# Top-quark pair production at NNLO: q<sub>T</sub> subtraction, differential distributions, and the MS mass

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Based on works in collaboration with S. Catani, S. Devoto, M. Grazzini, S. Kallweit, H. Sargsyan

# The top quark

- Heaviest particle in the SM
- Strongest coupling to Higgs boson
- Only quark that decays before hadronization
- Possible window to new physics
- Important background in many searches
- Standard candle at the LHC (triggering, tracking, b-tagging, energy and jet calibration)



# The top quark

Example: the top-quark mass

• Via the corrections to the Higgs quartic coupling, the precise value of its mass is crucial for the stability (or not) of the SM vacuum

Higgs quartic coupling  $\lambda$ 



- Relation between  $m_w$ ,  $m_t$  and  $m_H$  also allows to test the SM

# The top quark

#### More examples: searches in $t\bar{t}$ production

• New resonances



• Dark matter searches





#### The top quark at the LHC

Event display for a top quark pair production, all-hadronic final state candidate

Event display of a ft candidate event in the 2015 data. The large-R jets (reconstructed using the anti-kt algorithm with radius parameter R=1.0) are shown in blue while the remaining jets are smaller-radius, R=0.4 jets. The jets identified as containing b-hadrons are shown in magenta. The centers of magenta in the top right pad ellipses correspond to secondary vertices. The transverse momenta of the leading and second-leading large-R jets are 961 GeV and 824 GeV, respectively. The dijet invariant mass of the two large-R jets is 3.33 TeV. (Image: ATLAS Collaboration/CERN)



## The top quark at the LHC

Main source at the LHC: top-quark pair production



### **Theoretical status**

Precise theoretical predictions are needed to match experimental uncertainty:

#### NLO QCD

[Nason, Dawson, Ellis; '88], [Mangano, Nason, Ridolfi; '92], [Melnikov, Schulze; 0907.3090], [Bevilacqua et al.; 1012.4230], [Denner et al.; 1012.3975, 1207.5018], [Frederix; 1311.4893], [Cascioli et al.; 1312.0546], [Campbell et al.; 1204.1513, 1608.03356], ...

#### **NLO EW**

[Bernreuther et al.; hep-ph/0610335, 0804.1237, 0808.1142], [Kühn et al.; hep-ph/0508092, hep-ph/0610335], [Hollik, Kollar; 0708.1697], [Pagani et al.; 1606.01915]

#### **NNLO QCD**

[Moch et al.; 1203.6282], [Czakon et al.; 1303.6254, 1601.05375, 1606.03350], [Abelof et al.; 1506.04037], [Gao, Papanastasiou; 1705.08903], [Catani et al.; 1901.04005], [Catani et al.; 1906.06535]

#### NNLO QCD + NLO EW

[Czakon et al.; 1705.04105, 1711.03945]

#### **Resummation**

[Beneke et al.; 0907.1443], [Czakon et al.; 0907.1790, 1803.07623], [Ahrens et al.; 1003.5827], [Kidonakis; 0903.2561, 1009.4935], [Hu et al.; 1908.02179], [Ju et al.; 1908.02179]...

#### **NLO QCD matched to PS**

[Frixione et al.; hep-ph/0305252, 0707.3088], [Höche et al.; 1402.6293], [Garzelli et al.; 1405.5859], [Campbell et al.; 1412.1828], [Ježo et al.; 1607.04538]

# QCD corrections for on-shell $t\bar{t}$ production

#### NLO QCD

*Total cross section:* [Nason, Dawson, Ellis; '88] *Differential distributions:* [Mangano, Nason, Ridolfi; '92]

#### NNLO QCD

*Total cross section:* [Czakon, Fiedler, Mitov; 1303.6254] *Differential distributions:* [Czakon, Fiedler, Mitov; 1411.3007] (F-B assymetry), [Czakon, Heymes, Mitov; 1511.00549] (LHC), [Czakon, Fielder, Heymes, Mitov; 1601.05375] (Tevatron)

**NEW:**  $t\bar{t}$  production at NNLO using  $q_T$ -subtraction **Focus of this talk** 

- Independent check of a very complex calculation
- Public code to generate NNLO distributions available upon request
- Total cross section [Catani, Devoto, Grazzini, Kallweit, JM, Sargsyan; 1901.04005]
- Differential distributions [Catani, Devoto, Grazzini, Kallweit, JM; 1906.06535]
- NEW: NNLO distributions using the MS mass [Catani, Devoto, Grazzini, Kallweit, JM; 2005.00557]

