\bar{L}e\phi_2)(\phi_1^\dagger\phi_2)$	$\mathcal{O}_{Le}^{1(22)}$	$(\bar{L}e\phi_1)(\phi_2^\dagger\phi_2)$
$\mathcal{O}_{Le}^{2(21)}$	$(\bar{L}e\phi_2)(\phi_2^\dagger\phi_1)$	$\mathcal{O}_{Qd}^{1(11)}$	$(\bar{Q}d\phi_1)(\phi_1^\dagger\phi_1)$	$\mathcal{O}_{Qd}^{2(12)}$	$(\bar{Q}d\phi_2)(\phi_1^\dagger\phi_2)$
$\mathcal{O}_{Qd}^{1(22)}$	$(\bar{Q}d\phi_1)(\phi_2^\dagger\phi_2)$	$\mathcal{O}_{Qd}^{2(21)}$	$(\bar{Q}d\phi_2)(\phi_2^\dagger\phi_1)$	$\mathcal{O}_{Qu}^{\tilde{1}(11)}$	$(\bar{Q}u\tilde{\phi}_1)(\phi_1^\dagger\phi_1)$
$\mathcal{O}_{Qu}^{\tilde{2}(21)}$	$(\bar{Q}u\tilde{\phi}_2)(\phi_2^\dagger\phi_1)$	$\mathcal{O}_{Qu}^{\tilde{1}(22)}$	$(\bar{Q}u\tilde{\phi}_1)(\phi_2^\dagger\phi_2)$	$\mathcal{O}_{Qu}^{\tilde{2}(12)}$	$(\bar{Q}u\tilde{\phi}_2)(\phi_1^\dagger\phi_2)$

Table 8: 2HDM: $\psi^2\phi^3$ [$24\times2 = 48$] class of operators. Each of these operators also has a distinct Hermitian Conjugate.

2HDM: dim-6 effective operators in covariant form

$\mathcal{O}^{11[1]}_{L\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}_\mu \phi_1)(\bar{L}\gamma^\mu L)$	$\mathcal{O}^{22[1]}_{L\phi D}$	$i(\phi_2^\dagger \overleftrightarrow{D}_\mu \phi_2)(\bar{L}\gamma^\mu L)$	$\mathcal{O}^{12[1]}_{L\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}_\mu \phi_2)(\bar{L}\gamma^\mu L) + h.c.$
$\mathcal{O}^{11[1]}_{Q\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}_\mu \phi_1)(\bar{Q}\gamma^\mu Q)$	$\mathcal{O}^{22[1]}_{Q\phi D}$	$i(\phi_2^\dagger \overleftrightarrow{D}_\mu \phi_2)(\bar{Q}\gamma^\mu Q)$	$\mathcal{O}^{12[1]}_{Q\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}_\mu \phi_2)(\bar{Q}\gamma^\mu Q) + h.c.$
$\mathcal{O}^{11}_{e\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}_\mu \phi_1)(\bar{e}\gamma^\mu e)$	$\mathcal{O}^{22}_{e\phi D}$	$i(\phi_2^\dagger \overleftrightarrow{D}_\mu \phi_2)(\bar{e}\gamma^\mu e)$	$\mathcal{O}^{12}_{e\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}_\mu \phi_2)(\bar{e}\gamma^\mu e) + h.c.$
$\mathcal{O}^{11}_{d\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}_\mu \phi_1)(\bar{d}\gamma^\mu d)$	$\mathcal{O}^{22}_{d\phi D}$	$i(\phi_2^\dagger \overleftrightarrow{D}_\mu \phi_2)(\bar{d}\gamma^\mu d)$	$\mathcal{O}^{12}_{d\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}_\mu \phi_2)(\bar{d}\gamma^\mu d) + h.c.$
$\mathcal{O}^{11}_{u\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}_\mu \phi_1)(\bar{u}\gamma^\mu u)$	$\mathcal{O}^{22}_{u\phi D}$	$i(\phi_2^\dagger \overleftrightarrow{D}_\mu \phi_2)(\bar{u}\gamma^\mu u)$	$\mathcal{O}^{12}_{u\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}_\mu \phi_2)(\bar{u}\gamma^\mu u) + h.c.$
$\mathcal{O}^{11[3]}_{L\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}^I_\mu \phi_1)(\bar{L}\tau^I\gamma^\mu L)$	$\mathcal{O}^{22[3]}_{L\phi D}$	$i(\phi_2^\dagger \overleftrightarrow{D}^I_\mu \phi_2)(\bar{L}\tau^I\gamma^\mu L)$	$\mathcal{O}^{12[3]}_{L\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}^I_\mu \phi_2)(\bar{L}\tau^I\gamma^\mu L) + h.c.$
$\mathcal{O}^{11[3]}_{Q\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}^I_\mu \phi_1)(\bar{Q}\tau^I\gamma^\mu Q)$	$\mathcal{O}^{22[3]}_{Q\phi D}$	$i(\phi_2^\dagger \overleftrightarrow{D}^I_\mu \phi_2)(\bar{Q}\tau^I\gamma^\mu Q)$	$\mathcal{O}^{12[3]}_{Q\phi D}$	$i(\phi_1^\dagger \overleftrightarrow{D}^I_\mu \phi_2)(\bar{Q}\tau^I\gamma^\mu Q) + h.c.$
$\mathcal{O}^{\tilde{1}1}_{ud\phi D}$	$i(\tilde{\phi}_1{}^\dagger \overleftrightarrow{D}_\mu \phi_1)(\bar{u}\gamma^\mu d) + h.c.$	$\mathcal{O}^{\tilde{2}2}_{ud\phi D}$	$i(\tilde{\phi}_2{}^\dagger \overleftrightarrow{D}_\mu \phi_2)(\bar{u}\gamma^\mu d) + h.c.$	$\mathcal{O}^{\tilde{2}1}_{ud\phi D}$	$i(\tilde{\phi}_2{}^\dagger \overleftrightarrow{D}_\mu \phi_1)(\bar{u}\gamma^\mu d) + h.c.$

Table 9: 2HDM: $\psi^2\phi^2D$ [14 + 10×2 = 20] class of operators.

$\mathcal{O}^{11(11)}_{\Box}$	$(\phi_1^\dagger\phi_1)\Box(\phi_1^\dagger\phi_1)$	$\mathcal{O}^{22(22)}_{\Box}$	$(\phi_2^\dagger\phi_2)\Box(\phi_2^\dagger\phi_2)$
$\mathcal{O}^{11(22)}_{\Box}$	$(\phi_1^\dagger\phi_1)\Box(\phi_2^\dagger\phi_2)$	$\mathcal{O}^{22(11)}_{\Box}$	$(\phi_2^\dagger\phi_2)\Box(\phi_1^\dagger\phi_1)$
$\mathcal{O}^{21(21)}_{\Box}$	$(\phi_2^\dagger\phi_1)\Box(\phi_2^\dagger\phi_1) + h.c.$	$\mathcal{O}^{12(1)(2)}_{\phi D}$	$(\phi_1^\dagger\phi_2)\left[(D^\mu\phi_1)^\dagger(D_\mu\phi_2)\right] + h.c.$
$\mathcal{O}^{(1)11(1)}_{\phi D}$	$\left[(D^\mu\phi_1)^\dagger\phi_1\right]\left[\phi_1^\dagger(D_\mu\phi_1)\right]$	$\mathcal{O}^{(2)22(2)}_{\phi D}$	$\left[(D^\mu\phi_2)^\dagger\phi_2\right]\left[\phi_2^\dagger(D_\mu\phi_2)\right]$
$\mathcal{O}^{(1)22(1)}_{\phi D}$	$\left[(D^\mu\phi_1)^\dagger\phi_2\right]\left[\phi_2^\dagger(D_\mu\phi_1)\right]$	$\mathcal{O}^{(2)11(2)}_{\phi D}$	$\left[(D^\mu\phi_2)^\dagger\phi_1\right]\left[\phi_1^\dagger(D_\mu\phi_2)\right]$
$\mathcal{O}^{(2)22(1)}_{\phi D}$	$\left[(D^\mu\phi_2)^\dagger\phi_2\right]\left[\phi_2^\dagger(D_\mu\phi_1)\right] + h.c.$	$\mathcal{O}^{(1)21(1)}_{\phi D}$	$\left[(D^\mu\phi_1)^\dagger\phi_2\right]\left[\phi_1^\dagger(D_\mu\phi_1)\right] + h.c.$
$\mathcal{O}^{21(2)(2)}_{\phi D}$	$(\phi_2^\dagger\phi_1)\left[(D^\mu\phi_2)^\dagger((D_\mu\phi_2))\right] + h.c.$	$\mathcal{O}^{12(1)(1)}_{\phi D}$	$(\phi_1^\dagger\phi_2)\left[(D^\mu\phi_1)^\dagger((D_\mu\phi_1))\right] + h.c.$

Table 10: 2HDM: ϕ^4D^2 [8 + 6×2 = 20] class of operators. Note the presence of $O^{21(21)}_{\Box}$ operator instead of operators ($Q^{12(12)}_{\varphi D}$ and $Q^{(1)21(2)}_{\varphi D}$) as given in [112].

$\mathcal{O}_{QQQ}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jn}\epsilon_{km}\left[(Q^{\alpha j})^T C Q^{\beta k}\right]\left[(Q^{\gamma m})^T C L^n\right]$
$\mathcal{O}_{QQu}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[(Q^{\alpha j})^T C Q^{\beta k}\right]\left[(u^\gamma)^T C e\right]$
$\mathcal{O}_{duu}$	$\epsilon^{\alpha\beta\gamma}\left[(d^\alpha)^T C u^\beta\right]\left[(u^\gamma)^T C e\right]$
$\mathcal{O}_{duQ}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[(d^\alpha)^T C u^\beta\right]\left[(Q^{\gamma j})^T C L^k\right]$

Table 12: 2HDM: Baryon Number Violating ψ^4 [4×2 = 8] class of operators. Each operator has a distinct Hermitian Conjugate. Here, $C = i\gamma^2\gamma^0$ in Dirac representation. Here α, β, γ are $SU(3)$ indices and j, k are $SU(2)$ indices.

$\mathcal{O}_{dd}$	$(\bar{d}\gamma_\mu d)(\bar{d}\gamma^\mu d)$	$\mathcal{O}_{uu}$	$(\bar{u}\gamma_\mu u)(\bar{u}\gamma^\mu u)$
$\mathcal{O}_{Le}$	$(\bar{L}\gamma_\mu L)(\bar{e}\gamma^\mu e)$	$\mathcal{O}_{Qe}$	$(\bar{Q}\gamma_\mu Q)(\bar{e}\gamma^\mu e)$
$\mathcal{O}_{ee}$	$(\bar{e}\gamma_\mu e)(\bar{e}\gamma^\mu e)$	$\mathcal{O}_{LL}$	$(\bar{L}\gamma_\mu L)(\bar{L}\gamma^\mu L)$
$\mathcal{O}_{eu}$	$(\bar{e}\gamma_\mu e)(\bar{u}\gamma^\mu u)$	$\mathcal{O}_{ed}$	$(\bar{e}\gamma_\mu e)(\bar{d}\gamma^\mu d)$
$\mathcal{O}_{Lu}$	$(\bar{L}\gamma_\mu L)(\bar{u}\gamma^\mu u)$	$\mathcal{O}_{Ld}$	$(\bar{L}\gamma_\mu L)(\bar{d}\gamma^\mu d)$
$\mathcal{O}^{[1]}_{LQ}$	$(\bar{L}\gamma_\mu L)(\bar{Q}\gamma^\mu Q)$	$\mathcal{O}^{[3]}_{LQ}$	$(\bar{L}\gamma_\mu\tau^I L)(\bar{Q}\gamma^\mu\tau^I Q)$
$\mathcal{O}^{[1]}_{QQ}$	$(\bar{Q}\gamma_\mu Q)(\bar{Q}\gamma^\mu Q)$	$\mathcal{O}^{[3]}_{QQ}$	$(\bar{Q}\gamma_\mu\tau^I Q)(\bar{Q}\gamma^\mu\tau^I Q)$
$\mathcal{O}^{[1]}_{ud}$	$(\bar{u}\gamma_\mu u)(\bar{d}\gamma^\mu d)$	$\mathcal{O}^{[8]}_{ud}$	$(\bar{u}\gamma_\mu T^A u)(\bar{u}\gamma^\mu T^A u)$
$\mathcal{O}^{[1]}_{Qu}$	$(\bar{Q}\gamma_\mu Q)(\bar{u}\gamma^\mu u)$	$\mathcal{O}^{[8]}_{Qu}$	$(\bar{Q}\gamma_\mu T^A Q)(\bar{u}\gamma^\mu T^A u)$
$\mathcal{O}^{[1]}_{Qd}$	$(\bar{Q}\gamma_\mu Q)(\bar{d}\gamma^\mu d)$	$\mathcal{O}^{[8]}_{Qd}$	$(\bar{Q}\gamma_\mu T^A Q)(\bar{d}\gamma^\mu T^A d)$
$\mathcal{O}^{[1]}_{QuQd}$	$\epsilon_{jk}(\bar{Q}^j u)(\bar{Q}^k d) + h.c.$		
$\mathcal{O}^{[8]}_{QuQd}$	$\epsilon_{jk}(\bar{Q}^j T^A u)(\bar{Q}^k T^A d) + h.c.$		
$\mathcal{O}^{[1]}_{LeQu}$	$\epsilon_{jk}(\bar{L}^j e)(\bar{Q}^k u) + h.c.$		
$\mathcal{O}^{[3]}_{LeQu}$	$\epsilon_{jk}(\bar{L}^j\sigma_{\mu\nu} e)(\bar{Q}^k\sigma^{\mu\nu} u) + h.c.$		
$\mathcal{O}_{LedQ}$	$(\bar{L}^j e)(\bar{d}Q^j) + h.c.$		

Table 11: 2HDM: Baryon Number Conserving ψ^4 [20 + 5×2 = 30] class of operators. Here j, k are $SU(2)$ indices.

