



Durham, 09/09/2022

Entanglement in SMEFT: top pair production

arXiv:2203.05619

Luca Mantani

In collaboration with:
R. Aoude, E. Madge, F. Maltoni



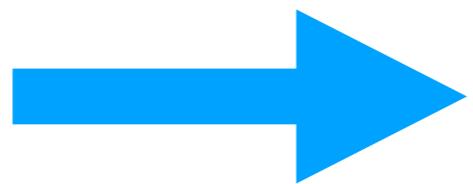
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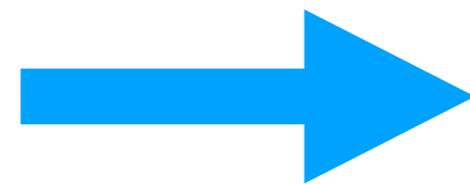
Quantum Information



Unveil the inner behaviour
of quantum mechanics.

Entanglement is a pure quantum phenomenon.
Measurement at high energies undertaken in recent years.

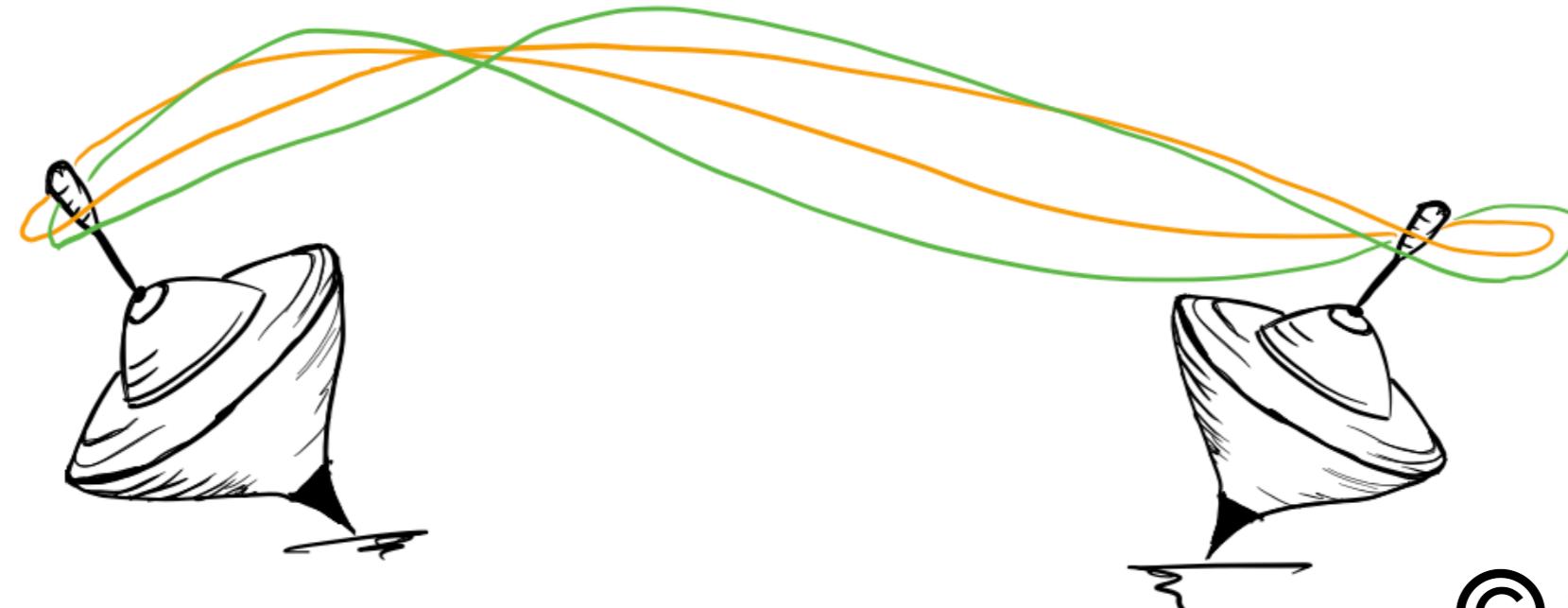
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Measurement at high energies undertaken in recent years.

Top pairs ideal probe: spin correlations preserved after decay



[arXiv:2003.02280]

[arXiv:2110.10112]

[arXiv:2102.11883]

[arXiv:2203.05582]

[arXiv:2205.00542]

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The fundamental object is the spin correlation matrix

$$R_{\alpha_1 \alpha_2, \beta_1 \beta_2}^I \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}_{\alpha_2 \beta_2}^* \mathcal{M}_{\alpha_1 \beta_1}$$

At LO in QCD
 $I = gg, q\bar{q}$

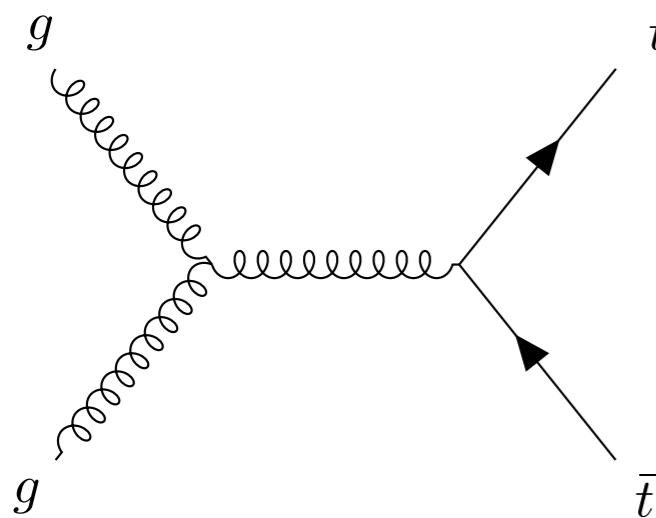
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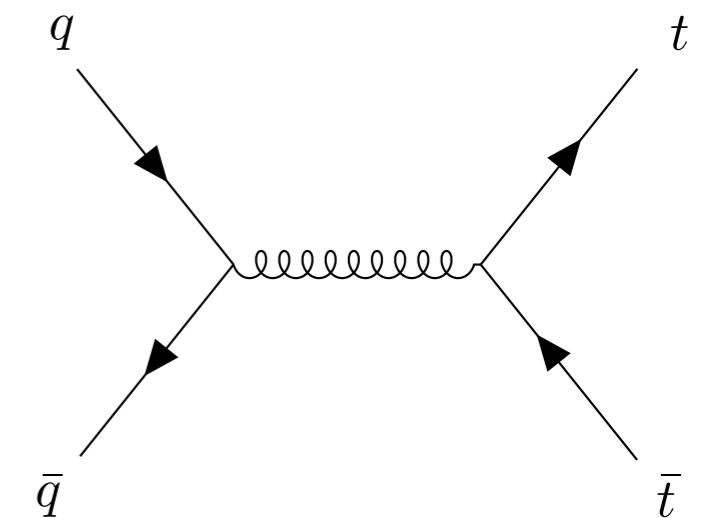
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We collide protons

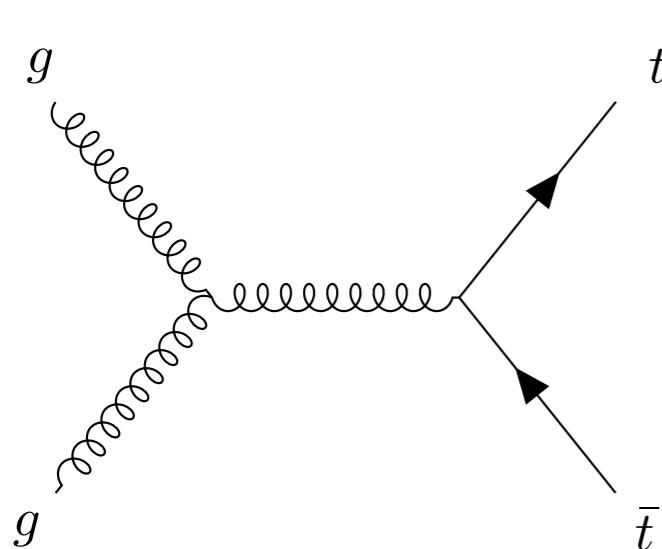


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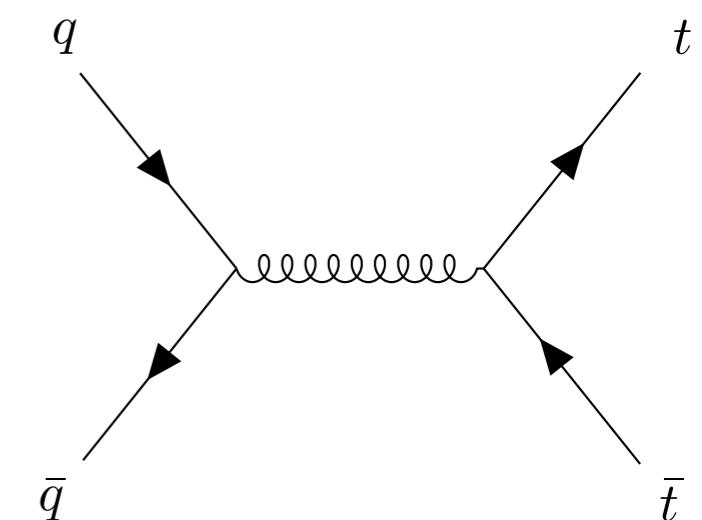
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$$R(\hat{s}, \mathbf{k}) = \sum_I L^I(\hat{s}) R^I(\hat{s}, \mathbf{k})$$



