15th International Workshop on Top-Quark Physics (TOP2022)

CP of the top quark Yukawa

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[see also talks in EFT session]

Prelude: SMEFT

At dim. 6:

- 59 operators for a single flavour
- 1350 CP-even and 1149 CP-odd C_i for three flavours
- Various flavour assumptions decrease these numbers, nonetheless many CPV sources waiting to be discovered

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \frac{1}{\Lambda^2}\sum_i C_i Q_i$$

B. Grzadkowski et al. - *JHEP* 10 (2010) 085
A. Greljo et al.- 2203.09561
D. A. Faroughy et al. - *JHEP* 08 (2020) 166
Q. Bonnefoy et al. - 2112.03889
...

In top sector:

- many operators impact top physics (dipoles $\psi^2 XH$, 4-fermion, current operators $\psi^2 H^2 D$, ...)
- In this talk: modified Higgs Yukawa

J. Brod et al. 2203.03736

$$\mathscr{L}_{\rm Yuk} = -\bar{Q}_L\tilde{H}Y_u u_E + \frac{1}{\Lambda^2}(H^\dagger H)\bar{Q}_L\tilde{H}C'_{uH}u_R$$ (similar for down)

broken phase, mass eigenbasis: $C = U^\dagger C' W$

The κ framework: $$\mathscr{L}_\kappa = -\sum_f \frac{y_f^{\rm SM}}{\sqrt{2}}\bar{f}(\kappa_f + i\tilde{\kappa}_f\gamma_5)fh$$

$$\kappa_f \sim 1 - \frac{v}{\sqrt{2}m_f}\frac{v^2}{\Lambda^2}{\rm Re}C_{fH} \qquad \tilde{\kappa}_f \sim \frac{v}{\sqrt{2}m_f}\frac{v^2}{\Lambda^2}{\rm Im}C_{fH} \longrightarrow$$ **CP violation**

[see also talk by J. Hermann]

CP nature of the top Yukawa

- Effective top quark Yukawa: $\mathscr{L}_{ht} = -\frac{y_t}{\sqrt{2}}\bar{t}(\kappa + i\tilde{\kappa}\gamma_5)th$ y_t is $\mathcal{O}(1) \rightarrow$ clear target at LHC

  SM limit with $\kappa = 1,\ \tilde{\kappa} = 0$

- Measurements of H(125) couplings to gauge bosons consistent with spin-0 CP even state
- 2HDM, composite Higgs, … models contain modified CP structures of Yukawas
- Also interesting in context of BAU, understanding the Higgs mechanism, …
- Still room for $\tilde{\kappa} \neq 0 \rightarrow$ clear sign of NP! **How can we best probe it?**

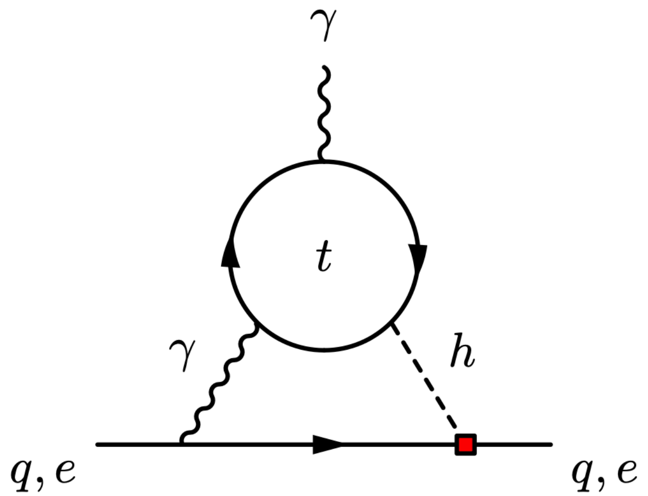
- **Indirect probes of $\tilde{\kappa}$:**

  Various collider cross sections, e.g.
  - gluon fusion $gg \rightarrow h$,
  - $h \rightarrow \gamma\gamma$ branching ratio,
  - various associated productions $pp \rightarrow t\bar{t}h, pp \rightarrow thj, \ldots$

  Indirect as $\sigma \sim \tilde{\kappa}^2$ and $\sigma(\tilde{\kappa} = 0) \neq 0$

  Various EDMs (electron, neutron, mercury)

  Indirect as $d(\tilde{\kappa} = 0) \neq 0$ (i.e. other $\tilde{\kappa}_f$ active)