<u>Part B</u>: Introducing "GrIP" and its utility.

Introduction to "GrIP"

"GrIP" is a Mathematica based scientific package that automatizes the computation of **Gr**oup **I**nvariant **P**olynomials

Download from: https://teamgrip.github.io/GrIP/

Package contains:

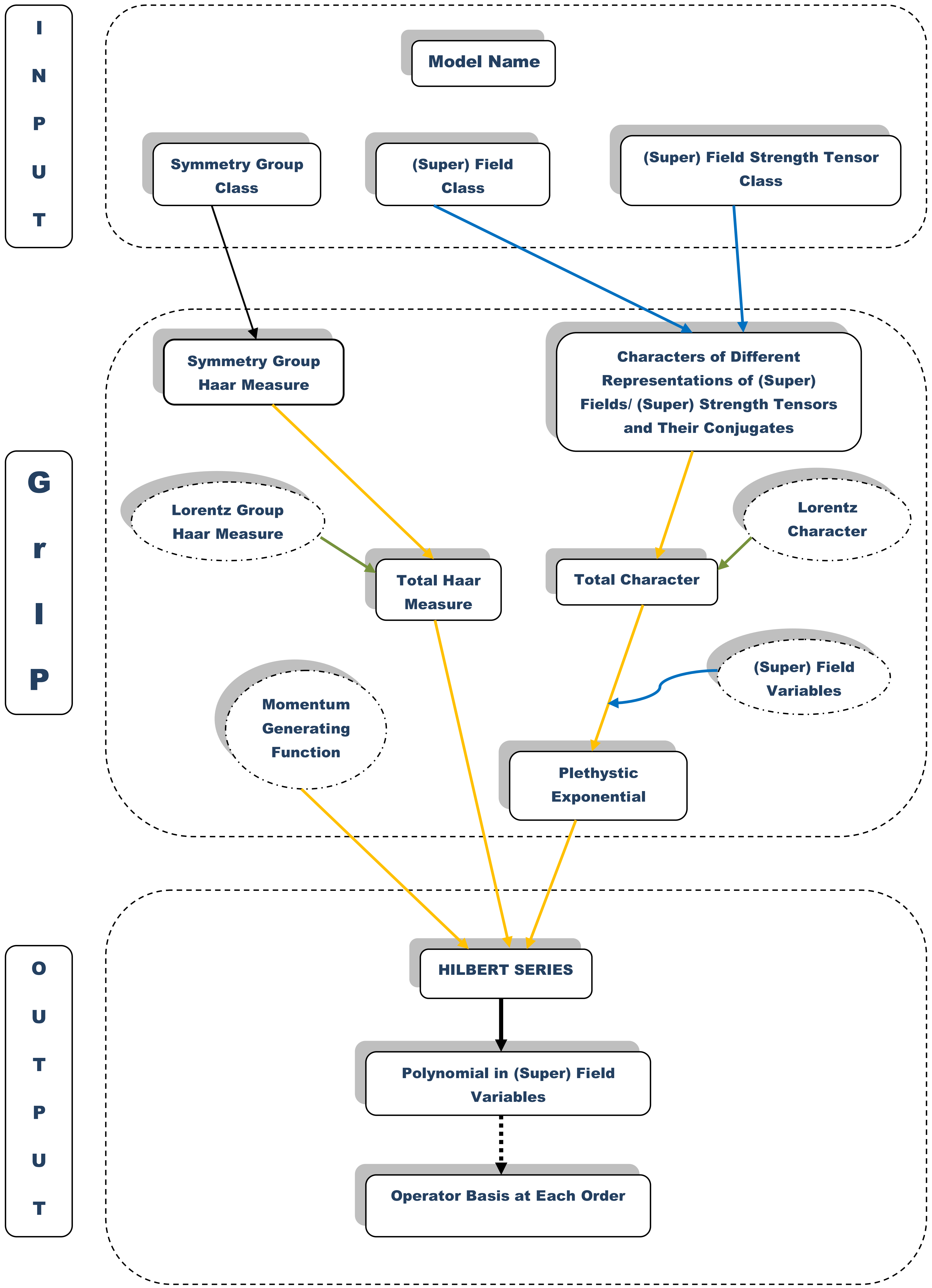
GrIP.m : main program

GrpInfo.m : representations of SU(N)

MODEL : folder containing example model input files

CHaar.m : sub-program to compute characters of representations and Haar measure of any connected compact groups.

Example_SM, Example_MSSM, Example_CHaar : example Mathematica Notebooks.



INPUT
Model Name
Symmetry Group Class
(Super) Field Class
(Super) Field Strength Tensor Class
GrIP
Symmetry Group Haar Measure
Characters of Different Representations of (Super) Fields/ (Super) Strength Tensors and Their Conjugates
Lorentz Group Haar Measure
Total Haar Measure
Total Character
Lorentz Character
(Super) Field Variables
Momentum Generating Function
Plethystic Exponential
OUTPUT
HILBERT SERIES
Polynomial in (Super) Field Variables
Operator Basis at Each Order

Paving the path to prepare the "input" model file

SM Fields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Lorentz Group $(SU(2)_l \otimes SU(2)_r)$	
H	1	2	1/2	Scalar	(0,0)
Q_l^p	3	2	1/6	Spinor	(1/2,0)
u_r^p	3	1	2/3	Spinor	(0,1/2)
d_r^p	3	1	-1/3	Spinor	(0,1/2)
L_l^p	1	2	-1/2	Spinor	(1/2,0)
e_r^p	1	1	-1	Spinor	(0,1/2)
Bl	1	1	0	Vector	(1,0)
Wl	1	3	0	Vector	(1,0)
Gl	8	1	0	Vector	(1,0)
Covariant Derivative	$\mathcal{D}_\mu$			Bi-spinor	(1/2,1/2)

Table 16: Quantum numbers of fields under the SM gauge groups and Lorentz group. $I = 1,2,3$; $a = 1,2,.....,8$; $p = 1,2,3$ denotes the flavor index. The color and isospin indices have been suppressed. l and r denote the chirality, i.e., the left or right handedness of the field.

Alternate provision: Providing Dynkin labels instead of dimension of the representation.

```
Field[2]={
"FieldName"-> Q,  "Self-Conjugate"-> False,  "Lorentz Behaviour"-> "FERMION",  "Chirality"-> "l",
"Baryon Number"-> 1/3,  "Lepton Number"-> 0,  "SU3Dyn"-> {1,0},  "SU2Dyn"-> {1},  "U1Dyn"-> 1/6},
```

```
ModelName="StandardModel"

SymmetryGroupClass ={
Group[1]={"GroupName"->"SU3", "N"->3},
Group[2]={"GroupName"->"SU2", "N"->2},
Group[3]={"GroupName"->"U1",  "N"->1} };
```

```
FieldClass={
Field[1]={
"FieldName"-> H,  "Self-Conjugate"-> False,  "Lorentz Behaviour"-> "SCALAR",  "Chirality"-> "NA",
"Baryon Number"-> 0,  "Lepton Number"-> 0,  "SU3Rep"-> "1",  "SU2Rep"-> "2",  "U1Rep"-> 1/2},

Field[2]={
"FieldName"-> Q,  "Self-Conjugate"-> False,  "Lorentz Behaviour"-> "FERMION",  "Chirality"-> "l",
"Baryon Number"-> 1/3,  "Lepton Number"-> 0,  "SU3Rep"-> "3",  "SU2Rep"-> "2",  "U1Rep"-> 1/6},
```

```
FieldTensorClass={
TensorField[1]={
"FieldName"-> Bl,  "Self-Conjugate"-> False,  "Lorentz Behaviour"-> "VECTOR",  "Chirality"-> "l",
"Baryon Number"-> 0,  "Lepton Number"-> 0,  "SU3Rep"-> "1",  "SU2Rep"-> "1",  "U1Rep"-> 0},

TensorField[2]={
"FieldName"-> Wl,  "Self-Conjugate"-> False,  "Lorentz Behaviour"-> "VECTOR",  "Chirality"-> "l",
"Baryon Number"-> 0,  "Lepton Number"-> 0,  "SU3Rep"-> "1",  "SU2Rep"-> "3",  "U1Rep"-> 0},

TensorField[3]={
"FieldName"-> Gl,  "Self-Conjugate"-> False,  "Lorentz Behaviour"-> "VECTOR",  "Chirality"-> "l",
"Baryon Number"-> 0,  "Lepton Number"-> 0,  "SU3Rep"-> "8",  "SU2Rep"-> "1",  "U1Rep"-> 0} };
```

Warnings to remember while preparing the "input" model file

```
Field[3]={
        "FieldName"-> Δ2,
        "Self-Conjugate"-> False,
        "Lorentz Behaviour"-> "SCALAR",
        "Chirality"-> "NA",
        "Baryon Number"-> 0,
        "Lepton Number"-> 0,
        "SU3Rep"-> "1",
        "SU2LRep"-> "1",
        "SU2RRep"-> "3",
        "U1Rep"-> 2},
Field[4]={
        "FieldName"-> Q1,
        "Self-Conjugate"-> False,
        "Lorentz Behaviour"-> "FERMION",
        "Chirality"-> "l",
        "Baryon Number"-> 1/3,
        "Lepton Number"-> 0,
        "SU3Rep"-> "3",
        "SU2LRep"-> "2",
        "SU2RRep"-> "1",
        "U1Rep"-> 1/3},
```

"Rep" is used consistently

```
Field[3]={
        "FieldName"-> Δ2,
        "Self-Conjugate"-> False,
        "Lorentz Behaviour"-> "SCALAR",
        "Chirality"-> "NA",
        "Baryon Number"-> 0,
        "Lepton Number"-> 0,
        "SU3Dyn"-> {0,0},
        "SU2LDyn"-> {0},
        "SU2RDyn"-> {2},
        "U1Dyn"-> 2},
Field[4]={
        "FieldName"-> Q1,
        "Self-Conjugate"-> False,
        "Lorentz Behaviour"-> "FERMION",
        "Chirality"-> "l",
        "Baryon Number"-> 1/3,
        "Lepton Number"-> 0,
        "SU3Dyn"-> {1,0},
        "SU2LDyn"-> {1},
        "SU2RDyn"-> {0},
        "U1Dyn"-> 1/3},
```

