Understanding the Covid 19 Epidemic

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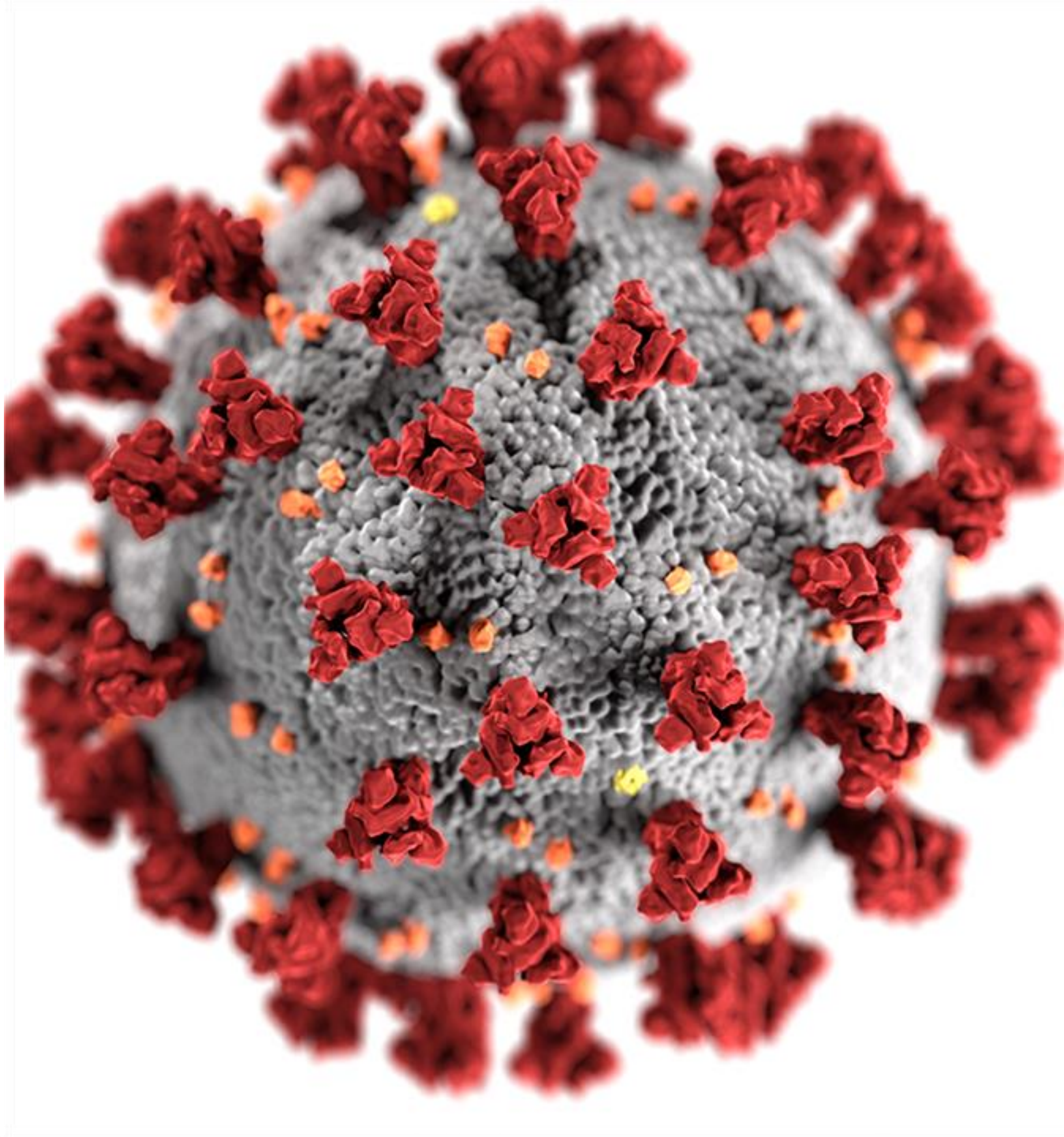
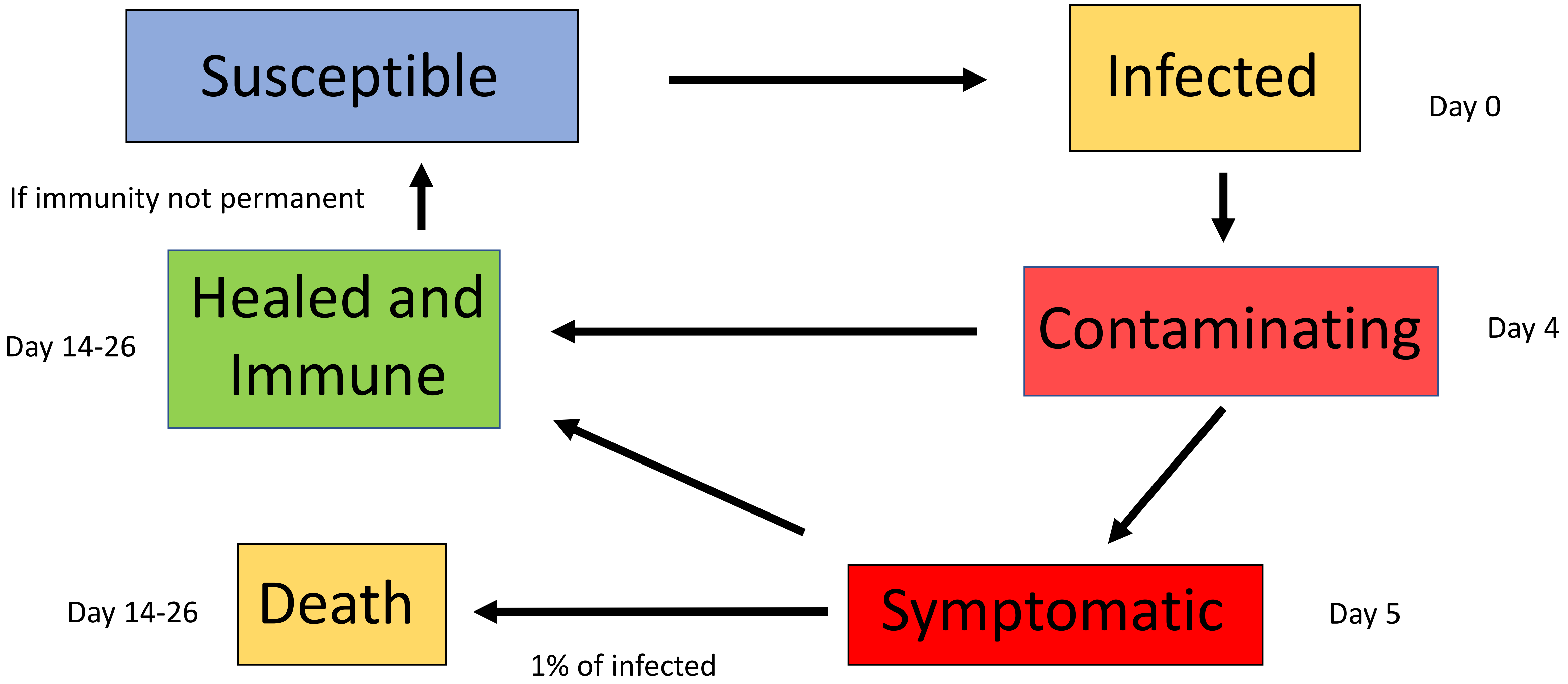
Content

- What is known about Covid 19
- A simple model
- A more complex model
- Analysing data for a few countries
- Possible evolution in the UK
- 2 other models
- Conclusions



Stages of Infection



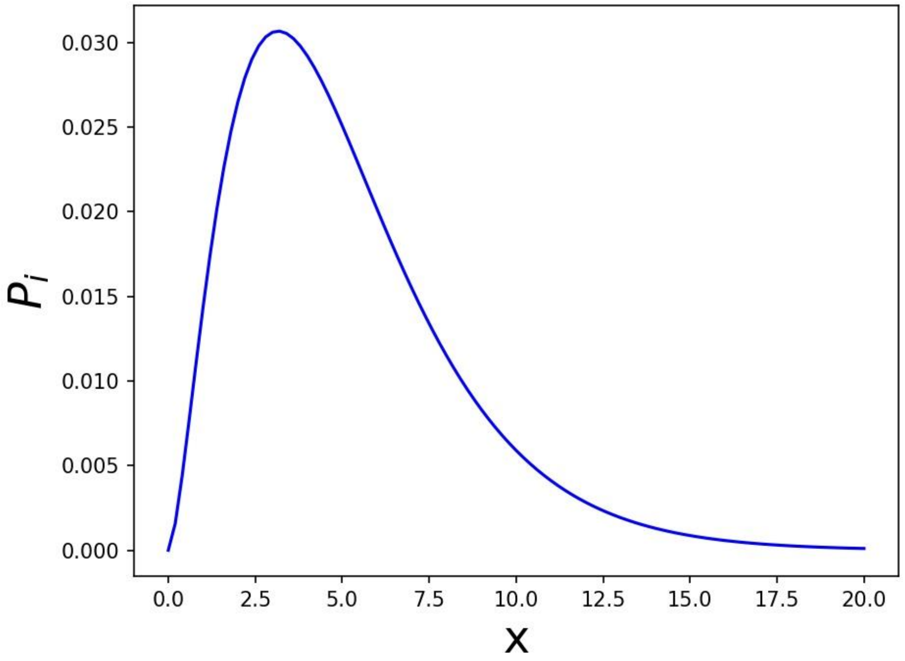
What is known about Covid 19?

- Incubation Period : average 5.1 days [**A. Lauer et al. :** doi:10.7326/M20-0504**]**

Day	2	3	4	5	6	7	8	9	10	11	12
Cumulative Prob	0.025	0.1	0.26	0.5	0.65	0.76	0.84	0.9	0.94	0.975	1
Prob Infection	0.025	0.075	0.16	0.24	0.15	0.11	0.08	0.06	0.04	0.035	0.025

$$P_i = Gamma(5.1, 0.86)$$

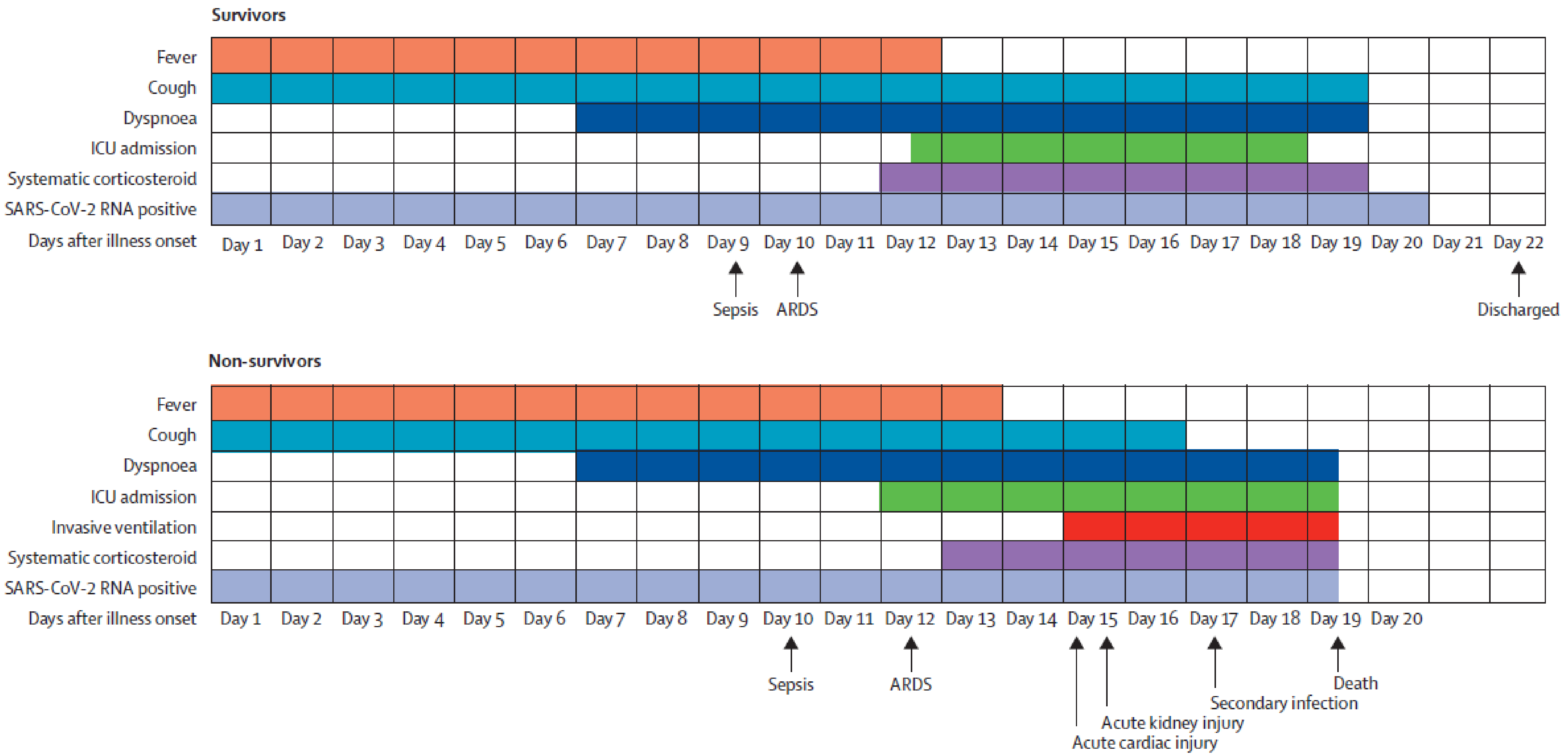
$\mu = 5.1$ $\sigma/\mu = 0.86$



https://www.imperial.ac.uk/mrc-global-infectious-disease-analysis/covid-19/report-13-europe-npi-impact/

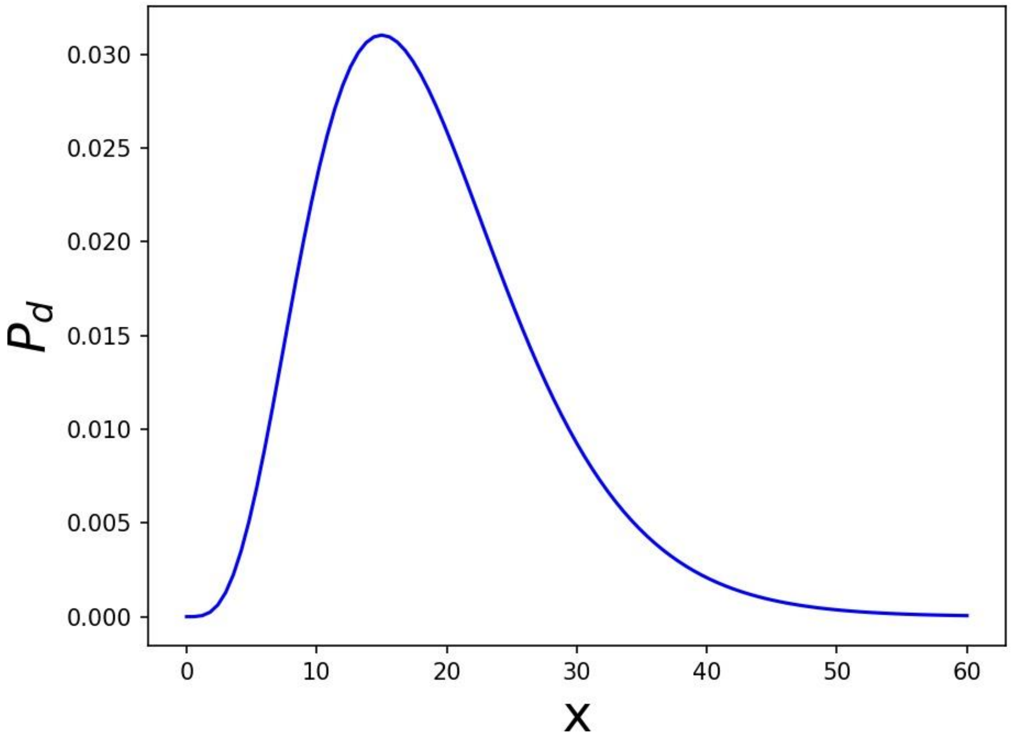
What is known about Covid 19?

