Hadronic top quark pair production near threshold

Wan-Li Ju

IPPP, Durham University

in collaboration with
Guoxing Wang, Xing Wang, Xiaofeng Xu, Yongqi Xu and Li Lin Yang

June 12, 2020

Overview

Outline

Experimental measurements

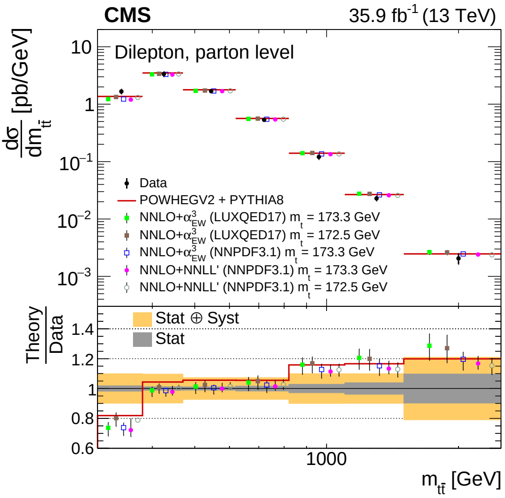
- In the recent years, a number of experimental measurements has been carried out on the top pair production at LHC
  1. Differential cross sections, $d\sigma/dM_{t\bar{t}}$, $d\sigma/dQ_{t\bar{t}}$, $d\sigma/dY_{t\bar{t}}$... [Aaboud:2018eqg,Sirunyan:2018ucr,Sirunyan:2018wem...]
  2. Double-differential cross-sections, $d\sigma/dM_{t\bar{t}}dQ_{t\bar{t}}$... [Aad:2019ntk,Sirunyan:2019zvx...]
  3. polarization & spin-correlation [CMS:2018jcg...]
  4. ...
- Particularly, this work investigates $d\sigma/dM_{t\bar{t}}$ and $d\sigma/dM_{t\bar{t}}dY_{t\bar{t}}$ near threshold of $t\bar{t}$, namely, $M_{t\bar{t}} \sim 2m_t$ or

$$\underbrace{\beta = \sqrt{1 - \frac{4m_t^2}{M_{t\bar{t}}^2}} \to 0}$$

$\to$Asymptotic Expansion$\to$EFT

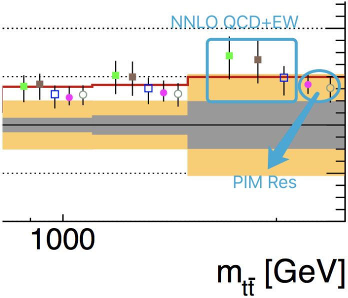
Experimental measurements

1. CMS ExpData absolute $M_{t\bar{t}}$ distribution [Sirunyan:2018ucr]
2. Theory
   - NNLO QCD+EW [Czakon:2016dgf,Catani:2019hi Czakon:2019txp]
   - NNLO+NNLL$'_{\mathrm{PIM}}$ [Czakon:2018nun]



Experimental measurements

1. Comparison
   - Large $M_{t\bar{t}}$ region:
     - Sizable uncertainties [Exp &Theor]
     - Seemingly correct direction [Soft resummation in the PIM limit]
     - What is PIM limit?



Is the resummation inevitable?

Briefly review PIM Resummation

QCD Factorization [Collins:1989gx]

$$\frac{d\sigma}{dM_{t\bar{t}}} \sim \sum \int \frac{dz}{z} \mathcal{L}(\tau/z)\hat{\sigma}(z).$$

where

- $\mathcal{L}$ PDF luminosity
- $z = M_{t\bar{t}}^2/\hat{s}, \quad \tau = M_{t\bar{t}}^2/s.$

PIM limit:

$$\sqrt{\hat{s}} \to M_{t\bar{t}} \quad \text{or} \quad z \to 1$$

Factorization in boosted HQET & SCET,

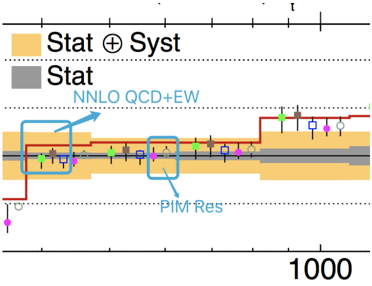
$$\hat{\sigma} \to \mathcal{H} \otimes \mathcal{S}$$

RGE $\Rightarrow$ Resumming singular terms $\delta[1-z]$, $\left[\frac{ln^n(1-z)}{1-z}\right]_+ \cdots$

Practically, $\sqrt{\hat{s}}(1-z) \ll m_t \ll M_{t\bar{t}}$ $\Rightarrow$Refactorization [Czakon:2018nun]

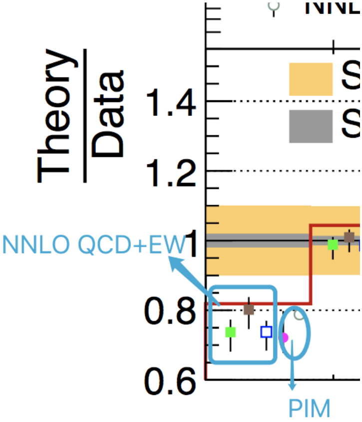
Experimental measurements

1. Comparison
   - Large $M_{t\bar{t}}$ region:
   - Medium $M_{t\bar{t}}$ region:
     - Obvious agreement [Exp &Theor]
     - Relatively small errors [Exp &Theor]



Stepping into the perturbative regime!!

1. Comparison
    - Large $M_{t\bar{t}}$ region:
    - Medium $M_{t\bar{t}}$ region:
    - Small $M_{t\bar{t}}$ region:
        - Theoretical agreement
        - Sizable discrepancies v.s. Exp



Any other singular terms beyond the soft regime????