Multiple instances of same group is properly distinguished

"Dyn" is used consistently

```
Field[3]={
        "FieldName"-> Δ2,
        "Self-Conjugate"-> False,
        "Lorentz Behaviour"-> "SCALAR",
        "Chirality"-> "NA",
        "Baryon Number"-> 0,
        "Lepton Number"-> 0,
        "SU3Rep"-> "1",
        "SU2LRep"-> "1",
        "SU2RRep"-> "3",
        "U1Rep"-> 2},
Field[4]={
        "FieldName"-> Q1,
        "Self-Conjugate"-> False,
        "Lorentz Behaviour"-> "FERMION",
        "Chirality"-> "l",
        "Baryon Number"-> 1/3,
        "Lepton Number"-> 0,
        "SU3Dyn"-> {1,0},
        "SU2LDyn"-> {1},
        "SU2RDyn"-> {0},
        "U1Dyn"-> 1/3},
```

"Dyn" or "Rep" both cannot be used in same input file. Either of them should be used consistently.

```
Field[3]={
        "FieldName"-> Δ2,
        "Self-Conjugate"-> False,
        "Lorentz Behaviour"-> "SCALAR",
        "Chirality"-> "NA",
        "Baryon Number"-> 0,
        "Lepton Number"-> 0,
        "SU3Rep"-> "1",
        "SU2Rep"-> "1",
        "SU2Rep"-> "3",
        "U1Rep"-> 2},
```

multiple occurence of same group must be distinguished

```
Field[3]={
        "FieldName"-> Δ2,
        "Self-Conjugate"-> False,
        "Lorentz Behaviour"-> "SCALAR",
        "Chirality"-> "NA",
        "Baryon Number"-> 0,
        "Lepton Number"-> 0,
        "SU3Dyn"-> {0,0},
        "SU2LRep"-> "1",
        "SU2RRep"-> "3",
        "U1Rep"-> 2},
```

```
TensorField[2]={
        "FieldName"-> W1l,
        "Self-Conjugate"-> False,
        "Lorentz Behaviour"-> "VECTOR",
        "Chirality"-> "l",
        "Baryon Number"-> 0,
        "Lepton Number"-> 0,
        "SU3Dyn"-> {0,0},
        "SU2LDyn"-> {2},
        "SU2RDyn"-> {0},
        "U1Dyn"-> 0},
```

```
TensorField[2]={
        "FieldName"-> W1^l,...}
```

```
TensorField[2]={
        "FieldName"-> W1_l,...}
```

```
TensorField[2]={
        "FieldName"-> W1L,...}
```

Details of the user interface for the Standard Model

Here, we provide an illustration of how to run **GrIP** and utilize its various commands to obtain specific outputs based on the Standard Model and how to further modify those results.

```
In[1]:= SetDirectory["~/home"]
```

```
In[2]:= Get["MODEL/SM_Rep.m"]
```

```
Model Name: Standard Model

Authors: Upalaparna Banerjee, Joydeep Chakrabortty, Suraj Prakash, Shakeel
Ur Rahaman

Institutes: Indian Institute of Technology Kanpur, India

Emails: upalab, joydeep, surajprk, shakel@iitk.ac.in
```

```
In[3]:= Get["GrIP.m"]
```

```
GrIP-V.1.0.0
```

```
Authors: Upalaparna Banerjee, Joydeep Chakrabortty, Suraj Prakash,
Shakeel Ur Rahaman
Indian Institute of Technology Kanpur, India
```

```
"GrIP is successfully loaded and ready to compute!
A folder, named [StandardModel] has been created in your working
directory and all the output will be saved in that folder.
Thank You!!"
```

SM Fields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Lorentz Group $(SU(2)_l \otimes SU(2)_r)$	
H	1	2	1/2	Scalar	(0,0)
Q_l^p	3	2	1/6	Spinor	(1/2,0)
u_r^p	3	1	2/3	Spinor	(0,1/2)
d_r^p	3	1	-1/3	Spinor	(0,1/2)
L_l^p	1	2	-1/2	Spinor	(1/2,0)
e_r^p	1	1	-1	Spinor	(0,1/2)
Bl	1	1	0	Vector	(1,0)
Wl	1	3	0	Vector	(1,0)
Gl	8	1	0	Vector	(1,0)
Covariant Derivative	$\mathcal{D}_\mu$			Bi-spinor	(1/2,1/2)

Table 16: Quantum numbers of fields under the SM gauge groups and Lorentz group. I = 1,2,3; a = 1,2,.....,8; p = 1,2,3 denotes the flavor index. The color and isospin indices have been suppressed. l and r denote the chirality, i.e., the left or right handedness of the field.

In[4]:= `DisplayUserInputTable`

Out[4]:=

Field Name	Self Conjugate	Lorentz Behaviour	Chirality	Baryon Number	Lepton Number	SU3Rep	SU2Rep	U1Rep
H	False	SCALAR	NA	0	0	1	2	1/2
Q	False	FERMION	l	1/3	0	3	2	1/6
u	False	FERMION	r	1/3	0	3	1	2/3
d	False	FERMION	r	1/3	0	3	1	-1/3
L	False	FERMION	l	0	-1	1	2	-1/2
el	False	FERMION	r	0	-1	1	1	-1
Bl	False	VECTOR	l	0	0	1	1	0
Wl	False	VECTOR	l	0	0	1	3	0
Gl	False	VECTOR	l	0	0	8	1	0

User input based on minimal knowledge:

In[5]:= `DisplayWorkingInputTable`

Out[5]:=

"Conjugate" fields are prepared internally and Returns the "complete" the set:

Field Name	Self Conjugate	Lorentz Behaviour	Chirality	Baryon Number	Lepton Number	SU3Dyn	SU2Dyn	U1Dyn
H	False	SCALAR	NA	0	0	{0,0}	{1}	1/2
Q	False	FERMION	l	1/3	0	{1,0}	{1}	1/6
u	False	FERMION	r	1/3	0	{1,0}	{0}	2/3
d	False	FERMION	r	1/3	0	{1,0}	{0}	-1/3
L	False	FERMION	l	0	-1	{0,0}	{1}	-1/2
el	False	FERMION	r	0	-1	{0,0}	{0}	-1
$H^\dagger$	False	SCALAR	NA	0	0	{0,0}	{1}	-1/2
$Q^\dagger$	False	FERMION	r	-1/3	0	{0,1}	{1}	-1/6
$u^\dagger$	False	FERMION	l	-1/3	0	{0,1}	{0}	-2/3
$d^\dagger$	False	FERMION	l	-1/3	0	{0,1}	{0}	1/3
$L^\dagger$	False	FERMION	r	0	1	{0,0}	{1}	1/2
$el^\dagger$	False	FERMION	l	0	1	{0,0}	{0}	1
Bl	False	VECTOR	l	0	0	{0,0}	{0}	0
Wl	False	VECTOR	l	0	0	{0,0}	{2}	0
Gl	False	VECTOR	l	0	0	{1,1}	{0}	0
Br	False	VECTOR	r	0	0	{0,0}	{0}	0
Wr	False	VECTOR	r	0	0	{0,0}	{2}	0
Gr	False	VECTOR	r	0	0	{1,1}	{0}	0

SM Fields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Lorentz Group $(SU(2)_l \otimes SU(2)_r)$	
H	1	2	1/2	Scalar	(0,0)
Q_l^p	3	2	1/6	Spinor	(1/2,0)
u_r^p	3	1	2/3	Spinor	(0,1/2)
d_r^p	3	1	-1/3	Spinor	(0,1/2)
L_l^p	1	2	-1/2	Spinor	(1/2,0)
e_r^p	1	1	-1	Spinor	(0,1/2)
Bl	1	1	0	Vector	(1,0)
Wl	1	3	0	Vector	(1,0)
Gl	8	1	0	Vector	(1,0)
Covariant Derivative	$\mathcal{D}_\mu$			Bi-spinor	(1/2,1/2)

Table 16: Quantum numbers of fields under the SM gauge groups and Lorentz group. $I = 1,2,3$; $a = 1,2,\ldots,8$; $p = 1,2,3$ denotes the flavor index. The color and isospin indices have been suppressed. l and r denote the chirality, i.e., the left or right handedness of the field.

$$SU(3) \otimes SU(2) \otimes U(1)$$

Characters of SM fields and their conjugates:

In[6]:= `DisplayCharacterTable`

Out[6]:=

Dyn	SU3	SU2	U1
{{0,0},{1},1/2}	1	$\frac{1}{G2z_1}+G2z_1$	$\sqrt{G3z}$
{{1,0},{1},1/6}	$G1z_1+\frac{1}{G1z_2}+\frac{G1z_2}{G1z_1}$	$\frac{1}{G2z_1}+G2z_1$	$G3z^{1/6}$
{{1,0},{0},2/3}	$G1z_1+\frac{1}{G1z_2}+\frac{G1z_2}{G1z_1}$	1	$G3z^{2/3}$
{{1,0},{0},-1/3}	$G1z_1+\frac{1}{G1z_2}+\frac{G1z_2}{G1z_1}$	1	$\frac{1}{G3z^{1/3}}$
{{0,0},{1},-1/2}	1	$\frac{1}{G2z_1}+G2z_1$	$\frac{1}{\sqrt{G3z}}$
{{0,0},{0},-1}	1	1	$\frac{1}{G3z}$
{{0,0},{1},-1/2}	1	$\frac{1}{G2z_1}+G2z_1$	$\frac{1}{\sqrt{G3z}}$
{{0,1},{1},-1/6}	$\frac{1}{G1z_1}+\frac{G1z_1}{G1z_2}+G1z_2$	$\frac{1}{G2z_1}+G2z_1$	$\frac{1}{G3z^{1/6}}$
{{0,1},{0},-2/3}	$\frac{1}{G1z_1}+\frac{G1z_1}{G1z_2}+G1z_2$	1	$\frac{1}{G3z^{2/3}}$
{{0,1},{0},1/3}	$\frac{1}{G1z_1}+\frac{G1z_1}{G1z_2}+G1z_2$	1	$G3z^{1/3}$
{{0,0},{1},1/2}	1	$\frac{1}{G2z_1}+G2z_1$	$\sqrt{G3z}$
{{0,0},{0},1}	1	1	$G3z$
{{0,0},{0},0}	1	1	1
{{0,0},{2},0}	1	$1+\frac{1}{G2z_1^2}+G2z_1^2$	1
{{1,1},{0},0}	$2+\frac{G1z_1}{G1z_2^2}+\frac{1}{G1z_1G1z_2}+\frac{G1z_1^2}{G1z_2}+\frac{G1z_2}{G1z_1^2}+G1z_1G1z_2+\frac{G1z_2^2}{G1z_1}$	1	1
{{0,0},{0},0}	1	1	1
{{0,0},{2},0}	1	$1+\frac{1}{G2z_1^2}+G2z_1^2$	1
{{1,1},{0},0}	$2+\frac{G1z_1}{G1z_2^2}+\frac{1}{G1z_1G1z_2}+\frac{G1z_1^2}{G1z_2}+\frac{G1z_2}{G1z_1^2}+G1z_1G1z_2+\frac{G1z_2^2}{G1z_1}$	1	1

Haar measures of gauge groups:

In[7]:= `DisplayHaarMeasure`

Out[7]:=

SU3	SU2	U1
$-\frac{\left(-G1z_1^4G1z_2+G1z_2^3+G1z_1^3\left(1+G1z_2^3\right)-G1z_1\left(G1z_2+G1z_2^4\right)\right)^2}{6G1z_1^5G1z_2^5}$	$-\frac{\left(-1+G2z_1^2\right)^2}{2G2z_1^3}$	$\frac{1}{G3z}$

Invariant operators having mass dimension 4 :

In[8]:= `DisplayHSOutput["MassDim"→4, "OnlyMassDimOutput"→True, "ΔB"→0, "ΔL"→0, "Flavours"→1]`

Out[8]:=

$$-Bl^2-Br^2-Gl^2-Gr^2-Wl^2-Wr^2-Bl\,\mathcal{D}^2-Br\,\mathcal{D}^2+d\,\mathcal{D}\,d^\dagger+el\,\mathcal{D}\,el^\dagger+H\,\mathcal{D}^2\,H^\dagger$$
$$-Q\,d^\dagger\,H^\dagger-L\,el^\dagger\,H^\dagger-H^2\left(H^\dagger\right)^2-el\,H\,L^\dagger+L\,\mathcal{D}\,L^\dagger-d\,H\,Q^\dagger+Q\,\mathcal{D}\,Q^\dagger-u\,H^\dagger\,Q^\dagger$$
$$-H\,Q\,u^\dagger+u\,\mathcal{D}\,u^\dagger-\mathcal{D}^4$$