### Outline

Introduction

•  $q_T$  subtraction and its extension to  $t\bar{t}$  production

- Differential results using MATRIX
- Predictions using the  $\overline{\text{MS}}$  mass

Conclusions and outlook

# **IR singularities - NLO**



# **IR singularities - NLO**



- Different possible approaches: local/non-local subtraction, slicing parameter, fully analytical, numerical, ...
- Solved problem at NLO: subtraction for arbitrary process well understood

# **IR singularities - NNLO**



# **Subtraction methods**

NLO: solved, Dipole subtraction, FKS, ...

NNLO:

- Antenna [Gehrmann-de Ridder, Gehrmann, Glover '05, ...]
- CoLoRFulNNLO [Somogyi, Trócsányi, Del Duca '05, ...]

•  $q_T$  subtraction [Catani, Grazzini '07, ...]

- STRIPPER [Czakon '10, '11]
- Projection-to-Born [Cacciari et al. '15]
- N-jettiness [Gaunt et al. '15; Boughezal et al. '15, ...]
- Nested soft-collinear [Caola, Melnikov, Roentsch '17]
- Geometric [Herzog '18]
- Local analytic sector [Magnea et al. '18]

#### $\mathbf{q}_{\mathrm{T}}$ subtraction

Originally developed for the hadroproduction of colourless final states Catani, Grazzini (2007)

Slicing method, slicing parameter:  $\mathbf{q}_{T}$  (transverse momentum of final state *F*)



#### $\mathbf{q}_{\mathrm{T}}$ subtraction

$$d\sigma_{\rm NNLO}^F = \mathcal{H}_{\rm NNLO}^F \otimes d\sigma_{\rm LO}^F + \left[ d\sigma_{\rm NLO}^{F+\rm jet} - d\sigma_{\rm NNLO}^{\rm CT} \right]$$

General form of hard-collinear function known at NNLO for colourless F

Implies knowledge of *correct* subtraction operator for virtual corrections

$$H \sim \langle \tilde{\mathcal{M}} | \tilde{\mathcal{M}} \rangle \text{ with } | \tilde{\mathcal{M}} \rangle = \left( 1 + \tilde{I} \right) | \mathcal{M} \rangle$$

q<sub>T</sub>-subtraction 'solved' for colourless final states

Method can be applied to the production of arbitrary colour singlets at NNLO once the relevant amplitudes are available



#### **The MATRIX project**



#### The MATRIX project



### **Extension to heavy-quark production**

Analogous formula, but with new contributions coming from **final state radiation** 

$$d\sigma_{\rm NNLO}^{t\bar{t}} = \mathcal{H}_{\rm NNLO}^{t\bar{t}} \otimes d\sigma_{\rm LO}^{t\bar{t}} + \left[ d\sigma_{\rm NLO}^{t\bar{t}+\rm jet} - d\sigma_{\rm NNLO}^{t\bar{t},\rm CT} \right]$$

- Modified subtraction counterterm fully known (ingredient: NNLO soft anomalous dimension Γ<sub>t</sub>)
- The structure of the hard-collinear function *H* also changes:



Additional radiative soft factor  $\Delta$  which includes **colour correlations** 

#### **Extension to heavy-quark production**

- Specifically, we have to compute  $d\sigma/d^2q_T$
- Only new soft singularities → integrate the (subtracted) **soft current**



• After integration the following NLO subtraction operator can be obtained: [Catani, Grazzini, Torre; 1408.4564]



We had to extend the above results to NNLO

# Final result - H<sup>(2)</sup>

#### $\tau = 4m^2/s$ , $\cos\theta$ scattering angle

- We have recently finished their computation Catani, Devoto, Grazzini, JM (to appear) See also Angeles-Martinez, Czakon, Sapeta (2018)
- Results mostly analytical, numerical integration for some pieces



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#### **First application: NNLO results using MATRIX**

Catani, Devoto, Grazzini, Kallweit, JM, Sargsyan, **1901.04005** Catani, Devoto, Grazzini, Kallweit, JM, **1906.06535** 

