Understanding template methods for top polarisation measurements

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Brief outline

- What is top polarisation?
- (A) Template method
- QFT is quantum!
- Right vs wrong templates
- Example for tt

Based on:

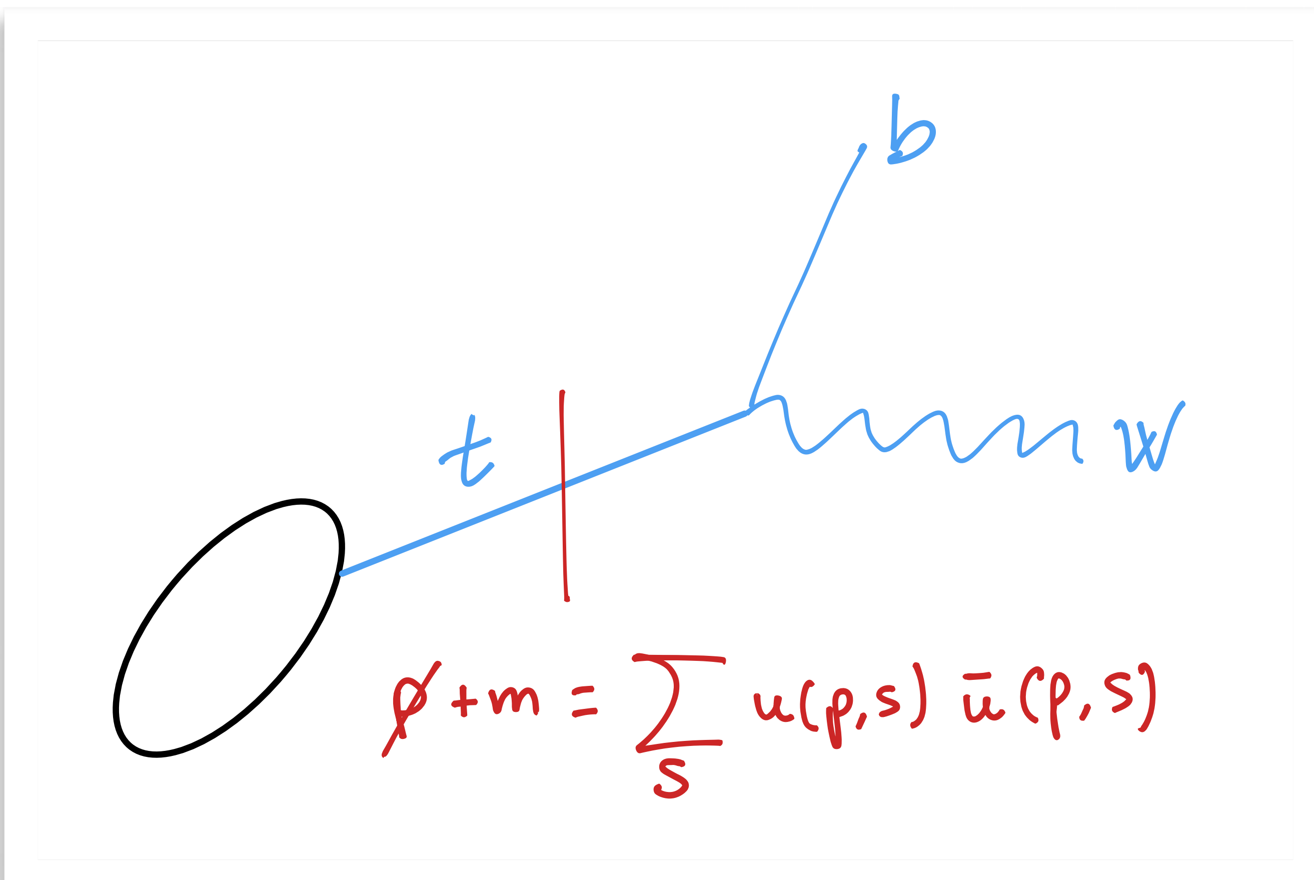
- JAAS, Fiolhais, Martín-Ramiro, Moreno, Onofre, *A template method to measure the tt polarisation*, 2111.10394
- JAAS, *Crafting polarisations for top, W and Z*, 2208.00424
- New material and discussions to appear on the proceedings

What is top polarisation?