Full correlation matrix is mixed state, weighted by parton luminosity

The R matrix can be decomposed in the spin space

$$R = \tilde{A} \mathbf{1}_2 \otimes \mathbf{1}_2 + \tilde{B}_i^+ \sigma^i \otimes \mathbf{1}_2 + \tilde{B}_i^- \mathbf{1}_2 \otimes \sigma^i + \tilde{C}_{ij} \sigma^i \otimes \sigma^j$$

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Cross section

$$\frac{d\sigma}{d\Omega d\hat{s}} = \frac{\alpha_s^2 \beta}{\hat{s}^2} \tilde{A}(\hat{s}, \mathbf{k})$$

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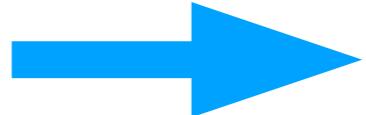
Spin correlations

If normalised, we define the density matrix of the system

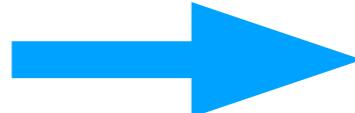
$$\rho = \frac{\mathbf{1}_2 \otimes \mathbf{1}_2 + B_i^+ \sigma^i \otimes \mathbf{1}_2 + B_i^- \mathbf{1}_2 \otimes \sigma^i + C_{ij} \sigma^i \otimes \sigma^j}{4}.$$

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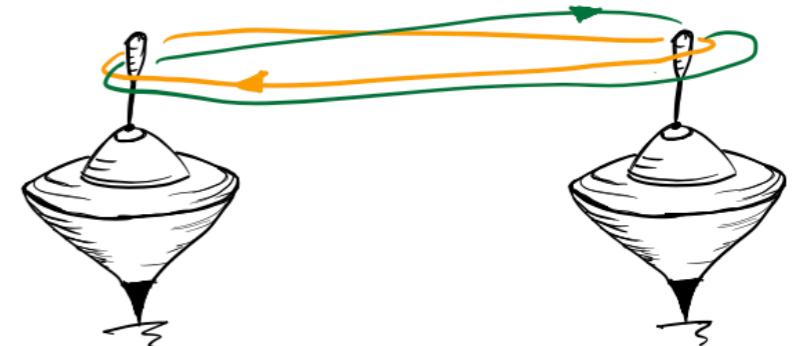
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Maximally entangled states

$$|\Phi^\pm\rangle = \frac{|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle}{\sqrt{2}} \quad |\Psi^\pm\rangle = \frac{|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle}{\sqrt{2}}$$

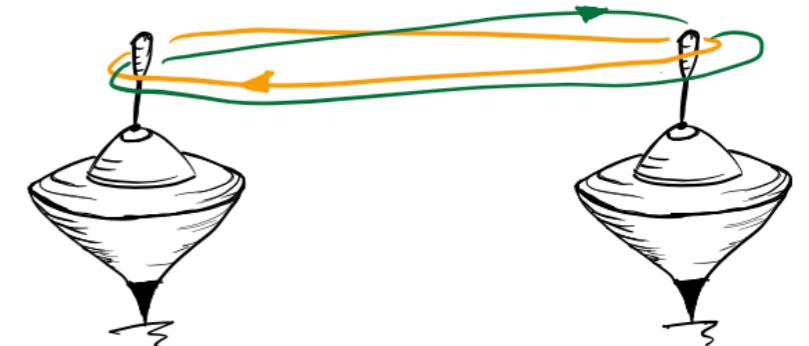


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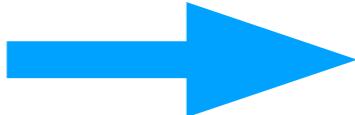


In the case of a statistical ensemble (mixed state)

$$\rho = \sum_k p_k \rho_k$$

entangled if $\rho_k \neq \rho_1 \otimes \rho_2$

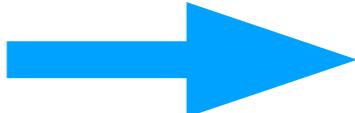
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Operative definition of entanglement: **Peres-Horodecki criterion**

$$\Delta = -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0 \quad \text{entangled}$$

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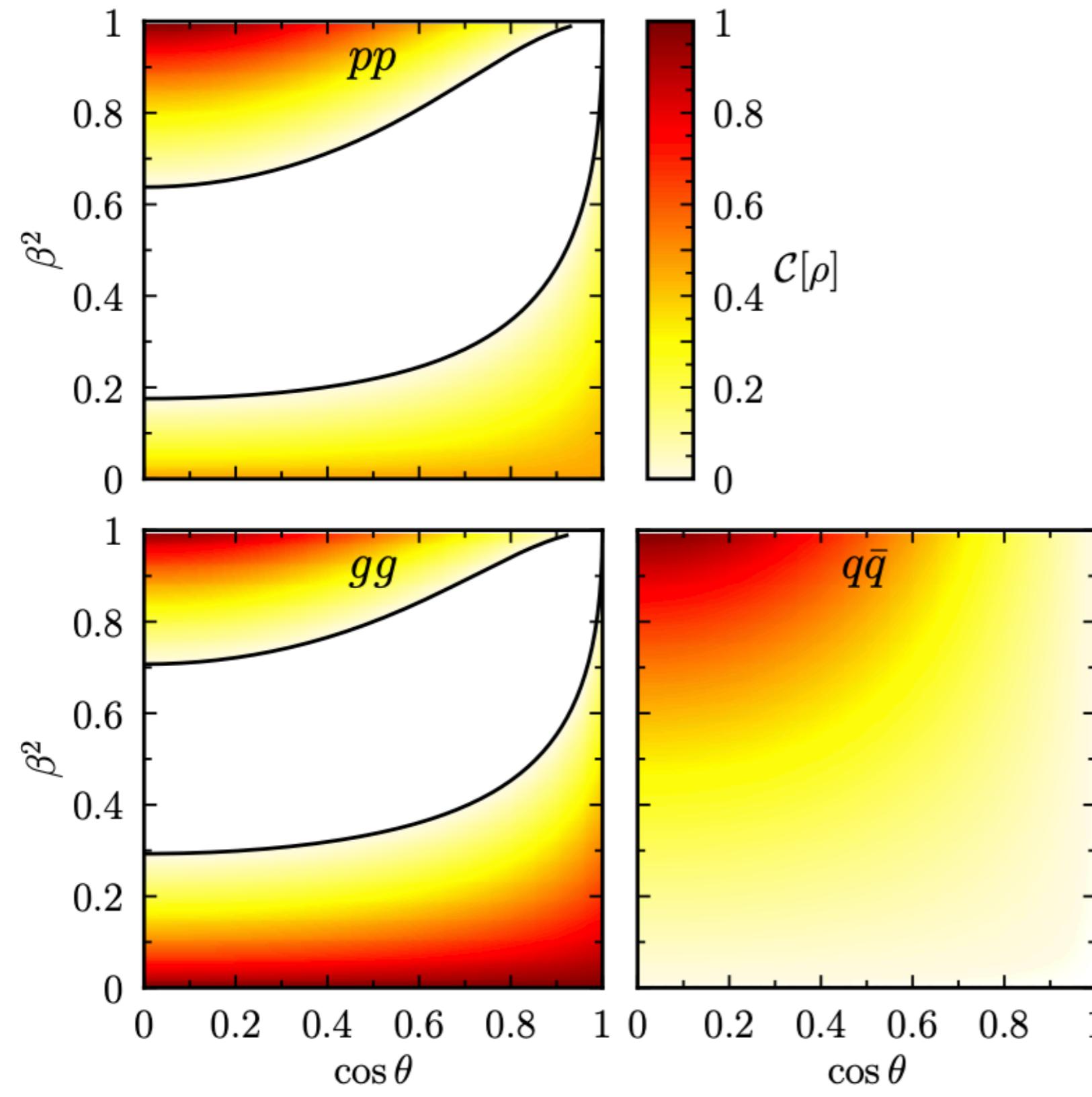
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We can then define the concurrence

