

## CP structure of the top-quark Yukawa interaction: NLO QCD corrections and off-shell effects

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#### LHC Higgs XS WG '16 2

 $\mathcal{BR}(H \rightarrow bb) \sim 58\%$ 

#### Introduction

#### ttH production:

- Observed for the first time in 2018, ATLAS <u>'18, CMS'18</u>
- Allows for direct probe of Yukawa interaction and it's CP nature at tree level
- Top is heaviest SM particle  $\rightarrow$  strongest Yukawa coupling
- Measurement of CP-odd component would indicate new physics

#### **Higgs production:**



#### **Higgs decay:**







### Introduction



#### SM-like interpretation:

Theoretical Particle Physic:

- Still freedom in the CP-state of the Higgs boson
- SM prediction: Higgs is CP-even
- CP-odd state excluded with 3.9  $\sigma$  (<u>ATLAS '20</u>) and 3.7  $\sigma$  (<u>CMS '22</u>)
- $\alpha_{CP} > 43^{\circ}$  excluded at 95% CL if CP-even and CP-odd couplings are equal (<u>ATLAS '20</u>),  $|f_{CP}^{\text{Htt}}| < 0.55$  at 68% (<u>CMS '22</u>)

#### **BSM** interpretations:

- Extended Higgs sector
- 2HDM

### Introduction



### Main Goal: $pp \to e^+ \nu_e \, \mu^- \bar{\nu}_\mu \, b\bar{b} \, H + X \text{ at } \mathcal{O}(\alpha_S^3 \alpha^5)$

Provide state-of-the-art predictions for ttH production with possible **CP-mixing** between CP-even (SM) and **CP-odd** Higgs states at **NLO in QCD** including **full off-shell effects** using **HELAC-NLO** 



- Which observables are sensitive to the CP-state?
- How are the different CP-states affected by NLO QCD corrections?
- How are the different CP-states affected by off-shell effects?



### **Theory status**



#### SM Higgs boson (stable tops):

• ttH @ NLO in QCD+EW with NNLL soft gluon resummation Broggio et al. '16, '17, '19, Kulesza, et al. '16, '18, '20

#### SM Higgs boson (with top quark decays):

- ttH @ NLO in QCD with full off-shell effects Denner, Feger '15
- ttH @ NLO in QCD+EW with full off-shell effects <u>Denner, Lang, Pellen, Uccirati '17</u>
- ttH @ NLO in QCD with full off-shell effects + Higgs decays in NWA <u>Stremmer, Worek '22</u>

#### Higgs boson with CP-odd admixture:

- ttX @ NLO in QCD with LO top decays matched to Parton Shower Demartin et al. '14
  - HC\_NLO\_X0 model Artoisenet et al. '13, Maltoni et al. '14, Demartin et al. '14, Demartin et al. '15
- ttX @ NLO in QCD with full off-shell effects JH, Stremmer, Worek '22

## The Higgs characterisation framework (HCF)

$$\begin{split} \textbf{SM:} \qquad \mathcal{L}_{t\bar{t}H} &= -\frac{Y_t}{\sqrt{2}} \bar{\psi}_t \psi_t H \\ \textbf{HCF:} \qquad \mathcal{L}_{t\bar{t}H} &= -\frac{Y_t}{\sqrt{2}} \bar{\psi}_t (\overbrace{\kappa_{Ht\bar{t}} \cos(\alpha_{CP}) + i\kappa_{At\bar{t}} \sin(\alpha_{CP}) \gamma_5} \psi_t H \\ \textbf{CP-even} \qquad \textbf{CP-odd} \\ \mathcal{L}_{HVV} &= \overbrace{\kappa_{HVV}} \left( \frac{g_{HZZ}}{2} Z_\mu Z^\mu + g_{HWW} W^+_\mu W^{-\mu} \right) H \\ \textbf{Additional coupling} \end{split}$$

#### **Coupling choices:**

Ensure consistency with current experimental bounds (ggF, VBF)









<u>Artoisenet et al. '13</u> <u>Maltoni et al. '14</u> <u>Demartin et al. '14</u> Demartin et al. '15