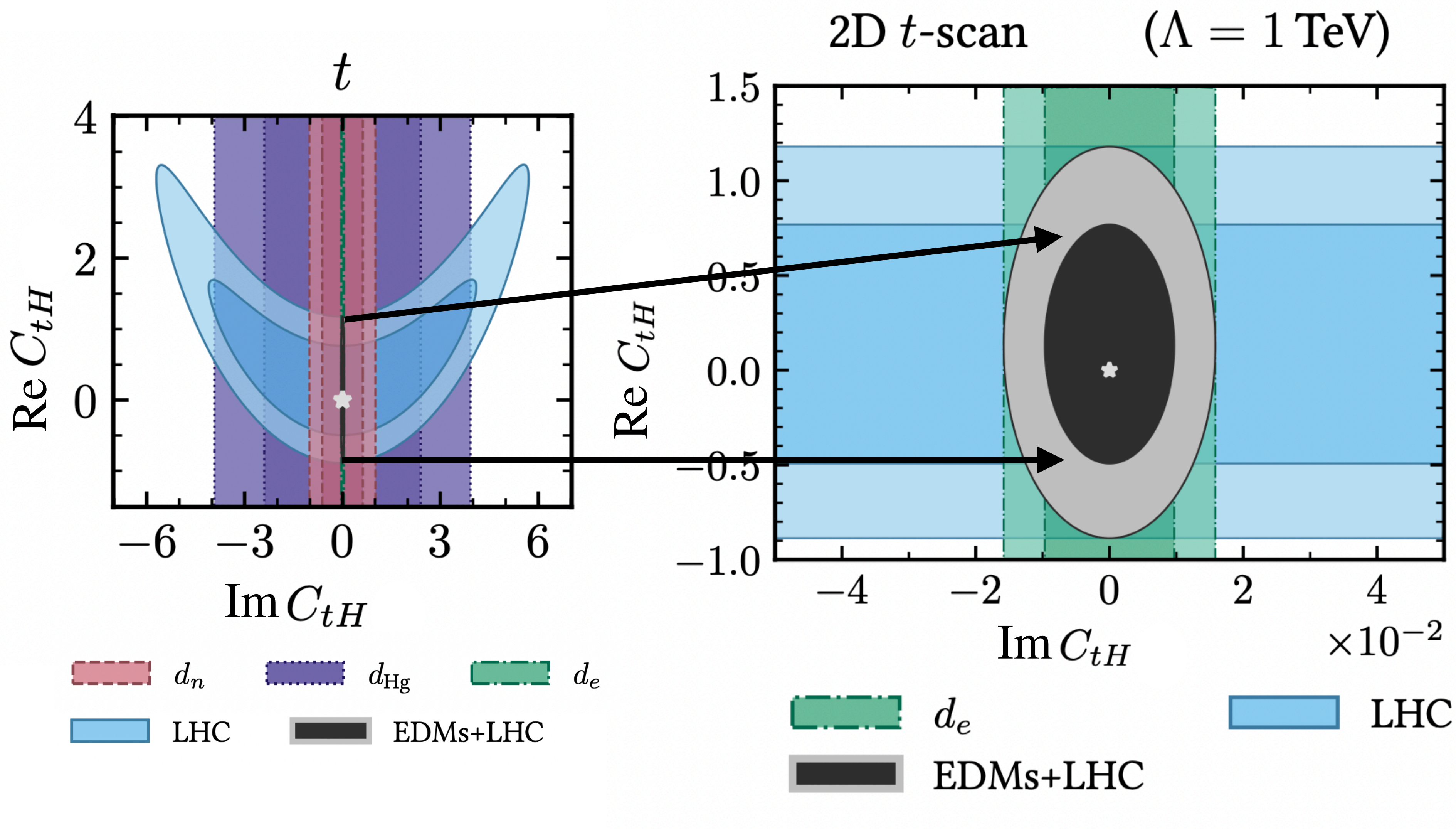


- **Direct probes of $\tilde{\kappa}$:**

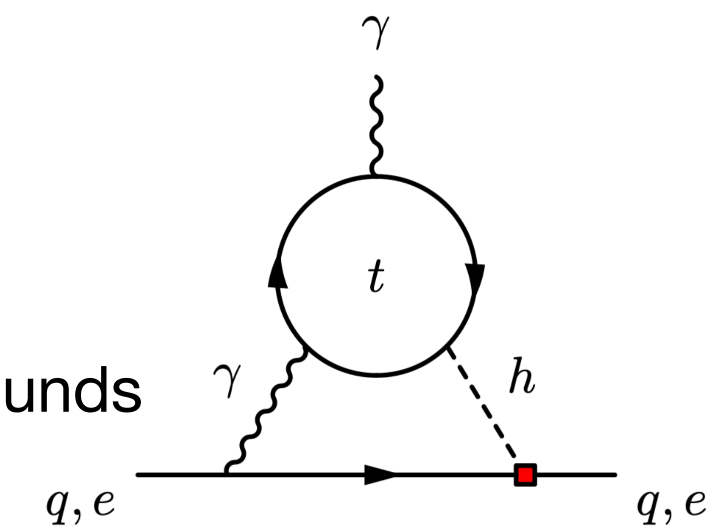
  Observables $\mathcal{O}$ with $\mathcal{O} \sim \tilde{\kappa}$ and $\mathcal{O}(\tilde{\kappa} = 0) = 0$