- **Distribution of death and recovery:** [F. Zhou et al. : doi:10.1016/S0140-6736(20)30566-3]

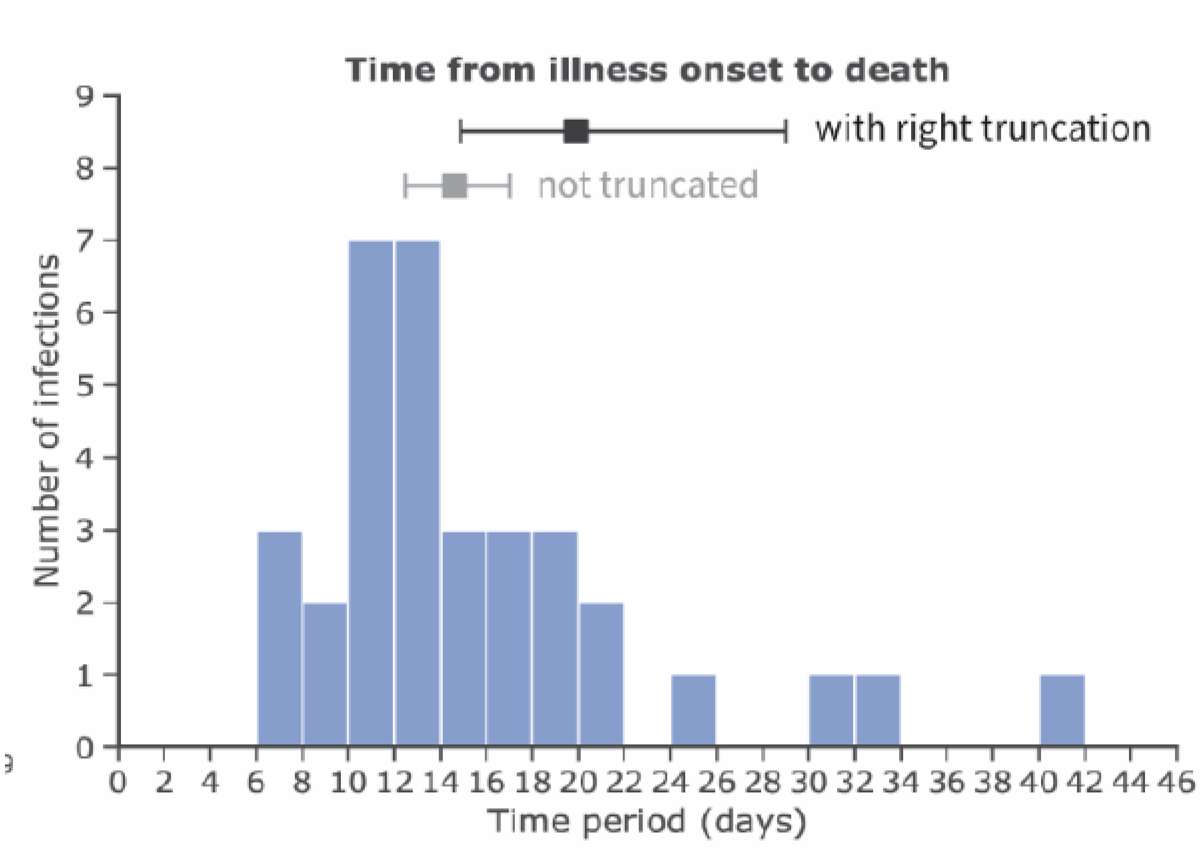


What is known about Covid 19?

- Distribution of death: [S. Jung et al. doi:10.3390/jcm9020523]



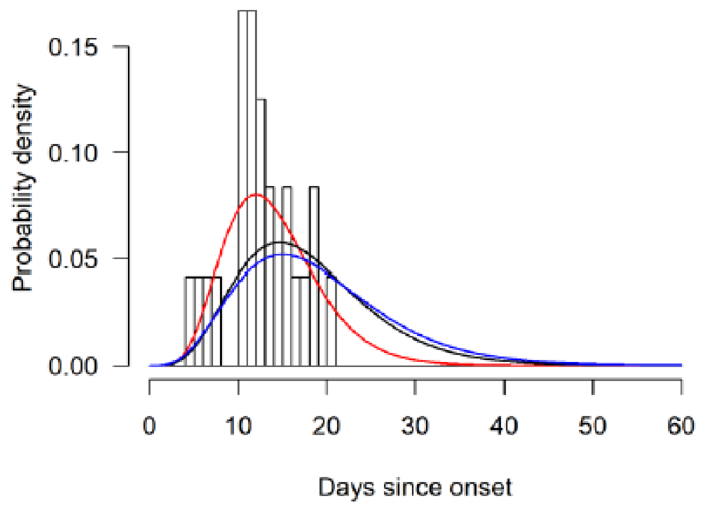
$$P_d = Gamma(18.8, 0.45)$$



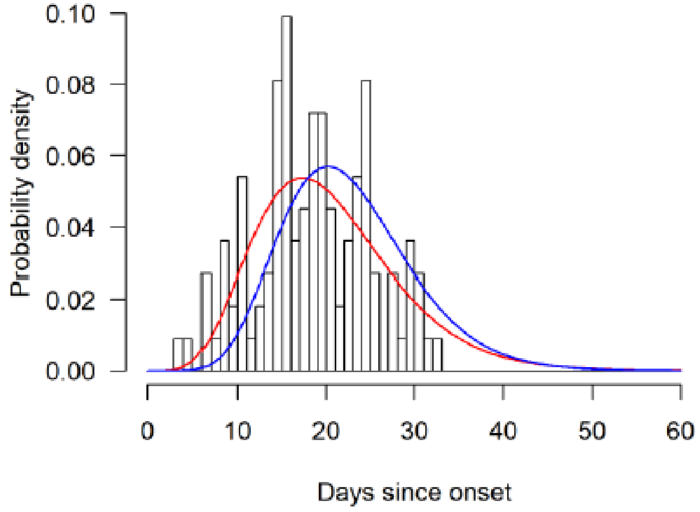
What is known about Covid 19?

- Distribution of death recovery and infectiousness

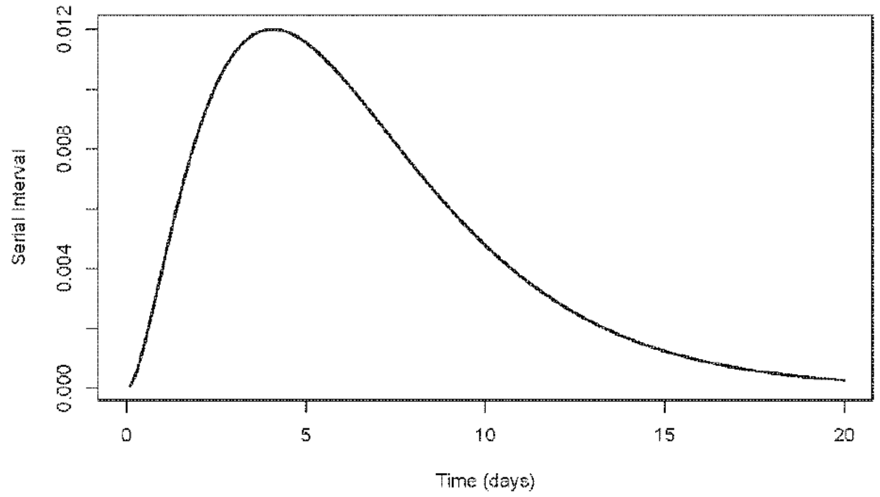
Death



Recovery



Infectiousness



R. Verity et al doi:10.1101/2020.03.09.20033357

Imperial report 13

Data

Daily data available for most countries from

https://covid.ourworldindata.org/data/ecdc/full_data.csv

- Number of recorded cases
- Number of deaths

Recorded Cases

Not every person with symptoms is tested

First symptoms appear after a few days

Tests performed several days after first symptoms

Many infected individuals do not have symptoms

Recorded infection cases not reliable data for analysis

Use number of fatalities (more systematically recorded)

Simple Mathematical Model

2 populations:

- Strong (survive)
- Weak (die)

4 states:

- Susceptible
- Infected
- Contagious
- Healed
- Dead

Simple Mathematical Model

- S : Strong Suceptible population
- S_i : Strong Infected population
- S_c : Strong Contaminating population
- S_h : Healed population
- W : Weak Suceptible population
- W_i: Weak Infected population
- W_c: Weak Contaminating population
- W_d: Deaths

Simple Mathematical Model

$$
\begin{aligned}
S(d+1) &= S(d) - \Delta N_{Si}(d) \\
S_i(d+1) &= S_i(d) + \Delta N_{Si}(d) - \Delta N_{Sc}(d) \\
S_c(d+1) &= S_c(d) + \Delta N_{Sc}(d) - \Delta N_{Sh}(d) \\
S_h(d+1) &= S_h(d) + \Delta N_{Sh}(d) \\
W(d+1) &= W(d) - \Delta N_{Wi}(d) \\
W_i(d+1) &= W_i(d) + \Delta N_{Wi}(d) - \Delta N_{Wc}(d) \\
W_c(d+1) &= W_c(d) + \Delta N_{Wc}(d) - \Delta N_{Wd}(d) \\
W_d(d+1) &= W_d(d) + \Delta N_{Wd}(d)
\end{aligned}
$$

Simple Mathematical Model

$$P_A(d) = S(d) + S_i(d) + S_c(d) + S_h(d) + W(d) + W_i(d) + W_c(d)$$

$$\Delta N_{Si}(d) = K_s \; S(d) \, \frac{S_c(d) + 0.5 * W_c(d)}{P_A(d)}$$

$$\Delta N_{Wi}(d) = K_w \; W(d) \, \frac{S_c(d) + 0.5 * W_c(d)}{P_A(d)}$$

$$\Delta N_{Sc}(d) = \Delta N_{Si}(d - t_i),$$

$$\Delta N_{Wc}(d) = \Delta N_{Wi}(d - t_i),$$

$$\Delta N_{Sh}(d) = \Delta N_{Sc}(d - t_h),$$

$$\Delta N_{Wd}(d) = \Delta N_{Wc}(d - t_d).$$

Characteristic of epidemics

Only one parameter to characterise an epidemic:

- Infection rate Rt : the average number of people infected by an infected person
- Doubling time: the number of days needed for the number of infected people to double
- K_S, K_W Set to different values when measures are introduced

A more realistic model

Delay: convolution with probability

$$
\begin{aligned}
\Delta N_{Sc}(d) &= \sum_{\tau} P_I(\tau - 1)\Delta N_{Si}(d-\tau), \\
\Delta N_{Wc}(d) &= \sum_{\tau} P_I(\tau - 1)\Delta N_{Wi}(d-\tau), \\
\Delta N_{Sh}(d) &= \Delta N_{Sc}(d - t_h), \\
\Delta N_{Wd}(d) &= \sum_{\tau} P_D(\tau + 1)\Delta N_{Wc}(d-\tau).
\end{aligned}
$$

Initial Conditions

- Fatality rate $F_R \in [0.01, 0.03]$
- $d = 0$: first recorded case
- $t_h = 7$ days

Fitting Data

Select value of F_R

Select country

Use death numbers for first part of data [until date 1st death]

Fit S_{I0} and K_S (=K_W)

Method : least square method and Metropolis Algorithm

Effect of uncertainty on parameters

P_i, P_d, (t_h):

- Smooth all the curves
- Different averages -> different delays between infection and death

F_R:

- Change the number of infected people inferred by the model
- Larger F_R -> longer epidemic

Interpretation of K_S, K_W and S_{I0}

K_S, K_W:

- Capture the dynamic of the epidemic
- $K_W \neq K_S$: Vulnerable people protected differently

S_{I0} :