What is top polarisation?

The top quark has a mean lifetime of 10^{-25} s. It appears as an intermediate resonance in processes.

Then, what do we mean by `top polarisation´?



In the NWA, the amplitudes can be decomposed into production × decay, summed over spins.

$$\mathcal{M} = \sum_{s=\pm 1/2} \mathcal{P}_s \mathcal{D}_s$$

This <u>is</u> the top polarisation that is measured by angular distributions

The production mechanism can be characterised by a spin density operator ρ whose matrix elements are related to production amplitudes $\mathcal{P}_s$

$$\rho_{ss'} = \mathrm{PS} \times \sum_{\text{subprocesses}} \mathrm{PDF} \times \mathcal{P}_s \mathcal{P}^*_{s'}$$

The goal of polarisation measurements is to measure matrix elements of ρ.

For the top quark,

$$\rho = \frac{1}{2} \begin{pmatrix} 1 + P_z & P_x - iP_y \\ P_x + iP_y & 1 - P_z \end{pmatrix} .$$

The polarisation vector $P_i = 2 \langle S_i \rangle$ satisfies $|\vec{P}| \leq 1$ for a physical ρ.

Template method

Template method

If we consider the charged lepton from the top decay to measure the polarisation, its angular distribution [in top rest frame] is

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{1}{4\pi}\left[1 + \alpha_\ell\, \vec{P} \cdot \hat{p}_\ell\right]$$

Not templatable

Imagine we want to measure P_z. Then, we can integrate over ϕ to obtain

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta} = \frac{1}{2}\left[1 + \alpha_\ell\, P_z \cos\theta\right]$$

which can be written as

Distribution for P_z = 1

Distribution for P_z = -1

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta} = a_+ \left[\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta}\right]_+ + a_- \left[\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta}\right]_-$$

with

$$a_+ = (1 + P_z)/2 = \rho_{\frac{1}{2}\frac{1}{2}} = \sigma_+/\sigma$$

$$a_- = (1 - P_z)/2 = \rho_{-\frac{1}{2}-\frac{1}{2}} = \sigma_-/\sigma$$

Template expansion

Template method

Discussion

- The template fit is well-founded on a theoretical equality

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta} = a_+ \left[\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta}\right]_+ + a_- \left[\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta}\right]_-$$

  and therefore the fit coefficients have a theoretical interpretation

- Side comment: not clear on which theoretical equality the template fit in ATLAS polarisation measurement 2202.11382 is based.

$\Rightarrow$ The theoretical interpretation of the fit coefficients is unclear to me

Template method

Discussion

- The template expansion looks classical:

  Sum over $+ , -$ without interference between polarisations.

- This is possible for the $\cos\theta$ distribution and after integration on ϕ. Also for the total cross section: σ_+ and σ_- are meaningful.

- Ignoring interference in other distributions leads to results that are not correct. Example: A_{FB}^{ℓ}

Template method

Reconstruction, detector effects and kinematical cuts make things more complicated.

To see this easily, we rewrite the template expansion [multiply by σ]

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta} = a_+\left[\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta}\right]_+ + a_-\left[\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta}\right]_- \quad \Longrightarrow \quad \frac{d\sigma}{d\cos\theta} = \frac{d\sigma_+}{d\cos\theta} + \frac{d\sigma_-}{d\cos\theta}$$

With cuts, etc. this is modified to

Quantities after cuts

Interference

$$\frac{d\bar\sigma}{d\cos\theta} = \frac{d\bar\sigma_+}{d\cos\theta} + \frac{d\bar\sigma_-}{d\cos\theta} + \ldots$$

Template method

In order to cast that sum in terms of normalised quantities, one can define \`efficiencies´ $\varepsilon = \bar{\sigma}/\sigma$, $\varepsilon_{\pm} = \bar{\sigma}_{\pm}/\sigma_{\pm}$ and obtain a template expansion

$$\varepsilon \left[\frac{1}{\bar{\sigma}} \frac{d\bar{\sigma}}{d\cos\theta} \right] = \varepsilon_{+} a_{+} \left[\frac{1}{\bar{\sigma}_{+}} \frac{d\bar{\sigma}_{+}}{d\cos\theta} \right] + \varepsilon_{-} a_{-} \left[\frac{1}{\bar{\sigma}_{-}} \frac{d\bar{\sigma}_{-}}{d\cos\theta} \right] + \Delta_{int}$$

Do experiments ever consider this term?

