Top-quark pair production at NNLO: q_T subtraction, differential distributions, and the MS mass

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Based on works in collaboration with S. Catani, S. Devoto, M. Grazzini, S. Kallweit, H. Sargsyan

The top quark

- Heaviest particle in the SM
- Strongest coupling to Higgs boson
- Only quark that decays before hadronization
- Possible window to new physics
- Important background in many searches
- Standard candle at the LHC (triggering, tracking, b-tagging, energy and jet calibration)



The top quark

Example: the top-quark mass

• Via the corrections to the Higgs quartic coupling, the precise value of its mass is crucial for the stability (or not) of the SM vacuum

Higgs quartic coupling λ



- Relation between m_w , m_t and m_H also allows to test the SM

The top quark

More examples: searches in $t\bar{t}$ production

• New resonances



• Dark matter searches





The top quark at the LHC

Event display for a top quark pair production, all-hadronic final state candidate

Event display of a ft candidate event in the 2015 data. The large-R jets (reconstructed using the anti-kt algorithm with radius parameter R=1.0) are shown in blue while the remaining jets are smaller-radius, R=0.4 jets. The jets identified as containing b-hadrons are shown in magenta. The centers of magenta in the top right pad ellipses correspond to secondary vertices. The transverse momenta of the leading and second-leading large-R jets are 961 GeV and 824 GeV, respectively. The dijet invariant mass of the two large-R jets is 3.33 TeV. (Image: ATLAS Collaboration/CERN)



The top quark at the LHC

Main source at the LHC: top-quark pair production



Theoretical status

Precise theoretical predictions are needed to match experimental uncertainty:

NLO QCD

[Nason, Dawson, Ellis; '88], [Mangano, Nason, Ridolfi; '92], [Melnikov, Schulze; 0907.3090], [Bevilacqua et al.; 1012.4230], [Denner et al.; 1012.3975, 1207.5018], [Frederix; 1311.4893], [Cascioli et al.; 1312.0546], [Campbell et al.; 1204.1513, 1608.03356], ...

NLO EW

[Bernreuther et al.; hep-ph/0610335, 0804.1237, 0808.1142], [Kühn et al.; hep-ph/0508092, hep-ph/0610335], [Hollik, Kollar; 0708.1697], [Pagani et al.; 1606.01915]

NNLO QCD

[Moch et al.; 1203.6282], [Czakon et al.; 1303.6254, 1601.05375, 1606.03350], [Abelof et al.; 1506.04037], [Gao, Papanastasiou; 1705.08903], [Catani et al.; 1901.04005], [Catani et al.; 1906.06535]

NNLO QCD + NLO EW

[Czakon et al.; 1705.04105, 1711.03945]

Resummation

[Beneke et al.; 0907.1443], [Czakon et al.; 0907.1790, 1803.07623], [Ahrens et al.; 1003.5827], [Kidonakis; 0903.2561, 1009.4935], [Hu et al.; 1908.02179], [Ju et al.; 1908.02179]...

NLO QCD matched to PS

[Frixione et al.; hep-ph/0305252, 0707.3088], [Höche et al.; 1402.6293], [Garzelli et al.; 1405.5859], [Campbell et al.; 1412.1828], [Ježo et al.; 1607.04538]

QCD corrections for on-shell $\ensuremath{t\bar{t}}$ production

NLO QCD

Total cross section: [Nason, Dawson, Ellis; '88] *Differential distributions:* [Mangano, Nason, Ridolfi; '92]

NNLO QCD

Total cross section: [Czakon, Fiedler, Mitov; 1303.6254] *Differential distributions:* [Czakon, Fiedler, Mitov; 1411.3007] (F-B assymetry), [Czakon, Heymes, Mitov; 1511.00549] (LHC), [Czakon, Fielder, Heymes, Mitov; 1601.05375] (Tevatron)

NEW: $t\bar{t}$ production at NNLO using q_T -subtraction **Focus of this talk**

- Independent check of a very complex calculation
- Public code to generate NNLO distributions available upon request
- Total cross section [Catani, Devoto, Grazzini, Kallweit, JM, Sargsyan; 1901.04005]
- Differential distributions [Catani, Devoto, Grazzini, Kallweit, JM; 1906.06535]
- NEW: NNLO distributions using the MS mass [Catani, Devoto, Grazzini, Kallweit, JM; 2005.00557]