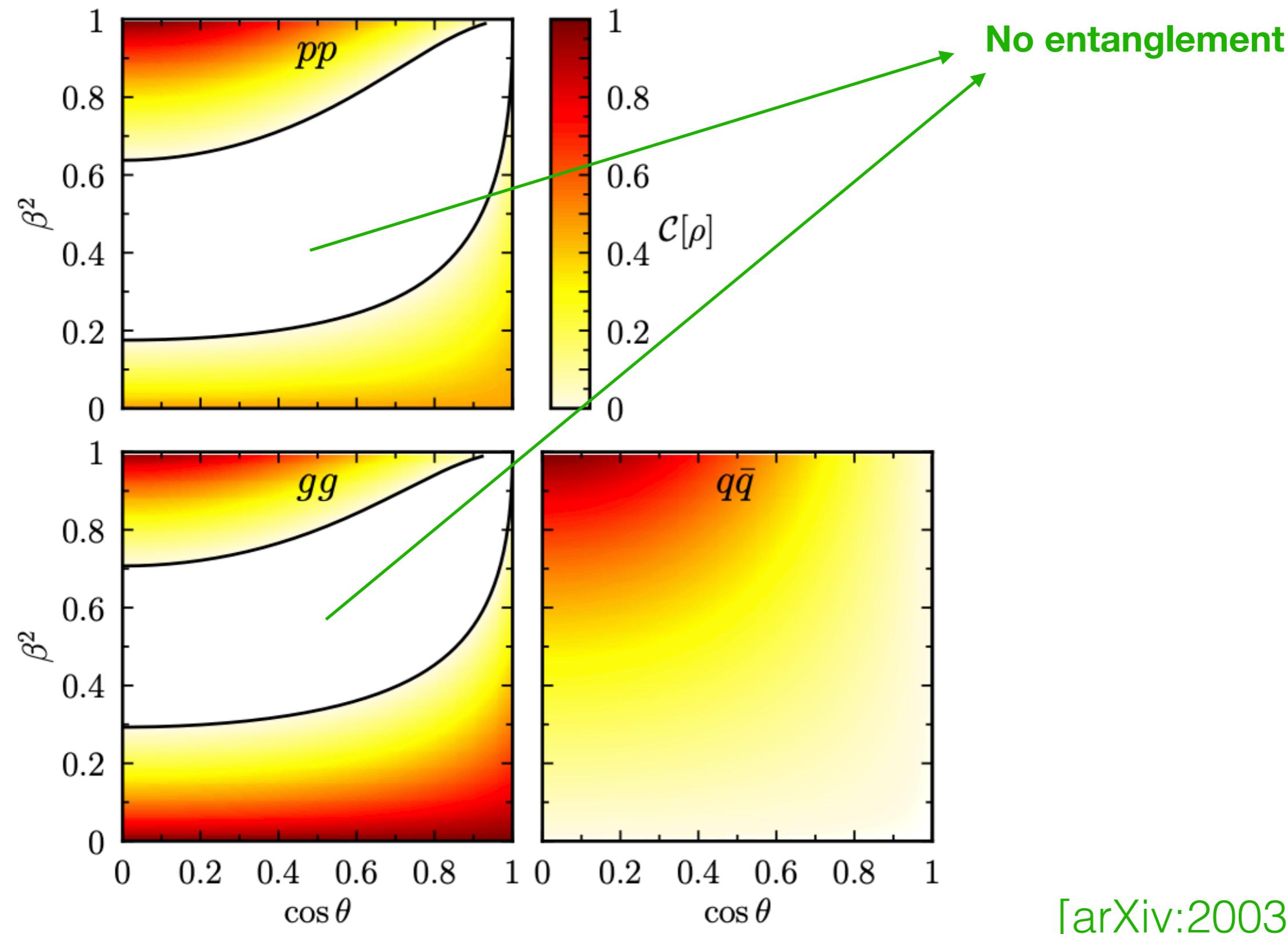
$$C[\rho] = \max(\Delta/2, 0)$$

$$C[\rho] = 1$$

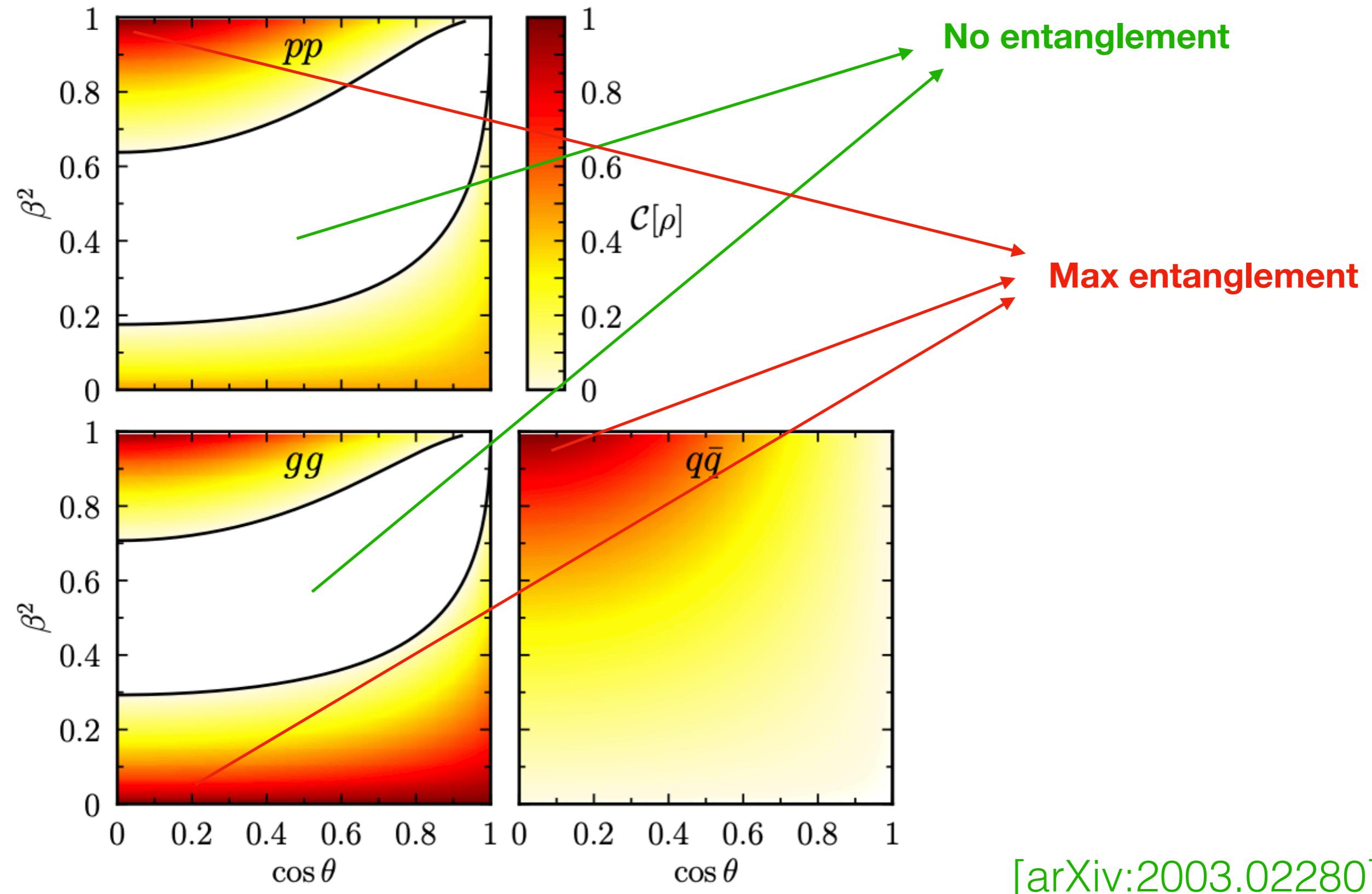
Max entanglement



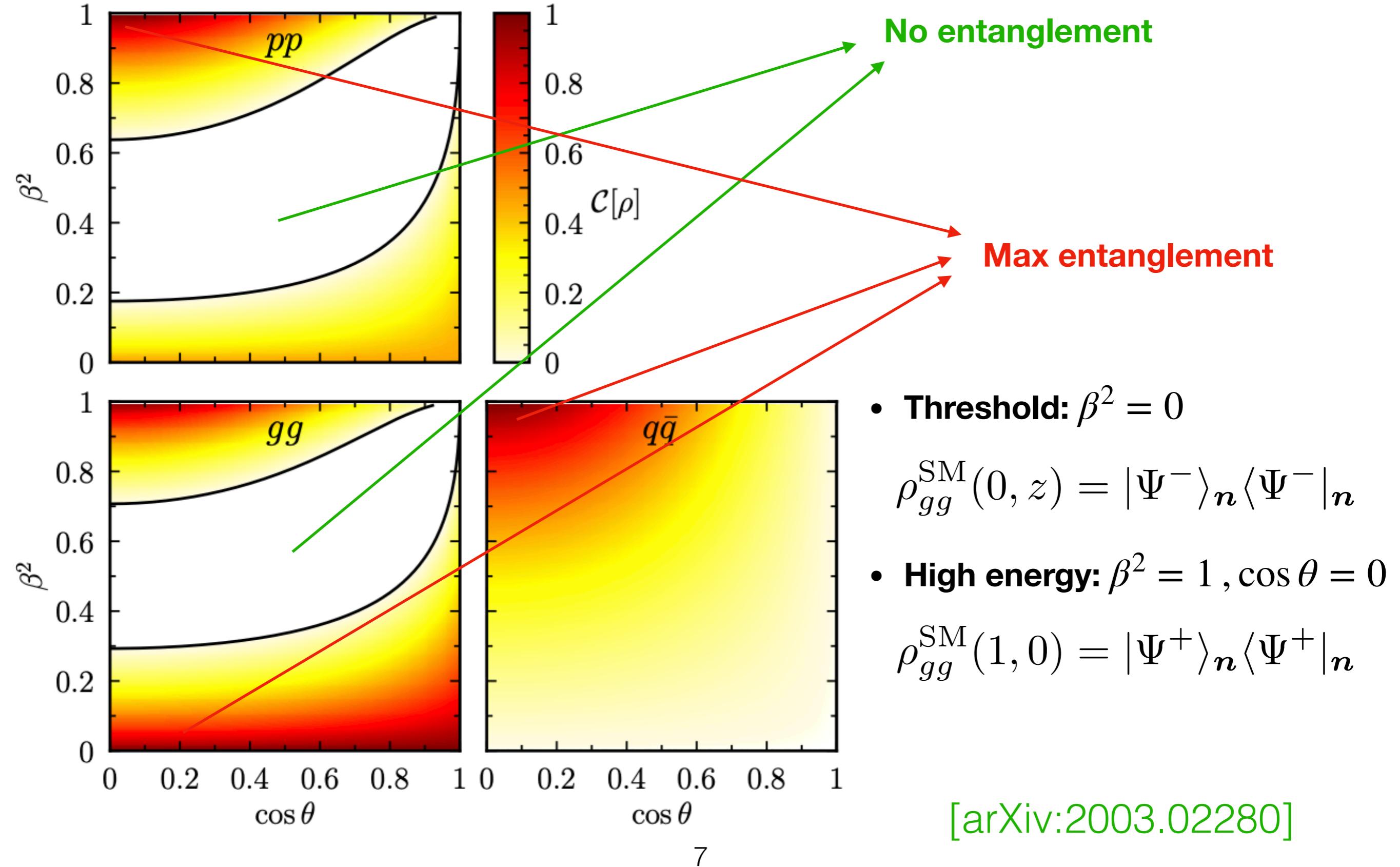
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$$\mathcal{O}_{\varphi G} = \left(\varphi^\dagger \varphi - \frac{v^2}{2} \right) G_A^{\mu\nu} G_{\mu\nu}^A$$

$$\mathcal{O}_{tG} = g_s (\bar{Q} \sigma^{\mu\nu} T^A t) \tilde{\varphi} G_{\mu\nu}^A + \text{h.c.}$$

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4-Fermion operators

$$\mathcal{O}_{Qq}^{(8,1)}, \mathcal{O}_{Qq}^{(8,3)}, \mathcal{O}_{tu}^{(8)}, \mathcal{O}_{td}^{(8)}, \mathcal{O}_{Qu}^{(8)}, \mathcal{O}_{Qd}^{(8)}, \mathcal{O}_{tq}^{(8)}$$

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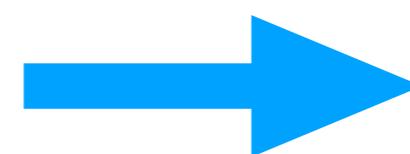
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What are the effects of NP on the entanglement regions?