Searching for "*specific*" operators:

```
In[9]:= DisplayBLviolatingOperators["HighestMassDim"→10, "ΔB"→ +1(−1),
                    "ΔL"→ −1(+1), "Flavours"→1]
```

Out[9]:= $L\,Q^3 + d\,L\,Q\,u + el\,Q^2\,u + d\,el\,u^2$ (corresponding hermitian conjugates)

"*Operators*" are multiplied by suitable couplings to form the "*Lagrangian*"

```
In[10]:= DisplayLagOutput["MassDim"→4, "OnlyMassDimOutput"→False,
                    "ΔB"→"NA", "ΔL"→"NA", "Flavours"→ Nf]
```

Out[10]:=
$$-Bl^2 - Br^2 - Gl^2 - Gr^2 - Wl^2 - Wr^2 + d\,N_f\,\mathcal{D}\,d^\dagger + el\,N_f\,\mathcal{D}\,el^\dagger + H\,\mathcal{D}^2 H^\dagger - N_f^2\,Q y_1\,d^\dagger\,H^\dagger - H^2\,\lambda\,\left(H^\dagger\right)^2 + L\,N_f\,\mathcal{D}\,L^\dagger - el\,H\,N_f^2\,y_2 L^\dagger + N_f\,Q\,\mathcal{D}Q^\dagger - d\,H\,N_f^2\,y_3\,Q^\dagger - N_f^2\,u\,y_4\,H^\dagger\,Q^\dagger + N_f\,u\,\mathcal{D}\,u^\dagger - H\,N_f^2\,Q\,y_5\,u^\dagger - L\,N_f^2 y_6\,el^\dagger H^\dagger$$

```
In[11]:= DisplayLagOutput["MassDim"→4, "OnlyMassDimOutput"→False,
                    "ΔB"→0, "ΔL"→0, "Flavours"→ 1]
```

Out[11]:=
$$-Bl^2 - Br^2 + d\,\mathcal{D}\,d^\dagger - Q\,y_1\,d^\dagger\,H^\dagger - d\,H\,y_4\,Q^\dagger + \mathcal{D}\,el\,el^\dagger - L\,y_2\,el^\dagger\,H^\dagger - el\,H\,y_3\,L^\dagger + \mathcal{D}^2 H\,H^\dagger - H m^2 H^\dagger - u\,y_5\,H^\dagger Q^\dagger - H^2\,\lambda\left(H^\dagger\right)^2 - H\,Q\,y_6\,u^\dagger + \mathcal{D}\,L\,L^\dagger + \mathcal{D}\,Q\,Q^\dagger + \mathcal{D}\,u\,u^\dagger - Gl^2 - Gr^2 - Wl^2 - Wr^2$$

Counting total number of operators:

```
In[12]:= PolyA=SaveHSOutput["MassDim"→4, "ΔB"→0, "ΔL"→0,
                    "Flavours"→1];

In[13]:= OpCounter[PolyA]
```

Out[13]:= 22

```
In[14]:= PolyB=SaveHSOutput["MassDim"→4, "ΔB"→"NA", "ΔL"→"NA",
                    "Flavours"→ Nf];

In[15]:= OpCounter[PolyB]
```

Out[15]:= $11N_f+11N_f^2$

Baryon and Lepton Number violating rare processes in the SM

In[1]:= DisplayBLviolatingOperators["HighestMassDim"→10, "ΔB"→ 0, "ΔL"→ -2, "Flavours"→ N_f]

Out[1]:= First instance of ΔB=0 and ΔL=-2 occurs at mass dimension 5,

Operators: $\frac{1}{2}H^2L^2N_f^2+\frac{1}{2}H^2L^2N_f$

In[2]:= DisplayBLviolatingOperators["HighestMassDim"→10, "ΔB"→ 1, "ΔL"→ -1, "Flavours"→ N_f]

Out[2]:= First instance of ΔB=1 and ΔL=-1 occurs at mass dimension 6,

Operators: $d\,el\,N_f^4\,u^2+d\,L\,N_f^4\,Q\,u+\frac{1}{2}el\,N_f^4\,Q^2\,u+\frac{1}{2}el\,N_f^3\,Q^2u+\frac{2}{3}\,LN_f^4\,Q^3+\frac{1}{3}L\,N_f^2\,Q^3$

ΔB	ΔL	Lowest Dimension	Operators
0	-2	5	H^2L^2
+1	-1	6	$LQ^3,\ LQdu,\ eluQ^2,\ elu^2d$
1	1	7	$el^\dagger d^3\mathcal{D},\ L^\dagger Qd^2\mathcal{D},\ H^\dagger L^\dagger dQ^2,\ H^\dagger L^\dagger ud^2$
+2	0	9	$d^2Q^4,\ d^3uQ^2,\ 2u^2d^4$
+1	-3	9	$L^3Qu^2,\ L^2elu^3$
+1	+3	10	$H^\dagger(L^\dagger)^3d^3$
0	-4	10	H^4L^4

External $U(1)_R$ Global Symmetry and extended MSSM

Superfields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_R$
H_u	1	2	1/2	0
H_d	1	2	-1/2	0
Q^i	3	2	1/6	1
U^i	$\overline{3}$	1	-2/3	1
D^i	$\overline{3}$	1	1/3	1
L^i	1	2	-1/2	2
E^i	1	1	1	0
R_u	1	2	1/2	2
R_d	1	2	-1/2	2
$\mathcal{S}$	1	1	0	0
$\mathcal{T}$	1	3	0	0
$\mathcal{O}$	8	1	0	0

Table 26: MSSM + Global $U(1)_R$: Quantum numbers of superfields under the gauge groups. Internal symmetry indices have been suppressed. $i=1,2,..,N_f$ is the flavour index.

Superpotential with $U(1)_R$ charge $=0$		
Canonical Dim.	**Operators**	**No. of Operators**
1	$\mathcal{S}$	1
2	$\mathcal{S}^2,\ \mathcal{T}^2,\ \mathcal{O}^2,\ H_dH_u$	4
3	$\mathcal{S}^3,\ \mathcal{S}\mathcal{T}^2,\ \mathcal{S}\mathcal{O}^2,\ \mathcal{O}^3,\ H_dH_u\mathcal{S},\ H_dH_u\mathcal{T}$	6
Superpotential with $U(1)_R$ charge $=2$		
Canonical Dim.	**Operators**	**No. of Operators**
2	$H_uL,\ H_uR_d,\ H_dR_u$	3
3	$LEH_d,\ QDH_d,\ QUH_u,\ EH_dR_d,\ H_uR_d\mathcal{S},H_uL\mathcal{S},\ H_dR_u\mathcal{S},\ H_uL\mathcal{T},\ H_uR_d\mathcal{T},\ H_dR_u\mathcal{T}$	10
Superpotential with $U(1)_R$ charge $=4$		
Canonical Dim.	**Operators**	**No. of Operators**
2	$R_dR_u,\ R_uL$	2
3	$QLD,\ LER_d,\ QDR_d,\ QUR_u,LR_u\mathcal{T},\ R_dR_u\mathcal{T},\ LR_u\mathcal{S},\ R_dR_u\mathcal{S}$	8

<u>Focussing on the utility of "GrIP"</u>

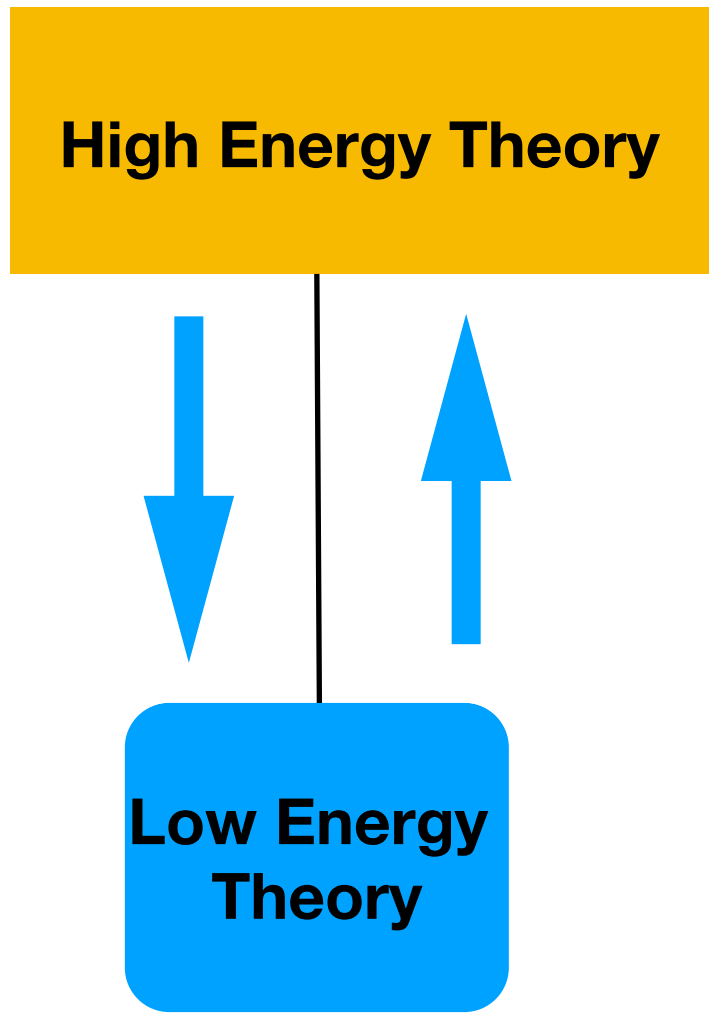
1. For a complicated framework, manual computation of group invariant operators can be tedious and error prone. Having an automatised tool would be relaxing :)

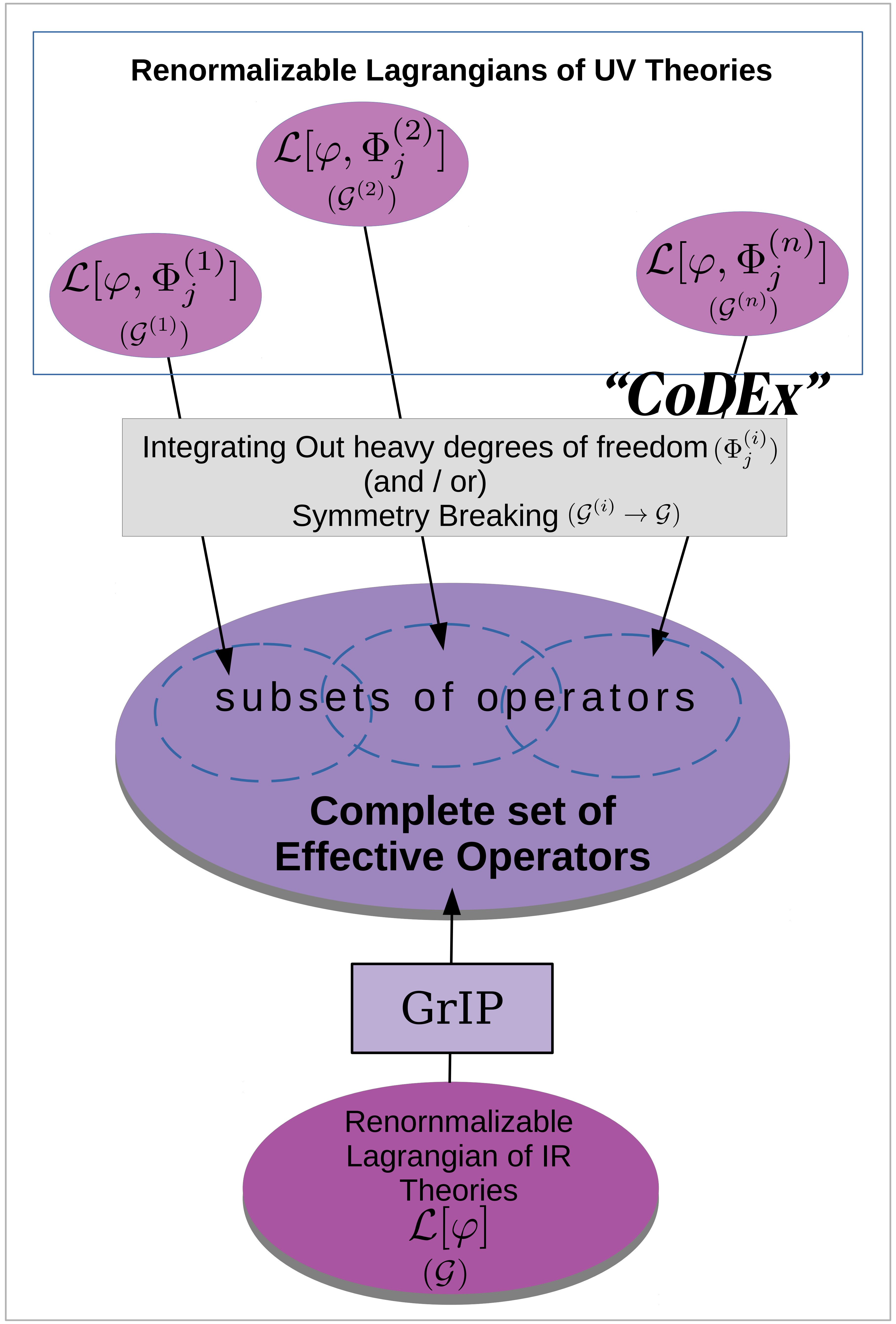
It can take care of <u>over-</u> and(or) <u>under-</u>estimation of invariants very easily.