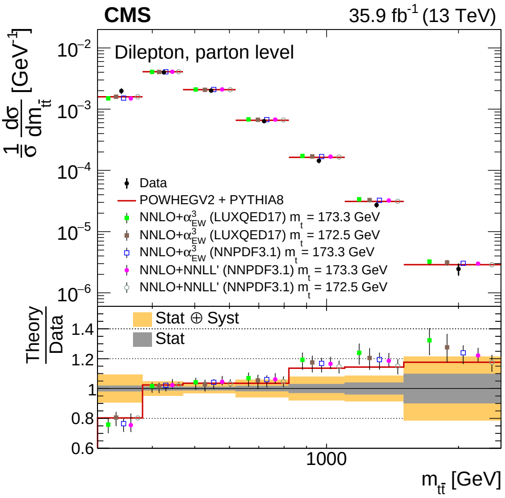
As will be shown, this would be partly answered by Coulomb resummation

Section of Numeric Results

Experimental measurements

1. Normalised CMS ExpData
   [Sirunyan:2018ucr]
2. Theory
   - NNLO QCD+EW [Czakon:2016dgf,Catani:2019hi Czakon:2019txp]
   - NNLO+NNLL$'_{\mathrm{PIM}}$ [Czakon:2018nun]

Similar caveat in 1st bin



Experimental measurements

1. Rapidity distribution& categories

   [Sirunyan:2019zvx]

2. Comparison NLO v.s. Data

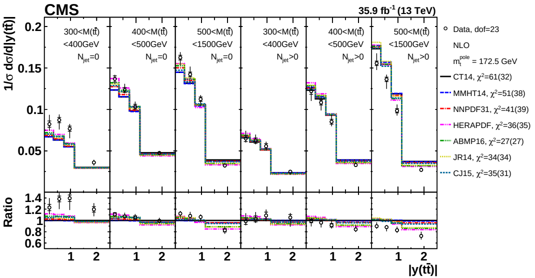
   - $N_{\text{jet}} = 0$

     Derivation near threshold

   - $N_{\text{jet}} > 0$

     Agreements near threshold

     Large recoil$\Rightarrow t\bar{t}_{[8]}$ [NGL?]



Outline

Coulomb Resummation

Generically, CR has been investigated in the following processes,

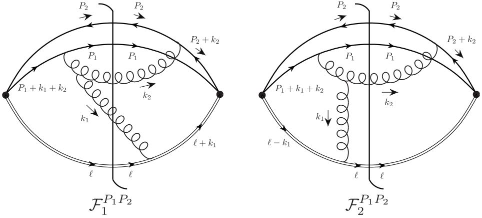
- Leptonic $Q\bar{Q}$ production $\beta \to 0$
  $[\alpha_s^m/\beta^m, \ \ln^n \beta \ ]$
  1. Non-QCD effects: EW&Higgs&non-resonance.....
     [Blumlein:2019pqb,Bach:2017ggt,Beneke:2017rdn,Beneke:2015qev,Beneke:2015lwa,Kuhn:2013zoa...]
  2. QCD NNNLO [Beneke:2015kwa]
  3. ...
- Hadronic $Q\bar{Q}$ production
  1. Soft & Coulomb combined resummation $\beta \to 0 \& z \to 1$
     $[\alpha_s^m/\beta^m, \ \ln^n \beta \ , [\ln^n(1-z)/(1-z)]_+ \ ]$
     [Kiyo:2008bv,Sumino:2010bv,Beneke:2011mq]
  2. Coulomb resummation $\beta \to 0$ $\Rightarrow$ 13 TeV LHC
     $[\alpha_s^m/\beta^m, \ \ln^n \beta \ ]$
     Topological Factorization[Bodwin:1994jh,Petrelli:1997ge]
- Decays......
- ...

Topological factorization and its breaking

- Topological Factorization[Bodwin:1994jh]

$$\beta \to 0,\ \frac{d\sigma}{dM_{t\bar{t}}} \sim \mathcal{H} \otimes \mathcal{J}_c$$

- Unabsorbed IR pole [Nayak:2005rw,Nayak:2005rt,Nayak:2005vp,Bodwin:2019bpf]



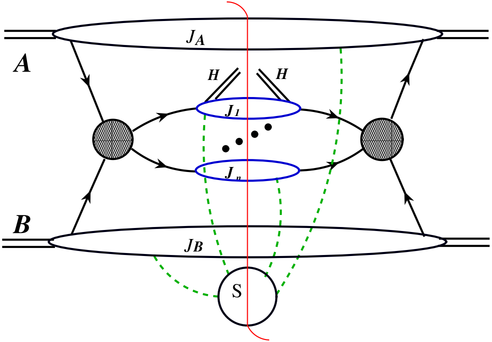
- Non-perturbative regime (J/ψ): $\mathcal{J}$ $\Leftrightarrow$Gauge complete LDME [Nayak:2005rw]
- Perturbative regime $t\bar{t}$: [rarely]

$$\underbrace{\text{EFT} \Leftrightarrow \text{Systematic Expansion}}$$

strategy of this work

Factorization

- Reduced diagrams⇔Generic PSS [Nayak:2005rt,Bodwin:2019bpf,etc]
IR of $\mathcal{H}$ ⇔Landau Equations[Landau:1959fi]&Norton-Coleman theorem[Coleman:1965xm]



Factorization

- To account for IR of $\mathcal{H}$, it is intuitive to incorporate into EFT the following regions,

$$\begin{aligned}
\text{hard:} \quad & k^\mu \sim M_{t\bar{t}}\,, \\
\text{potential:} \quad & k^0 \sim M_{t\bar{t}}\beta^2\,,\ \vec{k} \sim M_{t\bar{t}}\beta\,, \\
\text{soft:} \quad & k^\mu \sim M_{t\bar{t}}\beta\,, \\
\text{ultrasoft:} \quad & k^\mu \sim M_{t\bar{t}}\beta^2\,, \\
\text{collinear:} \quad & k^\mu = (\bar{n}_i \cdot k, n_i \cdot k, k_\perp) \sim M_{t\bar{t}}(1, \beta^2, \beta)\,.
\end{aligned} \tag{1}$$