### Full off-shell effects







 $\mathcal{O}(\Gamma_t/m_t) \sim 0.8\%$ 

NWA = DR with on-shell masses

 $\frac{\Gamma}{-} \rightarrow 0$ 



### Integrated fiducial cross-sections (NLO)

	$\alpha_{CP}$		Off-shell	NWA	Off-shell effects
CP-		$\sigma_{\rm LO}$ [fb] $\sigma_{\rm NLO}$ [fb] $\sigma_{\rm NLO_{LOdec}}$ [fb]	$\begin{array}{c} 2.0313(2)^{+0.6275(31\%)}_{-0.4471(22\%)}\\ 2.466(2)^{+0.027(1.1\%)}_{-0.112(4.5\%)}\\ -\end{array}$	$\begin{array}{l} 2.0388(2)^{+0.6290(31\%)}_{-0.4483(22\%)}\\ 2.475(1)^{+0.027(1.1\%)}_{-0.113(4.6\%)}\\ 2.592(1)^{+0.161(6.2\%)}_{-0.242(9.3\%)}\end{array}$	-0.37% -0.36%
		$\mathcal{K}=\sigma_{\rm NLO}/\sigma_{\rm LO}$	1.21	1.21 (LOdec: 1.27)	
CP- (	$\pi/4$	$\sigma_{\rm LO}$ [fb] $\sigma_{\rm NLO}$ [fb] $\sigma_{\rm NLO_{LOdec}}$ [fb]	$\begin{array}{c} 1.1930(2)^{+0.3742(31\%)}_{-0.2656(22\%)}\\ 1.465(2)^{+0.016(1.1\%)}_{-0.071(4.8\%)}\\ -\end{array}$	$\begin{array}{c} 1.1851(1) {}^{+0.3707(31\%)}_{-0.2633(22\%)} \\ 1.452(1) {}^{+0.015(1.0\%)}_{-0.069(4.8\%)} \\ 1.517(1) {}^{+0.097(6.4\%)}_{-0.144(9.5\%)} \end{array}$	0.66% 0.89%
		$\mathcal{K}=\sigma_{\rm NLO}/\sigma_{\rm LO}$	1.23	1.23 (LOdec: 1.28)	
CP- (	odd $\pi/2$	$\sigma_{\rm LO}$ [fb] $\sigma_{\rm NLO}$ [fb] $\sigma_{\rm NLO_{\rm LOdec}}$ [fb]	$\begin{array}{c} 0.38277(6)^{+0.13123(34\%)}_{-0.09121(24\%)} \\ 0.5018(3)^{+0.0083(1.2\%)}_{-0.0337(6.7\%)} \\ -\end{array}$	$\begin{array}{c} 0.33148(3)^{+0.11240(34\%)}_{-0.07835(24\%)}\\ 0.4301(2)^{+0.0035(0.8\%)}_{-0.0264(6.1\%)}\\ 0.4433(2)^{+0.0323(7.3\%)}_{-0.0470(11\%)}\end{array}$	13.4% 14.3%
		$\mathcal{K}=\sigma_{\rm NLO}/\sigma_{\rm LO}$	1.31	1.30 (LOdec: 1.34)	

JH. Stremmer. Worek '22

#### NLO corrections:

- 21% 31% corrections
- Increase with the mixing angle
- Reduced scale uncertainties
- NLO with LO decays overestimates NLO results by a few percent

#### **Off-shell effects:**

- Small for CP-even and CP-mixed Higgs boson
- Large effects for CP-odd Higgs boson



### Integrated fiducial cross-sections (LO)





### **Differential distributions - NLO corrections**



#### **General behaviour:**

- Larger corrections in distribution tails
- Corrections largest for CP-odd case
- Shape of K-factor similar between different CP-states
- Harder Higgs radiation in CP-odd case



### **Differential distributions - NLO corrections**



#### **Observables with top-quark decay products:**

- Corrections largest for CP-odd case only for large opening angles
- For small opening angles, CP-odd case receives smallest corrections -> smaller shape distortions
- Harder Higgs radiation in CP-odd case suppresses K-factor
- CP-even and CP-mixed very similar