2. Based on the current situation of particle physics, it's time to look for a "**method**" which can smell the "**unusual**" things even without knowing the exact "**source**" of it.

Effective Field Theory !

It's a tool to reveal the hidden things without unfolding it





CoDEx: Wilson coefficient calculator connecting SMEFT to UV theory

Supratim Das Bakshi, Joydeep Chakrabortty, Sunando Kumar Patra

- *Eur.Phys.J.C* 79 (2019) 1, 21 • e-Print: 1808.04403 [hep-ph]

Available at https://effexteam.github.io/CoDEx/

With "**CoDEx**" and "***GrIP***" in arsenal one can use effective field theory to connect **UV** and ***IR*** theories very easily and of course automatically.

Paving the path to BSM-EFT

SM extended by other IR DOFs

Model No.	Extra Particle	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Spin
1	δ^+	1	1	1	0
2	δ^{++}	1	1	2	0
3	Δ	1	3	1	0
4	Θ	1	4	3/2	0
5	Ω	1	5	0	0
6	Σ	1	3	0	1/2
7	$\mathcal{N}$	1	1	0	1/2

Table 28: Quantum numbers of various BSM fields under the SM gauge groups and their spins.

Model No.	Lepto-quark	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Spin	Baryon No.	Lepton No.
1	χ_1	3	2	1/6	0	1/3	-1
2	χ_2	3	2	7/6	0	1/3	-1
3	Φ_1	3	1	2/3	0	1/3	-1
4	Φ_2	3	1	-1/3	0	1/3	-1

Table 36: Quantum numbers of various lepto-quark fields under the SM gauge groups, their spins and baryon and lepton numbers.

BSM-EFT: examples

Right handed fermion singlet $(\mathcal{N})$

A couple of vector-like fermions (χ_E, χ_N)

A scalar (Φ)

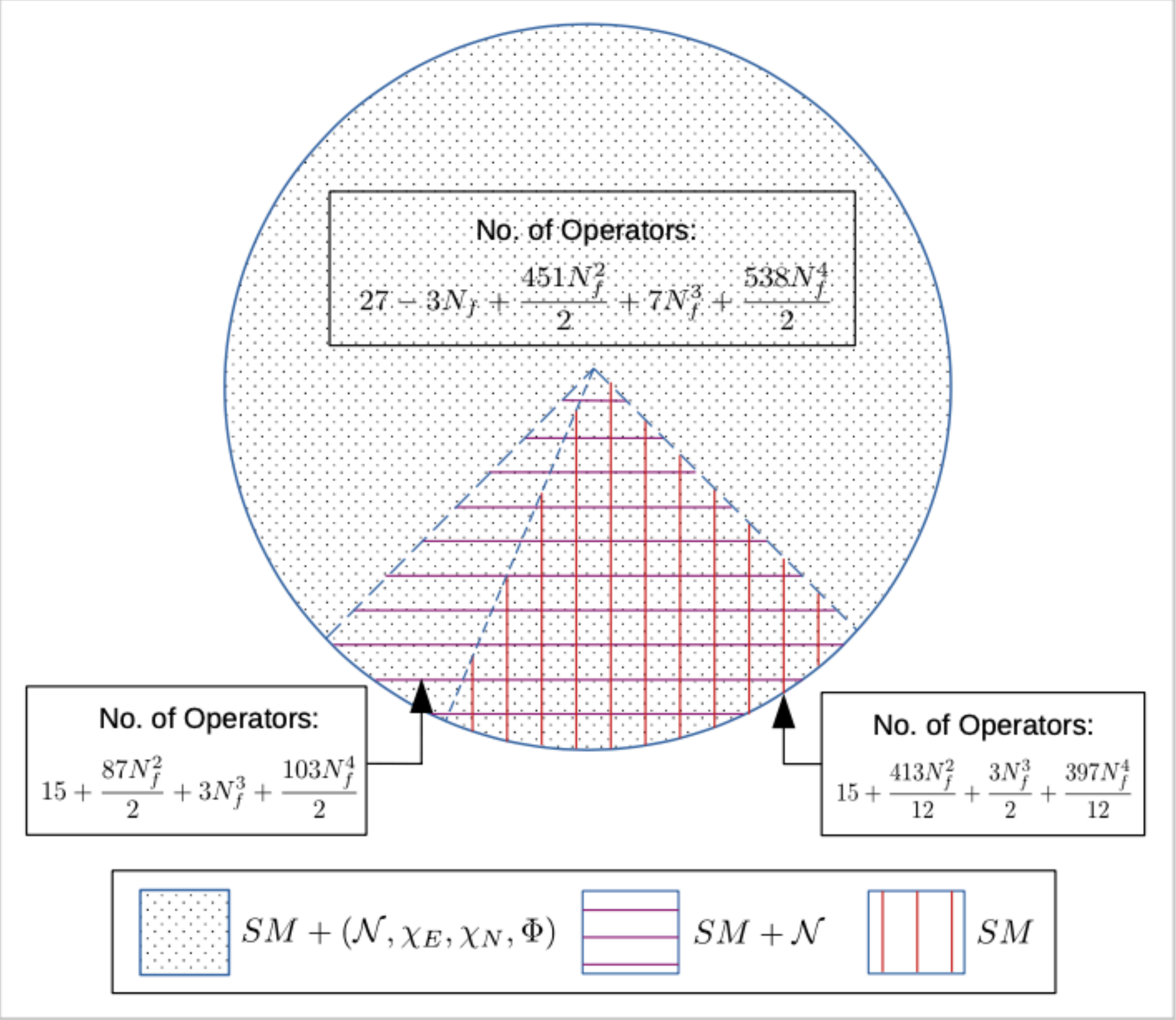


Figure 11: Pie-chart showing number of operators including their conjugates for 3 distinct scenarios.

Mass Dimension-6	
Operator Class	**Operators (in non-covariant form)**
Φ^6	$HH^\dagger\delta^2(\delta^\dagger)^2,\ H^2(H^\dagger)^2\delta\delta^\dagger,\ \delta^3(\delta^\dagger)^3$
Φ^2X^2	$Bl^2\delta\delta^\dagger,\ Gl^2\delta\delta^\dagger,\ Wl^2\delta\delta^\dagger$
$\Psi^2\Phi X$	$\boxed{\frac{1}{2}\left(N_f^2 - N_f\right)(el^\dagger)^2\delta^\dagger Bl}$
$\Psi^2\Phi^2\mathcal{D}$	$(N_f^2)QQ^\dagger\delta\delta^\dagger\mathcal{D},\ (N_f^2)uu^\dagger\delta\delta^\dagger\mathcal{D},\ (N_f^2)dd^\dagger\delta\delta^\dagger\mathcal{D},\ (N_f^2)LL^\dagger\delta\delta^\dagger\mathcal{D},\ (N_f^2)elel^\dagger\delta\delta^\dagger\mathcal{D},\ (N_f^2)\delta H^\dagger elL\mathcal{D}$
$\Psi^2\Phi^3$	$(N_f^2)\delta\delta^\dagger HL^\dagger el,\ (N_f^2)\delta\delta^\dagger HdQ^\dagger,\ (N_f^2)\delta\delta^\dagger H^\dagger uQ^\dagger,\ \frac{1}{2}\left(N_f^2 + N_f\right)L^2\delta(H^\dagger)^2,$ $\frac{1}{2}\left(N_f^2 + N_f\right)el^2\delta^2\delta^\dagger,\ \frac{1}{2}\left(N_f^2 + N_f\right)el^2\delta HH^\dagger$
$\Phi^4\mathcal{D}^2$	$\delta^2(\delta^\dagger)^2\mathcal{D}^2,\ 2HH^\dagger\delta\delta^\dagger\mathcal{D}^2$

Table 30: SM + Doubly Charged Scalar (δ^{++}): Operators of mass dimension-6 excluding pure SM operators. There are no mass dimension-5 operators except pure SM operators. Here, $\delta \to \delta^{++}$ and $\delta^\dagger \to \delta^{--}$.

Mass Dimension-5	
Operator Class	**Operators (in non-covariant form)**
$\Psi^2\Phi^2$	$(N_f^2)LQ\chi_1^2,\ (N_f^2)elu\chi_1^2,\ (N_f^2)ud(\chi_1^\dagger)^2,\ (N_f^2 + N_f)Q^2(\chi_1^\dagger)^2,\ (2N_f^2)LQH\chi_1^\dagger,\ (N_f^2)eluH\chi_1^\dagger,$ $(N_f^2)udH^\dagger\chi_1,\ (N_f^2)Q^2H^\dagger\chi_1,\ \boxed{\frac{1}{2}(N_f^2 - N_f)d^2H\chi_1}$

Mass Dimension-6	
Operator Class	**Operators (in non-covariant form)**
Φ^6	$2\chi_1^3(\chi_1^\dagger)^3,\ 3\chi_1^2(\chi_1^\dagger)^2HH^\dagger,\ 2\chi_1\chi_1^\dagger H^2(H^\dagger)^2$
Φ^2X^2	$Bl^2\chi_1^\dagger\chi_1,\ 2Gl^2\chi_1^\dagger\chi_1,\ BlGl\chi_1^\dagger\chi_1,\ BlWl\chi_1^\dagger\chi_1,\ GlWl\chi_1^\dagger\chi_1,\ Wl^2\chi_1^\dagger\chi_1$
$\Psi^2\Phi X$	$(N_f^2)BlLd^\dagger\chi_1,\ (N_f^2)GlLd^\dagger\chi_1,\ (N_f^2)WlLd^\dagger\chi_1$
$\Psi^2\Phi^2\mathcal{D}$	$(2N_f^2)uu^\dagger\chi_1\chi_1^\dagger\mathcal{D},\ (2N_f^2)dd^\dagger\chi_1\chi_1^\dagger\mathcal{D},\ (4N_f^2)QQ^\dagger\chi_1\chi_1^\dagger\mathcal{D},\ (2N_f^2)LL^\dagger\chi_1\chi_1^\dagger\mathcal{D},$ $(N_f^2)elel^\dagger\chi_1\chi_1^\dagger\mathcal{D},\ (N_f^2)eld^\dagger H\chi_1\mathcal{D},\ (2N_f^2)LQ^\dagger H\chi_1\mathcal{D}$
$\Psi^2\Phi^3$	$(4N_f^2)Q^\dagger dH\chi_1\chi_1^\dagger,\ (4N_f^2)Q^\dagger uH^\dagger\chi_1\chi_1^\dagger,\ (N_f^2)Q^\dagger d\chi_1^3,\ (N_f^2)Qu^\dagger\chi_1^3,\ (N_f^2)Q^\dagger elH^2\chi_1,$ $(2N_f^2)L^\dagger elH\chi_1\chi_1^\dagger,\ (2N_f^2)L^\dagger d\chi_1^\dagger HH^\dagger,\ (N_f^2)L^\dagger dH^\dagger\chi_1^2,\ (2N_f^2)L^\dagger d(\chi_1^\dagger)^2\chi_1,\ (N_f^2)Lu^\dagger H^2\chi_1$
$\Phi^4\mathcal{D}^2$	$4\chi_1^2(\chi_1^\dagger)^2\mathcal{D}^2,\ 4HH^\dagger\chi_1\chi_1^\dagger\mathcal{D}^2,\ \chi_1^3H^\dagger\mathcal{D}^2$

Table 37: Lepto-Quark Model 1 (χ_1): Operators of mass dimensions-5 and -6 excluding pure SM operators.