Is NP affecting the quantum state?

$$R_{\alpha_1 \alpha_2, \beta_1 \beta_2}^I \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}_{\alpha_2 \beta_2}^* \mathcal{M}_{\alpha_1 \beta_1}$$

$$\mathcal{M}_{\alpha \beta} = \mathcal{M}_{\alpha \beta}^{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{M}_{\alpha \beta}^{(\text{d6})}$$



$$\rho = \frac{R^{\text{SM}} + R^{\text{EFT}}}{\text{tr}(R^{\text{SM}}) + \text{tr}(R^{\text{EFT}})}$$

$$R^I_{\alpha_1 \alpha_2, \beta_1 \beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}_{\alpha_2 \beta_2}^* \mathcal{M}_{\alpha_1 \beta_1}$$

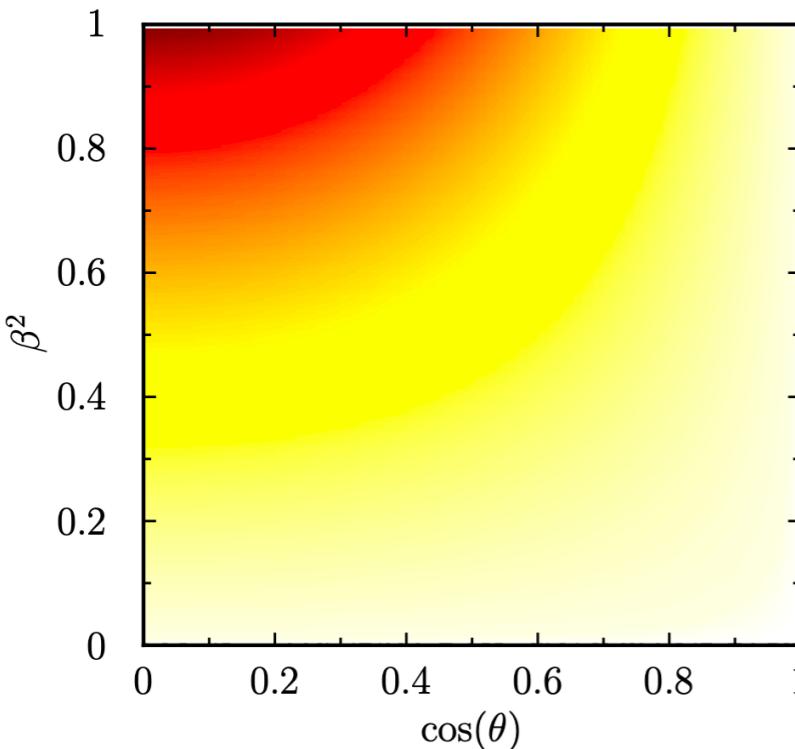
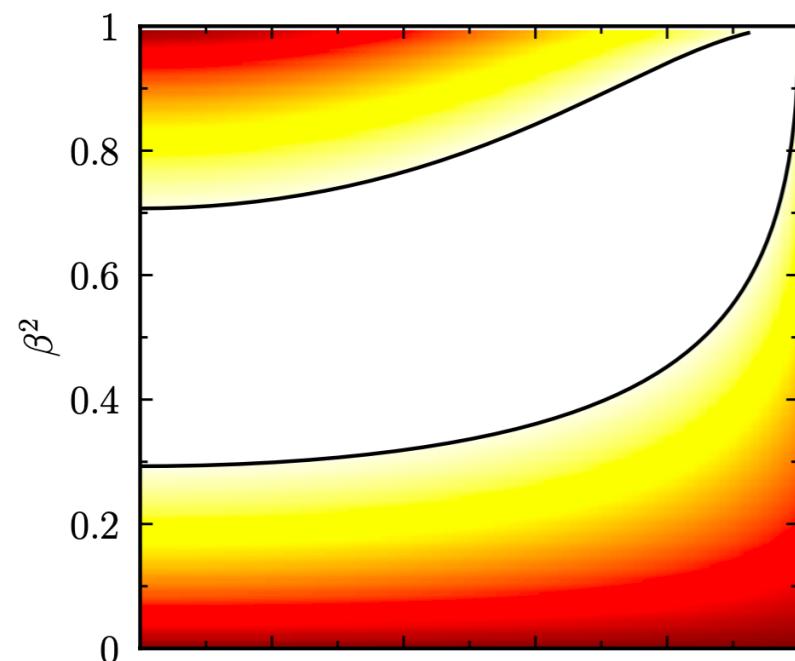
$$\mathcal{M}_{\alpha\beta} = \mathcal{M}_{\alpha\beta}^{\text{SM}} + \frac{1}{\Lambda^2} \mathcal{M}_{\alpha\beta}^{(\text{d6})} \quad \xrightarrow{\text{blue arrow}} \quad \rho = \frac{R^{\text{SM}} + R^{\text{EFT}}}{\text{tr}(R^{\text{SM}}) + \text{tr}(R^{\text{EFT}})}$$