### Differential distributions - Off-shell effects



#### Shape comparison:

- CP-even and CP-mixed similar, small difference in tails
- Tails much more pronounced in CP-odd case

#### Off-shell effects:

- Large effects on size and shape for CP-odd Higgs boson
- Only small effects for CP-even and CP-mixed



### **Differential distributions - Off-shell effects**



#### Shape comparison:

- CP-even and CP-mixed similar, large difference in tails
- In the tails, the CP-odd cross-section is actually the largest

#### **Off-shell effects:**

• Large effects for all CP-states above kinematic edge, largest for CP-odd



## What is causing the large off-shell effects?





off-shell calculation

Try to extract the different contributions from the full result



## What is causing the large off-shell effects?

Investigate different resonance regions:



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### What is causing the large off-shell effects?



*р*<sub>Т, Н</sub> [GeV]

#### JH, Stremmer, Worek '22



SR contributions (~ tWHb) lead to larger off-shell effects in CP-odd case (also at integrated level)

### Conclusions



- Provided predictions for ttH production at NLO in QCD with full off-shell effects including CP-mixing in the Yukawa coupling
- NLO corrections
  - Around **20 % 30 %** at integrated level, larger in distribution tails
  - Overall larger effects for CP-odd Higgs but smaller impact on distribution shapes
- Off-shell effects important
  - Large effects in distribution tails and around kinematic edges
  - Large effects already at integrated level for CP-odd Higgs
  - Break symmetry in mixing angle
  - Enhancement of single resonant contributions for CP-odd Higgs
- Many observables affected by CP-mixing, e.g.  $\sigma$ ,  $M_{T2,t}$ ,  $M_{e^+b}$ ,  $\cos \theta_{ll}^*$ , ...



# Thank you for your attention!



### Backup

### Outlook

How can these predictions be used?

- Comparison to data (with parton level unfolding) in fiducial phase-space regions
  - Has been done for tt <u>Czakon et al. '20</u>, <u>CMS '22</u>

and tty <u>Bevilacqua et al. '18 '19 '20</u>, <u>ATLAS '20</u>

- Combine with tt+X predictions matched to Parton showers to approximately take into account off-shell effects
  - Has been done for ttW <u>Bevilacqua et al. '22</u>
- Resonance-aware matching to Parton showers
  - Has been done for tt <u>Jezo et al. '16</u>





### **HELAC-NLO**

- Store events in Les Houches Event files Alwall et al. '06 or Root Ntuples Antcheva et al. '09, Bern et al. '14
- Use **HEPlot** <u>Bevilacqua (unpublished)</u> for histograms
  - **Flexible cuts** 0
  - Reweighting to different scales / PDF sets Ο

HELAC-NLO

Pittau, Worek '13





Bevilacqua, Czakon, Kubocz, Worek '13

### Inputs



#### PDF: NNPDF31-nlo-as-0118

**Parameters:** 

#### Inputs



Cuts: $p_{T,\,\ell} > 25 \,\,{\rm GeV}\,,$  $p_{T,\,b} > 25 \,\,{\rm GeV}\,,$  $|y_\ell| < 2.5\,,$  $|y_b| < 2.5\,,$ 

Jet-clustering:  $anti-k_T$  jet algorithm R = 0.4

Scale choice:  $\mu_0=\mu_R=\mu_F=H_T/2$ 

$$H_T = p_{T,b_1} + p_{T,b_2} + p_{T,e^+} + p_{T,\mu^-} + p_{T,miss} + p_{T,H}$$

Scale variation

ation: 
$$\left(\frac{\mu_R}{\mu_0}, \frac{\mu_F}{\mu_0}\right) = \left\{ (2,1), (0.5,1), (1,2), (1,1), (1,0.5), (2,2), (0.5,0.5) \right\}$$

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#### **Parameter choices**

#### $\underline{\kappa_{Ht\bar{t}}}$

• Choose  $\kappa_{Ht\bar{t}} = 1$  to recover SM results for  $\alpha_{CP} = 0$ 

#### $\underline{\kappa_{At\bar{t}}}$

- Choose  $\kappa_{At\bar{t}} = 1$  to have the same coupling as for CP-even part
- Choose  $\kappa_{At\bar{t}} = 2/3$  to be consistent with gluon-gluon fusion (ggF) measurements (ATLAS '21)