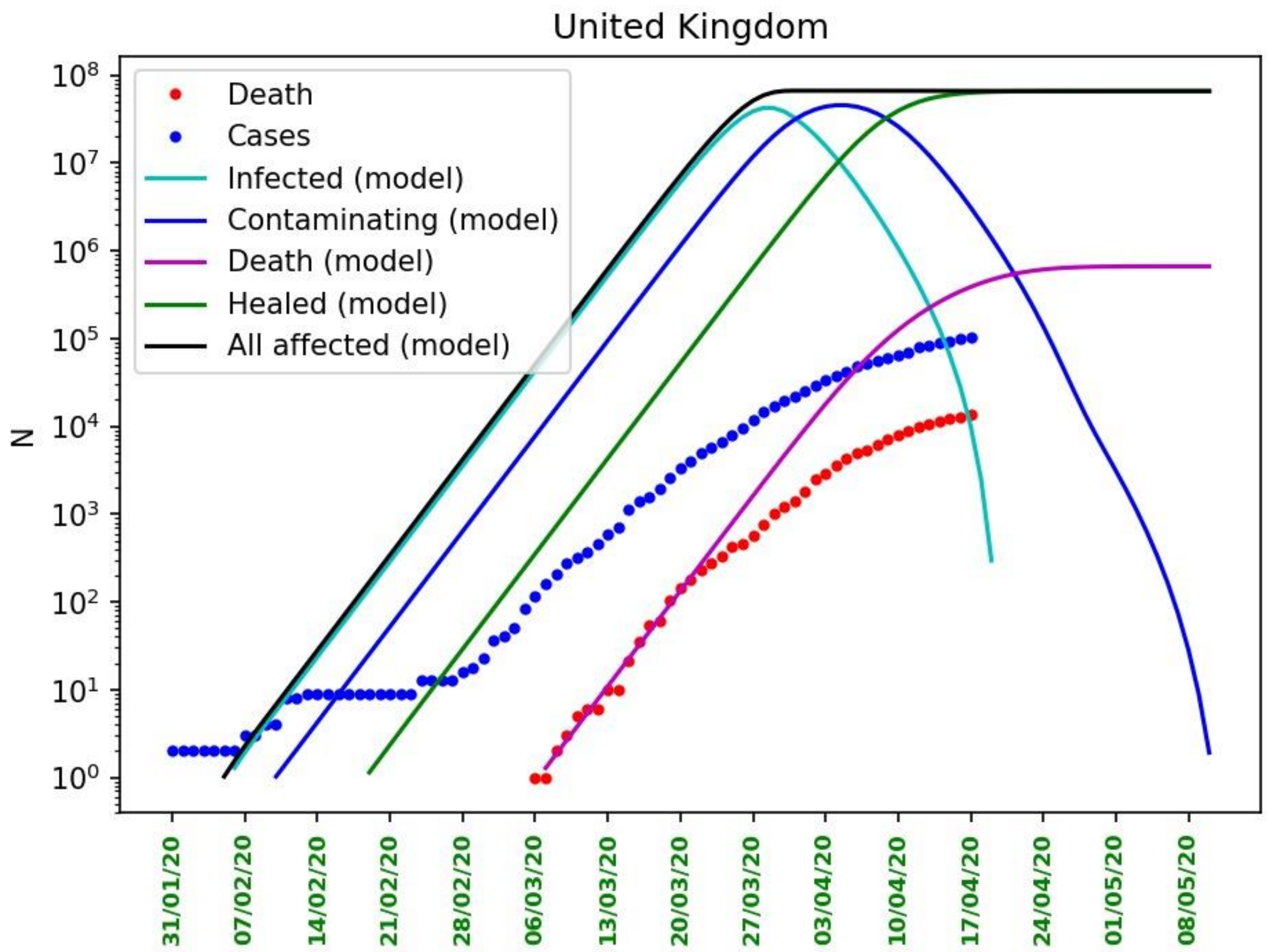
- $S_{I0} < 1$:first few cases well contained
- $S_{I0} > 1$: several/many “patient zero”

Model Strengths and Weaknesses

- Strengths
  - Very few parameters
  - All difficult unknowns hidden in Ks and Kw
- Weaknesses
  - Assume homogeneous population/evolution (especially near saturation)
  - Can't predict Ks and Kw

Country : UK (Pop : 66.4 million)

- First Case: 31 Jan 20
- First Death recorded: 6 March 20



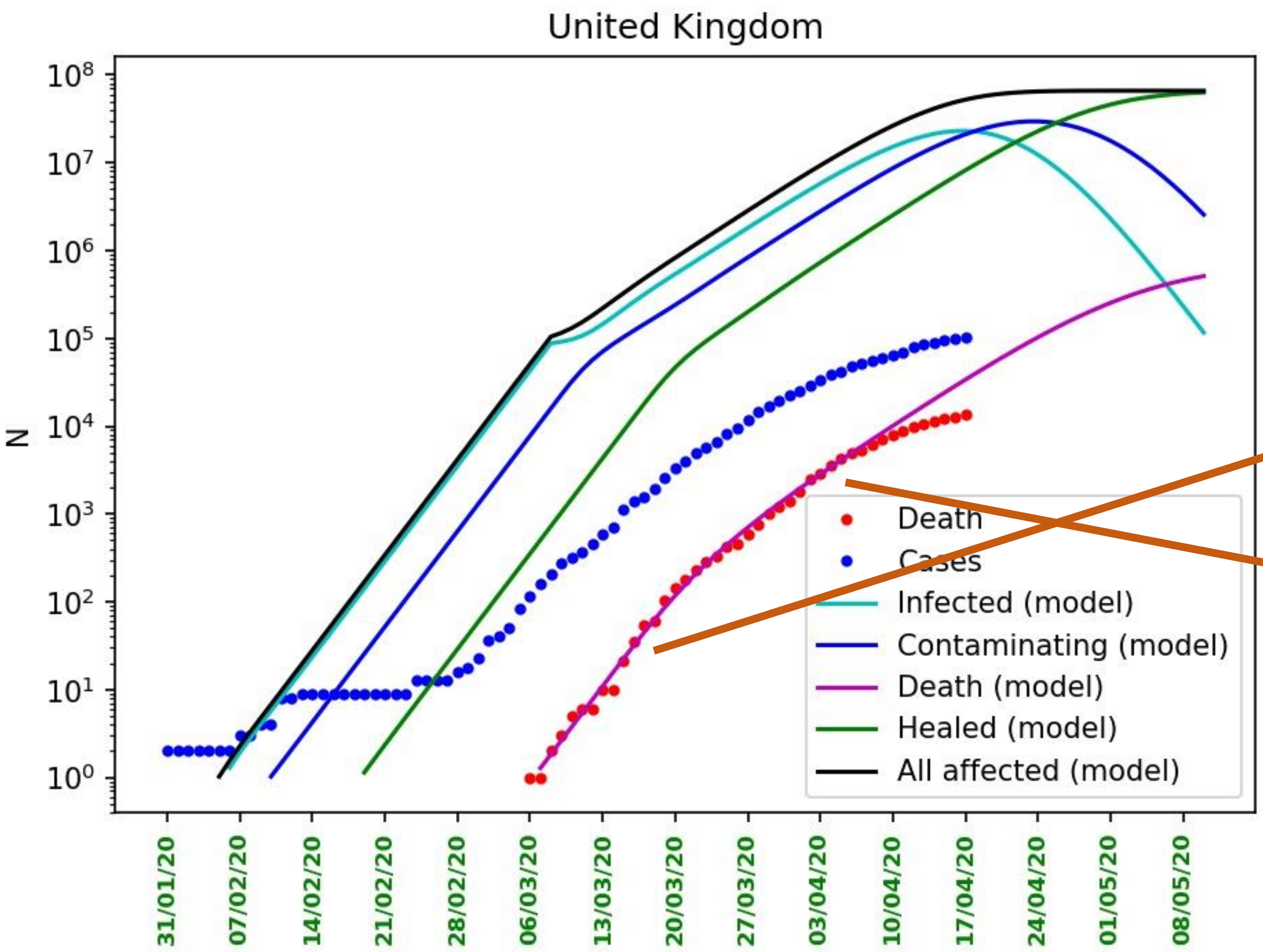
$F_R = 0.01$

$S_{I0} = 0.402$

$K_S = K_W = 2.840$

Country : UK (Pop : 66.4 million)

- First Case: 31 Jan 20
- First Death recorded: 6 March 20



$F_R = 0.01$

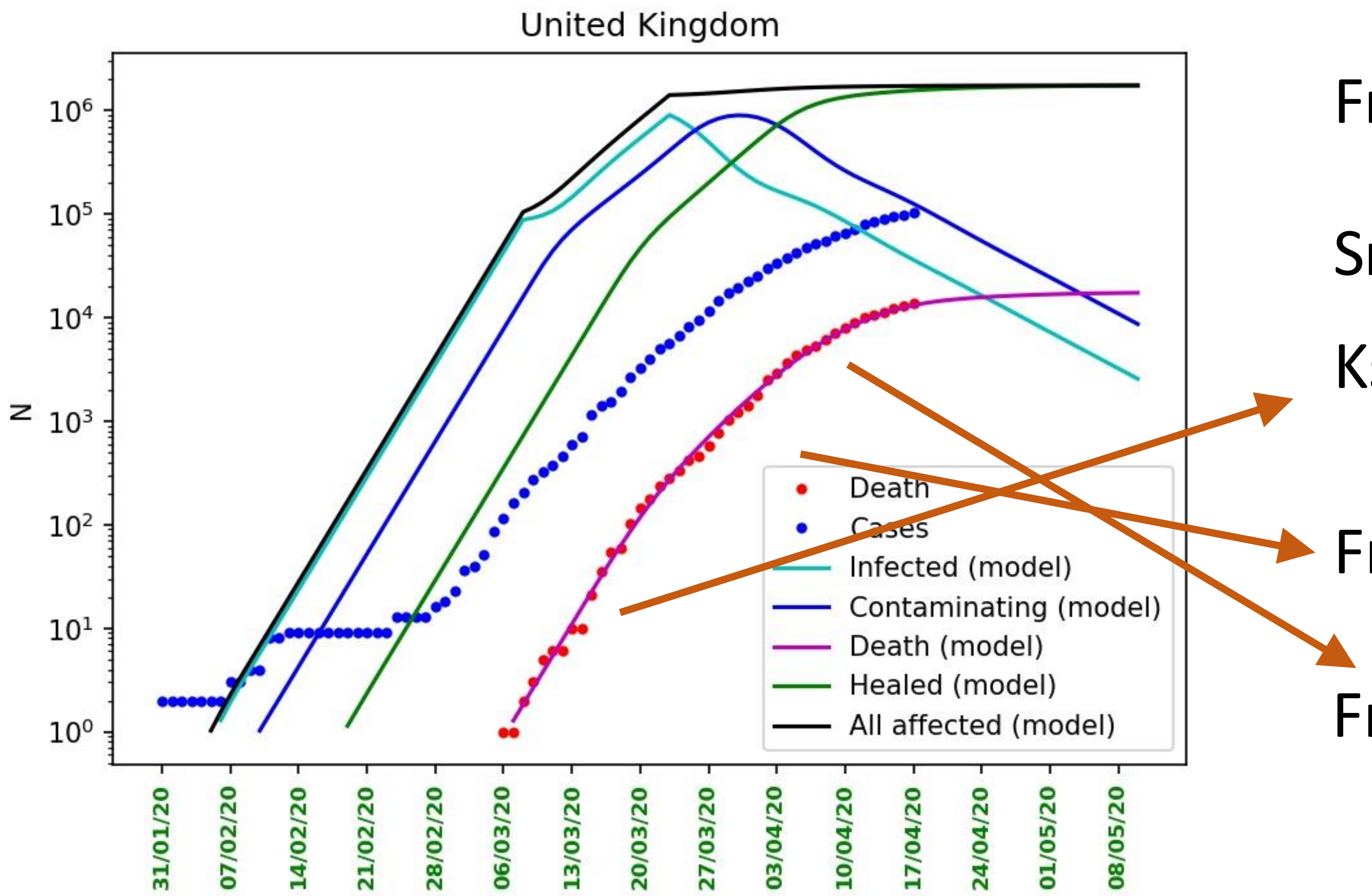
$S_{I0} = 0.402$

$K_S = K_W = 2.840$

From d= 37 : $K_S = K_W = 0.6818$

Country : UK (Pop : 66.4 million)

- First Case: 31 Jan 20
- First Death recorded: 6 March 20



$F_R = 0.01$

$S_{I0} = 0.402$

$K_s = K_w = 2.84$

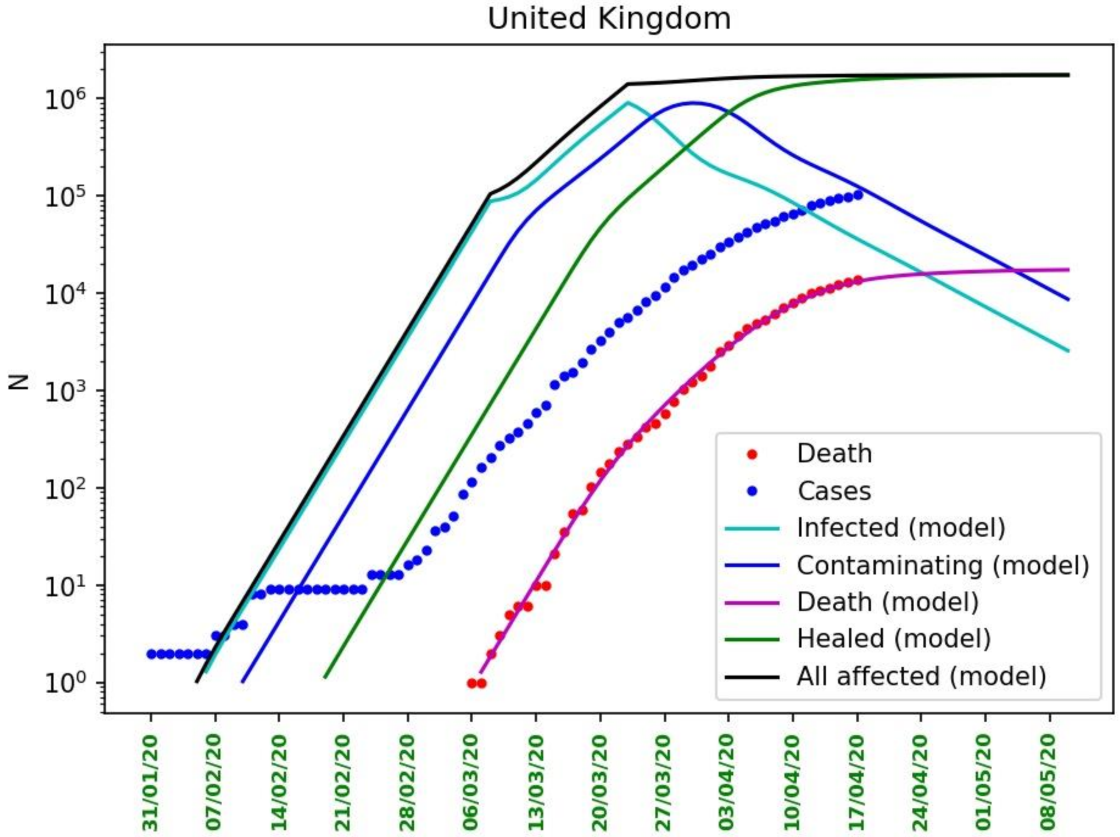
From d= 37 : $K_s = K_w = 0.682$

From d= 52 : $K_s = K_w = 0.026$

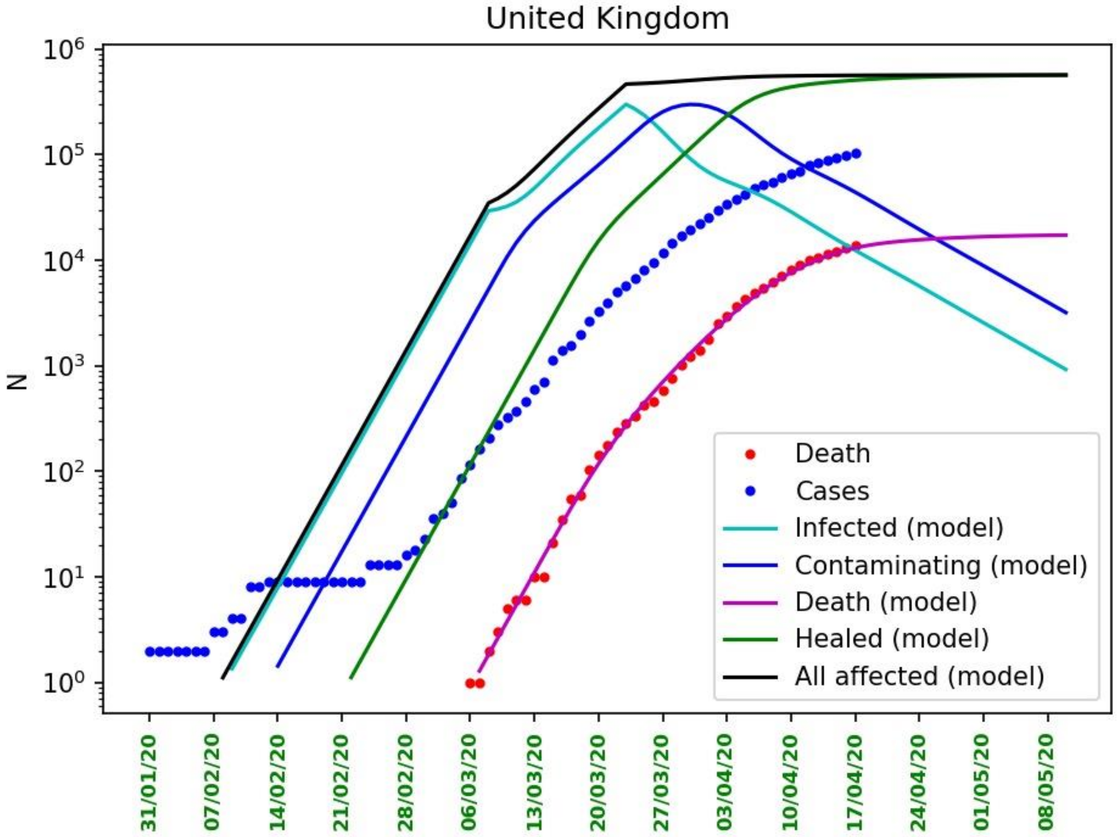
Country : UK (Pop : 66.4 million)

