Gegenbauer Goldstones
with some implications for top physics

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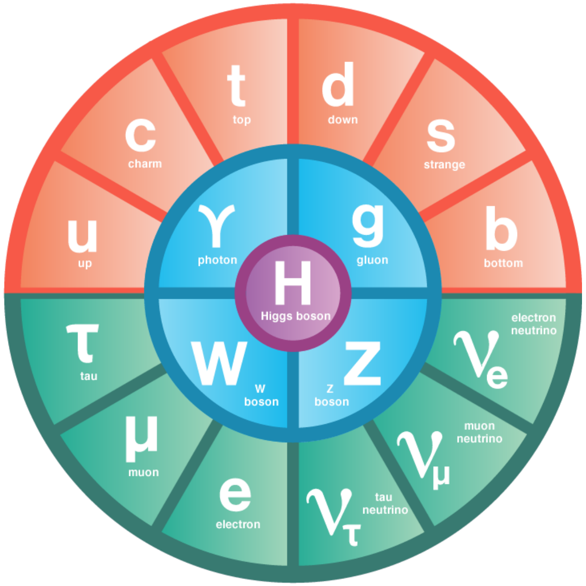
2110.06941 [JHEP] + 2202.01228 [JHEP] + 2209.00666

with Gauthier Durieux and Matthew McCullough (CERN)

symmetry

The Higgs mystery



$$V(H) = -m^2|H|^2 + \lambda|H|^4$$

Can we calculate it within a more fundamental theory?

Motivation

Other scalar particles we know: pions

They are composite pseudo Nambu-Goldstone bosons (pNGBs)

$$\Pi \sim (\bar{q}q)$$

$$\Pi \to \Pi + \theta f$$

Goldstone shift symmetry
(leading order transformation under broken generators)

$$f \sim 100 \text{ MeV}$$

Old question: could the Higgs field be a (composite) pNGB too?

[Kaplan, Georgi 1984]
[Kaplan 1992]
[Agashe, Contino, Pomarol 2004]
and many, many others

$$H \to H + \theta f \quad \xrightarrow{\text{at leading order}} \quad V(H) = 0$$

$$f \sim \text{TeV}$$

A light Higgs is natural

The need for $v \ll f$

Leading term of EFT for pNGBs:

$$\mathcal{L} = \frac{f^2}{2} D_\mu \phi^T D^\mu \phi$$

$SO(N+1)/SO(N)$

$$\phi = e^{i\Pi^a T^a/f} \begin{pmatrix} \vec{0}_N \\ 1 \end{pmatrix} = \begin{pmatrix} \sin\frac{\Pi}{f}\frac{\vec{\Pi}}{\Pi} \\ \cos\frac{\Pi}{f} \end{pmatrix} \qquad \Pi \equiv \sqrt{\vec{\Pi}^T \vec{\Pi}}$$

Introducing the SM weak interactions:

$$D_\mu \phi = \partial_\mu \phi - igW_\mu^a T_L^a \phi - ig' B_\mu T_R^3 \phi$$

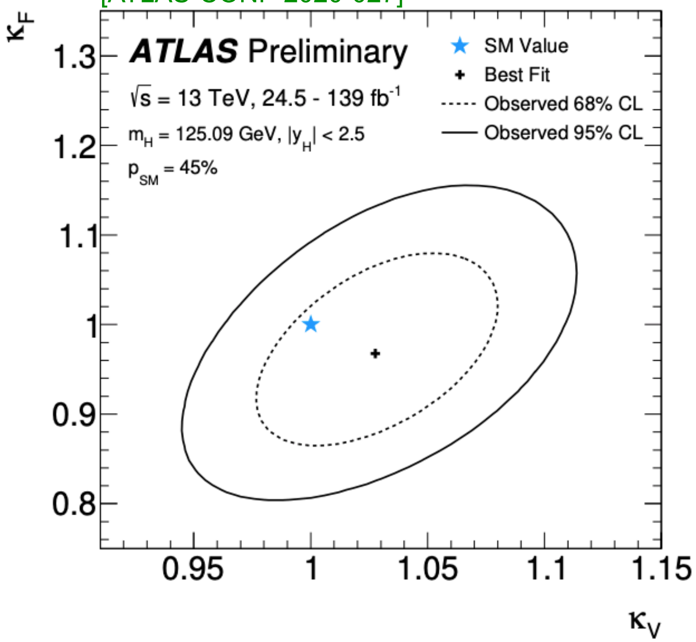
$$\frac{c_{hVV}}{c_{hVV}^{\text{SM}}} = \cos\frac{\langle \Pi \rangle}{f} = \sqrt{1 - \frac{v^2}{f^2}}$$

suppression
of Higgs couplings
to other SM particles

$v \approx 246 \text{ GeV}$

The need for $v \ll f$

[ATLAS-CONF-2020-027]



LHC Run 2:
Higgs couplings agree with SM to (10 ~ 20)%

Need $v \ll f$ by a factor 3 ~ 4 at least

$$\frac{c_{hVV}}{c_{hVV}^{\rm SM}} = \cos\frac{\langle\Pi\rangle}{f} = \sqrt{1-\frac{v^2}{f^2}}$$

suppression
of Higgs couplings
to other SM particles

$v \approx 246$ GeV

Realizing $v \ll f$

The vast majority of models require fine-tuning to achieve it

$$V_{1\ \mathrm{loop}} \sim \frac{y_t^2}{16\pi^2} M_T^2 f^2 \left(-\sin^2 \frac{\Pi}{f} + \sin^4 \frac{\Pi}{f} \right) \quad \Rightarrow \quad \Delta \sim \frac{v^2}{f^2} \lesssim 10\%$$

top partner mass

"minimal tuning" to get $v \ll f$

[Panico, Redi, Tesi, Wulzer 2012]