The term Δ_{int} arises because:

- The integration over ϕ is not complete because of cuts.
- The ϕ angle in the top rest frame is not well reconstructed and then, integration is not actually over the true ϕ.

In tt with dilepton decay, it was seen in 2111.10394 that the second effect is way more important.

QFT is quantum

QFT is quantum

The interference term is important for precision measurements.
Fortunately, a SM calculation may be sufficient.

Proposed method: JAAS, Fiolhais, Martín-Ramiro, Moreno, Onofre, 2111.10394

- Generate statistically independent templates (MC) and SM pseudo-data (also with MC)
- Use SM values for polarisation coefficients
- Use efficiencies from Monte Carlo
- Compute Δ_{int} in the SM as

$$\Delta_{int} = \varepsilon \left[\frac{1}{\bar{\sigma}} \frac{d\bar{\sigma}}{\mathrm{d}\cos\theta} \right] - \varepsilon_+ a_+ \left[\frac{1}{\bar{\sigma}_+} \frac{d\bar{\sigma}_+}{\mathrm{d}\cos\theta} \right] - \varepsilon_- a_- \left[\frac{1}{\bar{\sigma}_-} \frac{d\bar{\sigma}_-}{\mathrm{d}\cos\theta} \right]$$

➡ Checked in tt production that Δ_{int} computed in the SM is enough even with large [experimentally excluded] CMDM

QFT is quantum

Simple example for single top production

Axes: helicity (K), transverse (R) and normal (N)

$$\hat{k} = \hat{p}_t$$

top in CM frame

spectator jet in lab frame

$$\hat{r} = \mathrm{sign}(\vec{p}_j \cdot \hat{p}_p)(\hat{p}_p - \cos\theta\; \hat{k}) / \sin\theta$$

(0,0,1)

$\cos\theta = \hat{p}_t \cdot \hat{p}_p$

$$\hat{n} = \hat{k} \times \hat{r}$$

QFT is quantum

Simple example for single top production

No detector simulation, working at parton level but applying cuts $p_T \geq 30$ GeV for lepton and b from top decay.

This is sufficient to have large distortions on templates

From fit

	K			R			N		
	a_+	a_-	P	a_+	a_-	P	a_+	a_-	P
True	0.159	0.841	-0.682	0.615	0.385	0.230	0.501	0.499	0.002
no Δ_{int}	0.160	0.840	-0.680	0.580	0.420	0.160	0.500	0.500	0.000
with Δ_{int}	0.160	0.840	-0.680	0.616	0.384	0.232	0.500	0.500	0.000

At this level, the difference already is 1/4 of the uncertainty in ATLAS measurement

Right vs wrong templates

Right vs wrong templates

Q: Is any MC sample

with <u>correct</u> top decay distribution

appropriate as a template?

A: No

To see this subtle point, let us go back to

$$\frac{d\sigma}{d\cos\theta} = \frac{d\sigma_+}{d\cos\theta} + \frac{d\sigma_-}{d\cos\theta} \xrightarrow{\text{cuts}} \frac{d\bar{\sigma}}{d\cos\theta} = \frac{d\bar{\sigma}_+}{d\cos\theta} + \frac{d\bar{\sigma}_-}{d\cos\theta} + \ldots$$

The σ_+ and σ_- on the r.h.s. are the + and − components of σ on the l.h.s.

If σ [which will be data in the fit] is SM-like, σ_+ and σ_- must be too.

Therefore, σ_+ and σ_- must be obtained with spin projectors at the matrix-element level.