- First Case: 31 Jan 20
- First Death recorded: 6 March 20

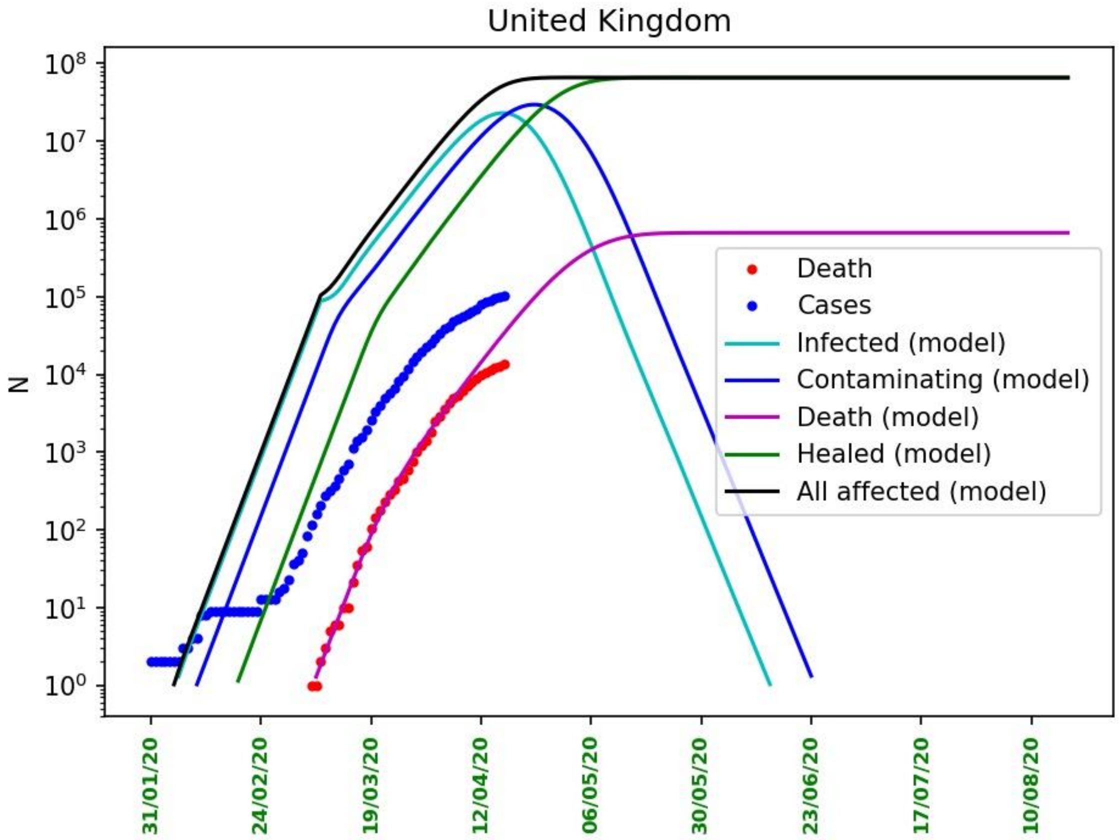
$F_R = 0.01$

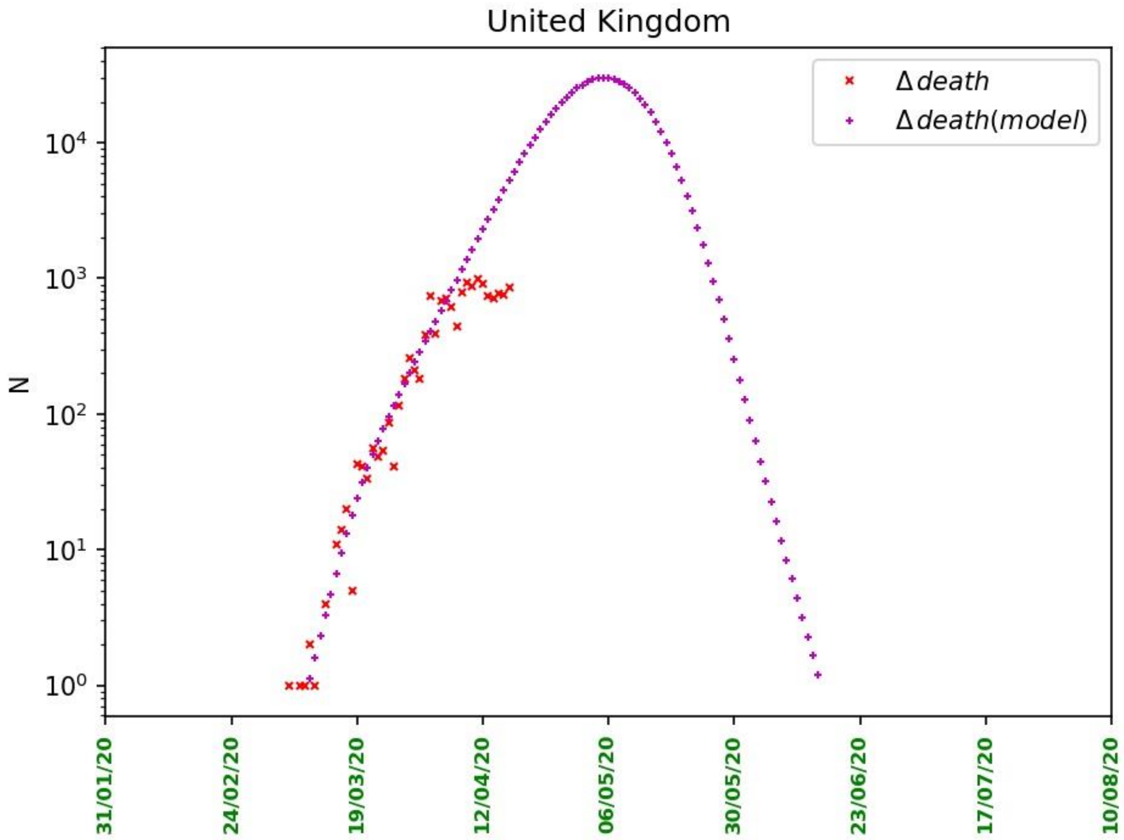


$F_R = 0.03$



Country : UK (without lockdown)



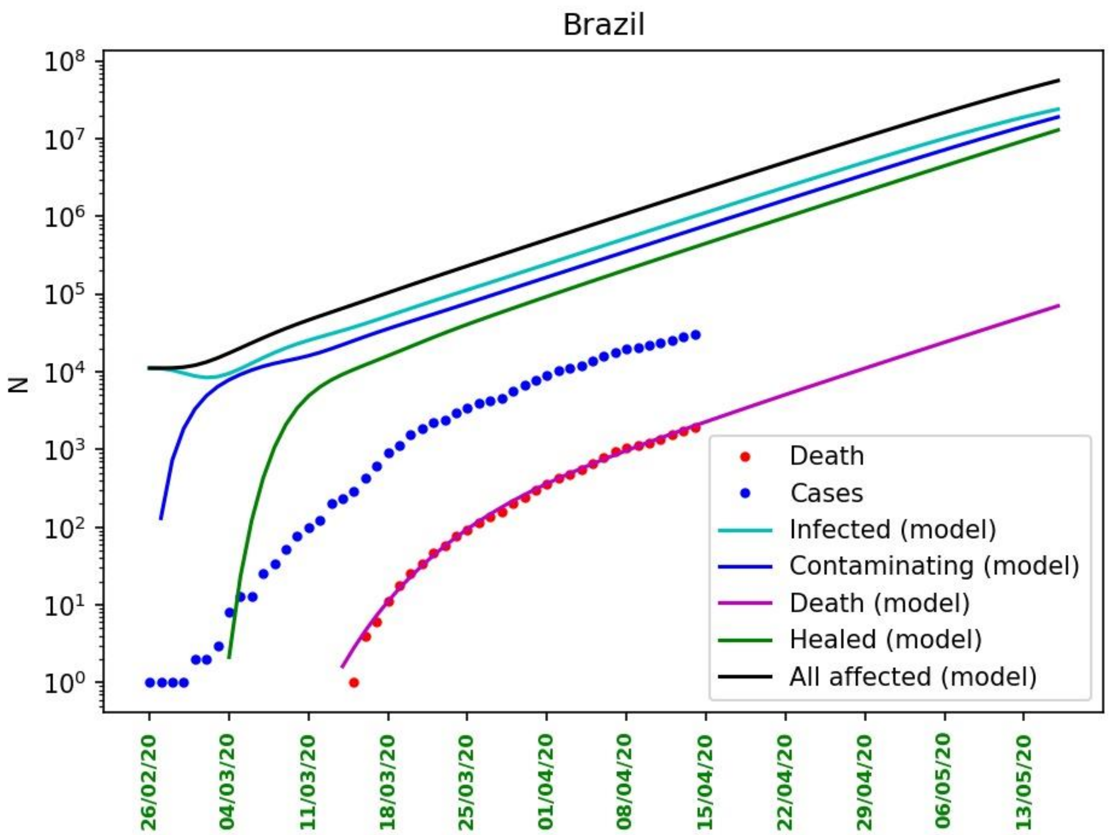


Total death toll : 663,000

Daily deaths : max : 30000
(10 times more than normal)

Country : Brazil (Pop : 209.3 million)

- First Case: 28 Feb 20
- First Death recorded: 18 March 20



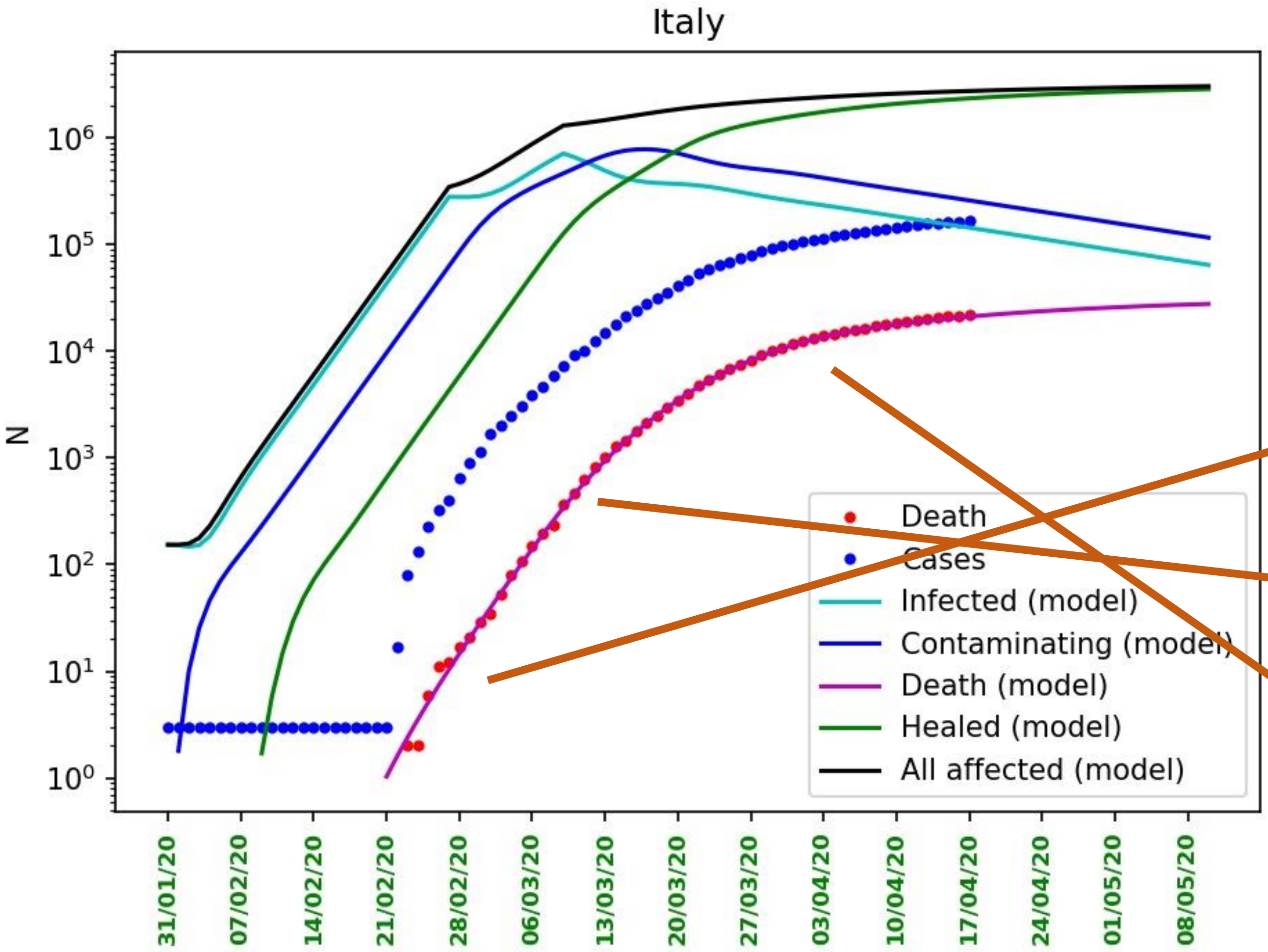
$F_R = 0.01$

$S_{I0} = 11155$

$K_S = K_W = 0.359$

Country : Italy (Pop : 60.5 million)