- pNRQCD: hard&potential&soft&us
- SCET: hard&collinear&us

Factorization

- In light of these dynamic regions, one can work out the asymptotic behavior $\beta \to 0$,

$$\frac{d\sigma}{dM_{t\bar{t}}} \sim \sum_m \left(\frac{\alpha_s}{\beta}\right)^m \left(\overbrace{\underbrace{\underbrace{\beta^0}_{lead} + \beta^1}_{sublead} + \beta^2}^{subsublead} \dots \right) \qquad (2)$$

Factorization at LC

- In the framework of pNRQCD&SCET, we have

$$\frac{\mathrm{d}\sigma_{\mathrm{LP}}}{\mathrm{d}M_{t\bar{t}}} = \sum_m \int \frac{dp_+^{(1)}}{2\pi} \frac{dp_+^{(2)}}{2\pi} \frac{d^4 p_{t\bar{t}}}{(2\pi)^4} (2\pi)\delta(\sqrt{p_{t\bar{t}}^2} - M_{t\bar{t}}) \prod_k^m \frac{d^3 \tilde{p}_{n_k}}{2|\tilde{p}_{n_k}|(2\pi)^4} \mathcal{H}_m(p_+^{(1)}, p_+^{(2)}, p_{t\bar{t}}, \tilde{p}_{n_k}) \delta_{MC}$$
$$\times \left\{ J_{coulomb}(M_{t\bar{t}}, m_t) \right\} \left\{ \int \frac{dp_-^{(1)} d^2 \vec{p}_\perp^{(1)}}{(2\pi)^4} B_{(1)}(p^{(1)}) \right\} \left\{ \frac{dp_-^{(2)} d^2 \vec{p}_\perp^{(2)}}{(2\pi)^4} B_{(2)}(p^{(2)}) \right\}$$
$$\times \prod_k^m \left\{ \int_0^\infty dm_{n_k}^2 J_{n_k}(\tilde{p}_{n_k}, m_{n_k}^2) \right\} \left\{ \int \frac{d^4 p_{us}}{(2\pi)^4} \mathcal{S}_m(p_{us}) \right\}, \tag{3}$$

- $p^{(1,2),\mu} = p_+^{(1,2)} \frac{n^\mu}{2} + p_-^{(1,2)} \frac{\bar{n}^\mu}{2} + \vec{p}_\perp^{(1,2),\mu}$ in light-cone coordinate
- Unlike the jettiness case, there is no power-suppression in the summation over $m \Leftarrow$NGL resummation [Becher:2016mmh,Becher:2015hka]

Factorization at LC

- Explicit definition

$$
\begin{aligned}
\delta_{MC} =&(2\pi)^4\delta^4\left\{p_+^{(1)}\frac{n_{B1}^\mu}{2}+p_+^{(2)}\frac{n_{B2}^\mu}{2}-\sum_k\tilde{p}_{n_k}n_k^\mu-p_{t\bar{t}}^\mu\right\},\\
J_{coulomb}(M_{t\bar{t}},m_t) =& \int\frac{\mathrm{d}^3\vec{P}_t}{(2\pi)^3}\frac{\mathrm{d}^3\vec{P}_{\bar{t}}}{(2\pi)^3}(2\pi)^4\delta(M_{t\bar{t}}-E_t-E_{\bar{t}})\delta^{(3)}(\vec{P}_t+\vec{P}_{\bar{t}})\\
&\times\sum_{\text{pol}}\sum_{\text{color}}P^\alpha_{\{a\}}\,\langle 0|\chi^\dagger_{a_2s_2}\psi_{a_1s_1}|t\bar{t}\rangle\,\langle t\bar{t}|\psi^\dagger_{a_3s_1}\chi_{a_4s_2}|0\rangle_{\mathrm{pNRQCD}}\,,\\
\mathcal{S}_m(p_{us}) =& \int d^4x\,e^{ip_{us}\cdot x}\langle 0|\bar{\mathbf{T}}\left[S_{B1}S_{B2}\prod_k^m S_{n_k}S_v\right](x)\mathbf{T}\left[S_v^\dagger\prod_k^m S_{n_k}^\dagger S_{B2}^\dagger S_{B1}^\dagger\right](0)|0\rangle,\\
B_{(1,2)}(p^{(1,2)}) =& \int d^4x\,e^{ip^{(1,2)}\cdot x}\langle i|\Xi(x)\Xi^\dagger(0)|i\rangle,\\
J_{n_k}(\tilde{p}_{n_k},m_{n_k}^2) =& \int d^4x\,e^{ip_c\cdot x}\langle 0|\Xi(x)\Xi^\dagger(0)|0\rangle\Big|_{p_c\cdot\bar{n}_k=2\tilde{p}_{n_k},\,p_c\cdot n_k=m_{n_k}/p_c\cdot\bar{n}_k,\,\vec{p}_c^\perp=\vec{0},}\,.
\end{aligned}
\tag{4}
$$

- $p^{(1,2),\mu}=p_+^{(1,2)}\frac{n^\mu}{2}+p_-^{(1,2)}\frac{\bar{n}^\mu}{2}+\vec{p}_\perp^{(1,2),\mu}$ in light-cone coordinate

Factorization at LC

- Scaleless sectors,
  1. The collinear function

$$J_{n_k}^{\mathbf{BARE}}(\tilde{p}_{n_k}, m_{n_k}^2) \sim \delta(m_{n_k}^2) + \sum_m \alpha_s^m \frac{\mathcal{F}(m^{n_k}, \tilde{p}_{n_k})}{m_{n_k}^{2\epsilon}}. \tag{5}$$

  2. The soft function

$$\int \frac{d^4 p_{us}}{(2\pi)^4} \mathcal{S}_m(p_{us}) = C_{some\ \ color\ \ indices}, \tag{6}$$