Indirect probes

Interplay of LHC and EDM constraints:



Translates to
$|\tilde{\kappa}| \lesssim 0.5$ (LHC),
$|\tilde{\kappa}| \lesssim 0.01$ (EDM)

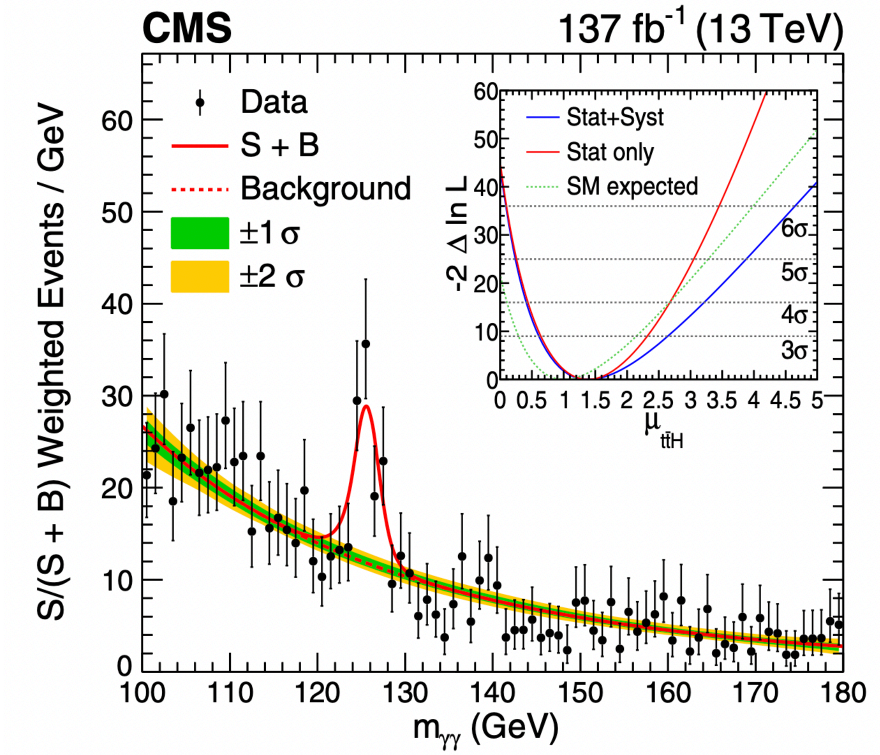


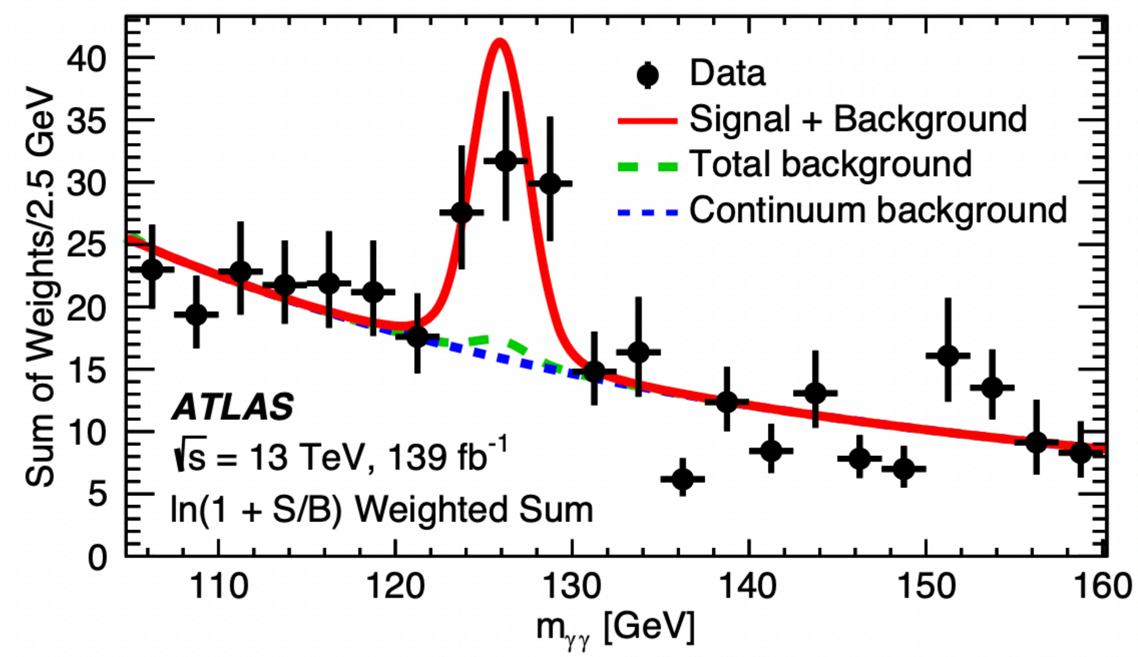
However, relaxing the assumption of SM y_e, y_q evades the EDM bounds