### **Inclusive cross section**



### **Differential results**

We compute single and double differential distributions

We compare our results with recent measurements from CMS in the lepton+jets channel [CMS-TOP-17-002]

CMS measurements are extrapolated to parton level in the inclusive phase space

we carry out our calculation without cuts

Perturbative results depend on the choice of scales  $\mu_F$ ,  $\mu_R$  which should be chosen of the order of the characteristic hard scale

- Total cross section and rapidity distribution: m<sub>t</sub>
- Invariant mass distribution:  $m_{\ensuremath{t\bar{t}}}$
- Transverse momentum distribution:  $m_{T}$

The dynamical scale  $\mu_0 = H_T/2 = (m_{T,t} + m_{T,t})/2$  is a good approximation to all these scales

#### **Differential results**



- Lower scale  $H_T/4$  (usually used as a benchmark) seems to lead to underestimation of perturbative uncertainties in certain  $m_{t\bar{t}}$  regions
- Good description of data except for first bin (m<sub>tī</sub><360GeV) Issues in extrapolation? Smaller m<sub>t</sub>? Resummation effects?

#### **Differential results**



• Kinematical boundary at LO:  $m_{t\bar{t}} > 2 m_{T,min}$ 

- NLO (NNLO) is effectively LO (NLO) below that threshold  $\rightarrow$  larger uncertainties
- NNLO nicely describes the data (except only close to the physical  $m_{t\bar{t}}$  threshold)

More distributions in backup slides

# NNLO predictions using the $\overline{MS}$ mass

Catani, Devoto, Grazzini, Kallweit, JM, 2005.00557

# **Top-quark mass renormalization scheme**

- The top-quark mass is subject to renormalization, and therefore it suffers from a scheme (and in general a scale) ambiguity
- Results presented so far correspond to the pole scheme —

Pole of the quark propagator is fixed to the same value, the **pole mass**  $M_t$ , at any order in perturbation theory

- 'Natural' choice when considering on-shell top quark production
- Still, we can re-express the pole mass in terms of a different mass parameter, e.g. the mass as defined in the **MS scheme**

Pole of the quark propagator receives corrections at any order

The **MS mass**  $m_t(\mu_m)$  differs from  $M_t$  and depends on arbitrary scale  $\mu_m$ 

Potential advantages of using the  $\overline{MS}$  mass:

- The pole mass is affected by a non-perturbative ambiguity of  $O(\Lambda_{QCD})$ , absent in the  $\overline{MS}$  mass
- The use of a dynamic scale for the  $\overline{\text{MS}}$  mass can potentially lead to a better theoretical description in certain kinematical regions
- It has been argued that the  $\ensuremath{t\bar{t}}$  cross section in the  $\overline{\mbox{MS}}$  scheme has a faster convergence

# Top-quark production using the $\overline{\mbox{MS}}$ mass

• Top pair production cross section using the MS mass has been considered -

At NLO for differential distributions -Dowling and Moch [1305.6422]

- Based on our MATRIX implementation, we extended the differential predictions to NNLO
- On previous studies the scale of the  $\overline{MS}$  mass was fixed to the value  $\mu_m = \overline{m}_t = m_t(\overline{m}_t)$  (variations around this central value only evaluated for a few observables)
- No dynamic scales were considered
- We consider (independent) variations of the scale at which the  $\overline{MS}$  mass is evaluated
  - Scale variations to estimate perturbative uncertainties
  - $\rightarrow$  Use of dynamic scales for the  $\overline{MS}$  mass
- Comparison with recent CMS measurement for the invariant mass distribution

# Cross section in pole and $\overline{\text{MS}}$ schemes

• Top mass in the pole scheme related to the  $\overline{MS}$  scheme by the perturbative relation:



• Starting from the pole-scheme XS  $\sigma$  we can define the  $\overline{\text{MS}}$  XS  $\overline{\sigma}$  as

$$\bar{\sigma}(\alpha_{\rm S}(\mu_R), \mu_R, \mu_F; \mu_m, m_t(\mu_m); X) = \sigma(\alpha_{\rm S}(\mu_R), \mu_R, \mu_F; M_t = m_t(\mu_m) \, d(m_t(\mu_m), \mu_m); X)$$

X denotes either a differential cross section *dσ/dX* or a set of acceptance cuts, absent for total XS