  3. The beam function

$$\int dp_\perp\, B^{\mathbf{TMD}}(p_+, p_\perp) \sim \int dp_\perp\, dp_-^{(1,2)} B_{(1,2)} \sim \int_0^\infty d|p_\perp| \left\{ \delta(p_\perp^2) + \sum_m \alpha_s^m \frac{\mathcal{F}(p_+, p_\perp)}{p_\perp^{2\epsilon}} \right\}. \tag{7}$$

Factorization at LC

- The summation over $m \Rightarrow$

$$\frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}M_{t\bar{t}} d\cos\theta_{t\bar{t}} d\cos\theta_t d\phi_t} \sim \frac{1}{2\pi (M_{t\bar{t}})^2} \left| C^{\theta}_{\alpha} \right|^2 J^{\theta}_{\alpha}(m_{t\bar{t}} - 2m_t, \vec{0}), \qquad (8)$$

- LP factorization retains the topological conjecture
- Natural to ask
  1. whether NLP still admits TF, or when and where the GS pole participates;
  2. In presence GS pole, what kind of new structure should be incorporated into EFT;
  3. whether the jet&beam&soft sectors will always be trivial.

Factorization at SLC

- Single insertions of SLC effective operators
  1. the effective Hamiltonians;

$$\mathcal{H}_{\rm SLC} = \sum_m \Bigg\{ C_1 \chi^\dagger \psi \, [\partial_\perp \Xi_{B1}] \, \Xi_{B_2} \prod_k^m \Xi_{n_k} + C_2 \chi^\dagger \psi \left[\mathcal{A}_\perp^{B1} \Xi_{B1} \right] \Xi_{B_2} \prod_k^m \Xi_{n_k} + , \\ C_3 \chi^\dagger \left[\vec{\partial} \psi \right] \Xi_{B1} \Xi_{B_2} \prod_k^m \Xi_{n_k} + permutations \Bigg\} \quad (9)$$

Forbidden by angular momentum conservation

  2. the subleading SCET and pNRQCD Lagrangians.

Factorization at SLC

- Single insertions of SLC effective operators
  1. the effective Hamiltonians;
  2. the subleading SCET and pNRQCD Lagrangians. [Bauer:2000ew, Bauer:2000yr, Bauer:2001yt, Beneke:2002ph, Beneke:2002ni]

$$\mathcal{L}^{1a}_{\text{SCET}} = \bar{\xi}_n \left(x^{\mu}_{\perp} n^{\nu} \overline{W}_n g_s F^{\text{us}}_{\mu\nu} \overline{W}^{\dagger}_n \right) \frac{\not{\bar{n}}}{2} \xi_n + (n \leftrightarrow \bar{n}) \,,$$

$$\mathcal{L}^{1b}_{\text{SCET}} = \text{Tr} \left\{ n^{\mu} F^{n}_{\mu\nu} \overline{W}_n i \left[x^{\rho}_{\perp} \bar{n}^{\rho} F^{\text{us}}_{\rho\sigma}, \overline{W}^{\dagger}_n \left(i D^{\nu}_{n\perp} \overline{W}_n \right) \right] \overline{W}^{\dagger}_n \right\} - \text{Tr} \left\{ n^{\mu} F^{\mu\nu\perp}_{n} \overline{W}_n \bar{n}^{\rho} F^{\text{us}}_{\rho\nu_{\perp}} \overline{W}^{\dagger}_n \right\} + (n \leftrightarrow \bar{n}) \,,$$

$$\mathcal{L}^{1c}_{\text{SCET}} = \bar{\xi}_n i \not{D}_{n\perp} \overline{W}_n q_{\text{us}} + \text{h.c.} + (n \leftrightarrow \bar{n}) \,,$$

Forbidden by angular momentum&baryonic number conservation

Factorization at SLC

- Single insertions of subleading effective operators
  1. the effective Hamiltonians;
  2. the subleading SCET and pNRQCD Lagrangians. [Pineda:1997bj, Brambilla:1999xf, Beneke:1999zr, Beneke:1999qg]

$$\mathcal{L}^{1a}_{\text{pNRQCD}}(x) = -\psi^\dagger(x)\, g_s\, \vec{x}\cdot\vec{E}_{\text{us}}(x^0,\vec{0})\,\psi(x) - \chi^\dagger(x)\, g_s\, \vec{x}\cdot\vec{E}_{\text{us}}(x^0,\vec{0})\,\chi(x)\,, \tag{10}$$

$$\mathcal{L}^{1b}_{\text{pNRQCD}}(x) = -\int \mathrm{d}^3\vec{r}\,\psi^\dagger T^a \psi(x^0,\vec{x}+\vec{r})\,\frac{\alpha_s^2}{4\pi r}\left[a_1 + 2\beta_0\,\ln\left(e^{\gamma_E}\mu r\right)\right]\chi^\dagger T^a \chi(x^0,\vec{x})\,, \tag{11}$$

$\mathcal{L}^{1a}_{\text{pNRQCD}}$ Forbidden by angular momentum

$\mathcal{L}^{1b}_{\text{pNRQCD}}$ survives

Factorization at SLC

- The summation over $m \Rightarrow$

$$\frac{\mathrm{d}\hat{\sigma}_{ij}^{\mathrm{SLC}}}{\mathrm{d}M_{t\bar{t}}d\cos\theta_{t\bar{t}}d\cos\theta_t d\phi_t} = \frac{1}{2\pi(M_{t\bar{t}})^2}\left|C_\alpha^\theta\right|^2\tilde{J}_\alpha^\theta(m_{t\bar{t}} - 2m_t, \vec{0}), \qquad (12)$$

- Subleading factorization also admits the topological conjecture

Factorization at SSLC (preliminary)