List of example models encoded in “GrIP”

SM and extended by :

- Singly Charged Scalar;
- Doubly Charged Scalar;
- Complex Triplet Scalar;
- SU(2) Quadruplet Scalar;
- SU(2) Quintuplet Scalar;
- Left-Handed Triplet Fermion;
- Right-Handed Singlet Fermion;
- Scalar Lepto-Quarks;
- SU(2) Doublet Scalar (with different hypercharge);
- Real Triplet Scalar;
- Color Triplet Scalars and Sterile Neutrino;
- SU(2) Triplet and Quadruplet Fermions.

MSSM and extended by :

- MSSM;
- NMSSM;
- Supersymmetric Pati-Salam;
- Minimal Supersymmetric Left-Right models.

Models below electro-weak scale.

$SU(3)_C \otimes U(1)_{em}$ and extended by additional :

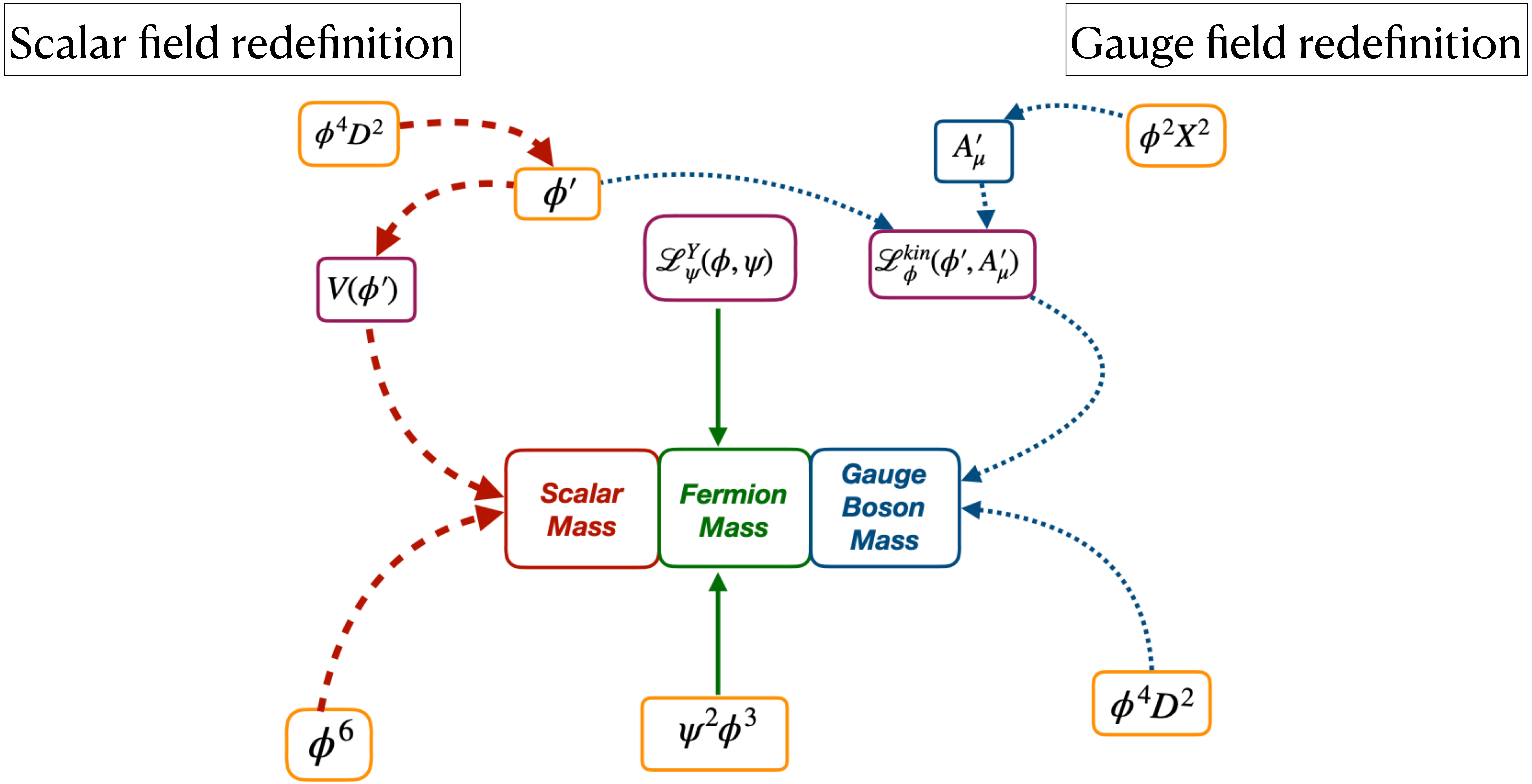
- Scalar Dark Matter;
- Vector-like Fermion Dark Matter.

UV models:

- Two Higgs doublet;
- Minimal Left-Right Symmetric;
- Pati-Salam;
- SU(5) Grand Unified models.

Part C: Impact of the effective operators

Impact of the effective operators



ϕ', A' are redefined scalar and gauge fields

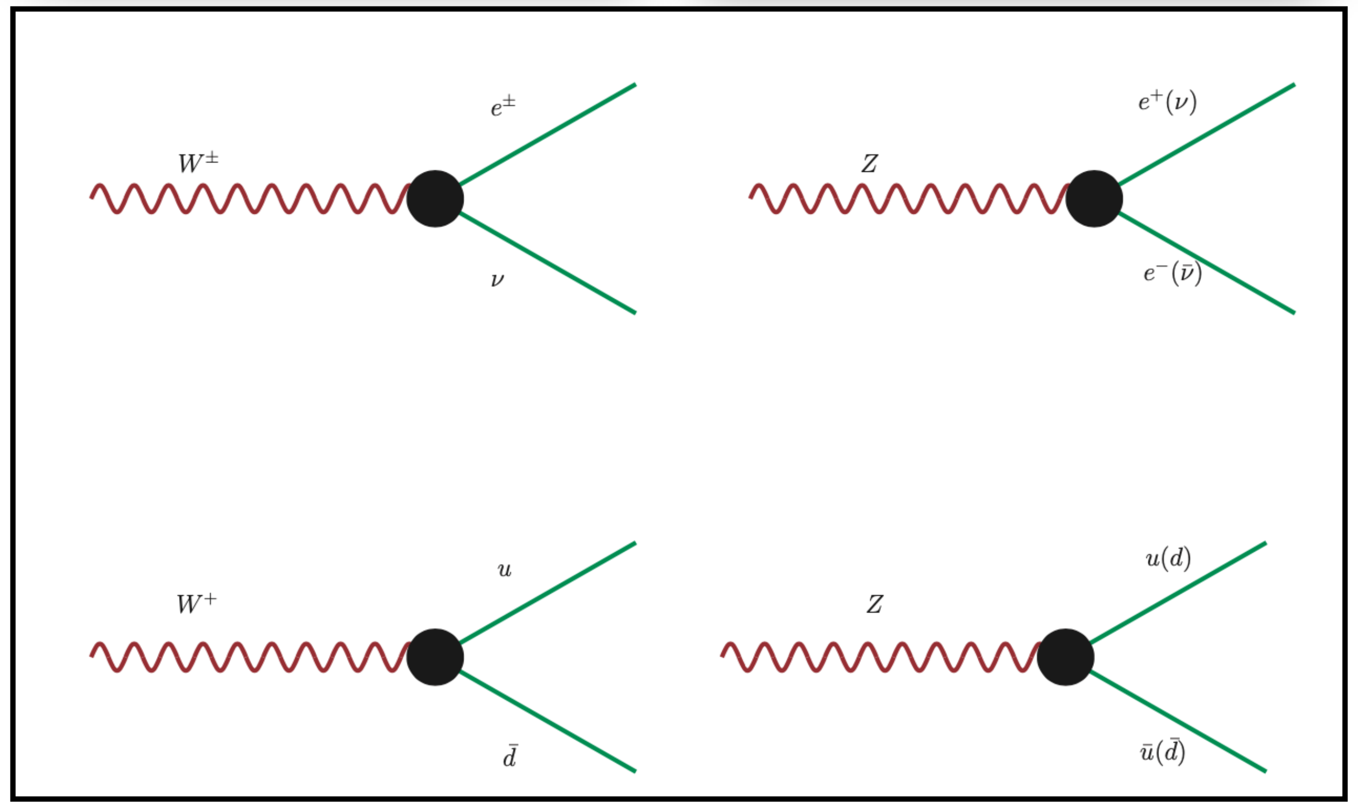
Effective operators and their impact on observables

Observables	Effective lagrangian terms
Weak mixing angle (θ_W)	$\mathcal{L}_{\psi}^{kin}(\psi, A'_\mu) + \frac{1}{\Lambda^2}[\psi^2\phi^2 D + \psi^4]$
Fermi constant (G_F)	
ρ parameter	
Oblique parameters (S, T, U)	$\mathcal{L}_{A}^{kin}(A'_\mu) + \mathcal{L}_{\phi}^{kin}(\phi', A'_\mu) + \frac{1}{\Lambda^2}[\phi^4 D^2 + \phi^2 X^2]$
Magnetic moment	$\mathcal{L}_{Majorana}^{Y}(\phi', \psi) + \frac{1}{\Lambda^2}[\psi^2\phi X]$
Lepton Flavor Violating (LFV) and Lepton Number Violating (LNV) processes	$\mathcal{L}_{Majorana}^{Y}(\phi', \psi) + \mathcal{L}_{\phi}^{kin}(\phi', A'_\mu) + \frac{1}{\Lambda^2}[\phi^4 D^2 + \psi^2\phi^3]$
Scalar Quartic couplings	$V(\phi') + \frac{1}{\Lambda^2}[\phi^6]$

Table 26: Low energy observables and their origin in effective lagrangian.

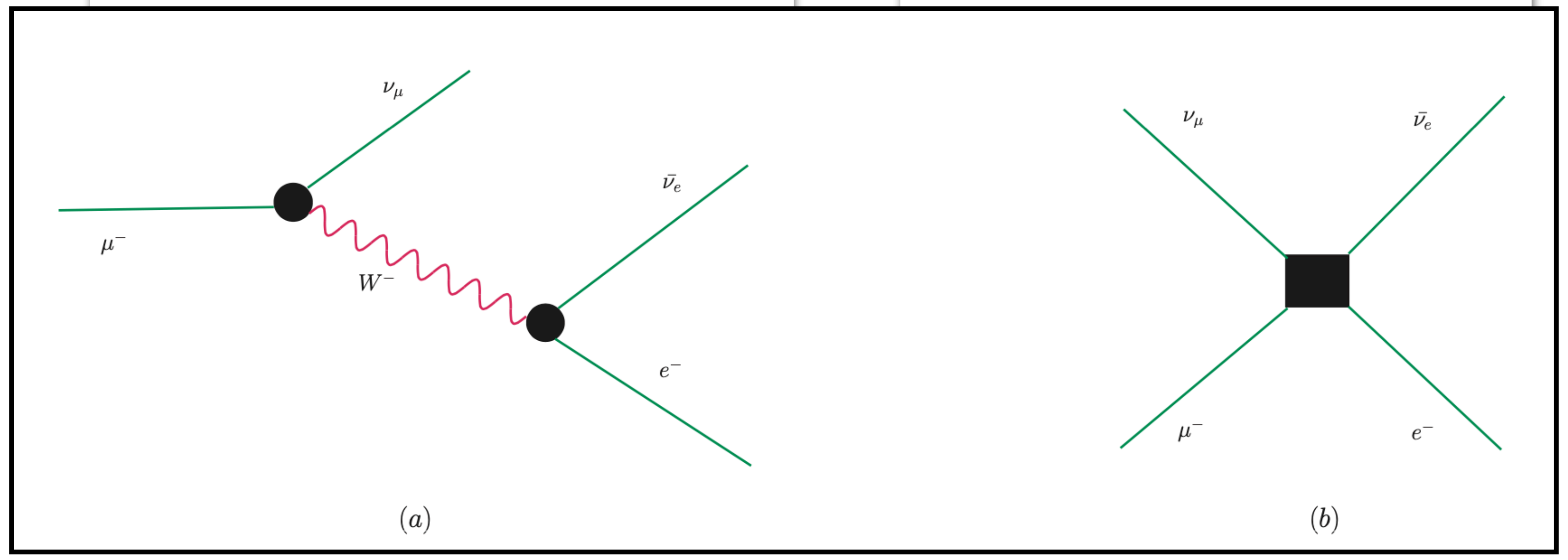
Weinberg's angle

$$\widehat{R} = \frac{\sigma^{\nu NC} - \sigma^{\bar{\nu} NC}}{\sigma^{\nu CC} - \sigma^{\bar{\nu} CC}}, \qquad \widehat{R} = \frac{1}{2} - \sin^2 \bar{\theta}_{\mathrm{w}}.$$