> Formally equivalent to the pole-scheme result to all orders, but different if expanded at fixed order in the strong coupling

### Cross section in pole and $\overline{\text{MS}}$ schemes

• MS XS can be obtained from the pole-scheme one and its derivatives w.r.t. the mass:

- Calculation of derivatives performed numerically in a bin-by-bin basis
- Obs: the replacement  $m \rightarrow m_t(\mu_m)$  also affects the kinematics. E.g. the threshold of the invariant mass distribution is located at  $m_{t\bar{t}} = 2m_t(\mu_m)$

### **Total cross section**

- Central scales  $\mu_0 = M_t$  and  $\mu_0 = \overline{m}_t$  (defined by  $m_t(\overline{m}_t) = \overline{m}_t$ )
- We use the values  $M_t=173.3$ GeV and  $\overline{m}_t=163.7$ GeV (related by 3-loop renormalization)
- We vary **all** scales independently ( $\mu_R$ ,  $\mu_F$  and  $\mu_m$ )

 $\mu_i = \xi_i \mu_0$  with  $\xi_i = \{1/2, 1, 2\}$  and  $\mu_i / \mu_j \le 2$  NEW: 15-point variation for DOE scheme

• For  $\mu_m = \mu_0$  we find perfect agreement with the results obtained with HATHOR

scheme	pole	$\overline{\mathrm{MS}}$			
variation	7-point	15-point	$\mu_m = \mu_0$	$\mu_{R/F} = \mu_0$	$\mu_{R/F} = \mu_m$
LO (pb)	$478.9  {}^{+29.6\%}_{-21.4\%}$	$625.7 \ ^{+29.4\%}_{-21.9\%}$	$^{+29.4\%}_{-21.3\%}$	$^{+24.7\%}_{-21.9\%}$	$^{+1.5\%}_{-1.5\%}$
NLO (pb)	$726.9\ ^{+11.7\%}_{-11.9\%}$	$826.4 \ ^{+7.6\%}_{-9.7\%}$	$+7.6\% \\ -9.6\%$	$+5.6\% \\ -9.7\%$	$^{+1.2\%}_{-1.2\%}$
NNLO (pb)	$794.0\ ^{+3.5\%}_{-5.7\%}$	$833.8 \ ^{+0.5\%}_{-3.1\%}$	$^{+0.4\%}_{-2.9\%}$	$^{+0.3\%}_{-3.1\%}$	$^{+0.0\%}_{-0.3\%}$

# **Total cross section**

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- Faster convergence and smaller uncertainties in the  $\overline{\text{MS}}$  scheme for this central scale
- $\mu_m$  and  $\mu_R$  dependence similar in size and in opposite direction

$$\rightarrow \mu_{R} \text{ and } \mu_{F} \text{ only } \text{Similar uncertainty estimations}$$

$$\rightarrow \text{ Almost no variation when setting } \mu_{R/F} = \mu_{m}$$

# **Total cross section**

• The 'faster convergence' of the  $\overline{\text{MS}}$  scheme strongly depends on the scale choice

scheme	pole	$\overline{\mathrm{MS}}$	$\overline{\mathrm{MS}}$	pole
central scale choice	$\mu_{R/F} = M_t$	$\mu_{R/F} = \overline{m}_t$ $\mu_m = \overline{m}_t/2$	$\mu_{R/F} = \overline{m}_t$ $\mu_m = \overline{m}_t$	$\mu_{R/F} = M_t/2$
LO (pb)	478.9	488.9	625.7	619.8
NLO (pb)	726.9	746.4	826.4	811.4
NNLO (pb)	794.0	808.0	833.8	822.4

'Worse convergence' in the  $\overline{MS}$  scheme for central scales  $\mu_{R/F} = \overline{m}_t$  and  $\mu_m = \overline{m}_t/2$ 

'Better convergence' in the pole scheme for central scales  $\mu_{\text{R/F}}=M_t/2$ 

# **Differential distributions**



- We present distributions for the invariant mass and rapidity of the pair, and the average of the transverse momentum and rapidity of the top and anti-top
- These results are obtained using the central scale  $\mu_0 = \overline{m}_t$
- Bands correspond to 15-point variation
- Similar pattern to the one observed for total cross section: large overlap of bands, better convergence than pole scheme with  $\mu_0=M_t$
- Difference between schemes largely reduced at NNLO