At $\mathcal{O}(1/\Lambda^2)$

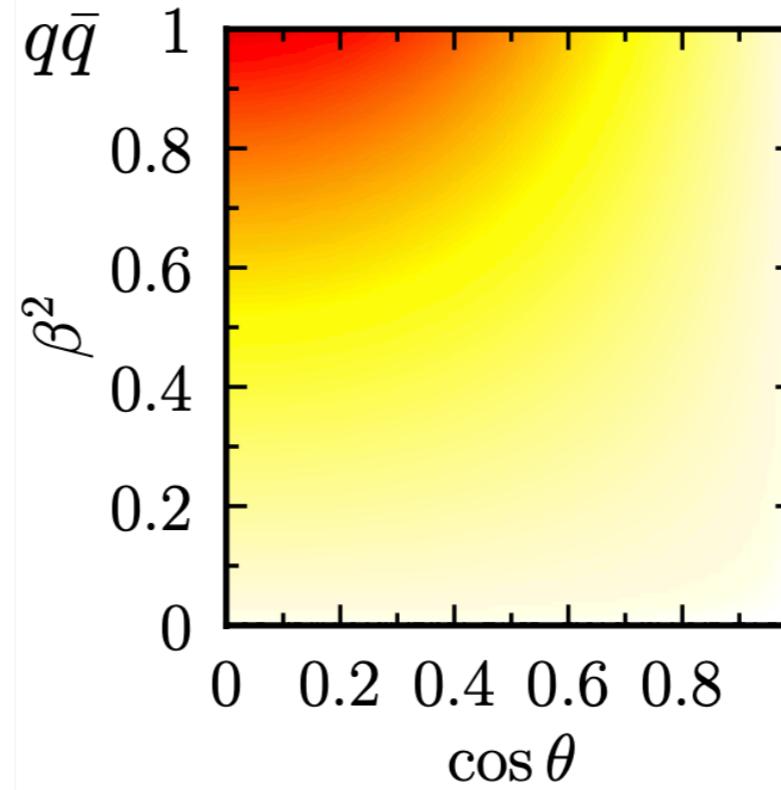
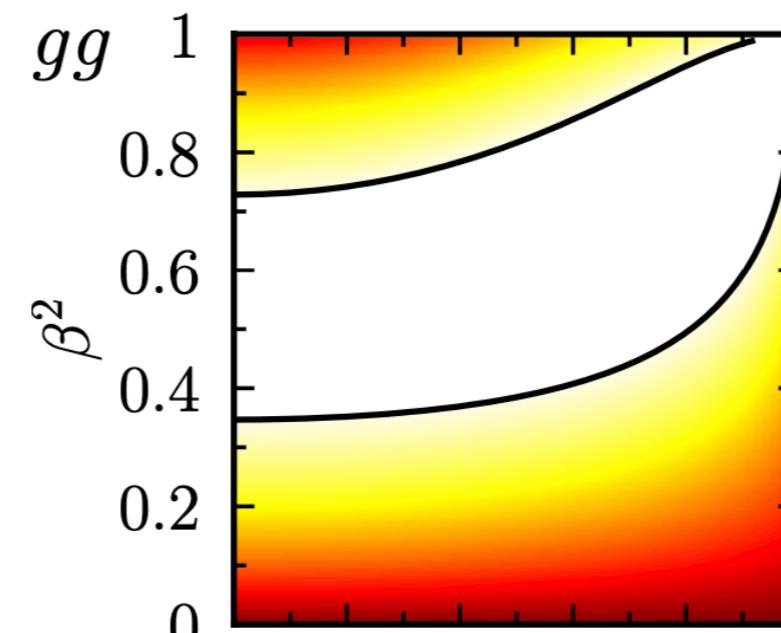
$$\begin{aligned} \tilde{A}^{gg,(1)} &= \frac{g_s^2}{\Lambda^2} \frac{1}{1 - \beta^2 z^2} \left[\frac{g_s^2 v m_t (9\beta^2 z^2 + 7)}{12\sqrt{2}} c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2 - (1 - \beta^2)m_h^2} c_{\varphi G} + \frac{9g_s^2 \beta^2 m_t^2 z^2}{8} c_G \right], \\ \tilde{C}_{nn}^{gg,(1)} &= \frac{g_s^2}{\Lambda^2} \frac{1}{1 - \beta^2 z^2} \left[\frac{-7g_s^2 v m_t}{12\sqrt{2}} c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2 - (1 - \beta^2)m_h^2} c_{\varphi G} + \frac{9g_s^2 \beta^2 m_t^2 z^2}{8} c_G \right], \\ \tilde{C}_{kk}^{gg,(1)} &= \frac{g_s^2}{\Lambda^2} \frac{1}{1 - \beta^2 z^2} \left[\frac{g_s^2 v m_t (9\beta^2 z^2 + 7) (\beta^2 (z^4 - z^2 - 1) + 1)}{12\sqrt{2} (\beta^2 z^2 - 1)} c_{tG} \right. \\ &\quad \left. + \frac{\beta^2 m_t^4}{4m_t^2 - (1 - \beta^2)m_h^2} c_{\varphi G} - \frac{9g_s^2 \beta^2 m_t^2 z^2}{8} c_G \right], \end{aligned}$$

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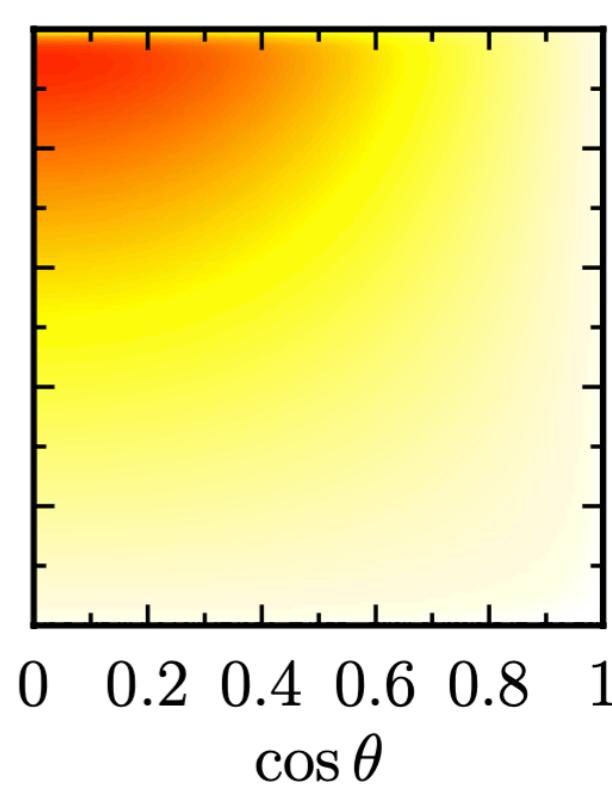
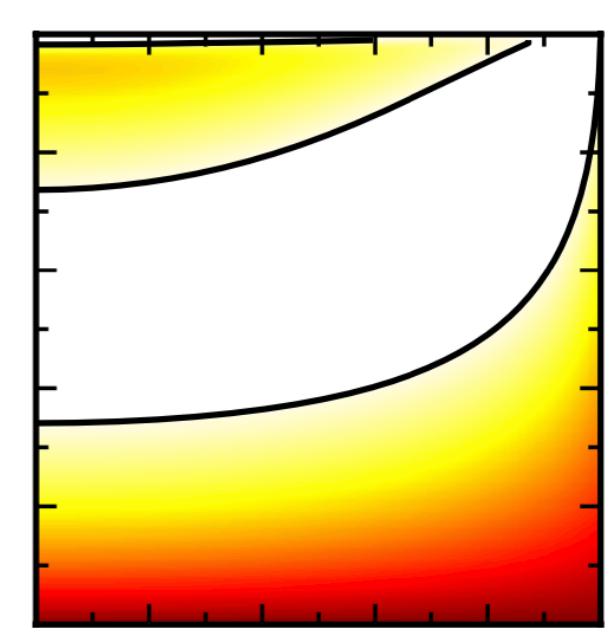
SM



Linear



Quad



$$\Delta = -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$$

$$\Delta_1 \equiv \Delta - \Delta_0 \quad \Delta \text{ computed up to } \mathcal{O}(1/\Lambda^2)$$

$$\Delta_2 \equiv \Delta - \Delta_1 - \Delta_0 \quad \Delta \text{ computed up to } \mathcal{O}(1/\Lambda^4)$$