Here I present a new class of potentials giving this naturally: "Gegenbauer Goldstones"

$V_{1\ \mathrm{loop}}$ is dominated by top sector → link with top phenomenology

Outline

- ✓ Gegenbauer Goldstones
- ✓ Application to QCD-charged Naturalness
- ✓ Application to Neutral Naturalness

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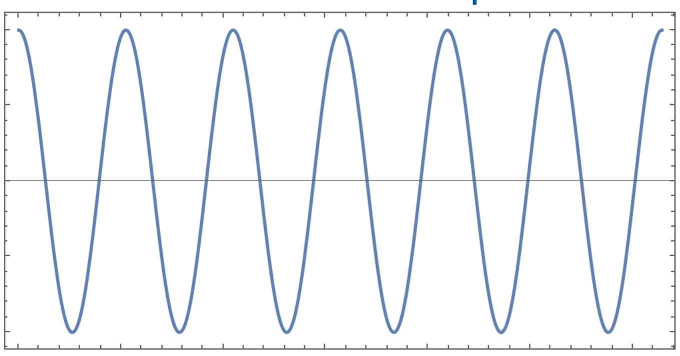
Warm-up: Abelian Goldstone

For a single $U(1)$ Goldstone,

we know a simple way to get $v/f \ll 1$



$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - \lambda \left(\Phi^* \Phi - f^2\right)^2$$

$$\frac{\langle \Pi \rangle}{f} = \frac{\pi}{n} \ll 1$$

Make it a pNGB: explicit breaking from operator of charge n

$$\delta V = \frac{\epsilon \lambda}{f^{n-4}} \Phi^n + \text{h.c.}$$

$$\Phi = f e^{i\Pi/f}$$

$$\delta V \sim \epsilon \lambda f^4 \cos\left(\frac{n\Pi}{f}\right)$$

$$\mathcal{Z}_n : \quad \Pi \to \Pi + \frac{2\pi}{n} f$$

Non-Abelian Goldstones

Consider N Goldstone bosons, from spontaneous breaking of global symmetry

$SO(N+1)/SO(N)$ (best studied pattern for pNGB Higgs)

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi - \lambda \left(\Phi^T \Phi - f^2\right)^2$$

How to get $v \ll f$ naturally?

Non-Abelian Goldstones

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$SO(N+1)/SO(N)$ (best studied pattern for pNGB Higgs)

$$\mathcal{L} = \frac{1}{2}\partial_\mu \Phi^T \partial^\mu \Phi - \lambda \left(\Phi^T \Phi - f^2\right)^2$$

Explicit breaking to $SO(N)$ by spurion in n-index symmetric tensor irrep of $SO(N+1)$

$$\delta V = \frac{\epsilon\,\lambda}{f^{n-4}} K_n^{i_1 \ldots i_n} \Phi_{i_1} \ldots \Phi_{i_n}$$

irrep $\rightarrow$ traceless

Radiatively stable at $O(\epsilon)$ and all loop orders, because only operator allowed.

Corrections at $O(\epsilon^2)$ and higher

[in d = 2: Brézin, Zinn-Justin, Le Guillou 1976]

Enter Gegenbauer

Parametrize

$$\Phi = f\phi \qquad \phi = e^{i\Pi^a T^a/f}\begin{pmatrix}\vec{0}_N \\ 1\end{pmatrix} = \begin{pmatrix}\sin\frac{\Pi}{f}\frac{\vec{\Pi}}{\Pi} \\ \cos\frac{\Pi}{f}\end{pmatrix} \qquad \Pi \equiv \sqrt{\vec{\Pi}^T\vec{\Pi}}$$

$$\delta V = \epsilon\lambda f^4 G_n^{(N-1)/2}\left(\cos\Pi/f\right)$$

potential is a Gegenbauer polynomial

$$\delta V = \frac{\epsilon\,\lambda}{f^{n-4}} K_n^{i_1\ldots i_n}\,\Phi_{i_1}\ldots\Phi_{i_n}$$

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Gegenbauer?

Leopold Gegenbauer
1849 - 1903

Gegenbauer?

Gegenbauer polynomials can be seen as generalization of Legendre polynomials to $D \neq 3$ spatial dimensions