# **Invariant mass distribution**



- Good perturbative behaviour of the  $\overline{\text{MS}}$  result except for the low  $m_{t\bar{t}}$  region
- Close to threshold: large *K*-factors and scale uncertainties (larger than pole scheme) associated to large derivatives w.r.t. the mass
- Scale uncertainty dominated by  $\mu_m$  variation, crucial for a correct assessment of the perturbative uncertainties close to threshold
- Shape differences between  $\overline{\text{MS}}$  and pole schemes very small at NNLO

# **CMS data and running**

In 1909.09193 the CMS collaboration performed an analysis using NLO predictions with the  $\overline{MS}$  mass

#### **Procedure:**

- $\bullet$  Measure the  $m_{t\,\bar{t}}$  distribution
- Compare data with NLO prediction computed setting all scales to  $\mu = \overline{m}_t$  (including  $\mu_m$ )
- Extract from each bin a value of  $\overline{m}_t$

At this point 4 values of  $\overline{m}_t$  are obtained, they are found to be consistent with each other

- Evolve these 4 values of  $\overline{m}_t$  to a characteristic scale of the corresponding bin ( $\mu_k$ )
- To do so, use the expected evolution dictated by the corresponding RGE
- Compare the evolved values to the expected running

A study of the running mass would crucially benefit from using a dynamic scale in the theory predictions...



# **CMS data and running**



- Difference between constant and dynamic scale reduced at NNLO
- In both cases NNLO improves the agreement with data
- Change between fixed and dynamic scales driven by change in  $\alpha_s$ , effect from changing  $m_t(\overline{m}_t)$  to  $m_t(\mu_k/2)$  rather small  $\rightarrow$  not much sensitivity to running effects
- In the tail resummation effects can be relevant (large logs of  $m_{t\bar{t}}/m_t$ )

### Summary

- We have presented a new computation of top-quark pair production at NNLO
- First complete application of  $q_{T}$  subtraction to colourful final states at NNLO
- Calculation fully implemented within the **MATRIX** framework, available upon request
- We are able to evaluate arbitrary IR safe observables for stable top quarks
  - multi-differential distributions
  - cross sections with cuts in the top quarks and jets kinematics
- NNLO differential distributions in 1000-2000 CPU days
- Nice description of parton level CMS data

### Summary

- We presented the **first NNLO differential results** for t  $\overline{t}$  production using the  $\overline{MS}$  mass
- We extend the usual 7-point to a **15-point scale variation** to include independent variations of the scale at which the MS mass is evaluated
- Variations of this scale are crucial for a reliable estimation of the perturbative uncertainties close to the  $m_{t\bar{t}}$  threshold
- We observe an excellent perturbative convergence and large reduction of scale uncertainties at NNLO
- We consider the use of **dynamic scales** for the MS mass and compare to CMS data for the invariant mass distribution
- The inclusion of NNLO corrections substantially improves the agreement with data

#### Thanks!

#### **Backup slides**

#### **Single-differential distributions**



Catani, Devoto, Grazzini, Kallweit, JM (2019)

Good perturbative behaviour, large
 overlap between NLO and NNLO bands

• As noted in previous analysis the measured  $p_{\tau}$  is slightly softer than the NNLO prediciton

• Data and theory consistent within uncertainties

#### **Single-differential distributions**



- Higher order corrections have a larger effect on the shape
- Low  $p_T(t_{high})$  region: FO instabilities associated with low  $p_T(t\bar{t})$
- Good agreement with data

#### **Double-differential distributions**



- Again some discrepancy in the low  $m_{t\bar{t}}$  region, smaller effect due to larger bin size
- Impact of radiative corrections relatively uniform in both variables

#### **Double-differential distributions**

### **New:** predictions for parton level CMS measurements using fully leptonic final state [CMS-TOP-18-004]



- Similar features in this decay channel (note these are normalized distributions)
- Using fitted top mass by CMS (170.81GeV) leads to a better agreement with data

#### **Double-differential distributions**



- $\bullet$  As for single differential distribution,  $p_{\scriptscriptstyle T}$  data softer than NNLO
- This feature holds in all the rapidity intervals

# **Comparison to existing results**



Excellent agreement even in extreme kinematical regions

# **Comparison to existing results**



Excellent agreement even in extreme kinematical regions