Another interesting interplay: allowing for $\tilde{\kappa}_b$ allows for some (but not total) cancelation between $\tilde{\kappa}_b$ and $\tilde{\kappa}_t$, in line with $h \to b\bar{b}$

CMS and ATLAS (2020)

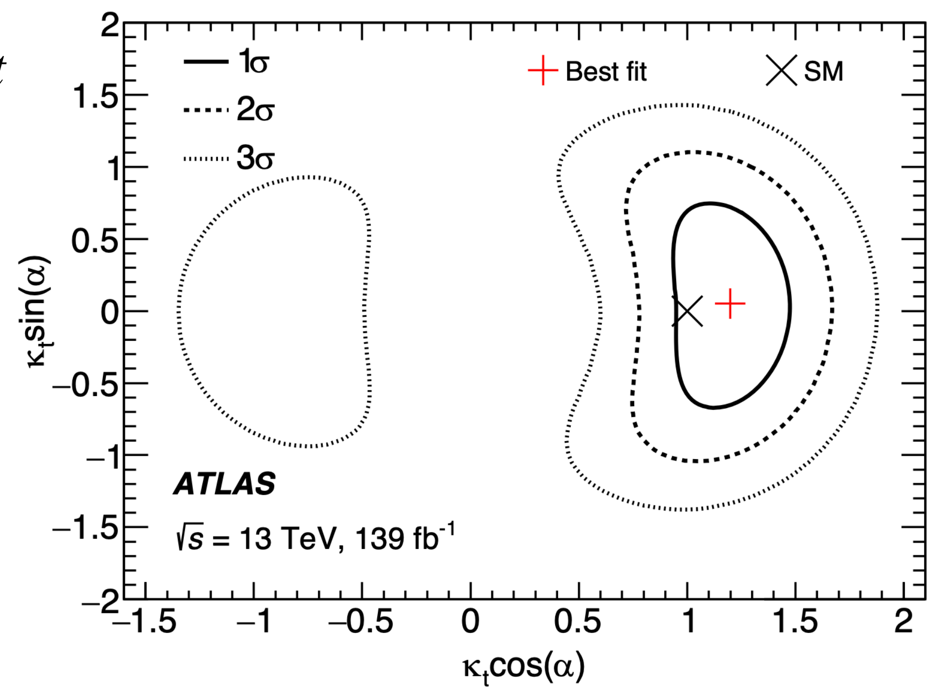
Both achieved a remarkable first observation of $pp \to t\bar{t}h$ in a single $h \to \gamma\gamma$ channel





Both carried out dedicated analyses of the CP structure of y_t

- ATLAS employing a CP BDT, trained to separate CP-even from CP-odd couplings
- CMS employing matrix element methods, however the discriminant $\mathscr{D}_{CP}$ requires tagging the flavour of light jets, thus has not been measured.



95% C.L. bounds:

CMS: $|\tilde{\kappa}| < 1.4$

ATLAS: $|\tilde{\kappa}| < 1.1$

Towards direct probes

A plethora of proposals to measure $\tilde{\kappa}$ in the literature

1507.07926, 1501.03157, 1312.5736, 2104.04277, 2208.14051, 2205.09983
1606.03107, 1503.07787, 1711.05292, 2208.04271, + many more..