Right vs wrong templates

Example: single top / helicity axis / cut $p_T \geq 30$ GeV for extra b

SM w/cut

Norm. distributions unchanged by cut

Cut keeps $\Delta_{\text{int}} = 0$

$$\varepsilon\left[\frac{1}{\bar{\sigma}}\frac{d\bar{\sigma}}{d\cos\theta}\right] = \varepsilon_+ a_+ \left[\frac{1}{\bar{\sigma}_+}\frac{d\bar{\sigma}_+}{d\cos\theta}\right] + \varepsilon_- a_- \left[\frac{1}{\bar{\sigma}_-}\frac{d\bar{\sigma}_-}{d\cos\theta}\right] + \Delta_{int}$$

Fit returns $a_+\varepsilon_+/\varepsilon = 0.286 \quad a_-\varepsilon_-/\varepsilon = 0.714$

SM eff: $\varepsilon = 0.292$

Templates with SM amplitudes and spin projectors in matrix element

$\varepsilon_+ = 0.524$ → $a_+ = 0.160$

$\varepsilon_- = 0.245$ → $a_- = 0.840$

✔

Templates with SM production kinematics and modified decay

$\varepsilon'_+ = \varepsilon$ → $a_+ = 0.286$

$\varepsilon'_- = \varepsilon$ → $a_- = 0.714$

✘

Right vs wrong templates

Discussion

- This example has been chosen so that $\Delta_{int} = 0$ for simplicity and to have the template distributions unaltered from parton level.
- It clearly shows that to fit SM (on the l.h.s.) we need SM polarised templates on the r.h.s.
- Because single top kinematics is SM-like, SM polarised templates must be used in measurements.
- Using templates with incorrect production mechanism, leads to [quite?] different ε and therefore, to [quite?] biased results.
- In addition, the normalised distributions are different [not in the previous example].
- In the lucky event that measurements deviate significantly from the SM, we should think about closure tests using SM polarised templates.

Right vs wrong templates

CAR: Custom Angle Replacement

JAAS, 2208.00424

Method to replace the decay kinematics of top, *W* and *Z* to tune desired polarisations [just noticed that the same method was used by ATLAS, PolManip].

In template measurements for top, it can be used to augment samples. In the previous example:

- Take a small $t_L \bar{t}_L$ polarised sample [50 K] to parameterise production kinematics.
- Use CAR to obtain large [1 M] $t_L \bar{t}_L$, $t_R \bar{t}_R$ statistically-independent samples with new top decays in less than a minute.

→ Especially useful for multi-top production, generation beyond LO, etc.

Example for tt

Example for tt

Top pair → two polarisations $t_L\bar{t}_L\,, t_L\bar{t}_R\,, t_R\bar{t}_L\,, t_R\bar{t}_R$

JAAS, Fiolhais, Martín-Ramiro, Moreno, Onofre, 2111.10394

- Simulation with Delphes.
- Reconstruction of dilepton decay by scan over neutrino momenta and fit of top and *W* masses.
- Three axes: K (helicity), R (transverse) and N (normal)
- No systematics study. A comparison with unfolding methods should be made by experiments.
- Background not considered: it is quite small for the dilepton decay
  [different flavour leptons / cut on same-flavour dilepton invariant mass]
- Fit results obtained by fluctuating templates assuming # events corresponding to 36.1 fb^{-1}.

Example for tt

As mentioned, there are two effects: distortion of the [normalised] templates and different efficiency factors for the different samples

Efficiency factors

Sample	ε
SM	0.174
CMDM	0.174

Template	ε K-axis	ε R-axis	ε N-axis
LL	0.177	0.175	0.178
RR	0.178	0.176	0.178
LR	0.160	0.188	0.171
RL	0.182	0.161	0.171