Outline

Introduction

• q_T subtraction and its extension to $t\bar{t}$ production

- Differential results using MATRIX
- Predictions using the $\overline{\text{MS}}$ mass

Conclusions and outlook

IR singularities - NLO



IR singularities - NLO



- Different possible approaches: local/non-local subtraction, slicing parameter, fully analytical, numerical, ...
- Solved problem at NLO: subtraction for arbitrary process well understood

IR singularities - NNLO



Subtraction methods

NLO: solved, Dipole subtraction, FKS, ...

NNLO:

- Antenna [Gehrmann-de Ridder, Gehrmann, Glover '05, ...]
- CoLoRFulNNLO [Somogyi, Trócsányi, Del Duca '05, ...]

• q_T subtraction [Catani, Grazzini '07, ...]

- STRIPPER [Czakon '10, '11]
- Projection-to-Born [Cacciari et al. '15]
- N-jettiness [Gaunt et al. '15; Boughezal et al. '15, ...]
- Nested soft-collinear [Caola, Melnikov, Roentsch '17]
- Geometric [Herzog '18]
- Local analytic sector [Magnea et al. '18]

\mathbf{q}_{T} subtraction

Originally developed for the hadroproduction of colourless final states Catani, Grazzini (2007)

Slicing method, slicing parameter: \mathbf{q}_{T} (transverse momentum of final state *F*)



\mathbf{q}_{T} subtraction

$$d\sigma_{\rm NNLO}^F = \mathcal{H}_{\rm NNLO}^F \otimes d\sigma_{\rm LO}^F + \left[d\sigma_{\rm NLO}^{F+\rm jet} - d\sigma_{\rm NNLO}^{\rm CT} \right]$$

General form of hard-collinear function known at NNLO for colourless F

Implies knowledge of *correct* subtraction operator for virtual corrections

$$H \sim \langle \tilde{\mathcal{M}} | \tilde{\mathcal{M}} \rangle \text{ with } | \tilde{\mathcal{M}} \rangle = \left(1 + \tilde{I} \right) | \mathcal{M} \rangle$$

q_T-subtraction 'solved' for colourless final states

Method can be applied to the production of arbitrary colour singlets at NNLO once the relevant amplitudes are available



The MATRIX project



The MATRIX project



Extension to heavy-quark production

Analogous formula, but with new contributions coming from **final state radiation**

$$d\sigma_{\rm NNLO}^{t\bar{t}} = \mathcal{H}_{\rm NNLO}^{t\bar{t}} \otimes d\sigma_{\rm LO}^{t\bar{t}} + \left[d\sigma_{\rm NLO}^{t\bar{t}+\rm jet} - d\sigma_{\rm NNLO}^{t\bar{t},\rm CT} \right]$$

- Modified subtraction counterterm fully known (ingredient: NNLO soft anomalous dimension Γ_t)
- The structure of the hard-collinear function *H* also changes:



Additional radiative soft factor Δ which includes **colour correlations**

Extension to heavy-quark production

- Specifically, we have to compute $d\sigma/d^2q_T$
- Only new soft singularities → integrate the (subtracted) **soft current**



• After integration the following NLO subtraction operator can be obtained: [Catani, Grazzini, Torre; 1408.4564]



We had to extend the above results to NNLO

Final result - H⁽²⁾

$\tau = 4m^2/s$, $\cos\theta$ scattering angle

- We have recently finished their computation Catani, Devoto, Grazzini, JM (to appear) See also Angeles-Martinez, Czakon, Sapeta (2018)
- Results mostly analytical, numerical integration for some pieces



Final result - H⁽²⁾

$\tau = 4m^2/s$, $\cos\theta$ scattering angle

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First application: NNLO results using MATRIX

Catani, Devoto, Grazzini, Kallweit, JM, Sargsyan, **1901.04005** Catani, Devoto, Grazzini, Kallweit, JM, **1906.06535**

Inclusive cross section



Differential results

We compute single and double differential distributions

We compare our results with recent measurements from CMS in the lepton+jets channel [CMS-TOP-17-002]