$$D = 3 \qquad\qquad SO(3) \to SO(2)$$

multipole expansion of axi-symmetric function of spacetime coordinates

$$f(\vec{r}) = \sum_{\ell=0}^{\infty} c_\ell P_\ell(\cos\theta)$$

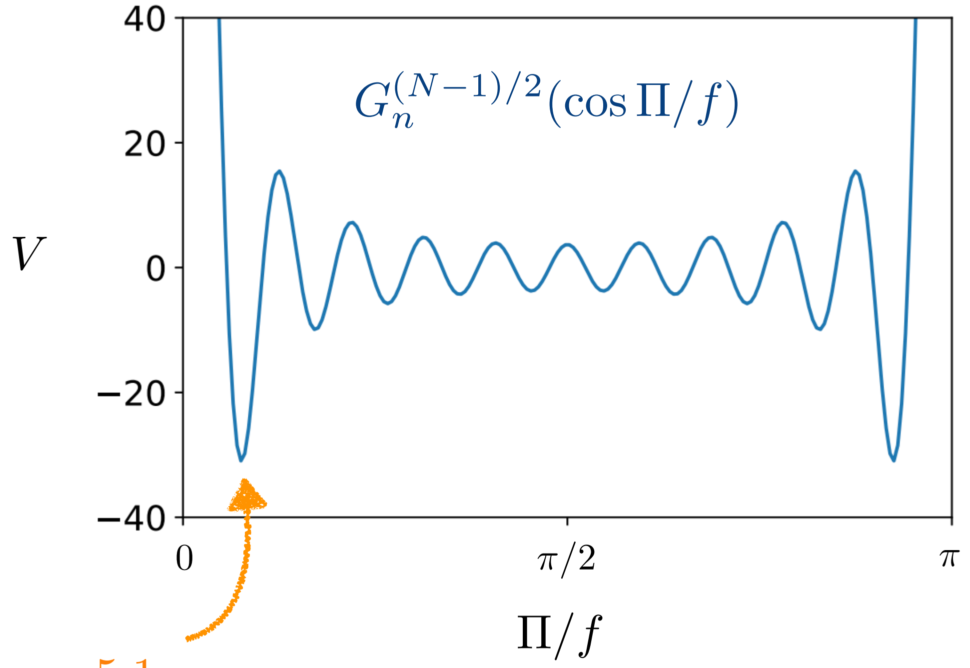
$$(m = 0)$$

They appear in many areas of physics, for example in the expansion of conformal blocks in CFT_d

[Hogervorst, Rychkov 2013]

Here, they arise from explicit breaking of internal symmetry $SO(N+1) \to SO(N)$, variables are pNGB fields

The shape of Gegenbauers



$N = 4$

$SO(5)/SO(4)$

Even n

$n = 20$

$$\frac{\langle\Pi\rangle}{f} \approx \frac{j_{N/2,1}}{n+\frac{N-1}{2}} \approx \frac{5.1}{n} \ll 1$$

for large n

Differently from Abelian case,

not periodic (only approximately)

A radiatively stable way to obtain $\langle\Pi\rangle \ll f$ for non-Abelian Goldstones

Outline

✓ Gegenbauer Goldstones

✓ Application to QCD-charged Naturalness

✓ Application to Neutral Naturalness

Composite Higgs potential: standard

In classic models, top sector loops break EW symmetry:

$$V = \frac{N_c y_t^2}{16\pi^2} M_T^2 f^2 \left(-\sin^2 \Pi/f + \sin^4 \Pi/f \right) + \delta_{\rm gauge} \sin^2 \Pi/f$$

"minimal tuning"
to get $v \ll f$

$$\Delta \sim \frac{v^2}{f^2}$$

(in unitary gauge, $\Pi = h$)

Composite Higgs potential: standard

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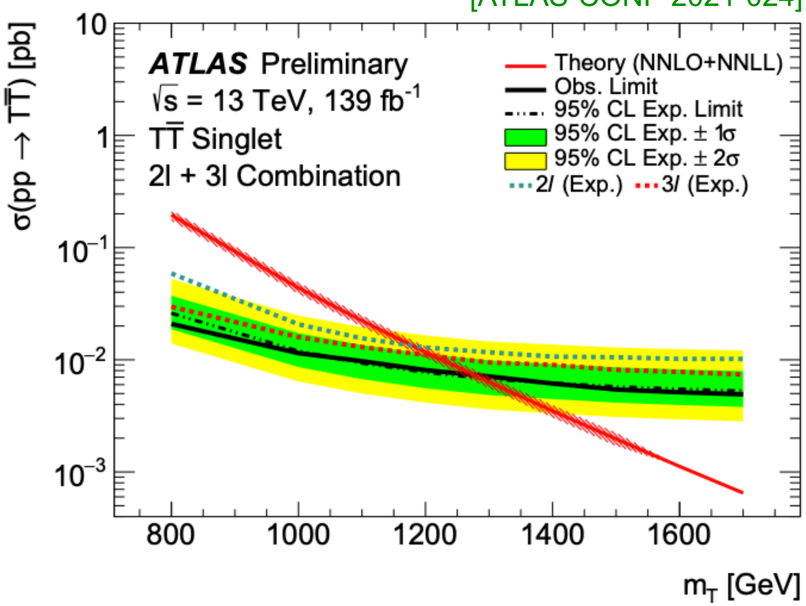
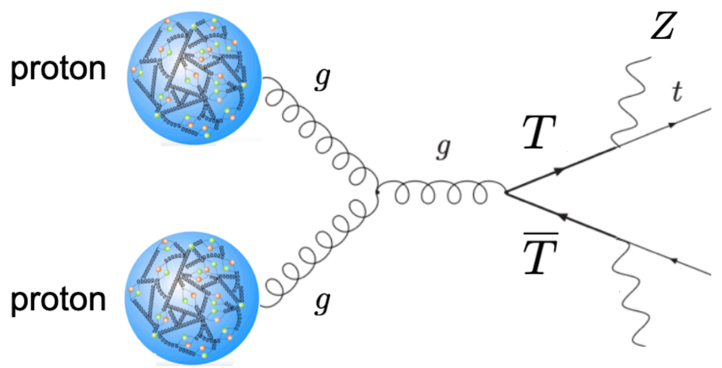
mass of QCD-charged top partners

LHC Run 2: $M_T \gtrsim 1.3\ \text{TeV}$

[ATLAS-CONF-2021-024]





Composite Higgs potential: Gegenbauer

Instead, we take EW preserving top contribution + Gegenbauer term:

$$V = \kappa \frac{N_c y_t^2}{16\pi^2} M_T^2 f^2 \left[+ \sin^2 \Pi/f \ + \gamma\, G_n^{(N-1)/2}(\cos \Pi/f) \right]$$

allows to tune overall size

positive sign

size of explicit breaking associated with Gegenbauer

The Gegenbauer is assumed to originate from UV breaking in non-minimal irrep

“Internal” to strong sector - think of quark masses in QCD

Composite Higgs potential: Gegenbauer

Instead, we take EW <u>preserving</u> top contribution + Gegenbauer term:

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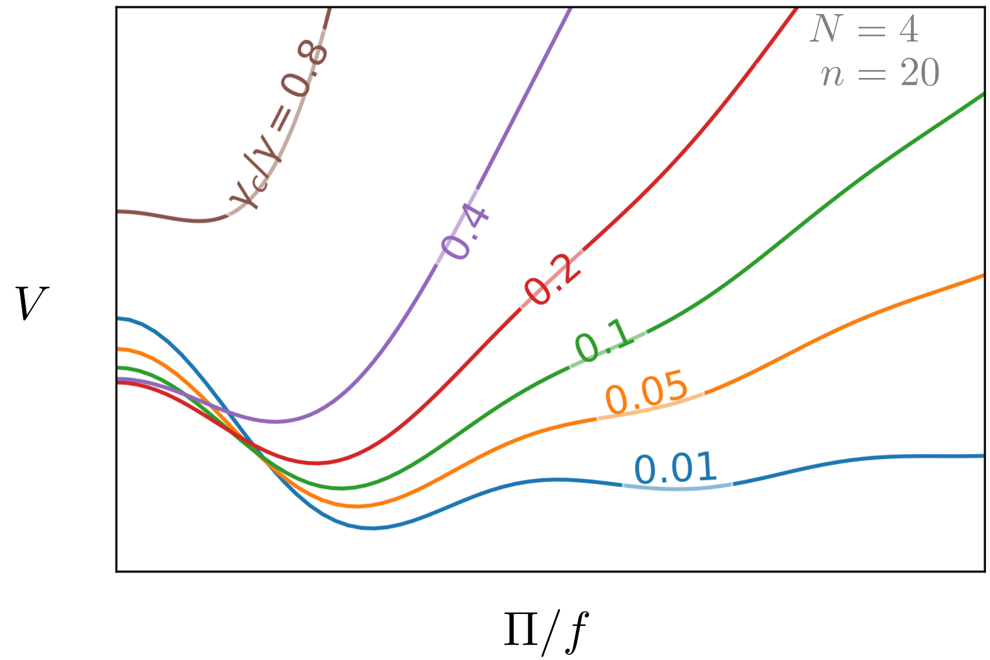

$\gamma_c \approx 8 \times 10^{-4}\, (10/n)^{3.6}$

for $\gamma < \gamma_c$, minimum is at origin

Composite Higgs potential: Gegenbauer

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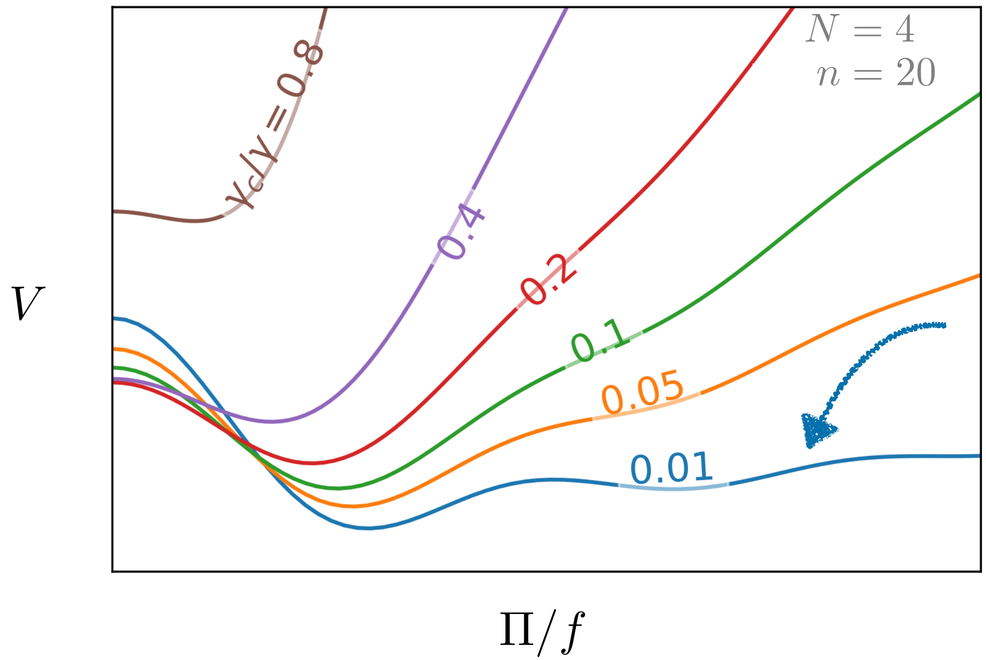


Gegenbauer dominates

no tuning to get $v \ll f$

but need $\kappa \ll 1$ tuning
for Higgs mass

for $\gamma < \gamma_c$, minimum is at origin

Composite Higgs potential: Gegenbauer

Instead, we take EW preserving top contribution + Gegenbauer term:

$$V = \kappa \frac{N_c y_t^2}{16\pi^2} M_T^2 f^2 \left[+ \sin^2 \Pi/f \ + \gamma\, G_n^{(N-1)/2}(\cos \Pi/f) \right]$$