Fermi constant and rho-parameter

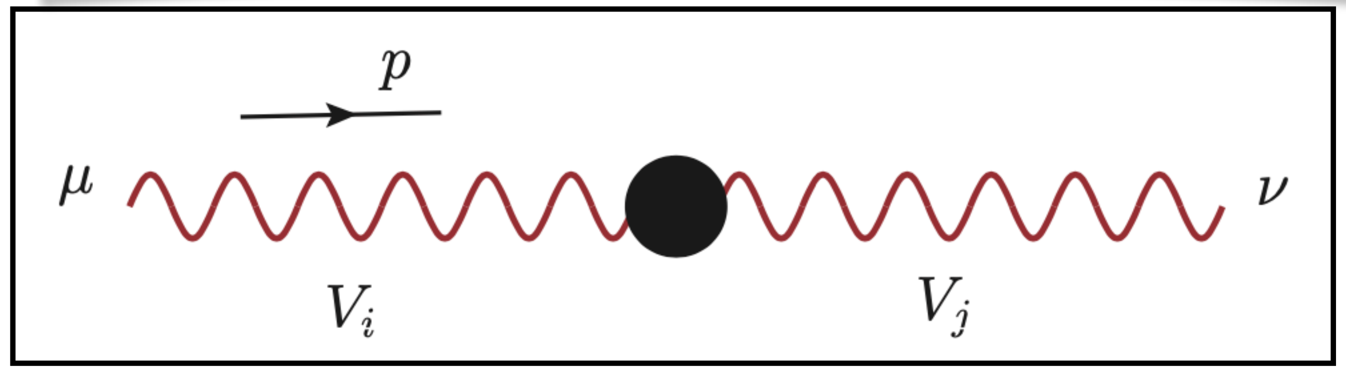
$$(\mathcal{G}_F)_{SM} = \frac{g^2}{4\sqrt{2} M_W^2}. \qquad \rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_{\mathrm{w}}}.$$



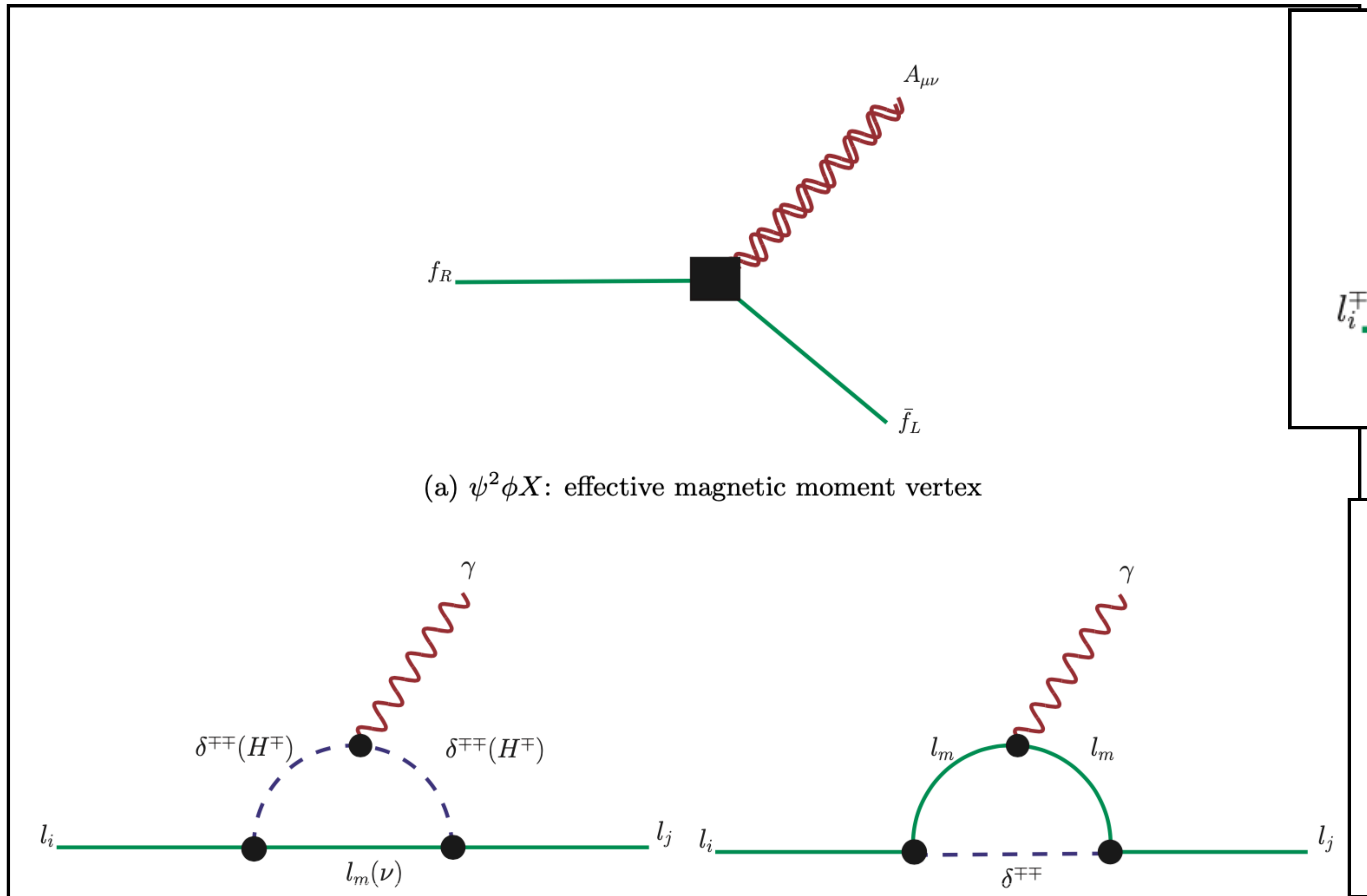
S, T, U parameters

$$i\Pi^{\mu\nu}_{V_i V_j}(p^2) = i\left(g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\Pi_{V_i V_j}(p^2) + \left(i\frac{p^\mu p^\nu}{p^2} \text{ terms}\right)$$

$$\Pi_{V_i V_j}(p^2) = [\Pi_0 + \Pi_2 p^2 + \Pi_4 p^4 + \mathcal{O}(p^6)]_{V_i V_j}.$$