CMS measurements are extrapolated to parton level in the inclusive phase space

we carry out our calculation without cuts

Perturbative results depend on the choice of scales μ_F , μ_R which should be chosen of the order of the characteristic hard scale

- Total cross section and rapidity distribution: m_t
- Invariant mass distribution: $m_{\ensuremath{t\bar{t}}}$
- Transverse momentum distribution: m_{T}

The dynamical scale $\mu_0 = H_T/2 = (m_{T,t} + m_{T,t})/2$ is a good approximation to all these scales

Differential results



- Lower scale $H_T/4$ (usually used as a benchmark) seems to lead to underestimation of perturbative uncertainties in certain $m_{t\bar{t}}$ regions
- Good description of data except for first bin ($m_{t\bar{t}}$ <360GeV) Issues in extrapolation? Smaller m_t ? Resummation effects?

Differential results



• Kinematical boundary at LO: $m_{t\bar{t}} > 2 m_{T,min}$

- NLO (NNLO) is effectively LO (NLO) below that threshold \rightarrow larger uncertainties
- NNLO nicely describes the data (except only close to the physical $m_{t\bar{t}}$ threshold)

More distributions in backup slides

NNLO predictions using the \overline{MS} mass

Catani, Devoto, Grazzini, Kallweit, JM, 2005.00557

Top-quark mass renormalization scheme

- The top-quark mass is subject to renormalization, and therefore it suffers from a scheme (and in general a scale) ambiguity
- Results presented so far correspond to the pole scheme —

Pole of the quark propagator is fixed to the same value, the **pole mass** M_t , at any order in perturbation theory

- 'Natural' choice when considering on-shell top quark production
- Still, we can re-express the pole mass in terms of a different mass parameter, e.g. the mass as defined in the **MS scheme**

Pole of the quark propagator receives corrections at any order

The **MS mass** $m_t(\mu_m)$ differs from M_t and depends on arbitrary scale μ_m

Potential advantages of using the \overline{MS} mass:

- The pole mass is affected by a non-perturbative ambiguity of $O(\Lambda_{QCD})$, absent in the \overline{MS} mass
- The use of a dynamic scale for the $\overline{\text{MS}}$ mass can potentially lead to a better theoretical description in certain kinematical regions
- It has been argued that the $\ensuremath{t\bar{t}}$ cross section in the $\overline{\mbox{MS}}$ scheme has a faster convergence

Top-quark production using the $\overline{\mbox{MS}}$ mass

• Top pair production cross section using the MS mass has been considered -

At NLO for differential distributions -Dowling and Moch [1305.6422]

- Based on our MATRIX implementation, we extended the differential predictions to NNLO
- On previous studies the scale of the \overline{MS} mass was fixed to the value $\mu_m = \overline{m}_t = m_t(\overline{m}_t)$ (variations around this central value only evaluated for a few observables)
- No dynamic scales were considered
- We consider (independent) variations of the scale at which the \overline{MS} mass is evaluated
 - Scale variations to estimate perturbative uncertainties
 - \rightarrow Use of dynamic scales for the \overline{MS} mass
- Comparison with recent CMS measurement for the invariant mass distribution

Cross section in pole and $\overline{\text{MS}}$ schemes

• Top mass in the pole scheme related to the \overline{MS} scheme by the perturbative relation:



• Starting from the pole-scheme XS σ we can define the $\overline{\text{MS}}$ XS $\overline{\sigma}$ as

$$\bar{\sigma}(\alpha_{\rm S}(\mu_R), \mu_R, \mu_F; \mu_m, m_t(\mu_m); X) = \sigma(\alpha_{\rm S}(\mu_R), \mu_R, \mu_F; M_t = m_t(\mu_m) \, d(m_t(\mu_m), \mu_m); X)$$

X denotes either a differential cross section *dσ/dX* or a set of acceptance cuts, absent for total XS

> Formally equivalent to the pole-scheme result to all orders, but different if expanded at fixed order in the strong coupling

Cross section in pole and $\overline{\text{MS}}$ schemes

• MS XS can be obtained from the pole-scheme one and its derivatives w.r.t. the mass:

- Calculation of derivatives performed numerically in a bin-by-bin basis
- Obs: the replacement $m \rightarrow m_t(\mu_m)$ also affects the kinematics. E.g. the threshold of the invariant mass distribution is located at $m_{t\bar{t}} = 2m_t(\mu_m)$

Total cross section

- Central scales $\mu_0 = M_t$ and $\mu_0 = \overline{m}_t$ (defined by $m_t(\overline{m}_t) = \overline{m}_t$)
- We use the values $M_t=173.3$ GeV and $\overline{m}_t=163.7$ GeV (related by 3-loop renormalization)
- We vary **all** scales independently (μ_R , μ_F and μ_m)

 $\mu_i = \xi_i \mu_0$ with $\xi_i = \{1/2, 1, 2\}$ and $\mu_i / \mu_j \le 2$ NEW: 15-point variation for DOE scheme

• For $\mu_m = \mu_0$ we find perfect agreement with the results obtained with HATHOR

scheme	pole	$\overline{\mathrm{MS}}$			
variation	7-point	15-point	$\mu_m = \mu_0$	$\mu_{R/F} = \mu_0$	$\mu_{R/F} = \mu_m$
LO (pb)	$478.9 {}^{+29.6\%}_{-21.4\%}$	$625.7 {}^{+29.4\%}_{-21.9\%}$	$^{+29.4\%}_{-21.3\%}$	$^{+24.7\%}_{-21.9\%}$	$^{+1.5\%}_{-1.5\%}$
NLO (pb)	$726.9 \ ^{+11.7\%}_{-11.9\%}$	$826.4 \ ^{+7.6\%}_{-9.7\%}$	$+7.6\% \\ -9.6\%$	$+5.6\% \\ -9.7\%$	$^{+1.2\%}_{-1.2\%}$
NNLO (pb)	$794.0 \ ^{+3.5\%}_{-5.7\%}$	$833.8 \ ^{+0.5\%}_{-3.1\%}$	$^{+0.4\%}_{-2.9\%}$	$^{+0.3\%}_{-3.1\%}$	$^{+0.0\%}_{-0.3\%}$

Total cross section

scheme	pole	MS			
variation	7-point	15-point	$\mu_m = \mu_0$	$\mu_{R/F}=\mu_0$	$\mu_{R/F} = \mu_m$
LO (pb)	$478.9 {}^{+29.6\%}_{-21.4\%}$	$625.7 {}^{+29.4\%}_{-21.9\%}$	$^{+29.4\%}_{-21.3\%}$	$^{+24.7\%}_{-21.9\%}$	$^{+1.5\%}_{-1.5\%}$
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- Faster convergence and smaller uncertainties in the $\overline{\text{MS}}$ scheme for this central scale
- μ_m and μ_R dependence similar in size and in opposite direction

$$\rightarrow \mu_{R} \text{ and } \mu_{F} \text{ only } Similar \text{ uncertainty estimations}$$

$$\rightarrow Almost \text{ no variation when setting } \mu_{R/F} = \mu_{m}$$

Total cross section

• The 'faster convergence' of the $\overline{\text{MS}}$ scheme strongly depends on the scale choice

scheme	pole	$\overline{\mathrm{MS}}$	$\overline{\mathrm{MS}}$	pole
central scale choice	$\mu_{R/F} = M_t$	$\mu_{R/F} = \overline{m}_t$ $\mu_m = \overline{m}_t/2$	$\mu_{R/F} = \overline{m}_t$ $\mu_m = \overline{m}_t$	$\mu_{R/F} = M_t/2$
LO (pb)	478.9	488.9	625.7	619.8
NLO (pb)	726.9	746.4	826.4	811.4
NNLO (pb)	794.0	808.0	833.8	822.4

'Worse convergence' in the \overline{MS} scheme for central scales $\mu_{R/F} = \overline{m}_t$ and $\mu_m = \overline{m}_t/2$

'Better convergence' in the pole scheme for central scales $\mu_{\text{R/F}}=M_t/2$