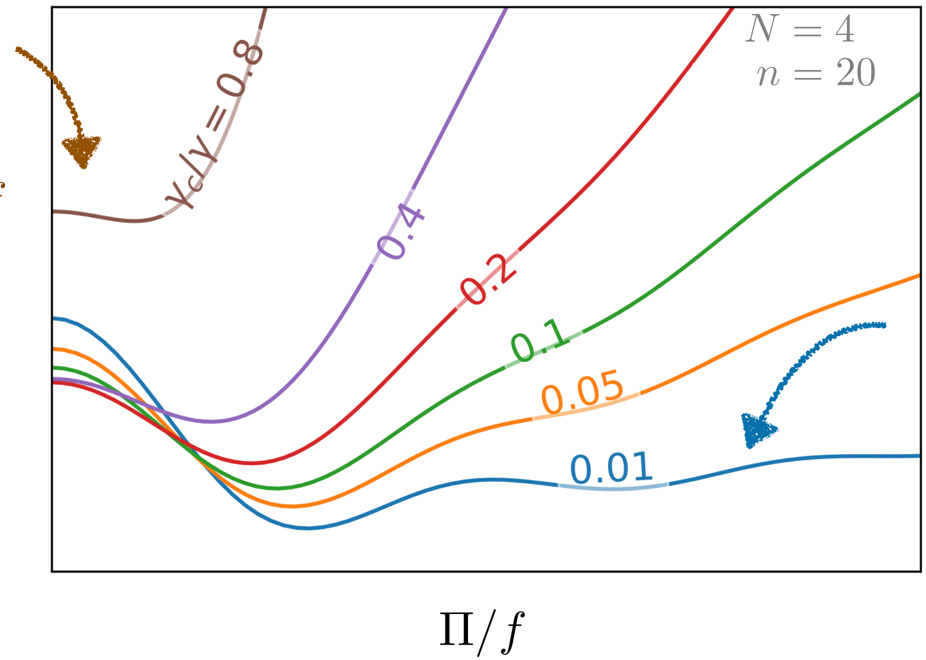
Gegenbauer is small perturbation

must tune to get $v \ll f$

😬

but $m_h = 125$ GeV for $\kappa \sim O(1)$

🙂



Gegenbauer dominates

no tuning to get $v \ll f$

🙂

but need $\kappa \ll 1$ tuning for Higgs mass

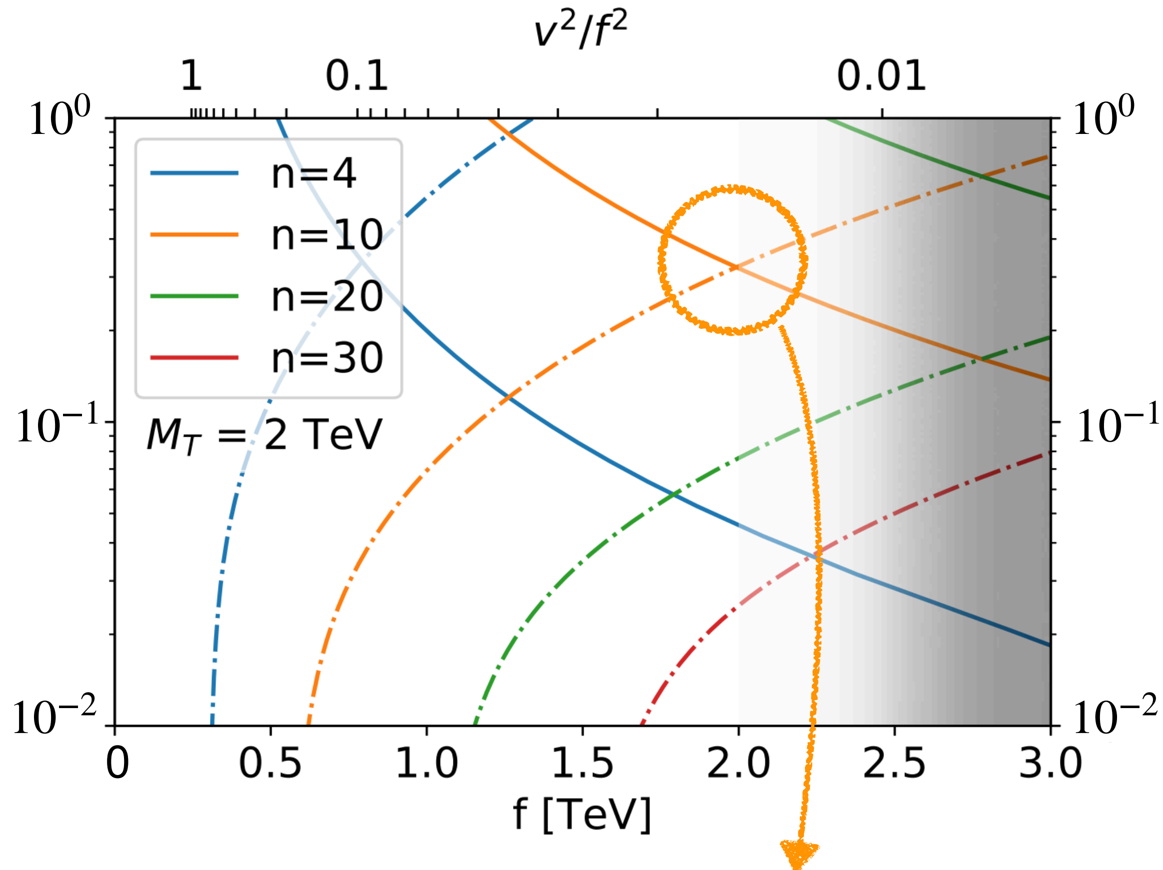
😬

Total tuning minimized in “intermediate” region

Quantifying the fine-tuning

$$\Delta = \left(\frac{\partial \log f/v}{\partial \log \gamma}\right)^{-1}$$

vev tuning
(solid)