- First Case: 31 Jan 20
- First Death recorded: 23 Feb 20



$F_R = 0.01$

$S_{I0} = 153.639$

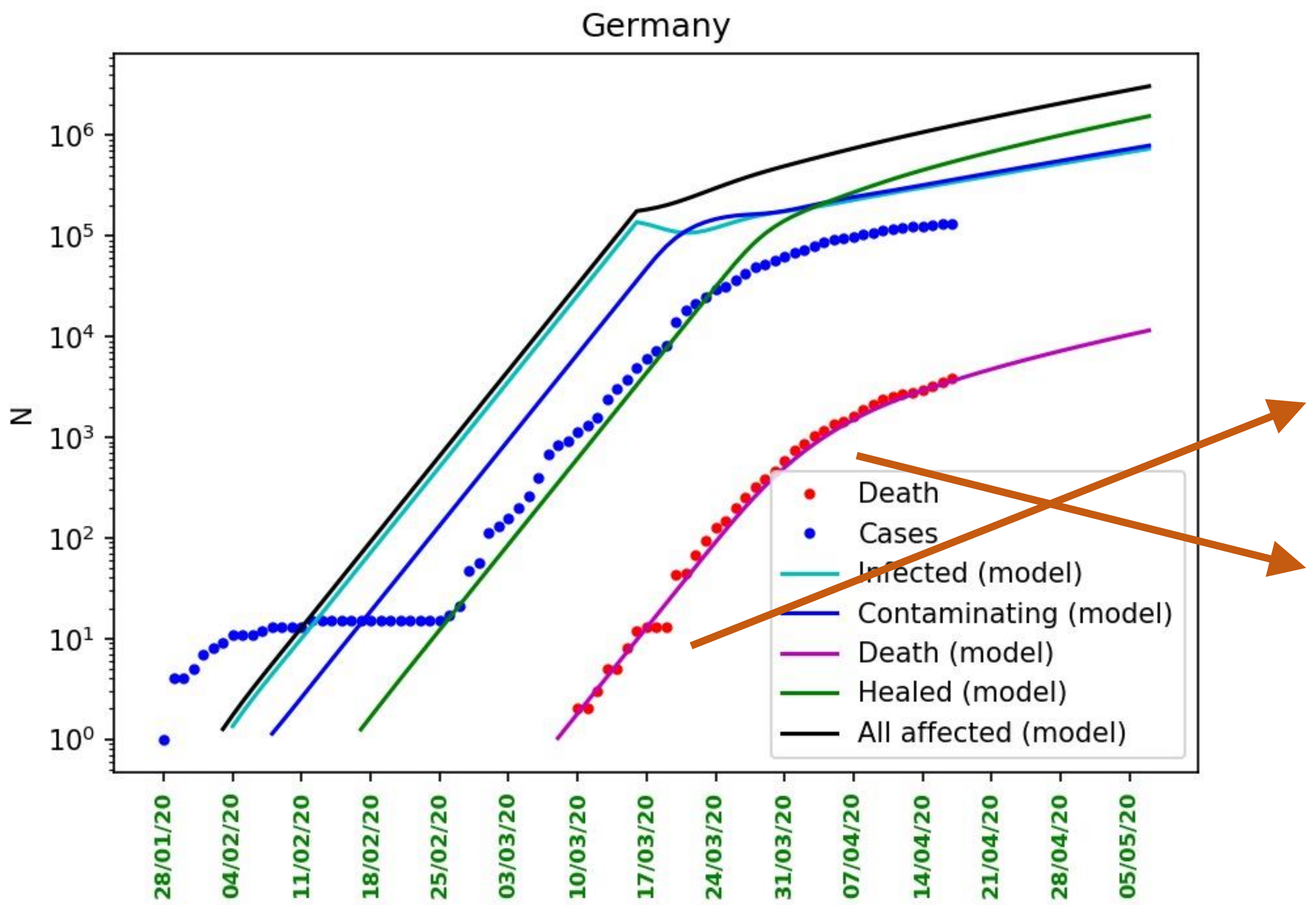
$K_S = K_W = 2.0376$

From d= 27 : $K_S = K_W = 0.3839$

From d= 38 : $K_S = K_W = 0.077$

Country : Germany (Pop : 82.8 million)

- First Case: 28 Jan 20
- First Death recorded: 10 March 20



$F_R = 0.01$

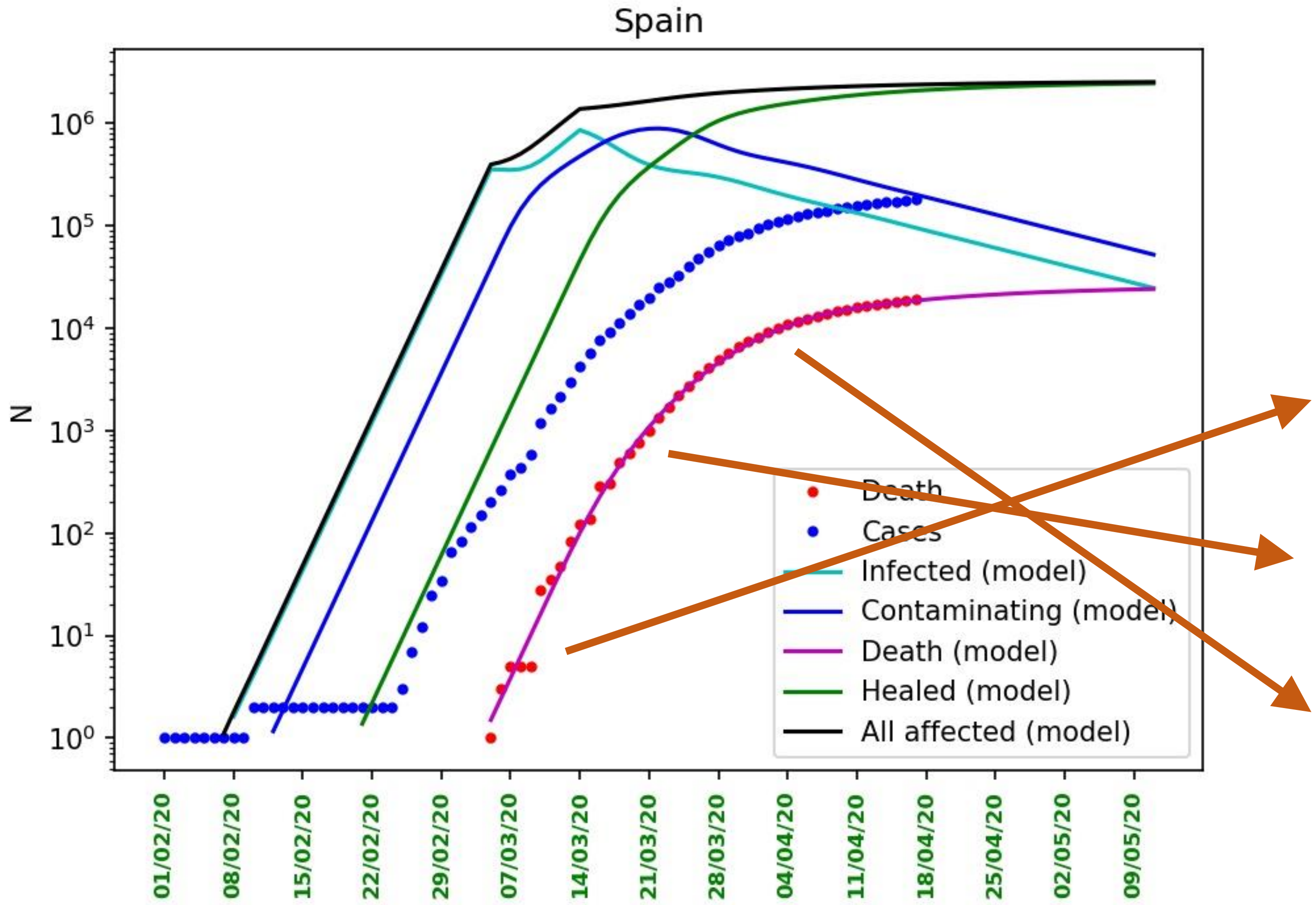
$S_{I0} = 0.485264$

$K_S = K_W = 1.600$

From d=48 : $K_S = K_W = 0.1764$

Country : Spain (Pop : 46.7 million)

- First Case: 1 Feb 20
- First Death recorded: 5 March 20



F_R = 0.01

S_{I0} = 0.1496

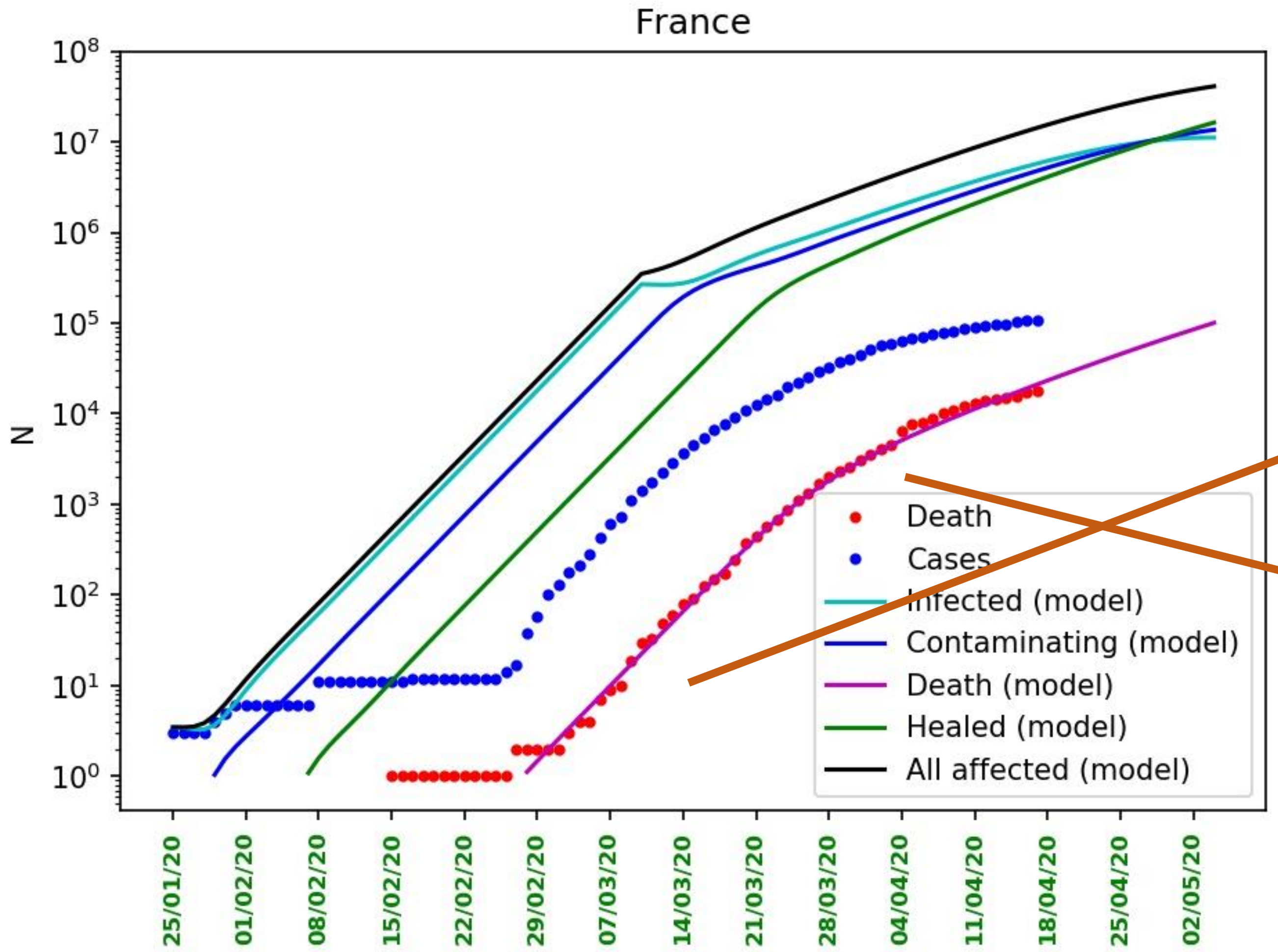
Ks= Kw = 6.1647

From d= 33 : Ks= Kw = 0.5422

From d= 42 : Ks= Kw = 0.0594

Country : France (Pop : 67 million)

- First Case: 25 Jan 20
- First Death recorded: 15 Feb 20



$F_R = 0.01$

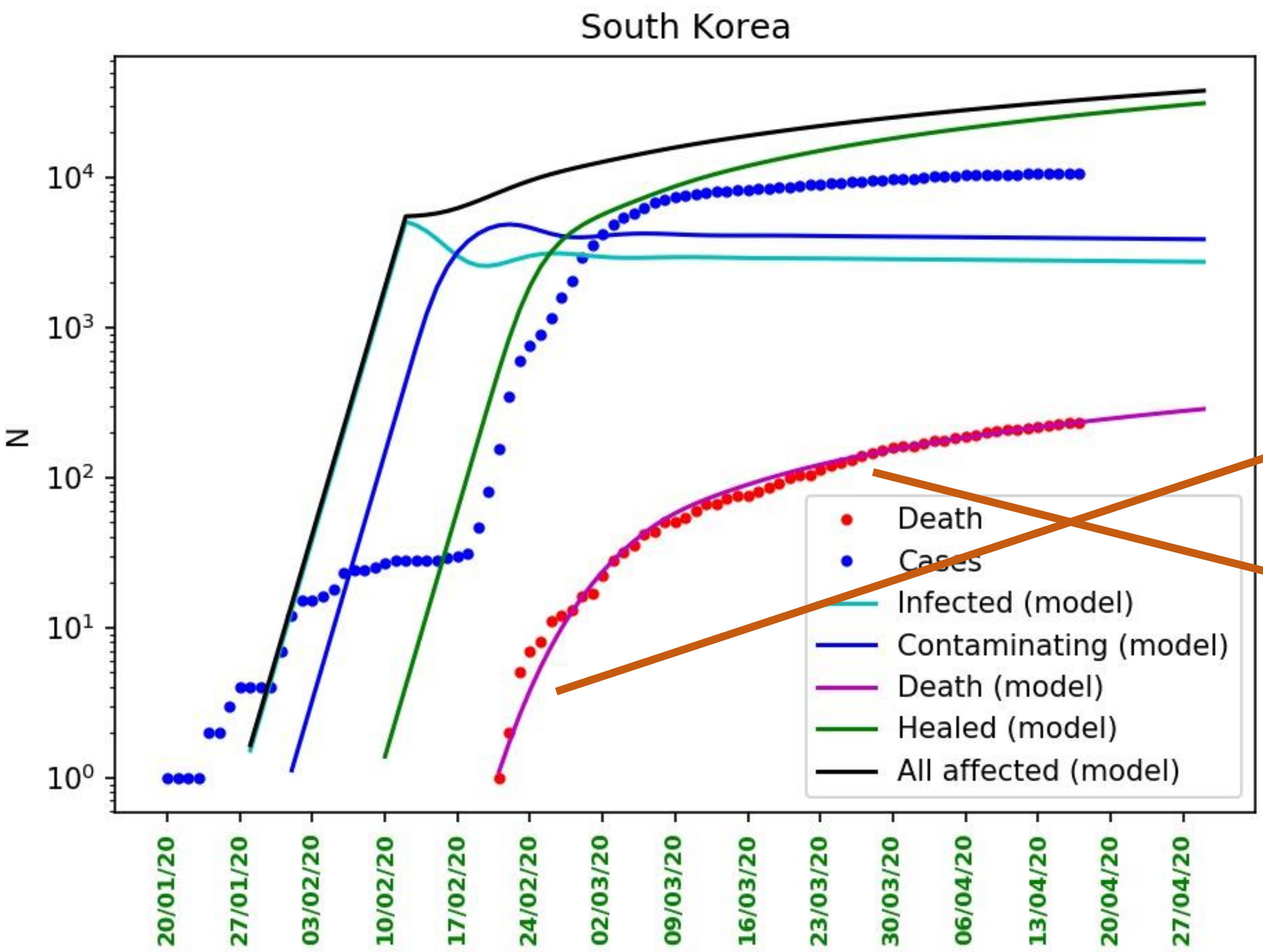
$S_{I0} = 3.4747$

$K_S = K_W = 1.476$

From d=45 : $K_S = K_W = 0.3147$

Country : South Korea (Pop : 51.5 million)