- Sub-subleading comprises:
  1. single insertion of sub-subleading ingredients
  2. double insertions of subleading ingredients
     - particularly, we will highlight the double insertions of $\mathcal{L}^{1a}_{\mathrm{pNRQCD}}(x)$

$$\frac{\mathrm{d}\hat{\sigma}^{1a}_{ij}}{\mathrm{d}M_{t\bar{t}}} \sim \sum_m \int \frac{d\vec{P}_{t\bar{t}}}{2\mathcal{E}_{t\bar{t}}(2\pi)^3} \mathcal{H}_m \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4} J_{1a}\left(m_{t\bar{t}} - 2m_t, \vec{0}, k_1, k_2\right) \int d^4z_1\, d^4z_2$$
$$\times\, e^{-i(k_1\cdot z_1 + k_2\cdot z_2)} \mathrm{Tr}\left[(\vec{z}_2\vec{z}_1) S^{1a}_{ij,n_{t\bar{t}},\bar{n}_{t\bar{t}}}(\infty, z_1^0, z_2^0, \mu)\right] + \text{h.c.} + \text{permutations}, \quad (13)$$

where

$$J_{1a}(E_q, \vec{q}, k_1, k_2) = -\int d\Phi_t\, d\Phi_{\bar{t}}\, (2\pi)^4 \delta^{(4)}(q - p_t - p_{\bar{t}})$$
$$\times \int d^4x_1\, d^4x_2\, e^{i(k_1\cdot x_1 + k_2\cdot x_2)} \langle 0|\chi^\dagger_{s_2}\psi_{s_1}(0)|t\bar{t}\rangle \langle t\bar{t}|\mathrm{T}\left[\left(\psi^\dagger(x_1)\psi(x_1)\right.\right.$$
$$\left.+ \chi^\dagger(x_1)\chi(x_1)\right)\left(\psi^\dagger(x_2)\psi(x_2) + \chi^\dagger(x_2)\chi(x_2)\right) \psi^\dagger_{s_1}\chi_{s_2}(0)\big]|0\rangle\,,$$
$$S^{1a}_{ij,n_{t\bar{t}},\bar{n}_{t\bar{t}}}(\infty, z_1^0, z_2^0, \mu) = g_s^2 \sum_X \langle 0|S^\dagger_v S_v S^j_{\bar{n}_{t\bar{t}}} S^{i\dagger}_{n_{t\bar{t}}}(0)|X\rangle$$
$$\times \langle X|\mathrm{T}\left[S_v \vec{E}_{\mathrm{us}} \prod_m S_m S^\dagger_v(z_1^0, \vec{0})\, S_v \vec{E}_{\mathrm{us}} S^\dagger_v(z_2^0, \vec{0})\, S^i_{n_{t\bar{t}}} S^{j\dagger}_{\bar{n}_{t\bar{t}}} \prod_m S^\dagger_m S^\dagger_v S_v(0)\right]|0\rangle\,. \quad (14)$$

$S^{1a}_{ij,n_{t\bar{t}},\bar{n}_{t\bar{t}}}$ reproduces GS poles !!!! Eq. [32] of hep-ph/0509021 or [Nayak:2005rt]. Convension ⇐[Ju:2019lwp]

Factorization at SSLC (preliminary)

- Answers from present work,
  1. whether subleading still admits TF, or when and where the GS pole participates;

     subleading⇒TF, the first appearance of GS pole⇒SSLC
  2. In presence of GS pole, what kind of new structure should be incorporated into EFT;

     $S^{1a}_{ij,n_{t\bar{t}},\bar{n}_{t\bar{t}}}$ could reproduce GS poles
  3. whether the jet&beam&soft sectors will always be trivial.

     Crucial to generate $S^{1a}_{ij,n_{t\bar{t}},\bar{n}_{t\bar{t}}}$

Outline

Implementation at NLP

- Even for the hard sector, it is impossible to exactly accomplish all perturbative orders
- truncations: $\beta \sim \alpha_s \sim \lambda$

$$\frac{d\sigma}{dM_{t\bar{t}}} \sim \sum_m \left(\frac{\alpha_s}{\beta}\right)^m \left(\underbrace{\overbrace{\underbrace{\lambda^0}_{LP} + \lambda^1 + \lambda^2}^{NNLP}}_{NLP} \ldots \right) \quad (15)$$

Implementation at NLP

- Subleadng Factorization formula is sufficient for NLP accuracy:

$$\frac{d\hat{\sigma}_{ij}}{dM_{t\bar{t}}\, d\Theta} = \frac{16\pi^2\alpha_s^2(\mu_r)}{M_{t\bar{t}}^5}\sqrt{\frac{M_{t\bar{t}}+2m_t}{2M_{t\bar{t}}}}\sum_{\alpha} c_{ij,\alpha}(\cos\theta_t) \times H_{ij,\alpha}(z, M_{t\bar{t}}, Q_T, Y, \mu_r, \mu_f)\, J^{\alpha}(E) + \mathcal{O}(\beta^3)\,, \quad (16)$$

where

$$c_{q\bar{q},8}(\cos\theta_t) = \frac{1}{4}\left[2-\beta^2(1-\cos^2\theta_t)\right]\,,$$

$$c_{gg,1}(\cos\theta_t) = \frac{1}{2(1-\beta^2\cos^2\theta_t)^2}\left[4-2(1-\beta^2)^2-2\beta^2(1-\beta^2\cos^2\theta_t)-(1+\beta^2\cos^2\theta_t)^2\right]\,,$$

$$c_{gg,8}(\cos\theta_t) = 2c_{gg,1}(\cos\theta_t)\left[\frac{16}{5}-\frac{9}{10}(3-\beta^2\cos^2\theta_t)\right]\,, \quad (17)$$

Here the kinematic variables contained in Θ include Q_T^2, Y, as well as θ_t and ϕ_t being the scattering angle and the azimuthal angle of the top quark in the $t\bar{t}$ rest frame.