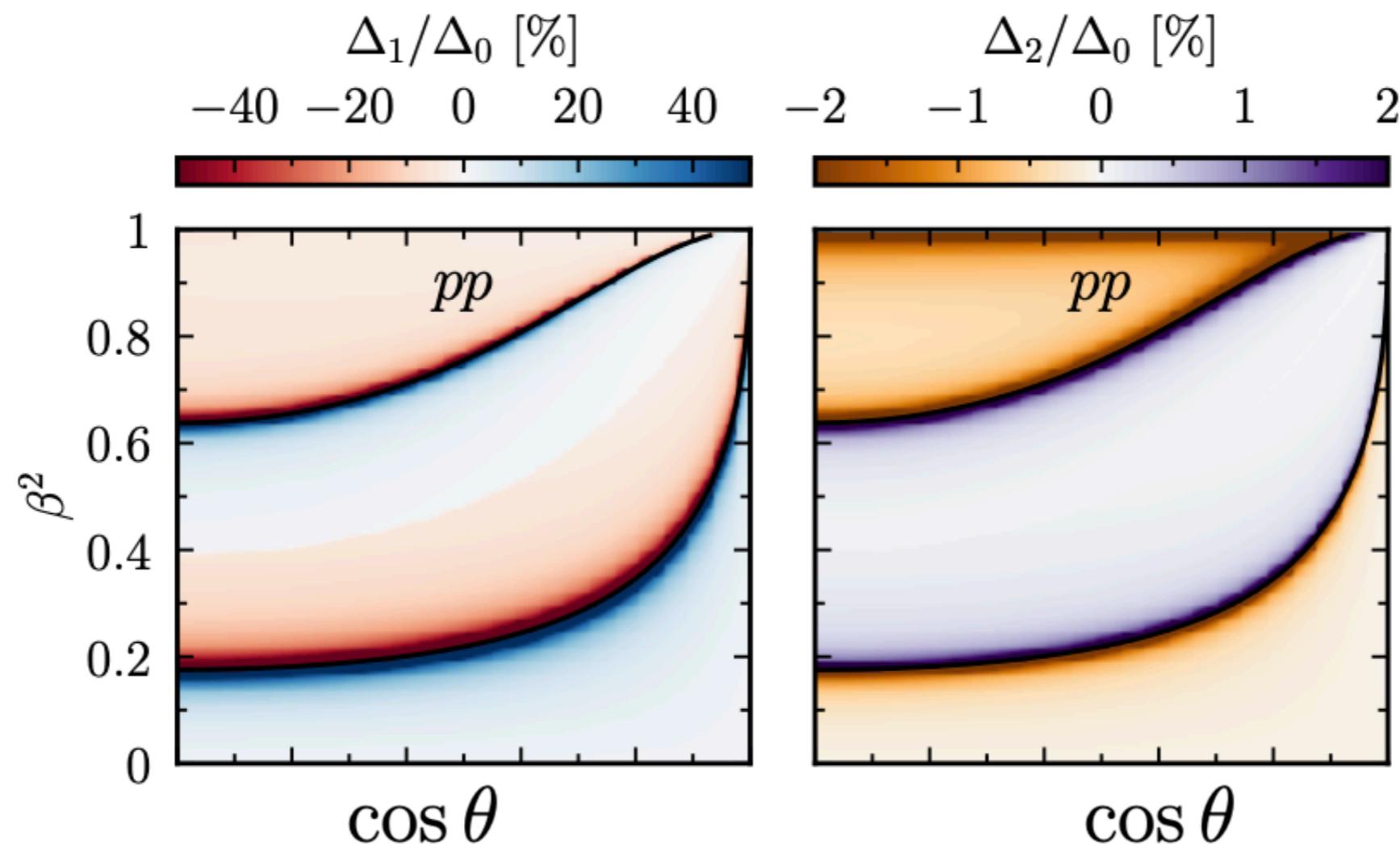
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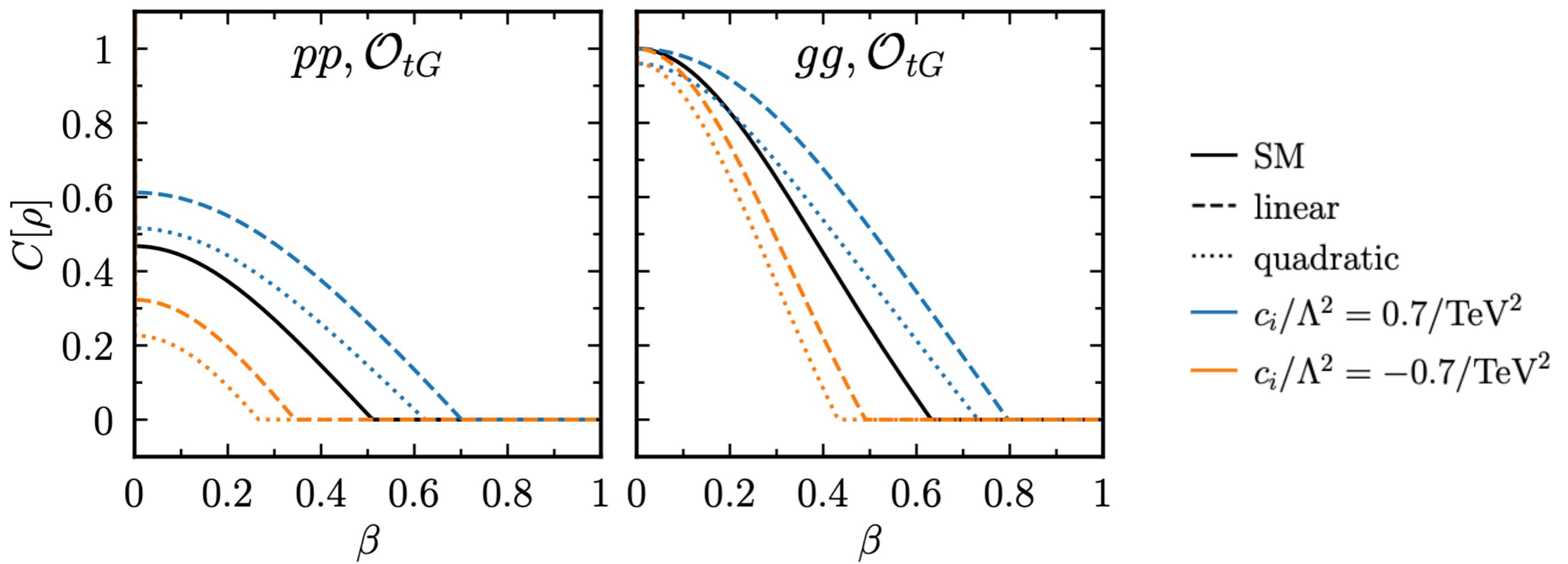


$$c_{tG}/\Lambda^2 = 0.1 \text{ TeV}^{-2}$$

$$\bar{R} = (4\pi)^{-1} \int d\Omega R(\hat{s}, \mathbf{k}), \quad \rightarrow \quad \delta \equiv -C_z + |2C_{\perp}| - 1 > 0$$
$$C[\rho] = \max(\delta/2, 0)$$

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gg-induced

$$\rho_{gg}^{\text{EFT}}(0, z) = p_{gg} |\Psi^+\rangle_{\mathbf{p}} \langle \Psi^+|_{\mathbf{p}} + (1 - p_{gg}) |\Psi^-\rangle_{\mathbf{p}} \langle \Psi^-|_{\mathbf{p}}$$

$$p_{gg} = \frac{72}{7\Lambda^4} m_t^2 (3\sqrt{2} m_t c_G + v c_{tG})^2 \quad \text{Only quadratic effects!}$$

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$$p_{q\bar{q}} = \frac{1}{2} - 4 \frac{c_{VA}^{(8),u}}{\Lambda^2} + \frac{8m_t^4}{\Lambda^4} \left(\frac{v\sqrt{2}}{m_t} c_{VA}^{(8),u} c_{tG} - 9c_{VA}^{(1),u} c_{VV}^{(1),u} + 2c_{VA}^{(8),u} c_{VV}^{(8),u} \right)$$