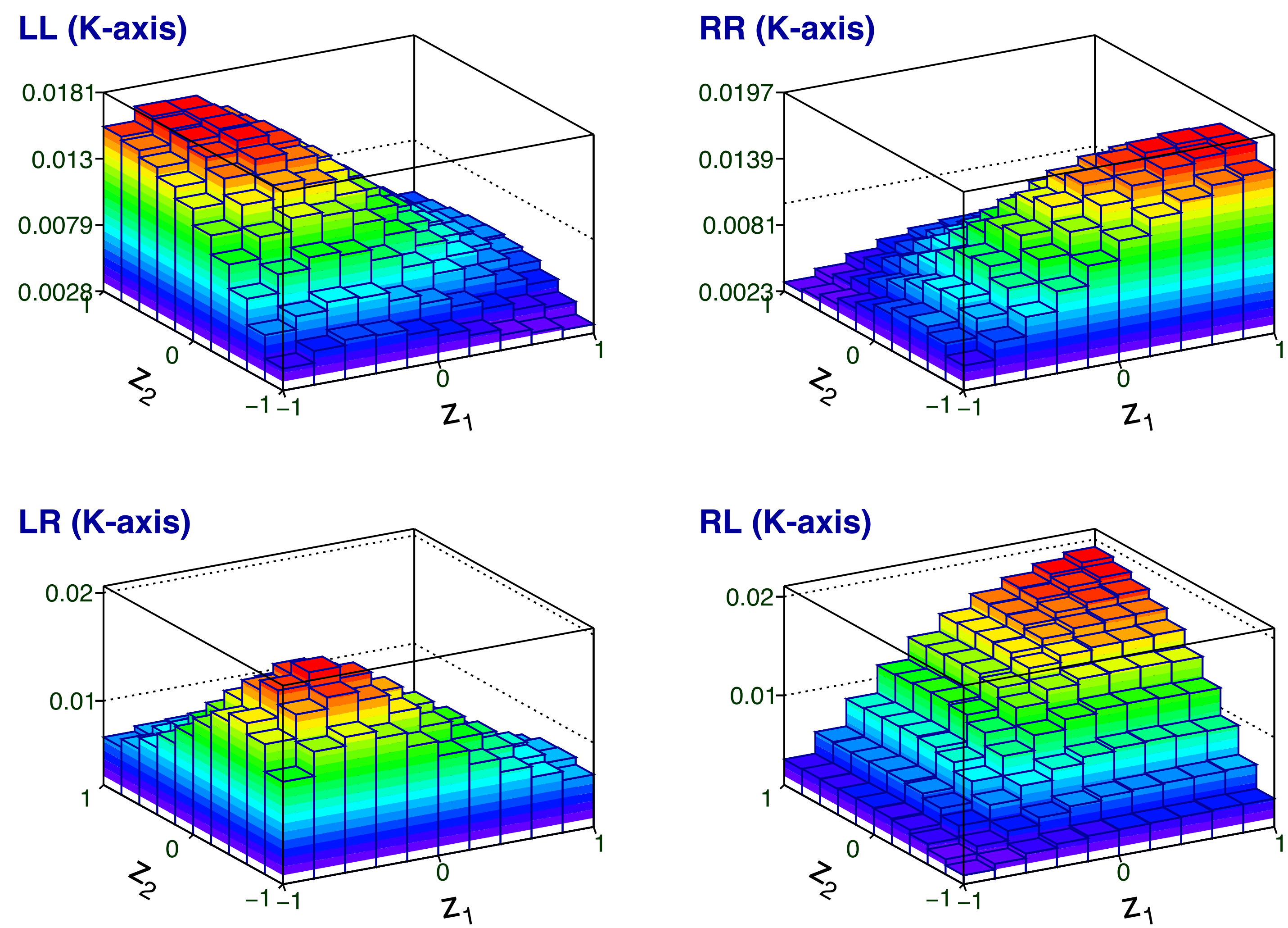
Values are similar but it is crucial to plug in the correct ones!