#### $\underline{\kappa_{HVV}}$

$$\mathcal{L}_{HVV} = \kappa_{HVV} \left( \frac{g_{HZZ}}{2} Z_{\mu} Z^{\mu} + g_{HWW} W^{+}_{\mu} W^{-\mu} \right) H$$





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#### $\kappa_{HVV}$

- Choose  $\kappa_{HVV} = 1$  to be consistent with vector-boson fusion (VBF) measurements (CMS '19)
- Choose  $\kappa_{HVV} = \cos(\alpha_{CP})$  to avoid coupling of pseudoscalar particle to vector bosons (e.g. 2HDM)



## Integrated fiducial cross-sections (LO)

#### Interpolation formula:

 $\sigma\left(\alpha_{CP}\right) = \cos^{2}\left(\alpha_{CP}\right) \kappa_{Ht\bar{t}}^{2} \sigma_{1} + \sin^{2}\left(\alpha_{CP}\right) \kappa_{At\bar{t}}^{2} \sigma_{2} + \cos\left(\alpha_{CP}\right) \sin\left(\alpha_{CP}\right) \kappa_{Ht\bar{t}} \kappa_{At\bar{t}} \sigma_{3} - +\cos\left(\alpha_{CP}\right) \kappa_{Ht\bar{t}} \kappa_{HVV} \left(\alpha_{CP}\right) \sigma_{4} + \sin\left(\alpha_{CP}\right) \kappa_{At\bar{t}} \kappa_{HVV} \left(\alpha_{CP}\right) \sigma_{5} + \kappa_{HVV}^{2} \left(\alpha_{CP}\right) \sigma_{6}.$ 

 No interference between diagrams with CP-even and CP-odd Yukawa interactions
 No interference between diagrams with HVV and CP-odd Yukawa interactions
 No HVV couplings in NWA





## Integrated fiducial cross-sections (LO)

Interpolation formula (without vanishing terms):

$$\sigma(\alpha_{CP}) = \cos^2(\alpha_{CP}) \kappa_{Ht\bar{t}}^2 \sigma_1 + \sin^2(\alpha_{CP}) \kappa_{At\bar{t}}^2 \sigma_2 + \cos(\alpha_{CP}) \kappa_{Ht\bar{t}} \kappa_{HVV} (\alpha_{CP}) \sigma_4 + \kappa_{HVV}^2 (\alpha_{CP}) \sigma_6$$

- First two terms are symmetric in  $\alpha_{CP}$
- Last term is either constant (  $\kappa_{HVV} = 1$  ) or symmetric ( $\kappa_{HVV} = \cos(\alpha_{CP})$ ) with respect to  $\alpha_{CP}$



Interference between diagrams with HVV and CP-even Yukawa interactions breaks the symmetry

	Off-shell	NWA	
$\sigma_1$ [fb]	2.0643(4)	2.0388(2)	
$\sigma_2$ [fb]	0.7800(1)	0.74583(7)	
$\sigma_3$ [fb]	-0.0002(8)	-0.0001(3)	
$\sigma_4$ [fb]	-0.0693(8)	) –	
$\sigma_5  [{\rm fb}]$	-0.0001(9)	_	
$\sigma_6$ [fb]	0.0363(9)	_	



## Integrated fiducial cross-sections (NLO)

Interpolation formula (without vanishing terms):

$$\sigma(\alpha_{CP}) = \cos^2(\alpha_{CP}) \kappa_{Ht\bar{t}}^2 \sigma_1 + \sin^2(\alpha_{CP}) \kappa_{At\bar{t}}^2 \sigma_2 + \cos(\alpha_{CP}) \kappa_{Ht\bar{t}} \kappa_{HVV} (\alpha_{CP}) \sigma_4 + \kappa_{HVV}^2 (\alpha_{CP}) \sigma_6$$