κ

mass tuning
(dashed)

total tuning better than 10%
and beyond LHC reach

$n = 10$ $\quad M_T = f = 2\ \text{TeV}$ $\quad \frac{c_{hVV}}{c_{hVV}^{\text{SM}}} \approx 0.8\%$

(compare to standard estimate: $2v^2/f^2 \approx 3\%$)

Summary for QCD-charged Naturalness

In principle, Gegenbauer potential could realize $v \ll f$ naturally
in composite pNGB models

Due to LHC lower bounds on QCD-charged top partners,
radiative top sector potential is too large and forces some tuning

Still, can obtain ~ 10% tuning for colored top partners as heavy as 2 TeV,
while Higgs coupling deviations are out of reach.
Strengthens further the motivation to search for QCD-charged vectorlike fermions
at LHC, in all plausible decay modes

Outline

- ✓ Gegenbauer Goldstones
- ✓ Application to QCD-charged Naturalness
- ✓ Application to Neutral Naturalness

Gegenbauer’s Twin

Gegenbauer potential can realize $v \ll f$ naturally

But for standard composite pNGB Higgs, top sector potential still forces some tuning:
QCD-charged top partner masses are constrained by LHC data

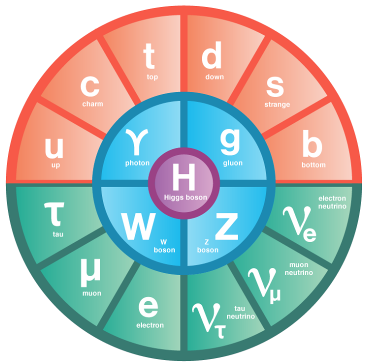
Twin Higgs models can reduce size of top contribution: top partners are QCD-neutral

[Chacko, Goh, Harnik 2005]

Could Gegenbauer’s Twin be fully natural?

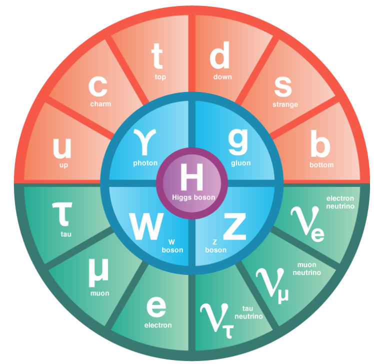
Twin Higgs

Standard Model



$$\xleftrightarrow{\mathcal{Z}_2}$$

Twin Standard Model



$$\mathcal{L} = y_t Q_A H_A t_A + \hat{y}_t Q_B H_B t_B$$

The top partners are neutral under whole SM (& charged under Twin QCD)
They can still be really light

The $\mathcal{Z}_2$ protects Higgs mass from <u>quadratic</u> corrections

Twin Higgs potential

Quartic terms do not cancel exactly, but resulting potential is not realistic:

$v = 0$ or $v = f$

$$V_t \approx \frac{3y_t^4 f^4}{64\pi^2}\left[\sin^4\frac{h}{f}\log\frac{a}{\sin^2 h/f} + \cos^4\frac{h}{f}\log\frac{a}{\cos^2 h/f}\right]$$

Need some form of $\mathcal{Z}_2$ breaking → again, tuning $\Delta = \frac{2v^2}{f^2}$

[Craig, Katz, Strassler, Sundrum 2015]
[Barbieri, Greco, Rattazzi, Wulzer 2015]

Gegenbauer's Twin

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Here, we introduce a Gegenbauer contribution:

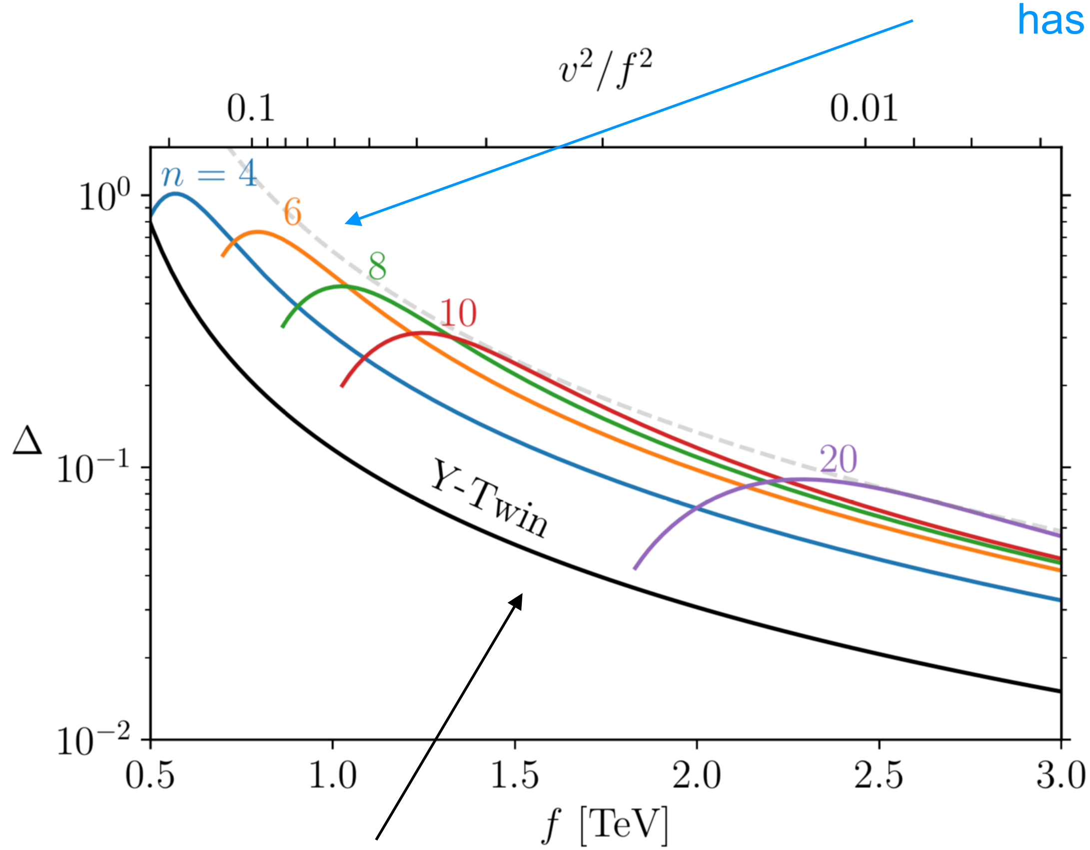
generalize construction to $SO(8) \to SO(4) \times SO(4)$ explicit breaking