Implementation at NLP

- Coulomb resummation kernels NLP
[Beneke:1999qg, Beneke:1999zr, Pineda:2006ri, Beneke:2011mq]

$$
\begin{aligned}
J^{\alpha}(E) =& M_{t\bar{t}}^2 \int \frac{d^4 p_t}{(2\pi)^4} \frac{d^4 p_{\bar{t}}}{(2\pi)^4} (2\pi)\delta(p_t^2 - m_t^2)\,(2\pi)\delta(p_{\bar{t}}^2 - m_t^2)\,(2\pi)^4 \delta^{(4)}(P_{t\bar{t}} - p_t - p_{\bar{t}}) \\
& \times P^{\alpha}_{\{a\}} \, \langle 0|\chi^{\dagger}\psi|t_{a_3}\bar{t}_{a_4}\rangle \, \langle t_{a_1}\bar{t}_{a_2}|\psi^{\dagger}\chi|0\rangle \;, \\
=& 2\,\mathrm{Im}\, G_0^{\alpha}(\vec{0},\vec{0};E) + 2\,\mathrm{Im}\, G_1^{\alpha}(\vec{0},\vec{0};E)
\end{aligned}
$$

where

$$
\begin{aligned}
G_0^{\alpha}(\vec{0},\vec{0};E) =& \frac{M_{t\bar{t}}^2}{16\pi}\left\{ -\sqrt{\frac{-2E}{M_{t\bar{t}}}} + \frac{\alpha_s(\mu_J)D_{\alpha}}{2}\Big[-2L_J + 2\psi(\lambda) + 2\gamma_E - 1\Big]\right\}, \\
G_1^{\alpha}(\vec{0},\vec{0};E) =& -\frac{M_{t\bar{t}}^2 D_{\alpha}\alpha_s^2(\mu_J)}{64\pi^2}\left\{ a_1\Big[L_J + (1-\lambda)\psi'(\lambda) - \psi(\lambda) - \gamma_E\Big]\right. \\
& + \beta_0\Big[L_J^2 + 2L_J\big((1-\lambda)\psi'(\lambda) - \psi(\lambda) - \gamma_E\big) + 4\,{}_4F_3(1,1,1,1;2,2,\lambda;1) \\
& \left. + (1-\lambda)\psi''(\lambda) - 2(1-\lambda)\big(\psi(\lambda)+\gamma_E\big)\psi'(\lambda) - \frac{\pi^2}{6} - 3\psi'(\lambda) + \big(\psi(\lambda)+\gamma_E\big)^2\Big]\right\}. \qquad (18)
\end{aligned}
$$

Here $a_1 = 31C_A/9 - 10N_l/9$, $D_1 = -C_F$, $D_8 = 1/(2N_c)$

$$
L_J = -\frac{1}{2}\ln\left(-\frac{2M_{t\bar{t}}E}{\mu_J^2}\right), \quad \lambda = 1 + \frac{\alpha_s(\mu_J)D_{\alpha}}{2\sqrt{-2E/M_{t\bar{t}}}}\,. \qquad (19)
$$

Implementation at NLP

- The hard sector

$$H_{ij,\alpha}(z, M_{t\bar{t}}, Q_T, Y, \mu_r, \mu_f) = \frac{zM_{t\bar{t}}^2}{32\pi^3\alpha_s^2} \sum_X \int \frac{d^4 P_{t\bar{t}}}{(2\pi)^4} (2\pi)\, \delta(P_{t\bar{t}}^2 - M_{t\bar{t}}^2)\, \delta(P_{T,t\bar{t}}^2 - Q_T^2)$$
$$\times\, \delta(Y_{t\bar{t}} - Y)\,(2\pi)^4 \delta^{(4)}(p_1 + p_2 - P_{t\bar{t}} - P_X)$$
$$\times\, \frac{1}{N_{ij}}\, P^{\alpha}_{\{a\}}\, C^{a_1 a_2}_{ij,X}(p_1, p_2, P_{t\bar{t}}, P_X)\, C^{a_3 a_4\,\dagger}_{ij,X}(p_1, p_2, P_{t\bar{t}}, P_X)\,, \qquad (20)$$

where

$$\left|\mathcal{M}(i + j \to t\bar{t} + X)\right|^2 = \frac{1}{N_{ij}}\, C^{a_1 a_2}_{ij,X}(p_1, p_2, P_{t\bar{t}}, P_X)\, C^{a_3 a_4\,\dagger}_{ij,X}(p_1, p_2, P_{t\bar{t}}, P_X)$$
$$\times\, \langle 0|\chi^\dagger \psi|t_{a_3} \bar{t}_{a_4}\rangle\, \langle t_{a_1} \bar{t}_{a_2}|\psi^\dagger \chi|0\rangle\;, \qquad (21)$$

To calculate the Wilson coefficients,

- pQCD: FeynArts [Hahn:2000kx],
  FeynCalc [Mertig:1990an, Shtabovenko:2016sxi]
  Package-X [Patel:2015tea, Patel:2016fam]
  FeynHelpers [Shtabovenko:2016whf]

Implementation at NLP

- Matching procedure

$$\frac{d\sigma^{\text{(N)NLO+NLP}}}{dM_{t\bar{t}}} = \frac{d\sigma^{\text{NLP}}}{dM_{t\bar{t}}} - \frac{d\sigma^{\text{(n)nLO}}}{dM_{t\bar{t}}} + \frac{d\sigma^{\text{(N)NLO}}}{dM_{t\bar{t}}} \,, \tag{22}$$

where

$$\frac{d\sigma^{\text{n}^k\text{LO}}}{dM_{t\bar{t}}} = \int_\tau^1 \frac{dz}{z} \int_{-1}^1 d\cos\theta_t \int_0^{2\pi} \frac{d\phi_t}{2\pi} \int_0^{Q^2_{T,\text{max}}} dQ_T^2 \int_{-Y_{\text{max}}}^{Y_{\text{max}}} dY \, \frac{16\pi^2\alpha_s^2(\mu_r)}{s\, M^3_{t\bar{t}}} \sqrt{\frac{M_{t\bar{t}}+2m_t}{2M_{t\bar{t}}}}$$
$$\times \sum_{ij,\alpha} c_{ij,\alpha}(\cos\theta_t)\, f\!f_{ij}(\tau/z,\mu_f)\, \frac{1}{z}\, \frac{M^2_{t\bar{t}}}{8\pi} \sqrt{\frac{2E}{M_{t\bar{t}}}} \sum_{n=0}^{k} \left(\frac{\alpha_s(\mu_r)}{4\pi}\right)^n K^{(n)}_{ij,\alpha} \,. \tag{23}$$