In this talk: laboratory-frame manifestly CP-odd observables

E.g. Boudjema et al (1501.03157):

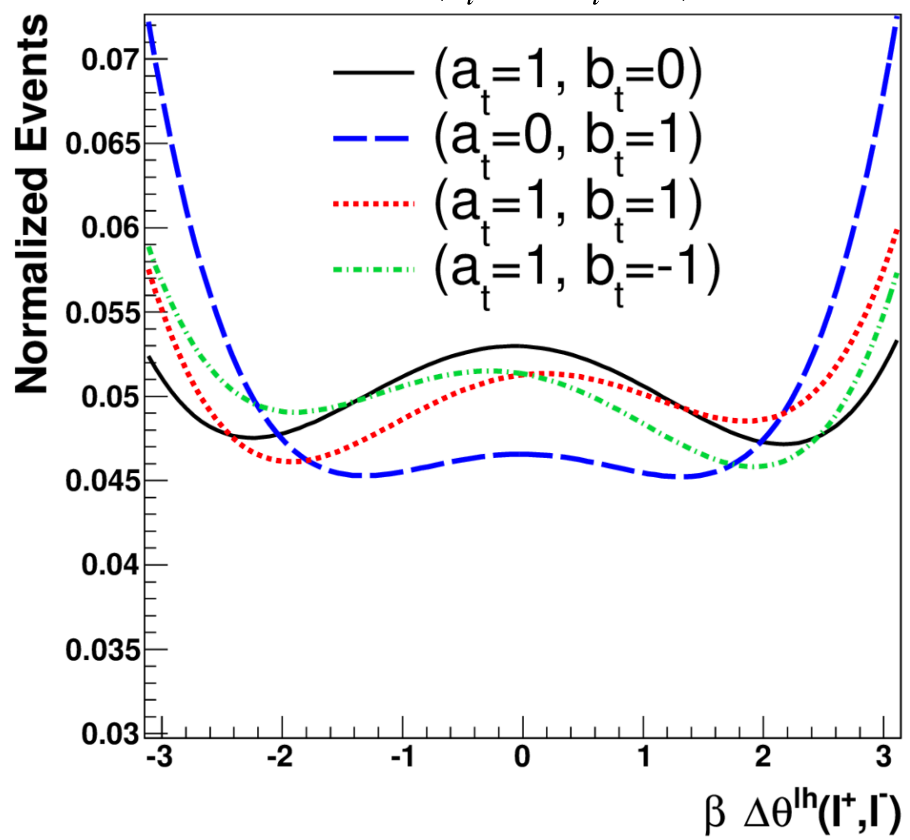
- study $pp \to t\bar{t}h$ with dileptonic $t\bar{t}$
- use $t\bar{t}$ decay product momenta $p_b, p_{\bar{b}}, p_{\ell^+}, p_{\ell^-}$ to construct a CP-odd variable in the lab. frame:

$$\beta \equiv \mathrm{sgn}\left((\vec{p}_b - \vec{p}_{\bar{b}}) \cdot (\vec{p}_{\ell^-} \times \vec{p}_{\ell^+})\right)$$

$$\cos(\Delta\theta^{\ell h}(\ell^+, \ell^-)) = \frac{(\vec{p}_h \times \vec{p}_{\ell^+}) \cdot (\vec{p}_h \times \vec{p}_{\ell^-})}{|\vec{p}_h \times \vec{p}_{\ell^+}||\vec{p}_h \times \vec{p}_{\ell^-}|}$$

(also Bernreuther et al 1993)

$(a_t \equiv \kappa, b_t \equiv \tilde{\kappa})$



- However, b-tagged jet charge discrimination is required

Remainder of the talk: focus on

B. Bortolato, J. F. Kamenik, N. Kosnik, A. S. - *Nucl.Phys.B* 964 (2021) 115328

D. Faroughy, J. F. Kamenik, N. Kosnik, A. S. - *JHEP* 02 (2020) 085

Lab. frame direct probes of $\tilde{\kappa}$

Assumptions:

- $pp \to t\bar{t}h$ with dileptonic $t\bar{t}$
- Higgs momentum reconstructed

Notice:

- No b-jet charge discrimination
- Any Higgs decay mode allowed

Construct CP-odd variables using the resolved $t\bar{t}$ decay products and the Higgs momentum in the lab. frame:

	$\boldsymbol{p}_h$	$\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+}$	$\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}$	$\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}}$	$\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}$
C	$+$	$+$	$-$	$+$	$-$
P	$-$	$-$	$-$	$-$	$-$
CP	$-$	$-$	$+$	$-$	$+$

The variables ω_i should be:

- C-even and P-odd
- Even under $b \leftrightarrow \bar{b}$

Examples:

$$\omega_{22} = \boldsymbol{p}_h \times \left(\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+}\right) \cdot \left(\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}}\right)$$

$$\omega_6 = \left[\left(\boldsymbol{p}_{\ell^-} \times \boldsymbol{p}_{\ell^+}\right) \cdot \left(\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}}\right)\right] \left[\left(\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}\right) \cdot \left(\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}}\right)\right]$$

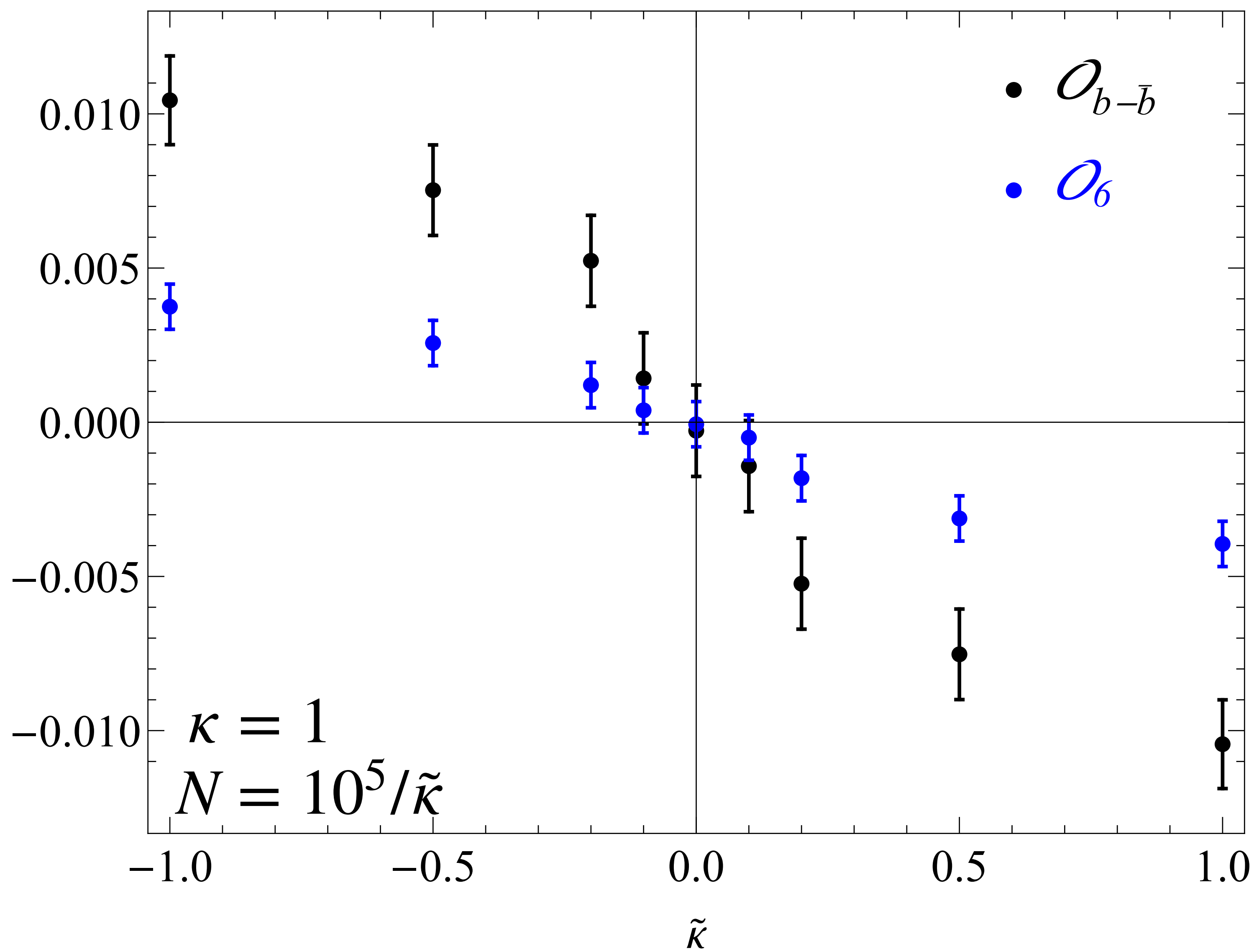
CP-odd observables simply mean over dataset: $\mathcal{O}_i = \langle \omega_i \rangle$