$$V_G^{(n)} = \epsilon f^4 G_n^{3/2}(\cos 2h/f)$$

Gegenbauer's Twin

Fine tuning:

Gegenbauer's Twin with
$n = 6$ or $n = 8$ and $f \sim 1$ TeV
has essentially no tuning



standard Twin model (Twin hypercharge not gauged)

Top phenomenology

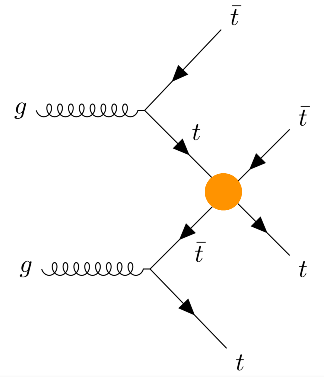
- Top partners are SM-neutral. Production only through Higgs portal, small rates

- But even if direct signatures are elusive, can probe corrections to Higgs/top couplings

- General preference for large compositeness of right-handed top

→ SMEFT operators built out of t_R (and H) expected with maximal size

Leading effect: $$\frac{1}{f^2}(\bar{t}_R \gamma^\mu t_R)^2$$



$f > (1.3 \text{ - } 1.7\ \text{TeV})$

[multi-lepton + jets projection, Banelli et al. 2010.05915]

$$\frac{c_{hXX}}{c_{hXX}^{\text{SM}}} = \sqrt{1 - \frac{v^2}{f^2}}$$

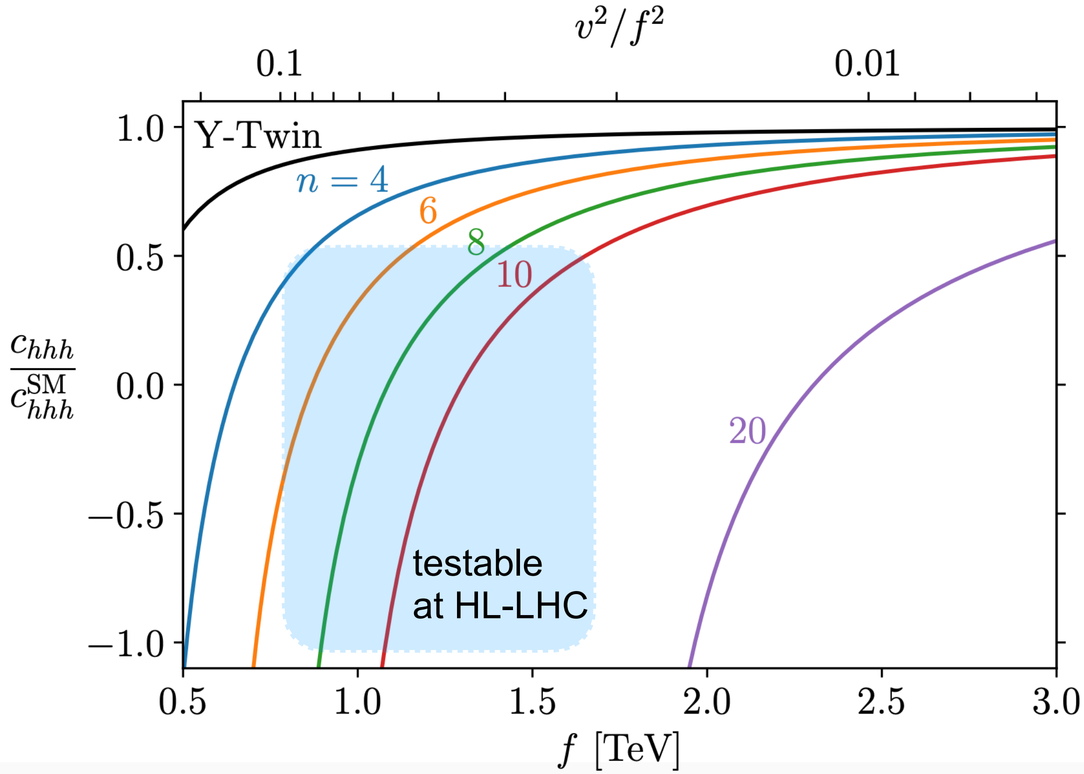
$f > 1.4\ \text{TeV}$

[de Blas et al. 1902.00134]

(HL-LHC)

Higgs cubic coupling

For Gegenbauer's Twin, corrections are parametrically enhanced



"Smoking gun" signal: could even be first deviation observed at LHC

[Durieux, McCullough, Salvioni 2202.01228 + 2209.00666]

Conclusions

Gegenbauer potentials can realize $v \ll f$ for a pNGB Higgs without fine-tuning

Require to drop often assumed "minimality criteria" about origin of (explicit) symmetry breaking

For QCD-charged Naturalness: irreducible size of top potential limits success, yet improvement over canonical models