Here K_{ij} stands for the coefficient.

$$K^{(0)}_{ij,\alpha} = H^{(0)}_{ij,\alpha} \,,$$
$$K^{(1)}_{ij,\alpha} = -2\pi^2 D_\alpha \sqrt{\frac{M_{t\bar{t}}}{2E}}\, H^{(0)}_{ij,\alpha} + H^{(1)}_{ij,\alpha} \,,$$
$$K^{(2)}_{ij,\alpha} = \frac{4\pi^4 D_\alpha^2}{3} \frac{M_{t\bar{t}}}{2E} H^{(0)}_{ij,\alpha} + 2\pi^2 D_\alpha \sqrt{\frac{M_{t\bar{t}}}{2E}} \left[(\beta_0 L_r - a_1) H^{(0)}_{ij,\alpha} - H^{(1)}_{ij,\alpha} \right] ,$$

Outline

Validity of the approximation

Comparison of the $M_{t\bar{t}}$ distributions in the range [340-380] GeV at NLO and approximate NLO.

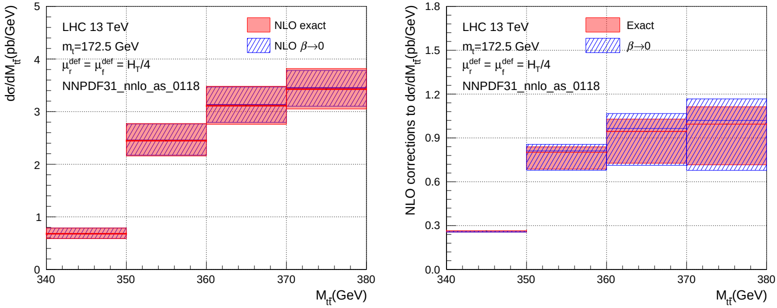


Figure: Comparison between the exact NLO distribution (red band) and the NLO expansion of our resummation formula (blue shaded band) in the range 340 GeV $\leq M_{t\bar{t}} \leq$ 380 GeV at the 13 TeV LHC. The left plot shows the differential cross sections, while the right plot shows the NLO corrections only.

Validity of the approximation

Numeric results for the averaged differential cross section in the four bins between 340 GeV and 380 GeV at the 13 TeV LHC

Scale	Region	LO	NLO	nLO	NLO+NLP
$H_T/2$	[340, 350]	0.332	0.587	0.588	1.173
	[350, 360]	1.318	2.157	2.173	2.296
	[360, 370]	1.729	2.758	2.796	2.844
	[370, 380]	1.937	3.050	3.103	3.111
$H_T/4$	[340, 350]	0.414	0.679	0.679	1.265
	[350, 360]	1.646	2.448	2.457	2.517
	[360, 370]	2.166	3.111	3.130	3.130
	[370, 380]	2.433	3.427	3.451	3.426
$H_T/8$	[340, 350]	0.524	0.789	0.788	1.323
	[350, 360]	2.087	2.774	2.766	2.707
	[360, 370]	2.755	3.482	3.466	3.374
	[370, 380]	3.103	3.819	3.778	3.700

- Perfect agreements in region [340,350]
- Slight derivation in region [370,380]
- Considerable enhancement from Coulomb resummation in [340,350]
- Mild corrections from Coulomb resummation in [370,380]

Validity of the approximation

Comparison of the $M_{t\bar{t}}$ distributions in the range [340-380] GeV at NLO and approximate NLO.

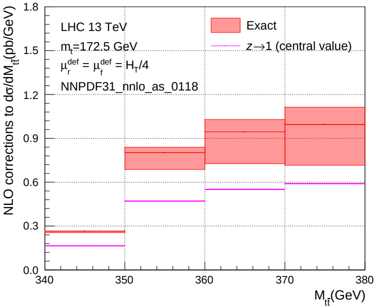
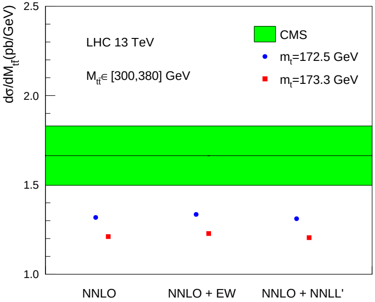


Figure: Comparison of the exact NLO corrections (red band) with the approximate results in the $z \to 1$ limit (pink line in the left plot) and in the double limit $\beta \to 0$ & $z \to 1$ (green line in the right plot). For the approximate results, only the central values are shown.

Reminder of the experimental data

1. Recent experimental measurements [Aaboud:2018eqg, Sirunyan:2018ucr, Sirunyan:2018wem, Sirunyan:2019ln]
2. NNLO QCD [Czakon:2015owf, Czakon:2016dgf, Catani:2019hip]
3. NNLO QCD + EW [Czakon:2017wor, Czakon:2019txp]
4. Soft Resummation NNLL′ [Pecjak:2016nee, Czakon:2018nun, Pecjak:2018lif]



The CMS measurements [Sirunyan:2018ucr] on $t\bar{t}$ invariant mass distribution in the [300-380] GeV range.