gg-induced

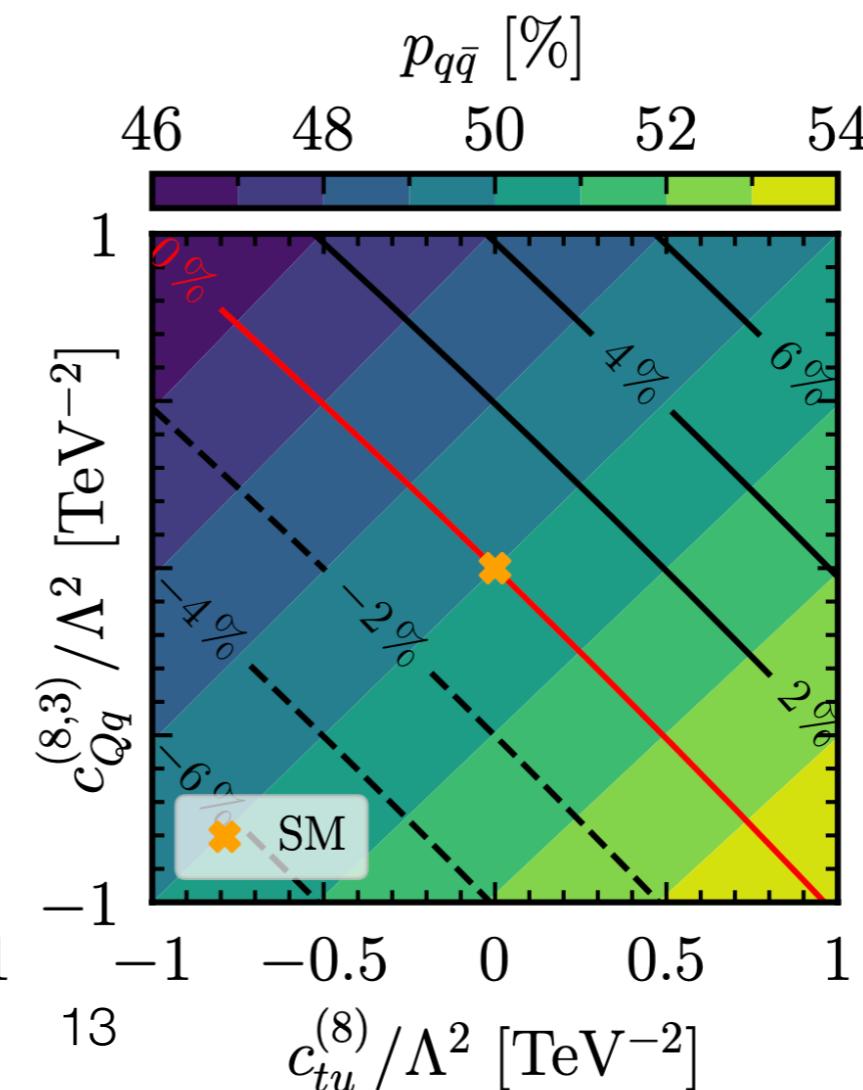
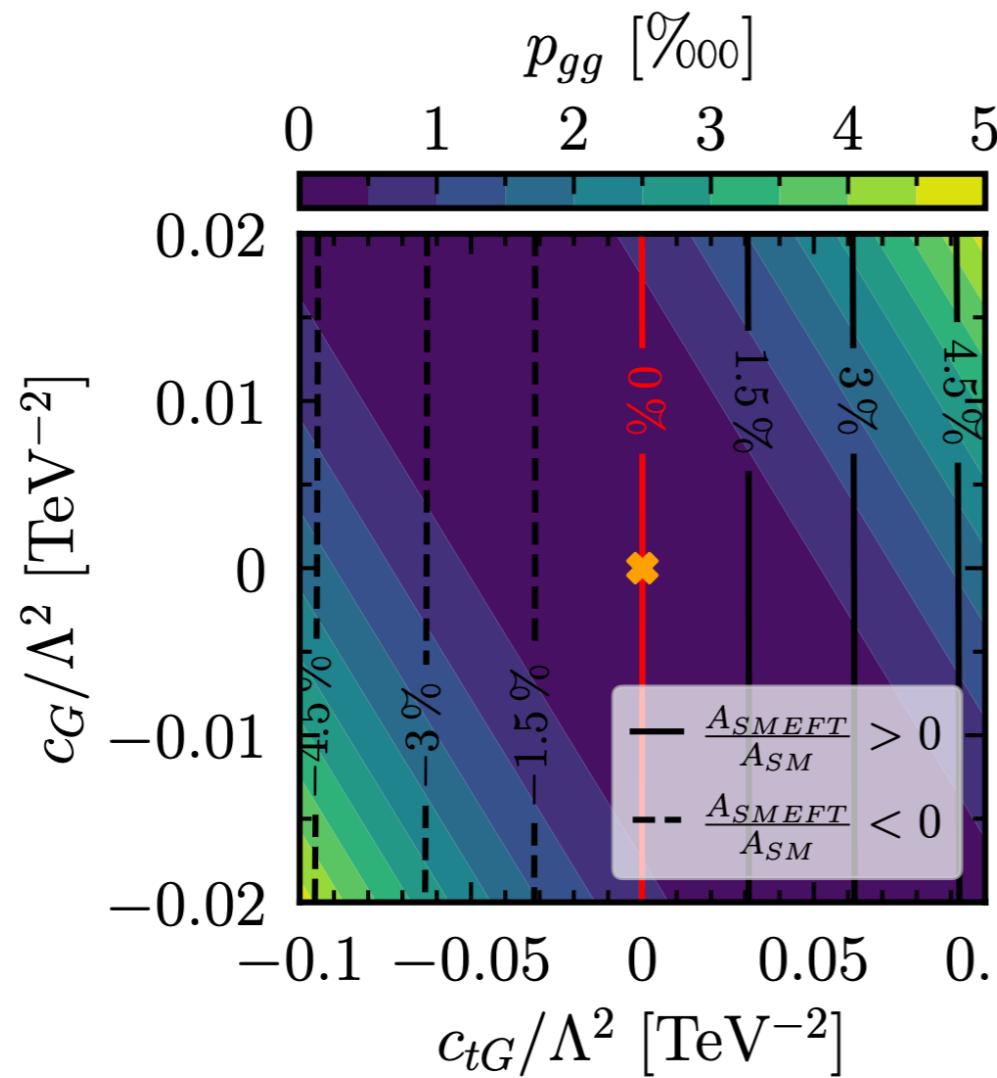
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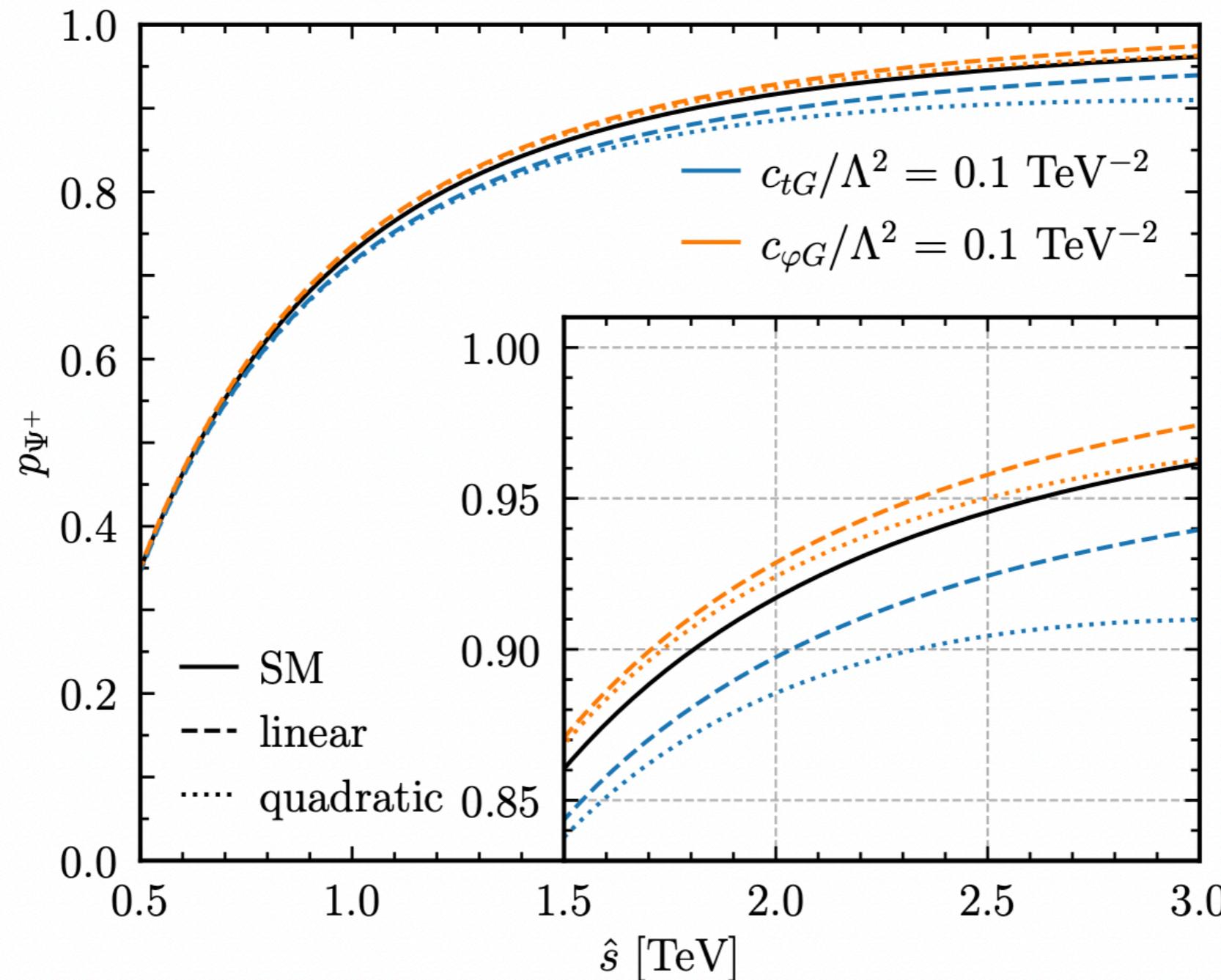
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$$p_{\Psi^+} = \langle \Psi^+ |_n \rho | \Psi^+ \rangle_n \quad \text{Probability triplet state}$$



- ❖ Possibility to exploit quantum observables as entanglement proposed.
- ❖ Measurement of top pair entanglement would be highest energy evidence ever.
- ❖ In the SM, top pairs are maximally entangled at threshold and very high energy.
- ❖ SMEFT effects induce presence of different quantum states, decreasing entanglement at threshold and modifying the overall pattern.
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- ❖ SMEFT effects induce presence of different quantum states, decreasing entanglement at threshold and modifying the overall pattern.
- ❖ Quantum observables probe complementary directions to the cross-section in EFT param space.