- First Case: 20 Jan 20
- First Death recorded: 21 Feb 20



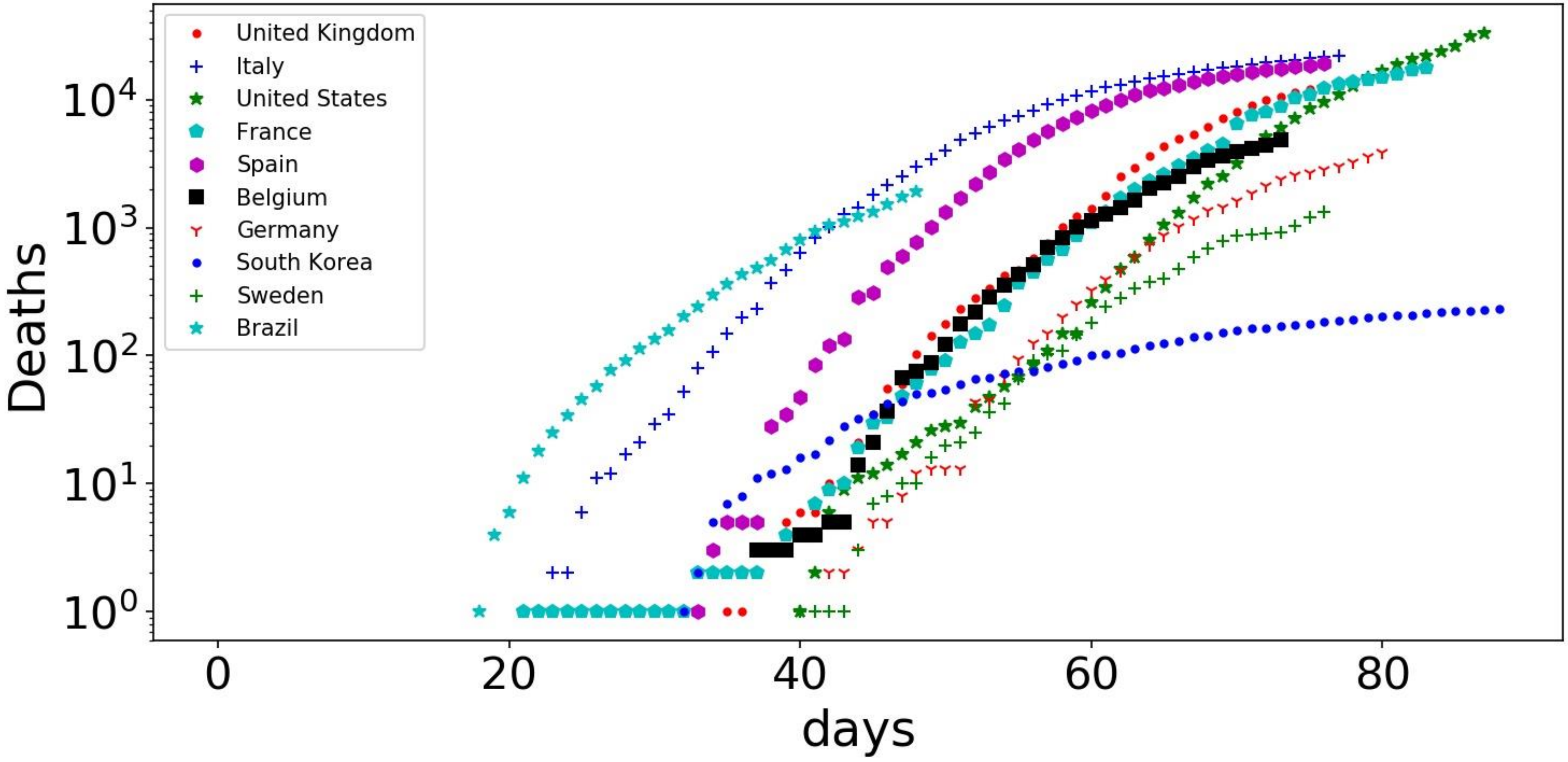
$F_R = 0.01$

$S_{I0} = 3.4747$

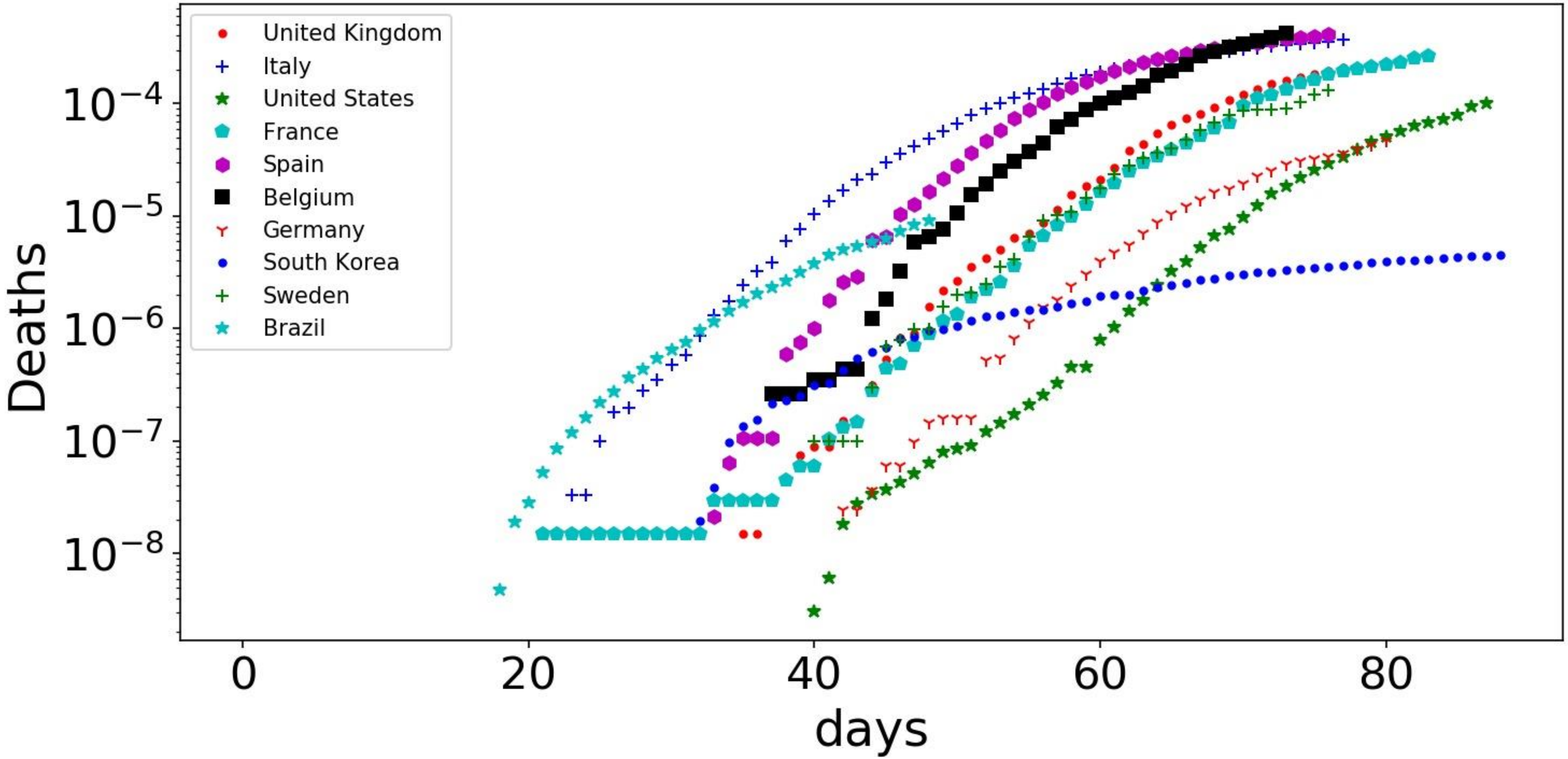
$K_S = K_W = 9.2156$

From d=23 : $K_S = K_W = 0.1095$

All Countries: fatalities



All Countries: fatalities/population



Origin of Differences between Countries

- Different sets of restriction measures
- Restriction measures taken at different times
- Isoform variation of angiotensin-converting enzyme (ACE) between populations
- Temperature/climat ?

How long will it last?

- Back of the envelope calculation:
- There are currently 20,000 people treated for Covid 19 in UK hospitals
- Assume:
  - We want to keep that number constant
  - 5% of Covid 19 patients go to hospital
  - Average hospital stay 1 week
  - 60% of UK population (40 millions) to become infected by the virus
- The epidemic will last : 40,000,000/(20,000*20) = 100 weeks

How can the epidemic end?

- Get rid of the virus:

  + Only a few weeks of confinement

  - Very strict confinement

  - How to prevent second wave?

+ No confinement,

+ Only a few weeks

- Huge number of dead in very short time ?

- Herd immunity:

  + Small number of death

  - Takes several months (long, less strict confinement)

  - Only works if immunity lasts

- Vaccine:

  +Long term solution

  - Not yet available. Will there be one?

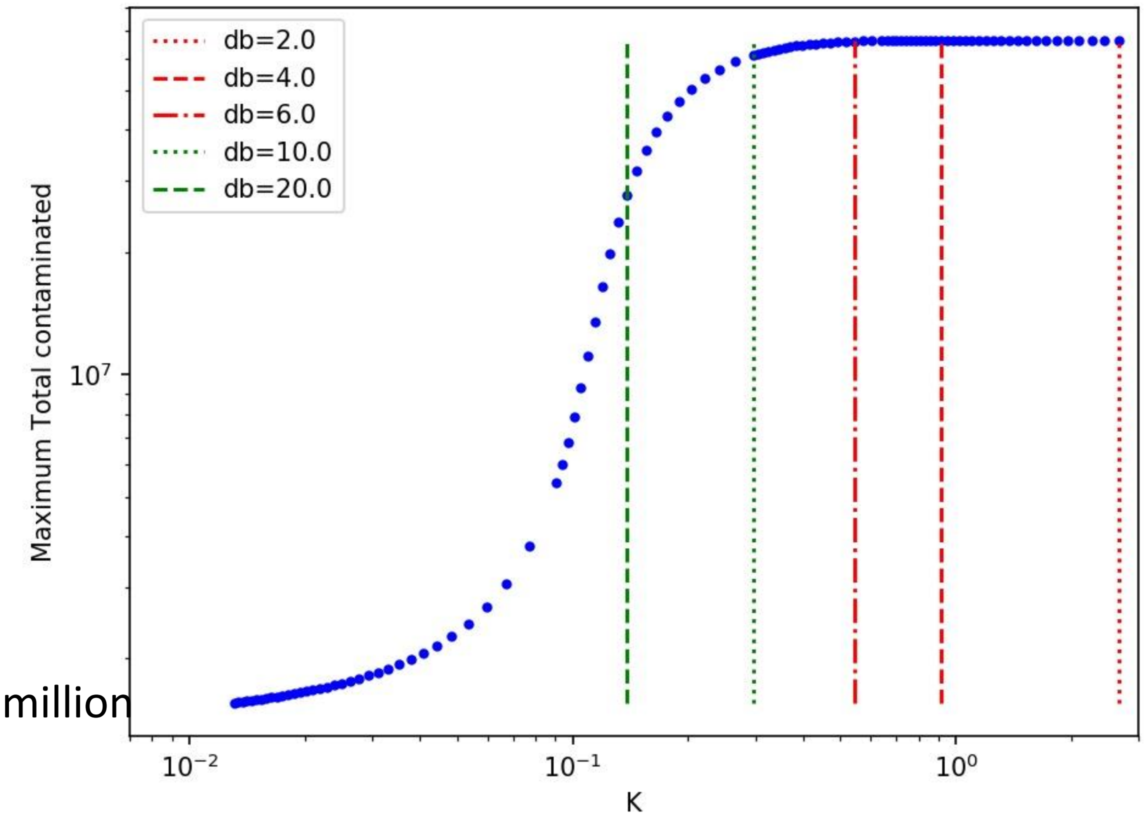
Simulating What Happens Next

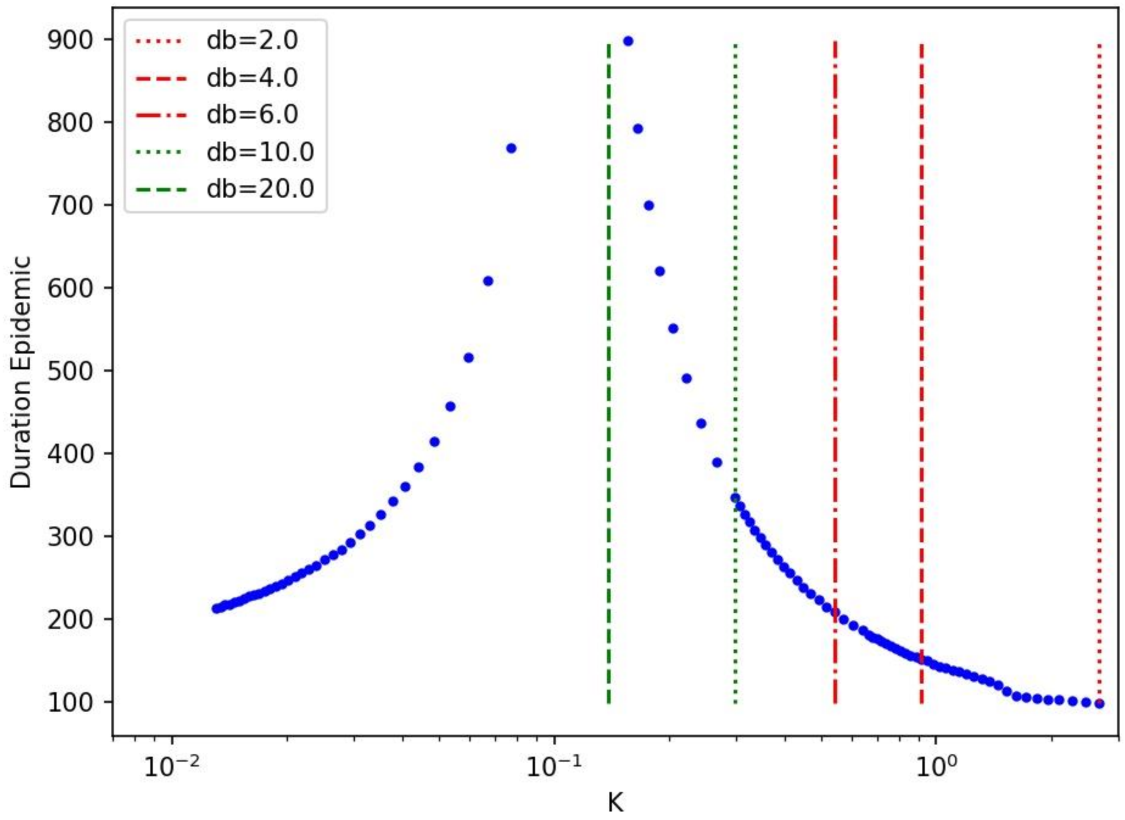
- Simulate Covid 19 in UK
  - $d \in (0,36)$: $Ks = Kw = 2.84$
  - $d \in (37,51)$: $Ks = Kw = 0.026$
  - $d > 51$: vary $Ks = Kw = K$
- Generate graphs as function of K