Example for tt

Also: distortion of [normalised] templates. Example: K axis

Normalised templates for K axis after simulation

$z_1 = \cos\theta_{\ell+} \quad z_2 = \cos\theta_{\ell-}$

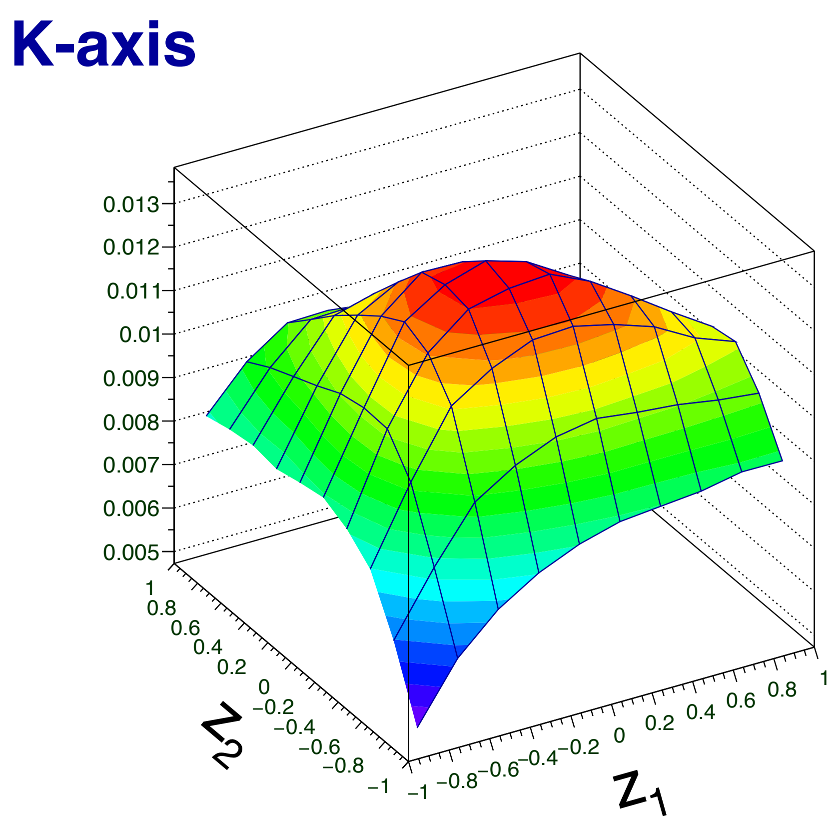


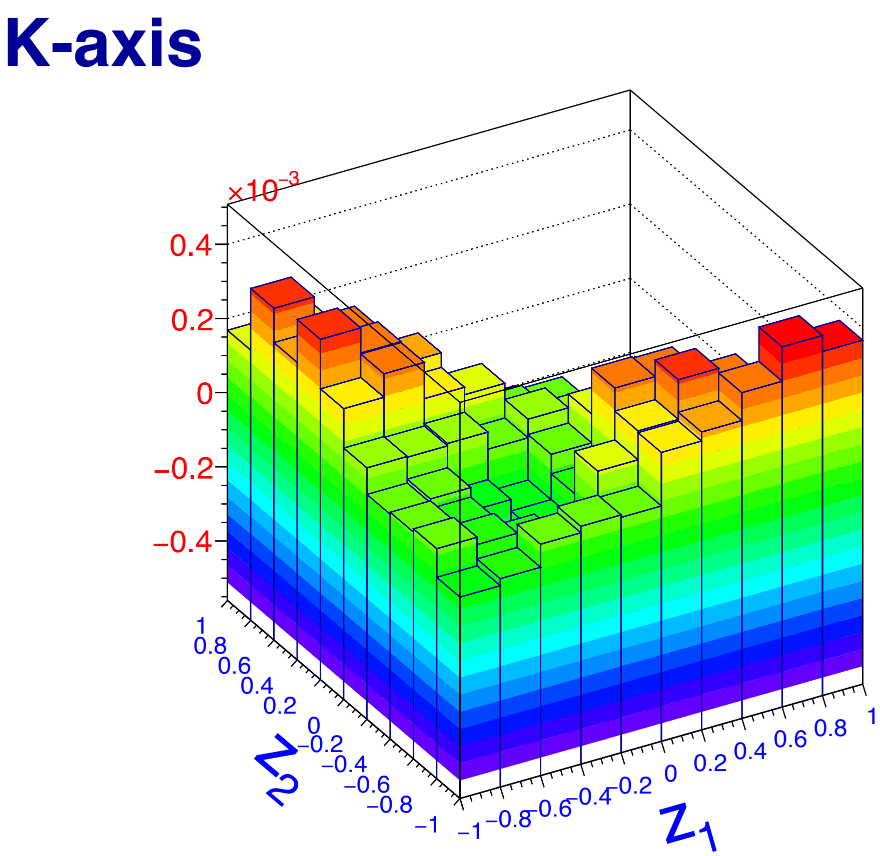
Example for tt

The interference term Δ_{int} is small but not negligible

SM distribution for K axis after simulation
$z_1 = \cos\theta_{\ell+} \quad z_2 = \cos\theta_{\ell-}$