Numerical Results after Coulomb resummation

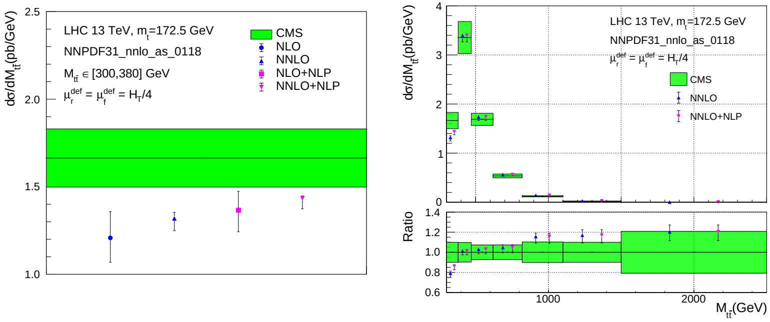


Figure: The NLO+NLP and NNLO+NLP predictions for the absolute $M_{t\bar{t}}$ distribution against the CMS data in the di-lepton channel [Sirunyan:2018ucr]. Fixed-order results are shown for comparison. The left plot shows the first bin $M_{t\bar{t}} \in [300, 380]$ GeV, while the right plot shows the full $M_{t\bar{t}}$ range.

Numerical Results after Coulomb resummation

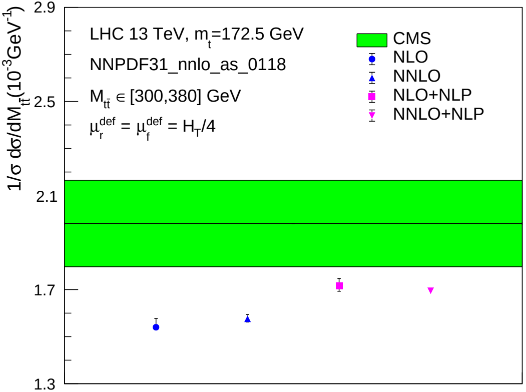


Figure: The NLO+NLP and NNLO+NLP predictions for the normalized $M_{t\bar{t}}$ distribution in the first bin $M_{t\bar{t}} \in [300, 380]$ GeV, against the CMS data in the di-lepton channel [Sirunyan:2018ucr]. Fixed-order results are shown for comparison.

Numerical Results after Coulomb resummation

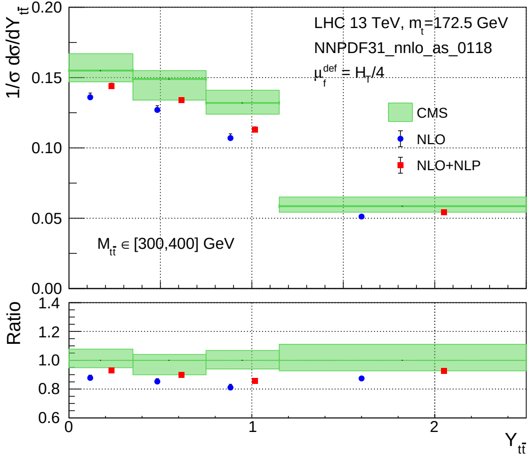


Figure: Normalized double differential distribution with respect to $M_{t\bar{t}}$ and and the rapidity $Y_{t\bar{t}}$ of the top quark pair in the threshold region. This plot corresponds to the first bin ([300, 400] GeV) in $M_{t\bar{t}}$ and four bins in $Y_{t\bar{t}}$. The NLO and NLO+NLP results are compared to the CMS data [Sirunyan:2018ucr].

Influence on the top quark mass determination

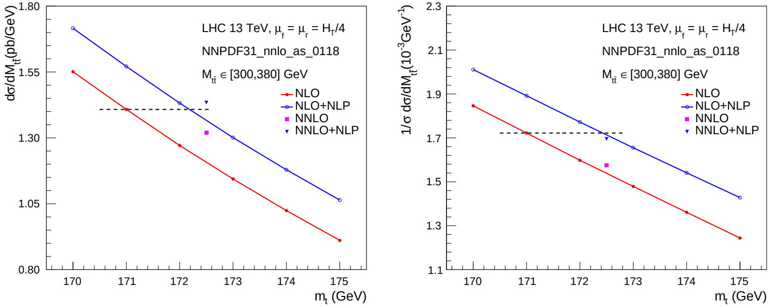


Figure: Top-quark-mass dependence of the absolute (left) and normalized (right) $M_{t\bar{t}}$ differential cross sections in the threshold region. Only central values of the NLO and NLO+NLP results are shown here. The NNLO and NNLO+NLP predictions at $m_t = 172.5$ GeV are given for reference. Word averaged $m_t^{pole} = 173.1 \pm 0.9\ GeV$ [PDG]. NLO fitting results in $m_t^{pole} \sim 171\ GeV$ [Sirunyan:2019zvx]. NLO+NLP gives $m_t^{pole} \sim 172.4\ GeV$.

Outline

Summary

Theoretical Framework

1. Revisiting the topological factorization within the EFT method
   1.1 TF holds up SLC$\Rightarrow$ the accuracy of this work
   1.2 GS pole will be properly reproduced at SSLC by a novel soft function
   1.3 Soft&collinear sectors are non-trivial starting from SSLC
2. Examining the validity
   2.1 The nLO expansions are in very good agreements with exact NLO.

Summary

Numerical Results

1. We calculate $d\sigma/dM_{t\bar{t}}$ at NNLO+NLP and compare them with the CMS measurements, finding that the Coulomb resummation is helpful to account for the discrepancy between the experimental detection and the fixed-order results.
2. We also investigate the double-differential observable $d\sigma/dY_{t\bar{t}}dM_{t\bar{t}}$, observing that the resummed results are more compatible with the experiment data.
3. Due to recent indirect measurements on m_t^{pole}, which is much smaller than the world average and mainly arises from the discrepancy between theory and data in the $\beta \to 0$ region, we investigate the Coulomb resummation in this area, demonstrating that the extracted results with NLO+NLP is closer to the world average.

The End