Differential distributions



- We present distributions for the invariant mass and rapidity of the pair, and the average of the transverse momentum and rapidity of the top and anti-top
- These results are obtained using the central scale $\mu_0 = \overline{m}_t$
- Bands correspond to 15-point variation
- Similar pattern to the one observed for total cross section: large overlap of bands, better convergence than pole scheme with µ₀=Mt
- Difference between schemes largely reduced at NNLO


Invariant mass distribution



- Good perturbative behaviour of the $\overline{\text{MS}}$ result except for the low $m_{t\bar{t}}$ region
- Close to threshold: large *K*-factors and scale uncertainties (larger than pole scheme) associated to large derivatives w.r.t. the mass
- Scale uncertainty dominated by μ_m variation, crucial for a correct assessment of the perturbative uncertainties close to threshold
- Shape differences between $\overline{\text{MS}}$ and pole schemes very small at NNLO

CMS data and running

In 1909.09193 the CMS collaboration performed an analysis using NLO predictions with the \overline{MS} mass

Procedure:

- \bullet Measure the $m_{t\,\bar{t}}$ distribution
- Compare data with NLO prediction computed setting all scales to $\mu = \overline{m}_t$ (including μ_m)
- Extract from each bin a value of \overline{m}_t

At this point 4 values of \overline{m}_t are obtained, they are found to be consistent with each other

- Evolve these 4 values of \overline{m}_t to a characteristic scale of the corresponding bin (μ_k)
- To do so, use the expected evolution dictated by the corresponding RGE
- Compare the evolved values to the expected running

A study of the running mass would crucially benefit from using a dynamic scale in the theory predictions...



CMS data and running



- Difference between constant and dynamic scale reduced at NNLO
- In both cases NNLO improves the agreement with data
- Change between fixed and dynamic scales driven by change in α_s , effect from changing $m_t(\overline{m}_t)$ to $m_t(\mu_k/2)$ rather small \rightarrow not much sensitivity to running effects
- In the tail resummation effects can be relevant (large logs of $m_{t\bar{t}}/m_t$)

Summary

- We have presented a new computation of top-quark pair production at NNLO
- First complete application of q_{T} subtraction to colourful final states at NNLO
- Calculation fully implemented within the **MATRIX** framework, available upon request
- We are able to evaluate arbitrary IR safe observables for stable top quarks
 - multi-differential distributions
 - cross sections with cuts in the top quarks and jets kinematics
- NNLO differential distributions in 1000-2000 CPU days
- Nice description of parton level CMS data

Summary

- We presented the **first NNLO differential results** for t \overline{t} production using the \overline{MS} mass
- We extend the usual 7-point to a **15-point scale variation** to include independent variations of the scale at which the MS mass is evaluated
- Variations of this scale are crucial for a reliable estimation of the perturbative uncertainties close to the $m_{t\bar{t}}$ threshold
- We observe an excellent perturbative convergence and large reduction of scale uncertainties at NNLO
- We consider the use of **dynamic scales** for the MS mass and compare to CMS data for the invariant mass distribution
- The inclusion of NNLO corrections substantially improves the agreement with data

Thanks!

Backup slides

Single-differential distributions



Catani, Devoto, Grazzini, Kallweit, JM (2019)

Good perturbative behaviour, large
overlap between NLO and NNLO bands

• As noted in previous analysis the measured p_{τ} is slightly softer than the NNLO prediciton

• Data and theory consistent within uncertainties

Single-differential distributions



- Higher order corrections have a larger effect on the shape
- Low $p_T(t_{high})$ region: FO instabilities associated with low $p_T(t\bar{t})$
- Good agreement with data

Double-differential distributions



- Again some discrepancy in the low $m_{t\bar{t}}$ region, smaller effect due to larger bin size
- Impact of radiative corrections relatively uniform in both variables

Double-differential distributions

New: predictions for parton level CMS measurements using fully leptonic final state [CMS-TOP-18-004]



- Similar features in this decay channel (note these are normalized distributions)
- Using fitted top mass by CMS (170.81GeV) leads to a better agreement with data

Double-differential distributions



- \bullet As for single differential distribution, $p_{\scriptscriptstyle T}$ data softer than NNLO
- This feature holds in all the rapidity intervals

Comparison to existing results



Excellent agreement even in extreme kinematical regions

Comparison to existing results



Excellent agreement even in extreme kinematical regions