For Neutral Naturalness: "Gegenbauer's Twin" shows that fully natural electroweak breaking is still compatible with LHC results.
Direct signatures could be elusive, but important deviations expected in Higgs and top interactions

Backup slides

More on radiative stability

Can also see Gegenbauers emerge from Coleman-Weinberg potential:

For general $SO(N)$ invariant potential $V = \epsilon\lambda f^4 \, G(\cos \Pi/f)$

quadratic piece of one-loop CW is

[Alonso, Jenkins, Manohar 2015]

$$' \equiv \frac{\partial}{\partial(\Pi/f)}$$

$$V_{\text{quantum}} = \epsilon\lambda f^4 \left[G + \frac{\Lambda^2}{32\pi^2 f^2}\left(G'' + (N-1)\cot\frac{\Pi}{f} G' \right) \right]$$

if $\propto G$, multiplicative renormalization!

More on radiative stability

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Indeed, Gegenbauers satisfy differential equation

$$G_n^{\alpha\,\prime\prime} + 2\alpha \cot\frac{\Pi}{f} G_n^{\alpha\,\prime} + n(n+2\alpha) G_n^\alpha = 0 \quad \longrightarrow \quad \alpha = \frac{N-1}{2}$$

Radiative corrections do not alter functional form

More on radiative stability

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if $\propto G$, multiplicative renormalization!

Abelian case is recovered for $N = 1$:

$$G'' \propto G \longrightarrow G = \cos\frac{n\Pi}{f}$$

Gegenbauers from irreps

$$\tilde{\phi} \equiv \begin{pmatrix} \vec{0}_N \\ 1 \end{pmatrix}$$

Consider scalar function

$$|t\phi - \tilde{\phi}|^{1-N} = \sum_{n=0}^{\infty} t^n K_n^{i_1 \ldots i_n} \phi^{i_1} \ldots \phi^{i_n}$$

Taylor expansion

$$K_n^{i_1 \ldots i_n} = \frac{1}{n!} \frac{\partial^n \phi^{1-N}}{\partial \phi_{i_1} \ldots \partial \phi_{i_n}} \bigg|_{\tilde{\phi}}$$

traceless, because Laplacian vanishes away from origin

//

$$\left(1 - 2t \cos \Pi/f + t^2\right)^{(1-N)/2}$$

||

$$\sum_{n=0}^{\infty} t^n G_n^{(N-1)/2}(\cos \Pi/f)$$

generating function for Gegenbauers is

$$\left(1 - 2tx + t^2\right)^{-\alpha} = \sum_{n=0}^{\infty} t^n G_n^{\alpha}(x)$$

Gegenbauers from irreps

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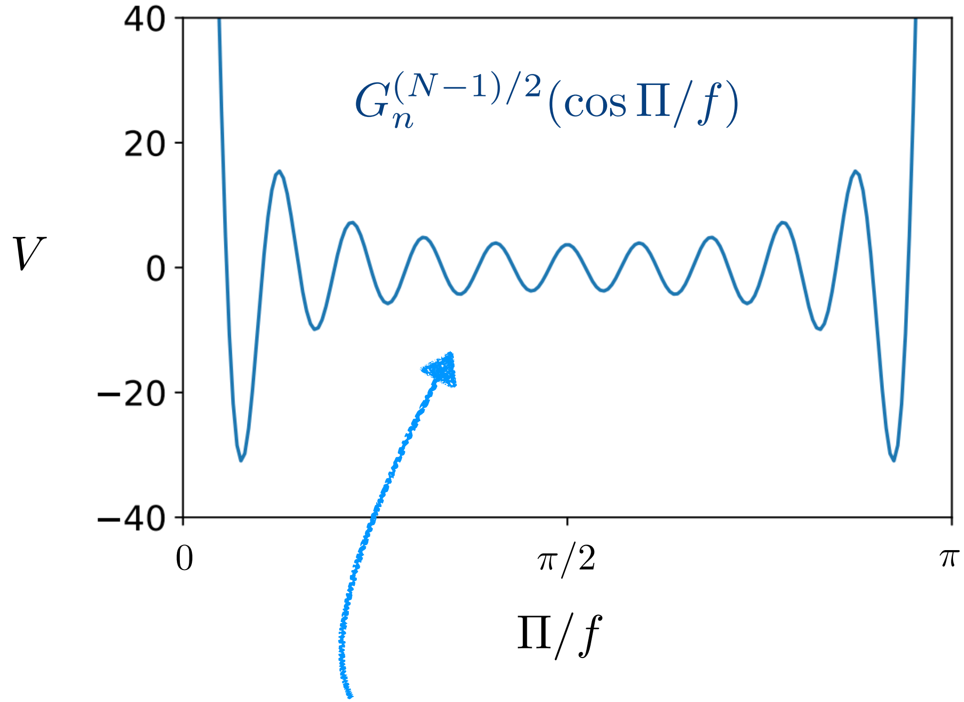
Gegenbauer polynomials from explicit $SO(N+1) \to SO(N)$ breaking

The shape of Gegenbauers



$N = 4$

$SO(5)/SO(4)$

Even n

$n = 20$

Differently from Abelian case, not periodic. Only approximately

$$G_n^\alpha\left(\cos\frac{\Pi}{f}\right) \xrightarrow{n\gg 1} \frac{J_{\alpha-1/2}\left((n+\alpha)\frac{\Pi}{f}\right)}{\Pi^{\alpha-1/2}} \xrightarrow{\frac{\Pi}{f}\gg\frac{1}{n}} \frac{\cos\left((n+\alpha)\frac{\Pi}{f}-\alpha\frac{\pi}{2}\right)}{\Pi^\alpha}$$