Backup

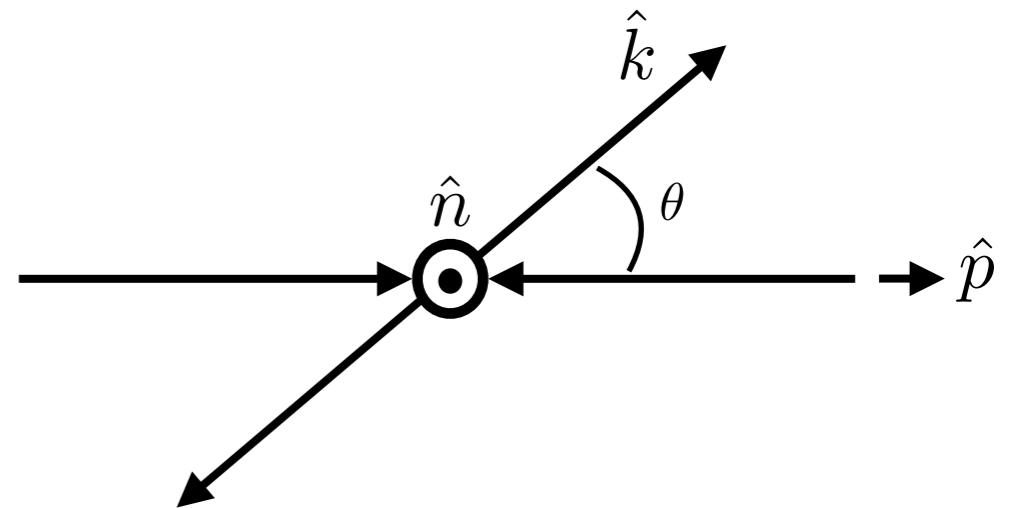
LO coefficients - gg channel

$$\begin{aligned}
\tilde{A}^{gg,(1)} &= \frac{g_s^2}{\Lambda^2} \frac{1}{1 - \beta^2 z^2} \left[\frac{g_s^2 v m_t (9\beta^2 z^2 + 7)}{12\sqrt{2}} c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2 - (1 - \beta^2)m_h^2} c_{\varphi G} + \frac{9g_s^2 \beta^2 m_t^2 z^2}{8} c_G \right], \\
\tilde{C}_{nn}^{gg,(1)} &= \frac{g_s^2}{\Lambda^2} \frac{1}{1 - \beta^2 z^2} \left[\frac{-7g_s^2 v m_t}{12\sqrt{2}} c_{tG} - \frac{\beta^2 m_t^4}{4m_t^2 - (1 - \beta^2)m_h^2} c_{\varphi G} + \frac{9g_s^2 \beta^2 m_t^2 z^2}{8} c_G \right], \\
\tilde{C}_{kk}^{gg,(1)} &= \frac{g_s^2}{\Lambda^2} \frac{1}{1 - \beta^2 z^2} \left[\frac{g_s^2 v m_t (9\beta^2 z^2 + 7) (\beta^2 (z^4 - z^2 - 1) + 1)}{12\sqrt{2} (\beta^2 z^2 - 1)} c_{tG} \right. \\
&\quad \left. + \frac{\beta^2 m_t^4}{4m_t^2 - (1 - \beta^2)m_h^2} c_{\varphi G} - \frac{9g_s^2 \beta^2 m_t^2 z^2}{8} c_G \right], \\
\tilde{C}_{rr}^{gg,(1)} &= \frac{g_s^2}{\Lambda^2} \frac{1}{1 - \beta^2 z^2} \left[\frac{g_s^2 v m_t (-9\beta^4 (z - z^3)^2 - 7\beta^2 (z^4 - z^2 + 1) + 7)}{12\sqrt{2} (\beta^2 z^2 - 1)} c_{tG} \right. \\
&\quad \left. - \frac{\beta^2 m_t^4}{4m_t^2 - (1 - \beta^2)m_h^2} c_{\varphi G} + \frac{9g_s^2 \beta^2 m_t^2 z^2}{8} c_G \right], \\
\tilde{C}_{rk}^{gg,(1)} &= \frac{g_s^2}{\Lambda^2} \frac{1}{1 - \beta^2 z^2} \left[\frac{g_s^2 v m_t \beta^2 z (1 - z^2) (9\beta^2 + (\beta^2 - 2) z^2 (9\beta^2 (z^2 - 1) + 7) - 2)}{24\sqrt{2} \sqrt{(\beta^2 - 1)(z^2 - 1)} (\beta^2 z^2 - 1)} c_{tG} \right. \\
&\quad \left. + \frac{9g_s^2 \beta^2 m_t^2 z}{8} \sqrt{\frac{1 - z^2}{1 - \beta^2}} c_G \right].
\end{aligned}$$

LO coefficients - qq channel

$$\begin{aligned}
\tilde{A}^{q\bar{q},(1)} &= \frac{4g_s^2 m_t^2}{9\Lambda^2(1-\beta^2)} \left[\sqrt{2}g_s^2 \frac{v}{m_t} (1-\beta^2) c_{tG} + (2 - (1-z^2)\beta^2) c_{VV}^{(8),u} + 2z\beta c_{AA}^{(8),u} \right], \\
\tilde{C}_{nn}^{q\bar{q},(1)} &= -\frac{g_s^2 m_t^2}{\Lambda^2} \frac{4\beta^2(1-z^2)}{9(1-\beta^2)} c_{VV}^{(8),u}, \\
\tilde{C}_{kk}^{q\bar{q},(1)} &= \frac{2g_s^2 m_t^2}{9\Lambda^2(1-\beta^2)} \left[2\sqrt{2}g_s^2 \frac{v}{m_t} (1-\beta^2) z^2 c_{tG} + (2 + \beta^2 - (2-\beta^2)(1-2z^2)) c_{VV}^{(8),u} + 4\beta z c_{AA}^{(8),u} \right] \\
\tilde{C}_{rr}^{q\bar{q},(1)} &= \frac{4g_s^2 m_t^2 (1-z^2)}{9\Lambda^2(1-\beta^2)} \left[\sqrt{2}g_s^2 \frac{v}{m_t} (1-\beta^2) c_{tG} + (2-\beta^2) c_{VV}^{(8),u} \right], \\
\tilde{C}_{rk}^{q\bar{q},(1)} &= -\frac{2g_s^2 m_t^2}{9\Lambda^2} \sqrt{\frac{1-z^2}{1-\beta^2}} \left[\sqrt{2}g_s^2 \frac{v}{m_t} (2-\beta^2) z c_{tG} + 4z c_{VV}^{(8),u} + 2\beta c_{AA}^{(8),u} \right], \\
B_k^{\pm,q\bar{q},(1)} &= 4g_s^2 \frac{m_t^2}{9\Lambda^2} \frac{1}{1-\beta^2} \left(\beta(z^2+1) c_{AV}^{(8),u} + 2z c_{VA}^{(8),u} \right), \\
B_r^{\pm,q\bar{q},(1)} &= -4g_s^2 \frac{m_t^2}{9\Lambda^2} \sqrt{\frac{1-z^2}{1-\beta^2}} \left(\beta z c_{AV}^{(8),u} + 2c_{VA}^{(8),u} \right). \\
c_{VV}^{(8),u} &= (c_{Qq}^{(8,1)} + c_{Qq}^{(8,3)} + c_{tu}^{(8)} + c_{tq}^{(8)} + c_{Qu}^{(8)})/4, & c_{AA}^{(8),u} &= (c_{Qq}^{(8,1)} + c_{Qq}^{(8,3)} + c_{tu}^{(8)} - c_{tq}^{(8)} - c_{Qu}^{(8)})/4, \\
c_{AV}^{(8),u} &= (-c_{Qq}^{(8,1)} - c_{Qq}^{(8,3)} + c_{tu}^{(8)} + c_{tq}^{(8)} - c_{Qu}^{(8)})/4, & c_{VA}^{(8),u} &= (-c_{Qq}^{(8,1)} - c_{Qq}^{(8,3)} + c_{tu}^{(8)} - c_{tq}^{(8)} + c_{Qu}^{(8)})/4,
\end{aligned}$$

$$\{\mathbf{k}, \mathbf{n}, \mathbf{r}\} : \mathbf{r} = \frac{(\mathbf{p} - z\mathbf{k})}{\sqrt{1 - z^2}}, \quad \mathbf{n} = \mathbf{k} \times \mathbf{r},$$



To expand in this basis, e.g.

$$C_{nn} = \text{tr}[C_{ij} \mathbf{n} \otimes \mathbf{n}]$$

Phase-space parametrized by:

$$\begin{aligned} \beta^2 &= (1 - 4m_t^2/\hat{s}) \\ \cos \theta &\end{aligned}$$