**Problem:** The virtual contributions do not factorise in this manner

 $\rightarrow$  Interpolation much more complicated



### Integrated fiducial cross-sections (NLO)

 $\sigma_{NLO}$ 

$$_{,expanded} = \left(\frac{\Gamma_{NLO}}{\Gamma_{LO}}\right)^2 \cdot \sigma_{NLO} - 2\frac{\Gamma_{NLO} - \Gamma_{LO}}{\Gamma_{LO}} \cdot \sigma_{LO}$$

$\alpha_{CP}$		Off-shell	NWA	Off-shell effects
0 (SM)	$ \begin{aligned} \sigma_{\rm LO} ~ [\rm fb] \\ \sigma_{\rm NLO} ~ [\rm fb] \\ \sigma_{\rm NLO_{\rm LOdec}} ~ [\rm fb] \\ \\ \mathcal{K} = \sigma_{\rm NLO}/\sigma_{\rm LO} \end{aligned} $	$\begin{array}{c} 2.0313(2)^{+0.6275(31\%)}_{-0.4471(22\%)}\\ 2.466(2)^{+0.027(1.1\%)}_{-0.112(4.5\%)}\\ -\\ 1.21\end{array}$	$2.0388(2)^{+0.6290}_{-0.4483}(22\%)$ $2.475(1)^{+0.027}_{-0.113}(4.6\%)$ $2.592(1)^{+0.161}_{-0.242}(9.3\%)$ 1.21  (LOdec: 1.27)	-0.37% -0.36%
π/4	$\sigma_{ m LO}$ [fb] $\sigma_{ m NLO}$ [fb] $\sigma_{ m NLO_{ m LOdec}}$ [fb]	$\begin{array}{c} 1.1930(2)^{+0.3742(31\%)}_{-0.2656(22\%)}\\ 1.465(2)^{+0.016(1.1\%)}_{-0.071(4.8\%)}\\ -\end{array}$	$\begin{array}{c} 1.1851(1)^{+0.3707(31\%)}_{-0.2633(22\%)}\\ 1.452(1)^{+0.015(1.0\%)}_{-0.069(4.8\%)}\\ 1.517(1)^{+0.097(6.4\%)}_{-0.144(9.5\%)}\end{array}$	0.66% 0.89%
π/2	$\begin{split} \mathcal{K} &= \sigma_{\rm NLO}/\sigma_{\rm LO} \\ \\ \sigma_{\rm LO} ~ [\rm fb] \\ \\ \sigma_{\rm NLO} ~ [\rm fb] \\ \\ \\ \sigma_{\rm NLO_{\rm LOdec}} ~ [\rm fb] \end{split}$	$\begin{array}{c} 1.23\\ 0.38277(6)^{+0.13123(34\%)}_{-0.09121(24\%)}\\ 0.5018(3)^{+0.0083(1.2\%)}_{-0.0337(6.7\%)}\\ -\end{array}$	$\begin{array}{c} 1.23 \text{ (LOdec: } 1.28 \text{)} \\ \\ 0.33148(3) {}^{+0.11240(34\%)}_{-0.07835(24\%)} \\ 0.4301(2) {}^{+0.0035(0.8\%)}_{-0.0264(6.1\%)} \\ 0.4433(2) {}^{+0.0323(7.3\%)}_{-0.0470(11\%)} \end{array}$	13.4% 14.3%
,	$\mathcal{K} = \sigma_{ m NLO}/\sigma_{ m LO}$	1.31	1.30 (LOdec: 1.34)	

#### **Expanded NWA:**

- CP-even: 2.418 fb (-2.3 %)
- CP-mixed: 1.417 fb (-2.4%)
- CP-odd: 0.416 fb (-3.2 %)



### **Differential distributions - NLO corrections**



#### **Observables with top-quark decay products:**

- Corrections largest for CP-odd case only for small transverse momenta
- For large momenta, CP-odd case receives smallest corrections -> smaller shape distortions
- Harder Higgs radiation in CP-odd case suppresses K-factor
- CP-even and CP-mixed very similar

### **Differential distributions**





• Shape comparison:

- CP-even and CP-mixed similar, small differences around 1 and -1
- Significant differences for CP-odd case