Δ_{int} for K axis after simulation





Omitting Δ_{int} we get results with a bias of the order of the statistical uncertainty or larger

Example for tt

Fit results with statistical uncertainties for 36.1 fb^{-1}

K	SM		CMDM	
	Prediction	Fit	Prediction	Fit
a_{LL}	0.335 ± 0.001	0.337 ± 0.006	0.349 ± 0.001	0.350 ± 0.006
a_{RR}	0.336 ± 0.003	0.330 ± 0.005	0.349 ± 0.001	0.339 ± 0.005
a_{LR}	0.165 ± 0.003	0.167 ± 0.007	0.151 ± 0.001	0.175 ± 0.007
a_{RL}	0.165 ± 0.002	0.160 ± 0.004	0.151 ± 0.001	0.131 ± 0.004
C_{kk}	0.340 ± 0.002	0.340 ± 0.019	0.394 ± 0.004	0.383 ± 0.019
P_t	0.001 ± 0.002	-0.014 ± 0.008	0.000 ± 0.001	-0.058 ± 0.008
$P_{\bar{t}}$	0.001 ± 0.002	0.000 ± 0.008	0.001 ± 0.002	0.033 ± 0.008

R	SM		CMDM	
	Prediction	Fit	Prediction	Fit
a_{LL}	0.258 ± 0.001	0.254 ± 0.006	0.290 ± 0.002	0.291 ± 0.006
a_{RR}	0.259 ± 0.002	0.264 ± 0.006	0.289 ± 0.002	0.290 ± 0.006
a_{LR}	0.242 ± 0.001	0.236 ± 0.006	0.210 ± 0.001	0.210 ± 0.006
a_{RL}	0.241 ± 0.002	0.241 ± 0.006	0.211 ± 0.001	0.201 ± 0.006
C_{rr}	0.036 ± 0.002	0.041 ± 0.019	0.159 ± 0.002	0.170 ± 0.019
P_t	0.0004 ± 0.0005	0.015 ± 0.010	-0.001 ± 0.004	-0.011 ± 0.010
$P_{\bar{t}}$	0.002 ± 0.002	0.006 ± 0.010	-0.001 ± 0.003	0.008 ± 0.009

N	SM		CMDM	
	Prediction	Fit	Prediction	Fit
a_{LL}	0.333 ± 0.001	0.329 ± 0.004	0.358 ± 0.001	0.363 ± 0.004
a_{RR}	0.334 ± 0.002	0.329 ± 0.004	0.358 ± 0.001	0.352 ± 0.004
a_{LR}	0.166 ± 0.001	0.164 ± 0.004	0.142 ± 0.0003	0.138 ± 0.004
a_{RL}	0.167 ± 0.002	0.169 ± 0.004	0.142 ± 0.001	0.136 ± 0.004
C_{nn}	0.336 ± 0.002	0.325 ± 0.010	0.433 ± 0.002	0.442 ± 0.010
P_t	0.002 ± 0.001	0.005 ± 0.009	-0.001 ± 0.002	-0.014 ± 0.009
$P_{\bar{t}}$	0.000 ± 0.002	-0.005 ± 0.008	0.000 ± 0.001	-0.009 ± 0.009

Example for tt

Discussion

- The template method presented allows to access quantities that are not directly measured yet [individual a's].
- The fit converges well, the only deviations found may have a statistical origin [samples have ~ 200 K events]
- The calculation of Δ_{int} is valid not only for the SM but also for a sample with a large CMDM that yields quite different polarisation coefficients.

End

Top Quark Production Asymmetries A_{FB}^{t} and A_{FB}^{ℓ}

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