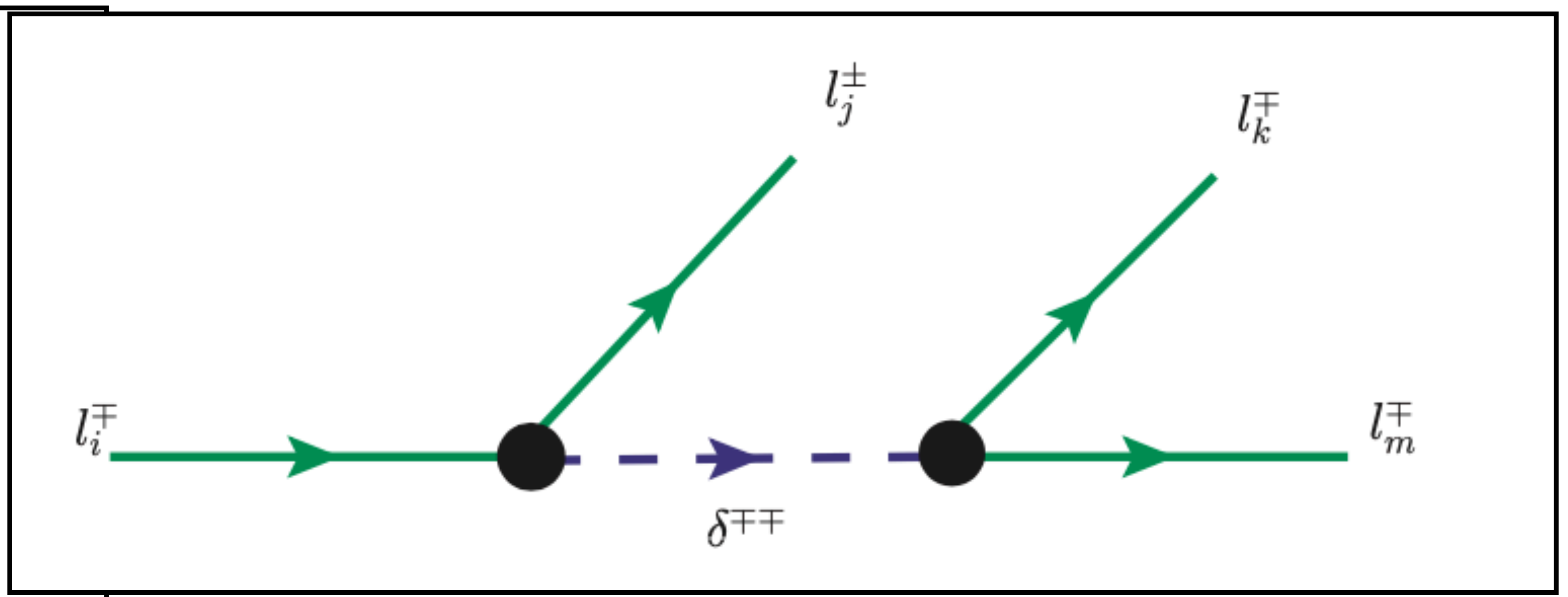
Rare processes : LNV and CLFV

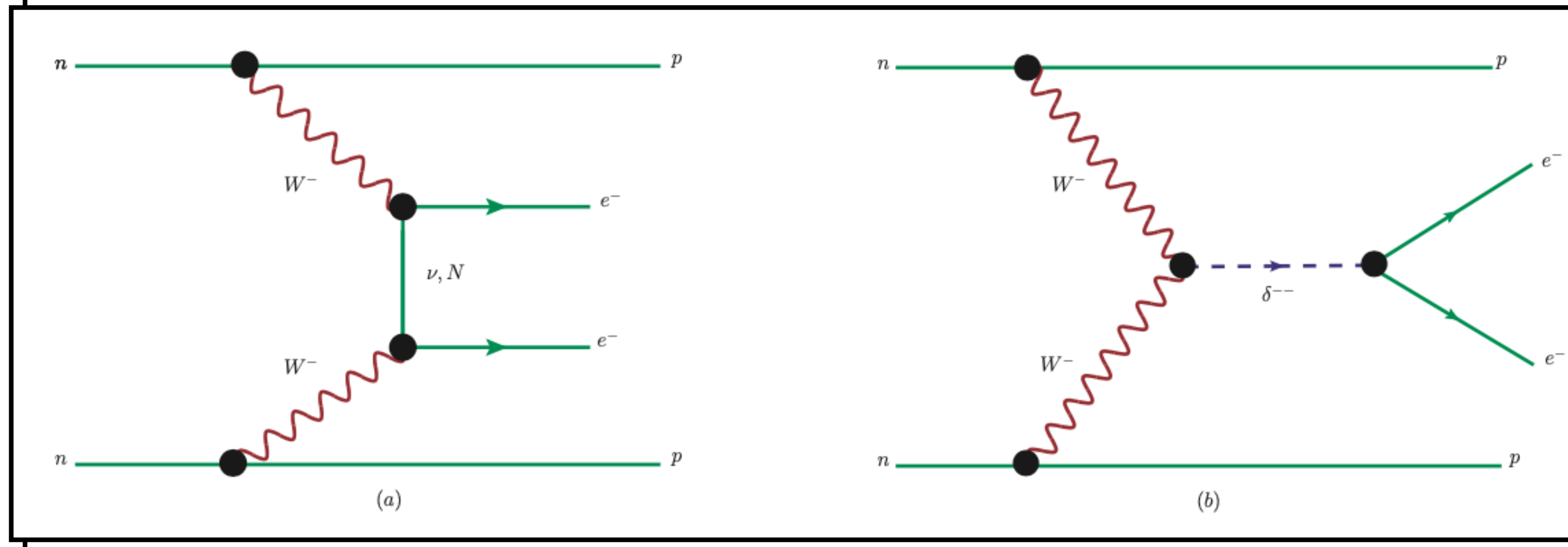


(a) $\psi^2\phi X$: effective magnetic moment vertex

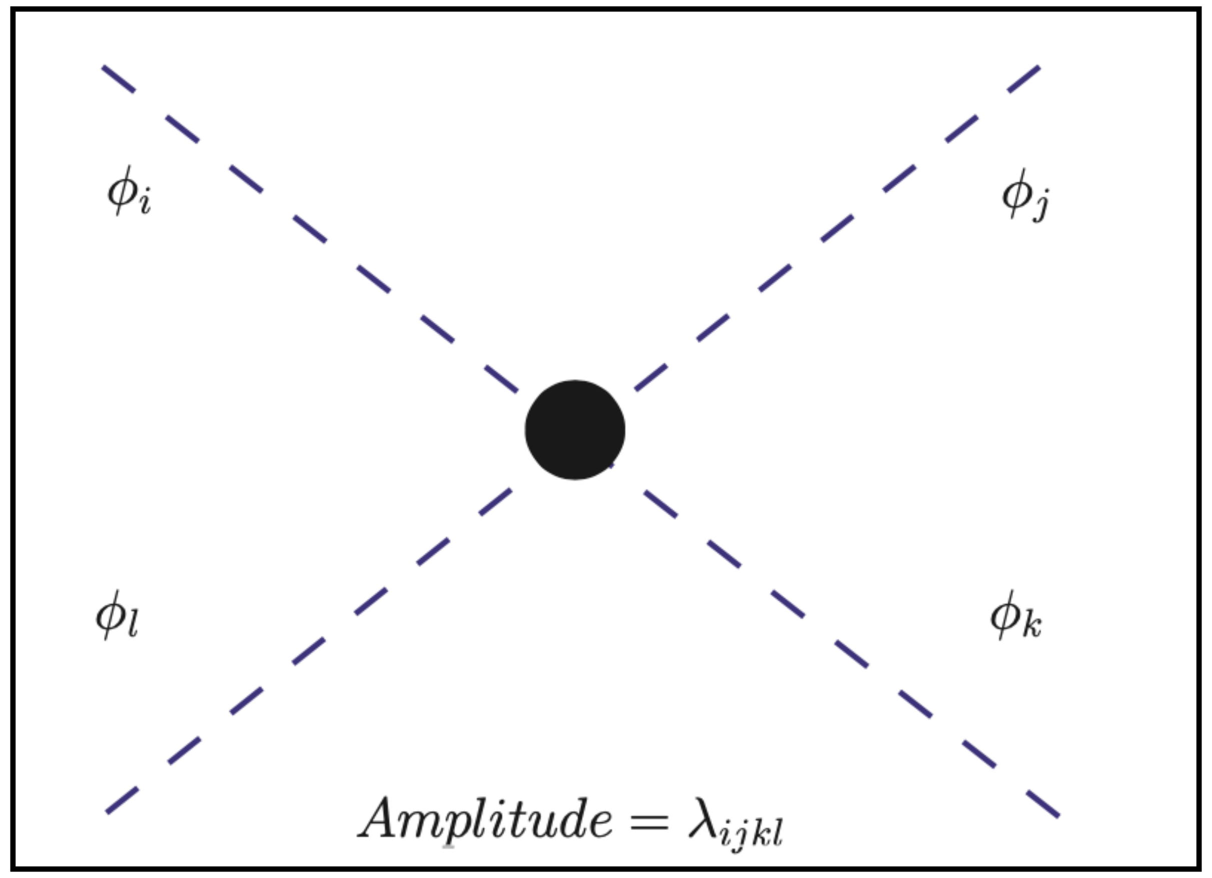
(b) $l_i \to l_j\gamma$: singly and doubly charged scalars

(c) $l_i \to l_j\gamma$: doubly charged scalars



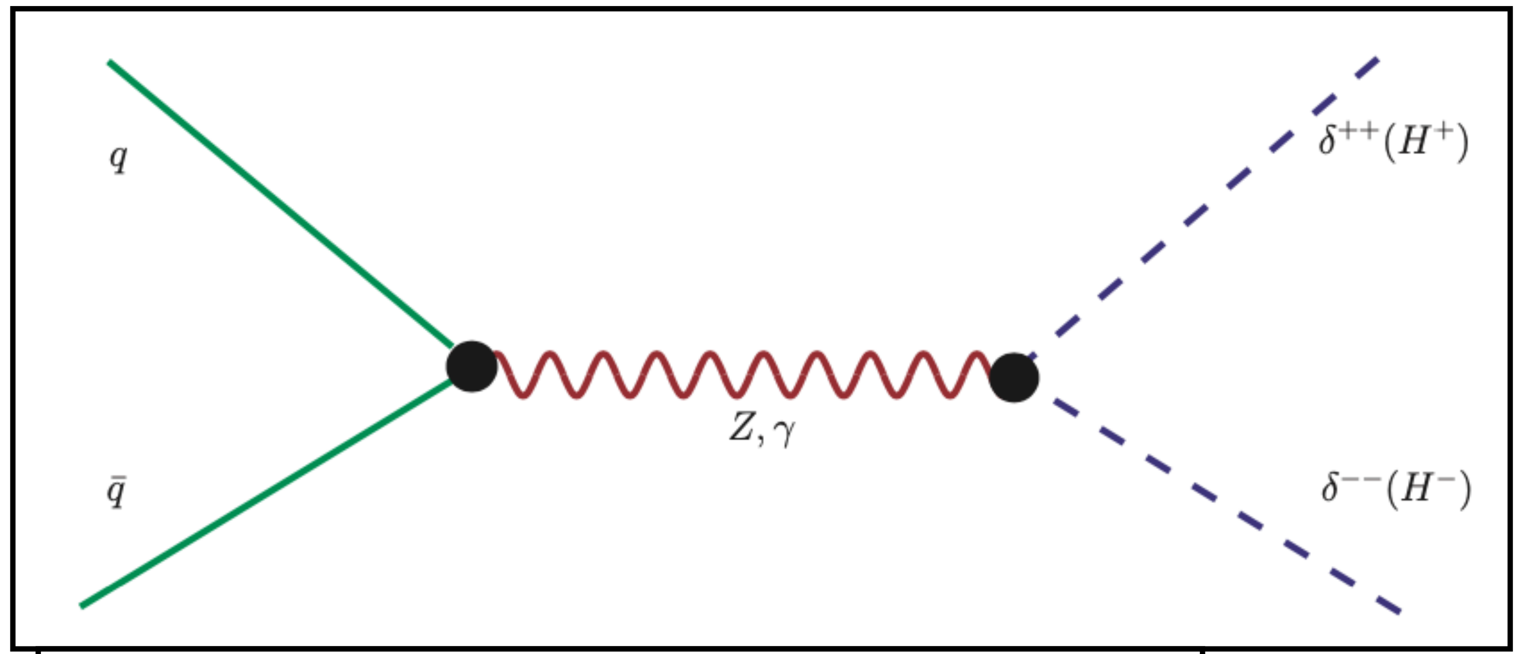


Tree - unitarity



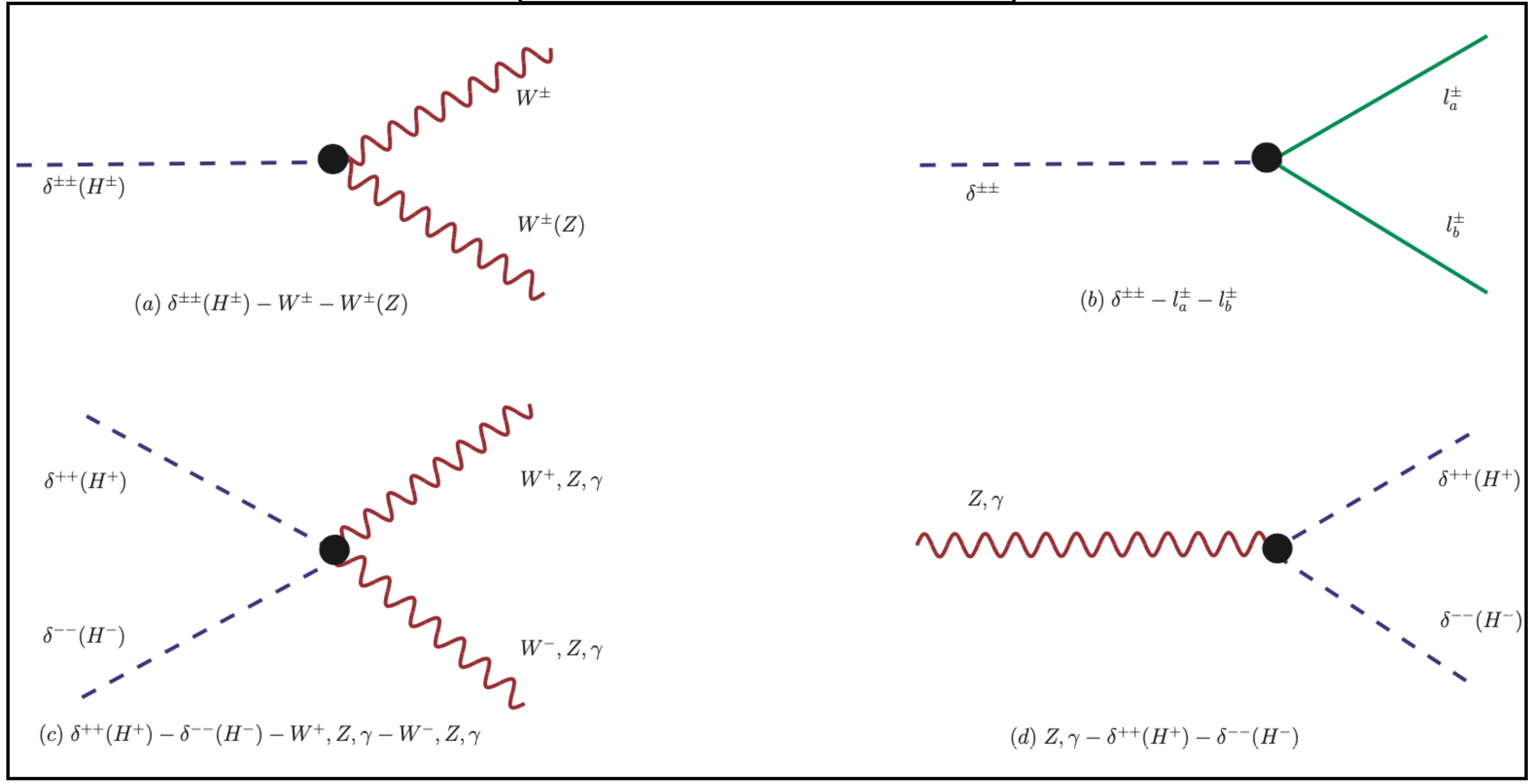
How does the collider phenomenology get affected ?

Examples: production and decay of charged scalars



production @ LHC

decay modes



What else...

Formalism

1. Scattering amplitudes and invariant structures.
2. Implementation of space-time symmetry beyond 3+1.
3. Application in GR-EFT.
4. Methods for invariant polynomial computation apart from HS.

Phenomenology

1. Covariant formulation of BSM-EFT.
2. EWPO and Low energy observables in the light of effective operators.
3. Readdressing the exclusion limits of BSM parter space in presence of effective operators.
4. Addressing “semi-inverse” problem using “CoDEx” and “GrIP”.