UK: Ks = Kw constant after lockdown

66 million

1 million

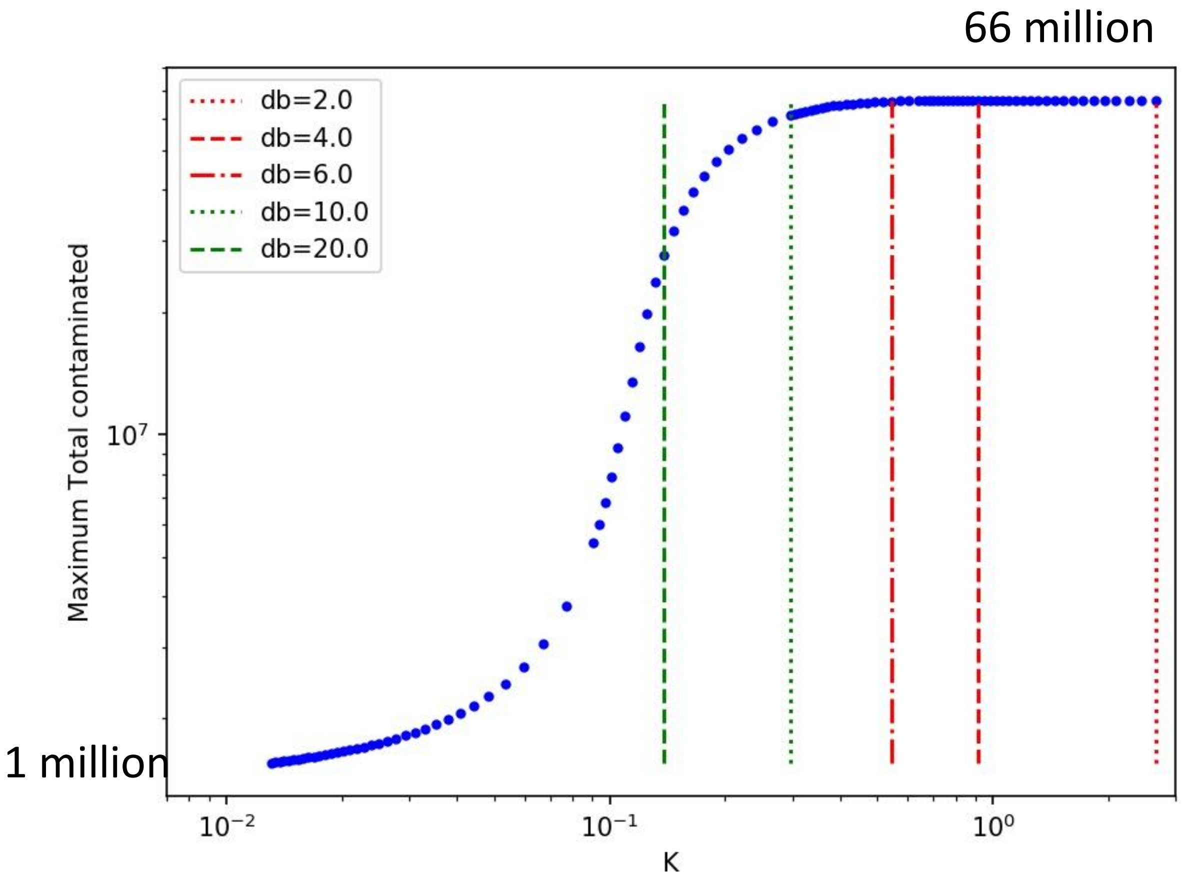




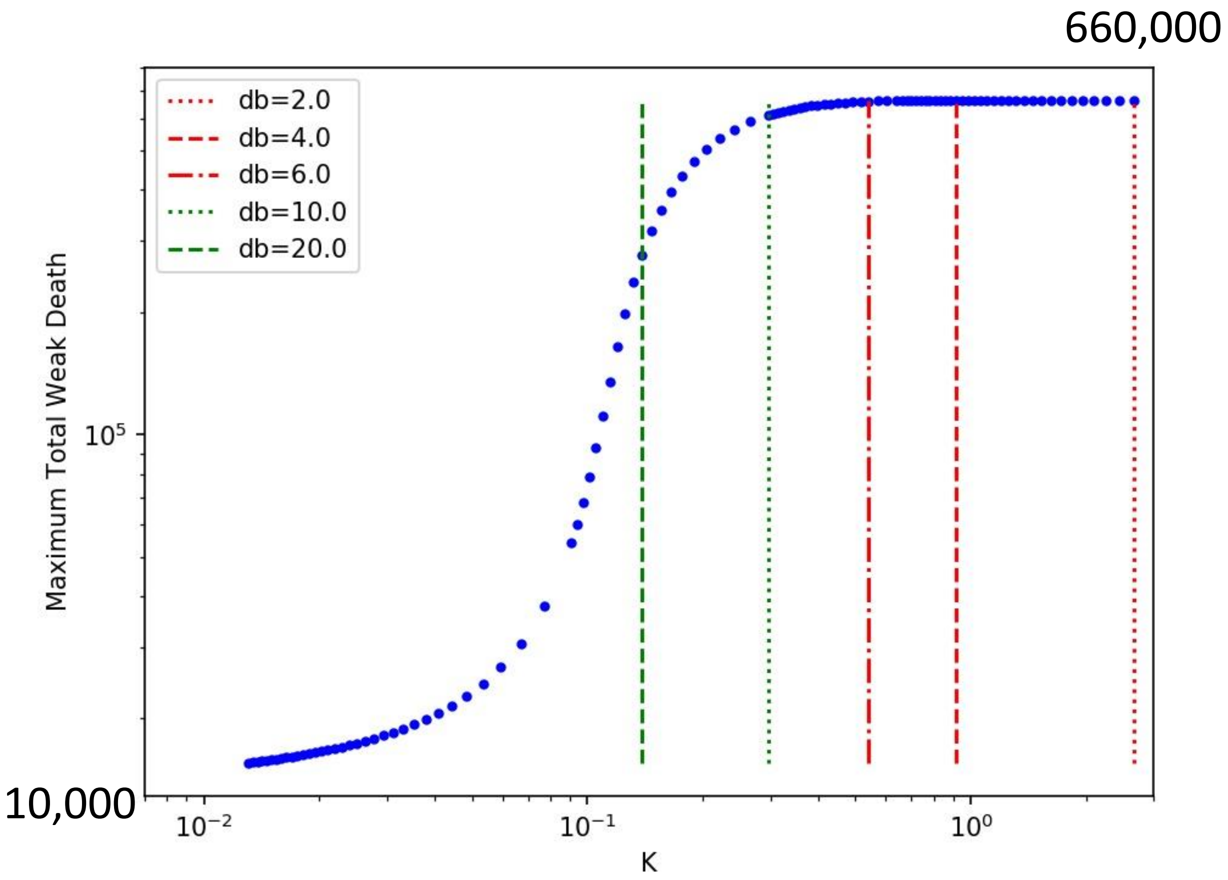
Total contaminated

Epidemic duration

UK: Ks=Kw constant after lockdown

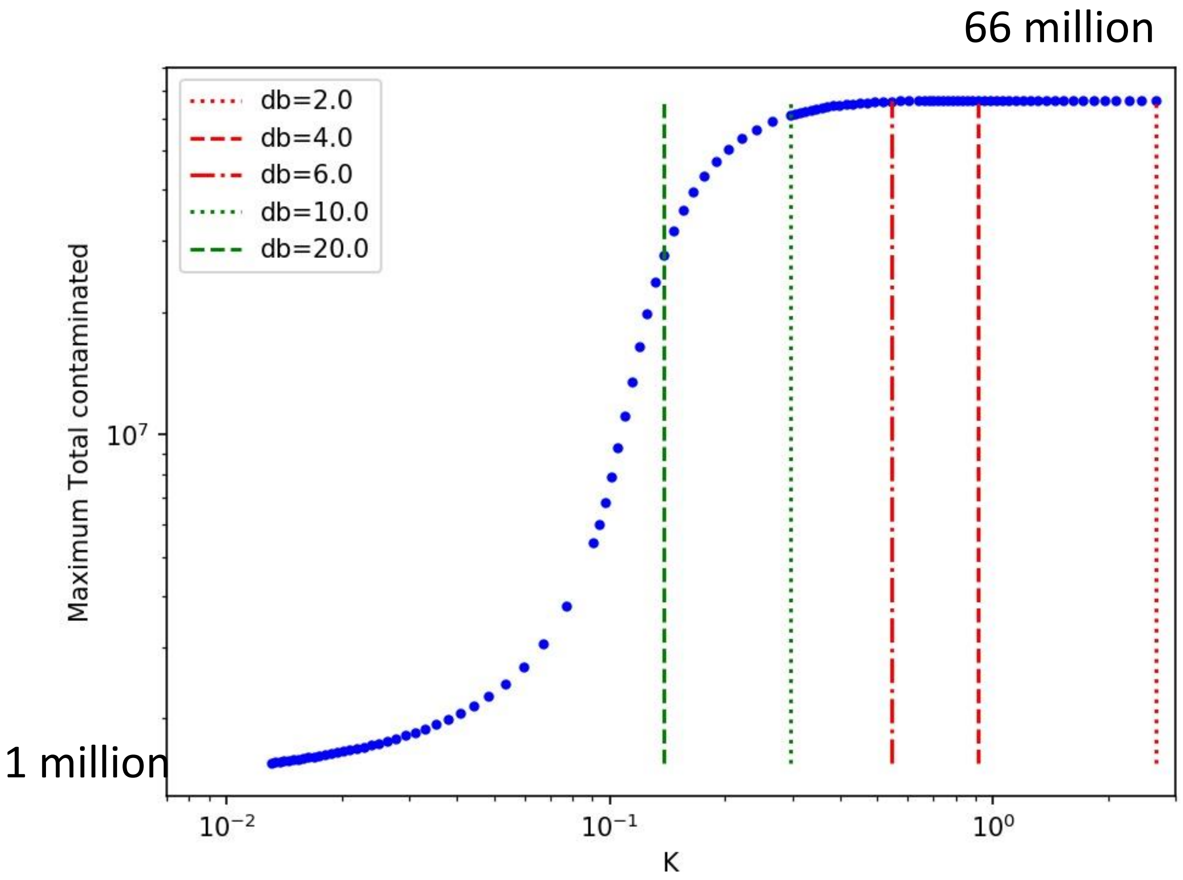


Total contaminated

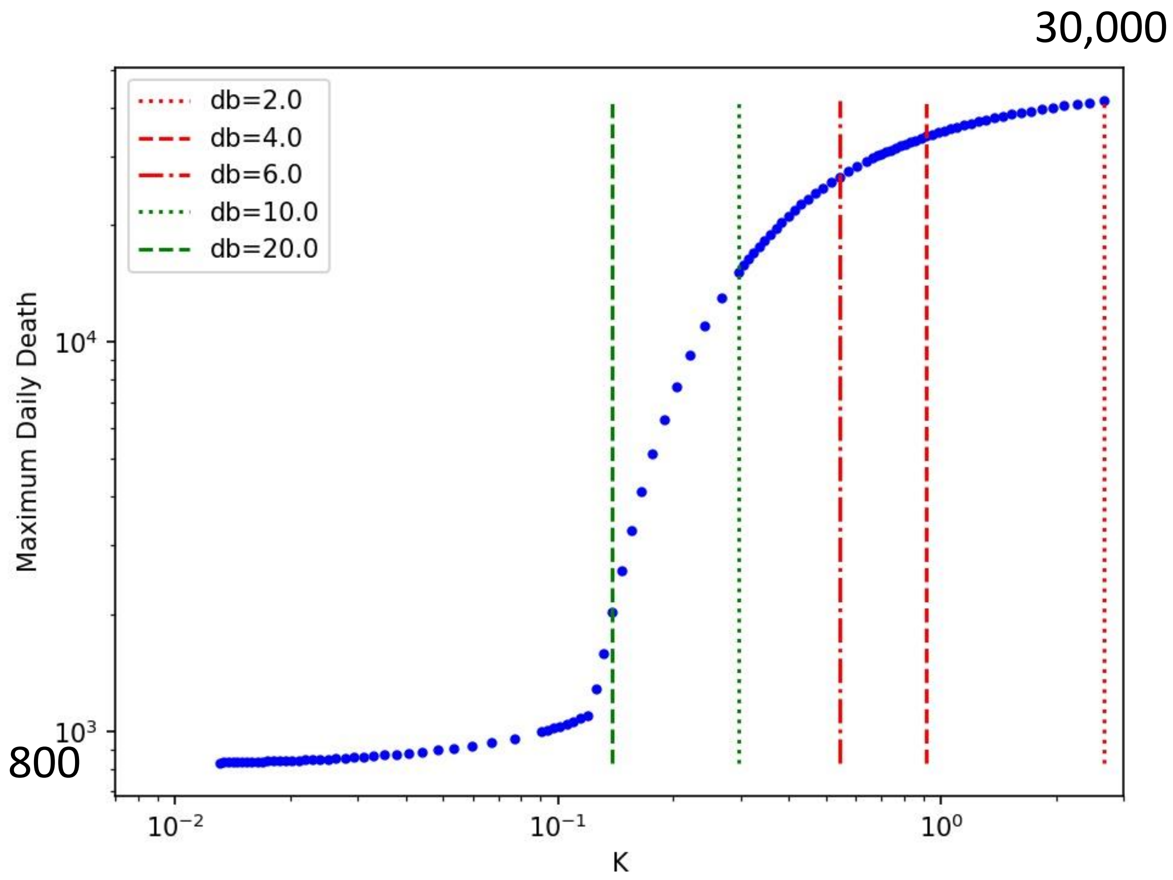


Total number of deaths

UK: Ks=Kw constant after lockdown



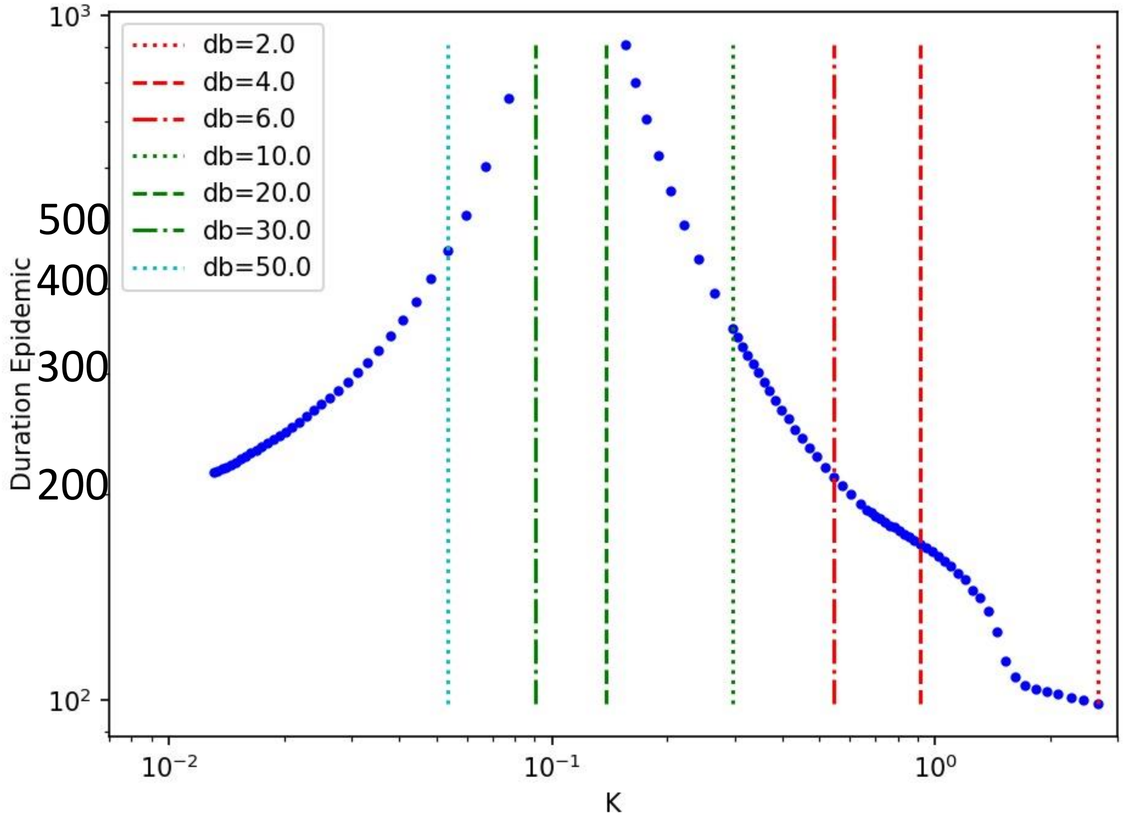
Total contaminated



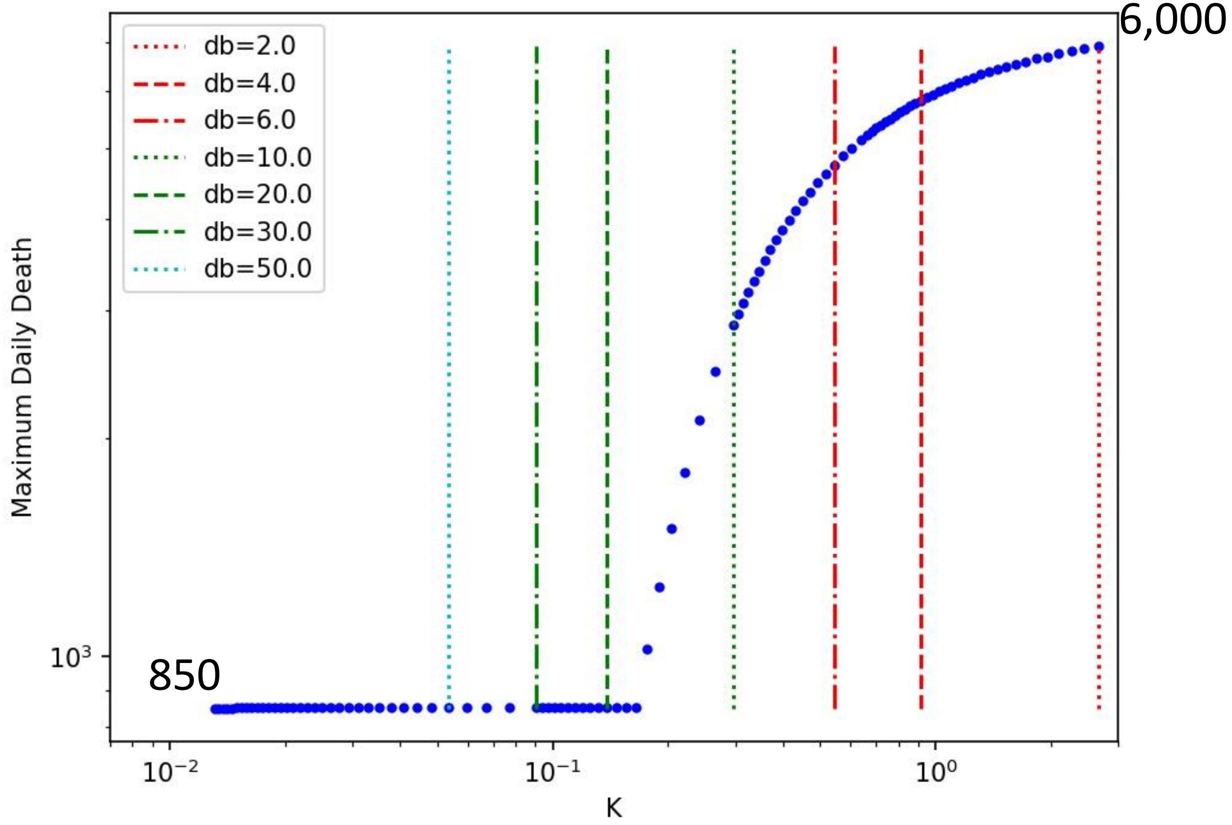
Max number of daily deaths

UK: Stricter confinement for the vulnerable

Kw = 0.025



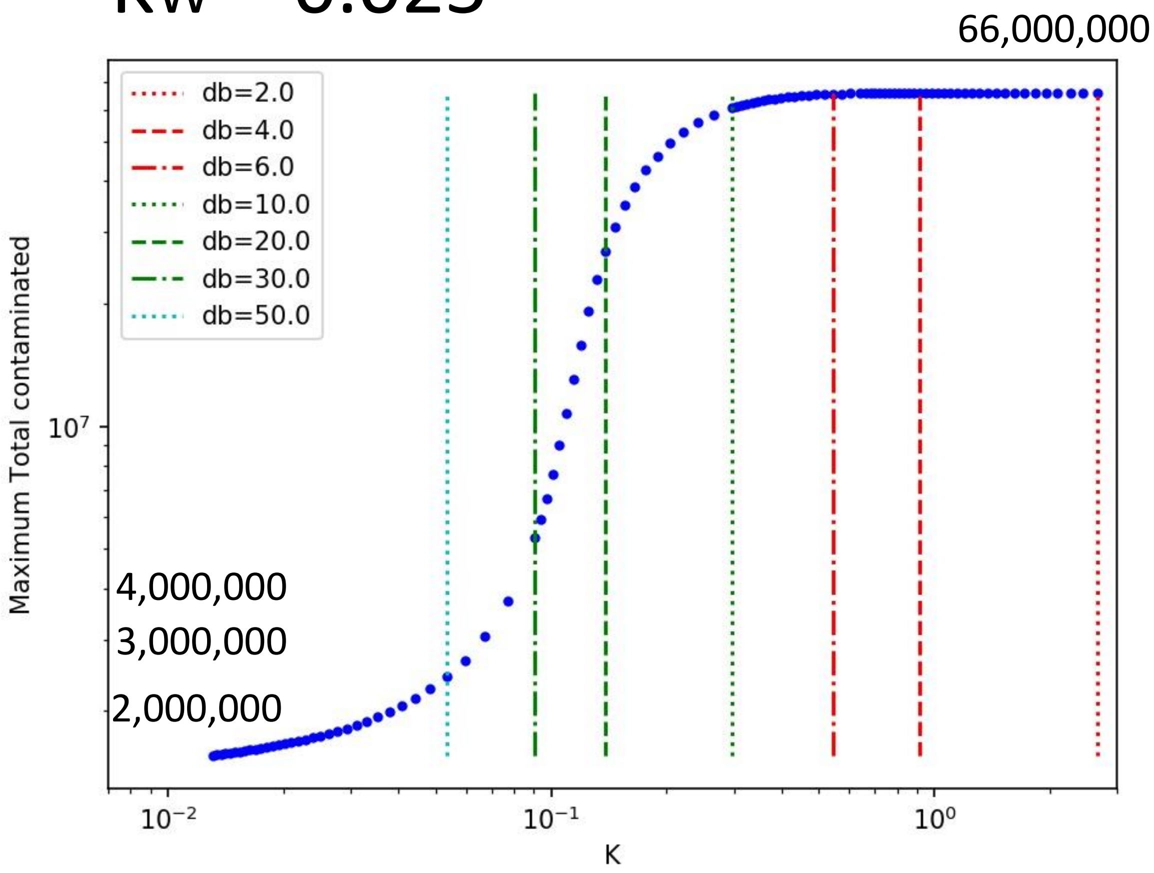
Length of epidemic (days)



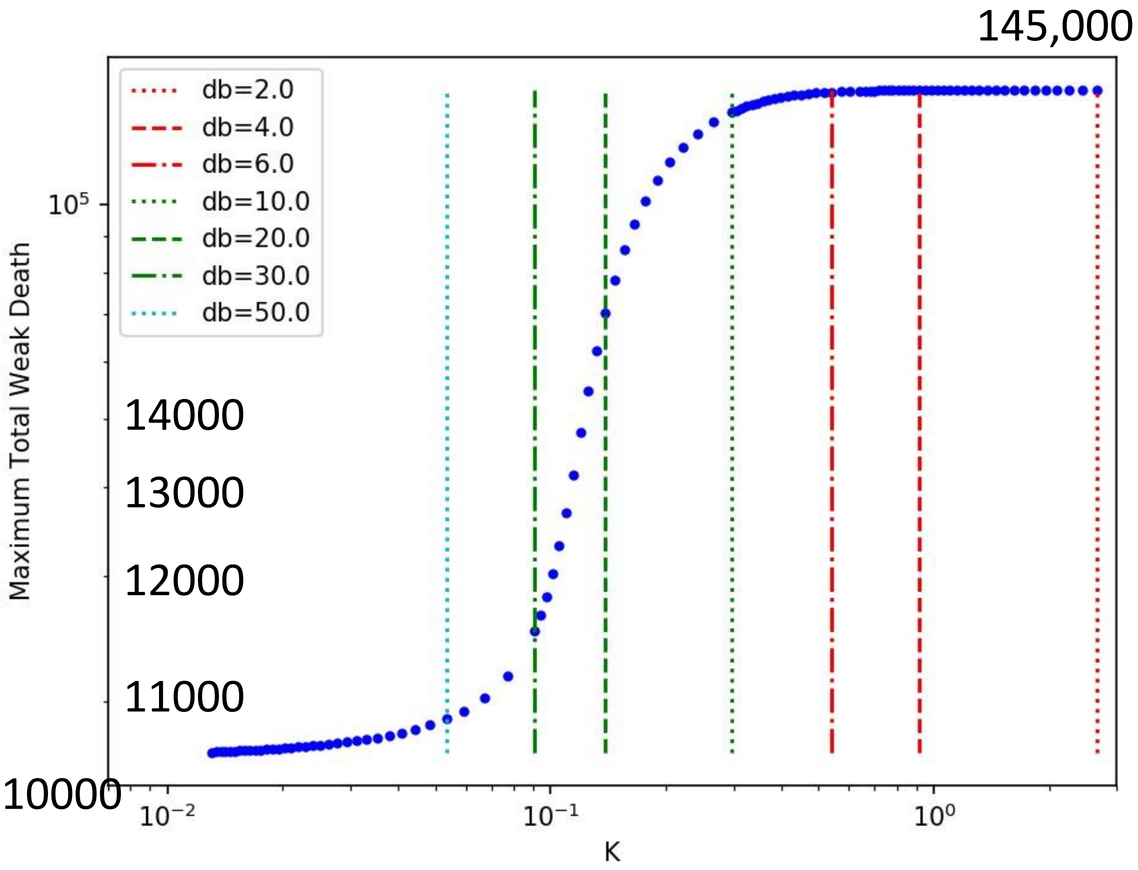
Maximum daily death

UK: Stricter confinement for the vulnerable

Kw = 0.025



Total affected



Total death

Infection Methods

- Direct : when people cough or sneeze near us
- Indirect : spay of cough or sneeze lands on objects

N. van Doremalen et al. **DOI: 10.1056/NEJMc2004973**

- Computed half time of various “spray” on a few surfaces
- Half survival time : Cardboard: 3.5h ; Stainless Steel 6h ; Plastic 7h
- Experiment performed at 22^{o}C and 40% humidity

- Longer half time at lower T higher humidity?
- Problem in fridges and deep freezers?

Other Models

- S Flaxman et al. (Imperial)

https://www.imperial.ac.uk/mrc-global-infectious-disease-analysis/covid-19/report-13-europe-npi-impact/

Assume all confinement effect have same impact in different European countries

- Complete lockdown
- Public event banned
- Closing schools
- Self Isolation
- Social distancing

Infer from data the impact of each measure on Rt

Unfortunately error bars are large

Other Models

Oliver Linton (University of Cambridge)

Predict the peak the epidemic using a quadratic function on log(#cases)

On 2nd April 2020:

"We find that for the UK, this peak is mostly likely to occur within the next two weeks. The total number of cases per day will peak at around 8000 yielding a little more than 255,000 cases in total."

- Number reported cases peaked at 8719 on 12th February 2020
- Total number reported cases on 16th February 2020: 103093

Conclusions

- The finite difference model is simple
- It captures the dynamics into 1 main parameter
- The epidemic is likely to last several months

Look after yourselves!