#### Off-shell effects:

Significant effects on size and shape for

CP-odd Higgs boson

- Only small effects for CP-even and
  - **CP-mixed**



### **Differential distributions - NLO corrections**



#### NLO corrections to top-quark decays:

- Almost no difference between the CP-states
- Significant shape distortions

## SM Higgs boson decays



- Include SM Higgs boson decays in NWA (only Higgs on-shell)
- Decay events generated from LHEF in Higgs boson rest frame
- NLO QCD corrections to Higgs decays included

$$d\sigma = d\sigma_{t\bar{t}H} \frac{d\Gamma_{H\to X}}{\Gamma_{H}}$$
$$= d\sigma_{t\bar{t}H}^{0} \frac{d\Gamma_{H\to X}^{0}}{\Gamma_{H}} + d\sigma_{t\bar{t}H}^{1} \frac{d\Gamma_{H\to X}^{0}}{\Gamma_{H}} + d\sigma_{t\bar{t}H}^{0} \frac{d\Gamma_{H\to X}^{1}}{\Gamma_{H}}$$

• Four decay channels

(i)  $H \to b\bar{b}$ (ii)  $H \to \tau^+ \tau^-$ (iv)  $H \to Z^*Z^* \to e^+e^-e^+e^-$ 



## SM Higgs boson decays

	$\sigma_{ m LO}$	$\sigma_{ m NLO}$	$\mathcal{K}$
	[fb]	[fb]	
Stable Higgs	$2.2130(2)^{+30.1\%}_{-21.6\%}$	$2.728(2)^{+1.1\%}_{-4.7\%}$	1.23
$H  o b \overline{b}$	$0.8304(2)^{+44.4\%}_{-28.7\%}$	$0.9456(8)^{+2.5\%}_{-9.5\%}$	1.14
$H \to \tau^+ \tau^-$	$0.11426(2)^{+30.0\%}_{-21.6\%}$	$0.1418(1)^{+1.2\%}_{-4.8\%}$	1.24
$H \to \gamma \gamma$	$0.0037754(8)^{+30.0\%}_{-21.6\%}$	$0.004552(4)^{+0.9\%}_{-4.1\%}$	1.21
$H \to e^+ e^- e^+ e^-$	$1.0083(7) \cdot 10^{-5+30.2\%}_{-21.6\%}$	$1.313(4) \cdot 10^{-5+1.8\%}_{-6.2\%}$	1.30

- Integrated cross-sections ordered according to branching ratio
- Most distribution shapes similar to stable Higgs case
- Cuts on leptons reduce cross-section and affect distribution shapes for  $H \rightarrow e^+e^-e^+e^-$



### The 'stransverse' mass - idea





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### The 'stransverse' mass - idea & definition



### The 'stransverse' mass - distribution



• Not a 'hard' cut-off but drop-off is clearly visible





### The 'stransverse' mass - distribution



- Use b-jet + lepton instead of lepton as visible, massive 'particle'
  - Problem: which jet is associated with which lepton?
    - take minimum of invariant
       b-jet + lepton mass
       combinations
    - minimize the sum of the two invariant masses to avoid combining one lepton with both b-jets



### The 'stransverse' mass - definition

$$M_{T2}^{2} = \min_{\mathbf{p}_{T}^{\nu_{1}} + \mathbf{p}_{T}^{\nu_{2}} = \mathbf{p}_{T,\text{miss}}} \left[ \max\{M_{T}^{2}\left(\mathbf{p}_{T}^{(lb)_{1}}, \mathbf{p}_{T}^{\nu_{1}}\right), M_{T}^{2}\left(\mathbf{p}_{T}^{(lb)_{2}}, \mathbf{p}_{T}^{\nu_{2}}\right)\} \right]$$

where 
$$M_T^2 \left( \mathbf{p}_T^{(lb)_i}, \mathbf{p}_T^{\nu_i} \right) = M_{(lb)_i}^2 + 2 \left( E_T^{(lb)_i} E_T^{\nu_i} - \mathbf{p}_T^{(lb)_i} \mathbf{p}_T^{\nu_i} \right)$$

Lepton + b-jet combinations chosen such that  $\ M_{(lb)_1} + M_{(lb)_2}$  is minimal