Example behaviour of manifestly CP-odd observables:

$$\omega_{b-\bar{b}} = (\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_{\ell^-} \times \boldsymbol{p}_{\ell^+})$$ **(1501.03157)**

$$\omega_6 = \left[(\boldsymbol{p}_{\ell^-} \times \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})\right]\left[(\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})\right]$$

$$\mathcal{O}_i = \langle \omega_i \rangle$$

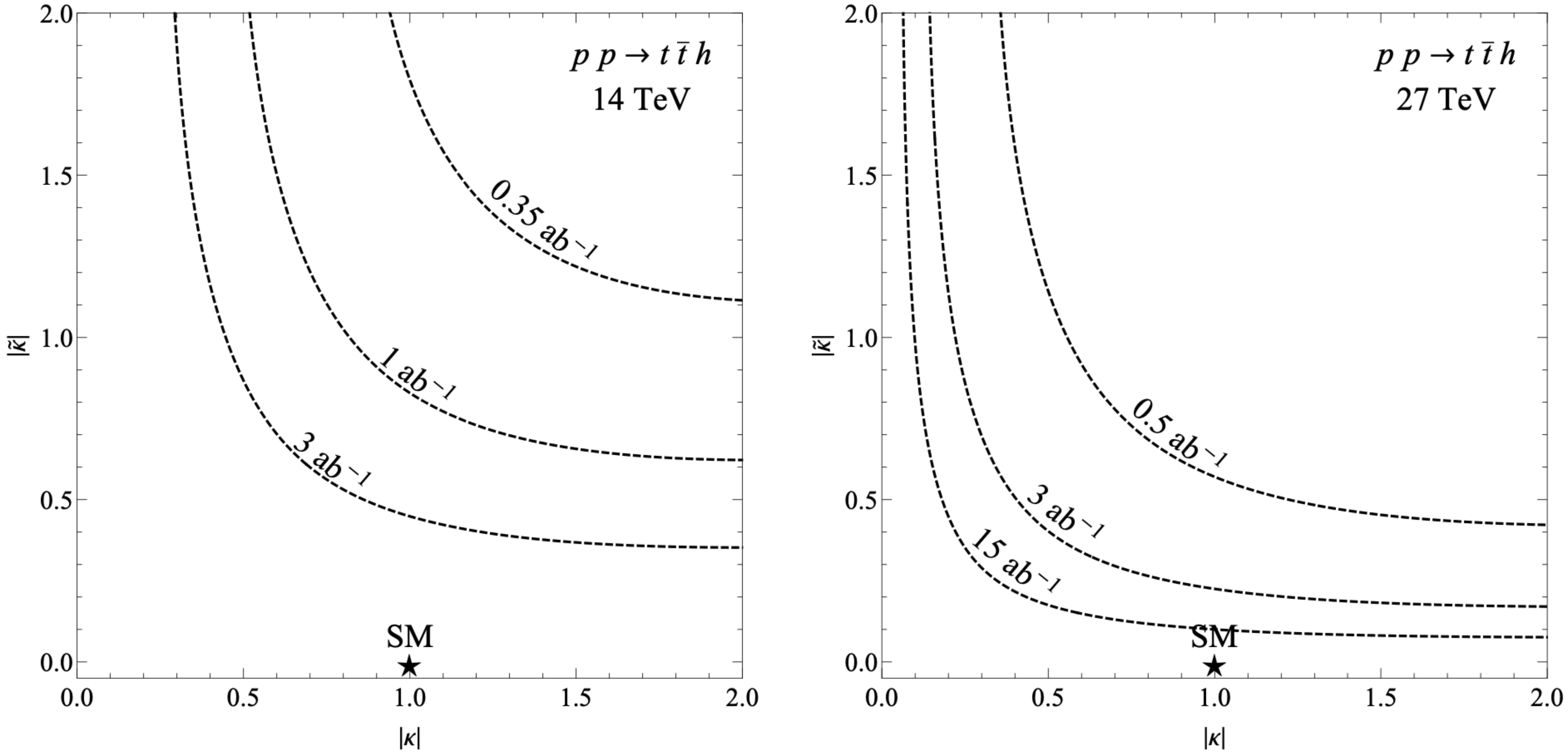


Key points:

- $\mathcal{O}_i$ sensitive to the sign of $\tilde{\kappa}$, zero when $\tilde{\kappa} = 0$
- Measurable from lab. frame quantities - no need to reconstruct the $t\bar{t}$ c.o.m. frame
- Our proposed ω_i do not depend on b-jet charge discrimination

The properties of e.g. $\mathcal{O}_6$ make it straightforward to measure, even after accounting for showering, hadronization, and detector effects

Example 2σ exclusion bounds after Pythia and Delphes:



Here:

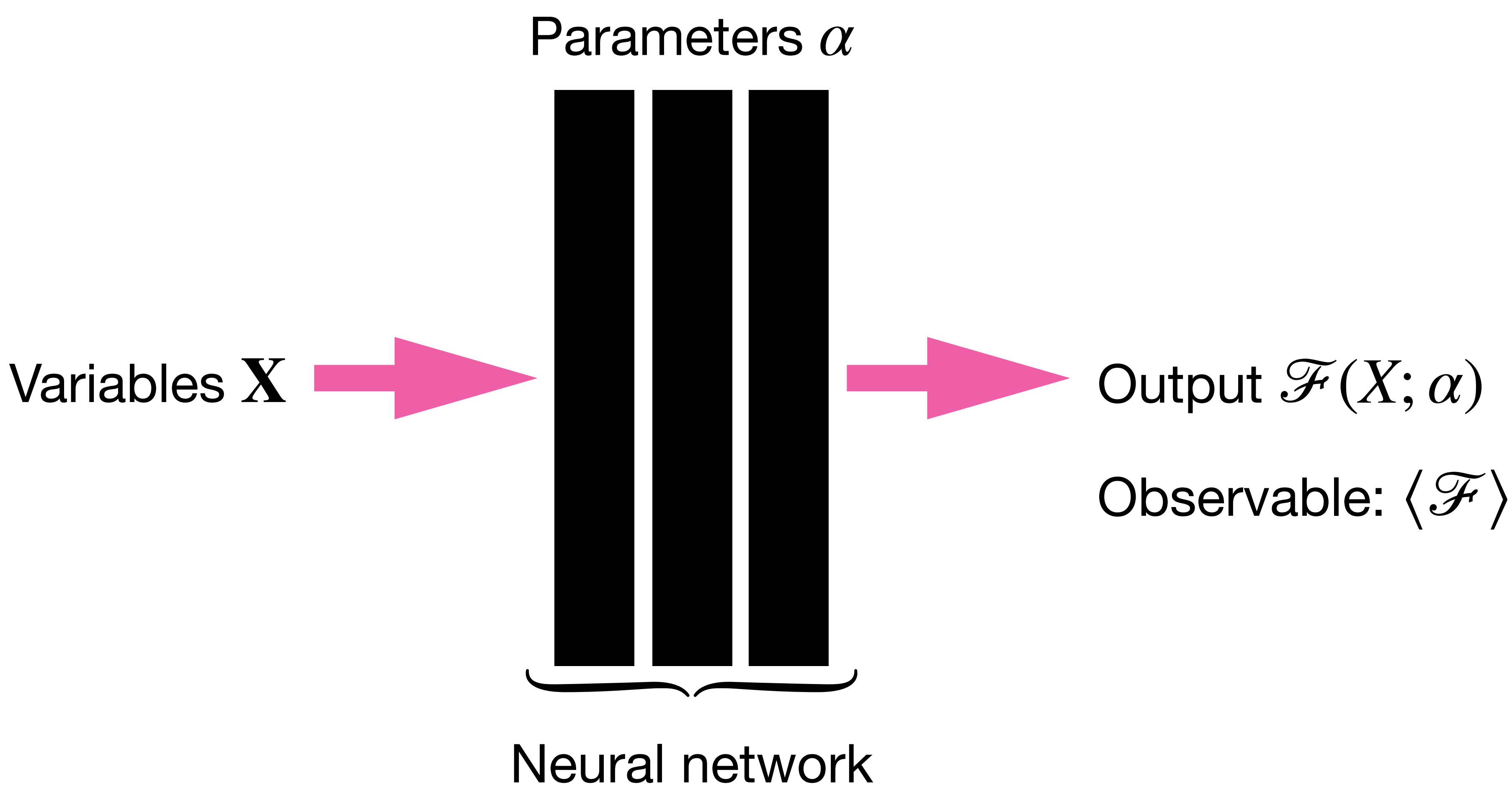
- $h \to b\bar{b}$ channel assumed
- main irreducible background $pp \to t\bar{t}b\bar{b}$ included
- require 4 or more jets of which at least 3 are b-tagged + 2 oppositely charged leptons
- identify two b-jets or b-jet + jet belonging to h with a window of (125 ± 15) GeV
- remaining b-jets + two leptons assumed to belong to top quarks
- assume to measure 0 with estimated statistical uncertainty for each luminosity

Note: if a non-zero value is measured, the bounds are asymmetric in the $\kappa - \tilde{\kappa}$ plane (will show an example later)

Further optimisation

We found 22 CP-odd variables ω_i satisfying all conditions, up to mass-dimension 5

To optimise the sensitivity to $\tilde{\kappa}$ we employ neural networks as universal function approximators



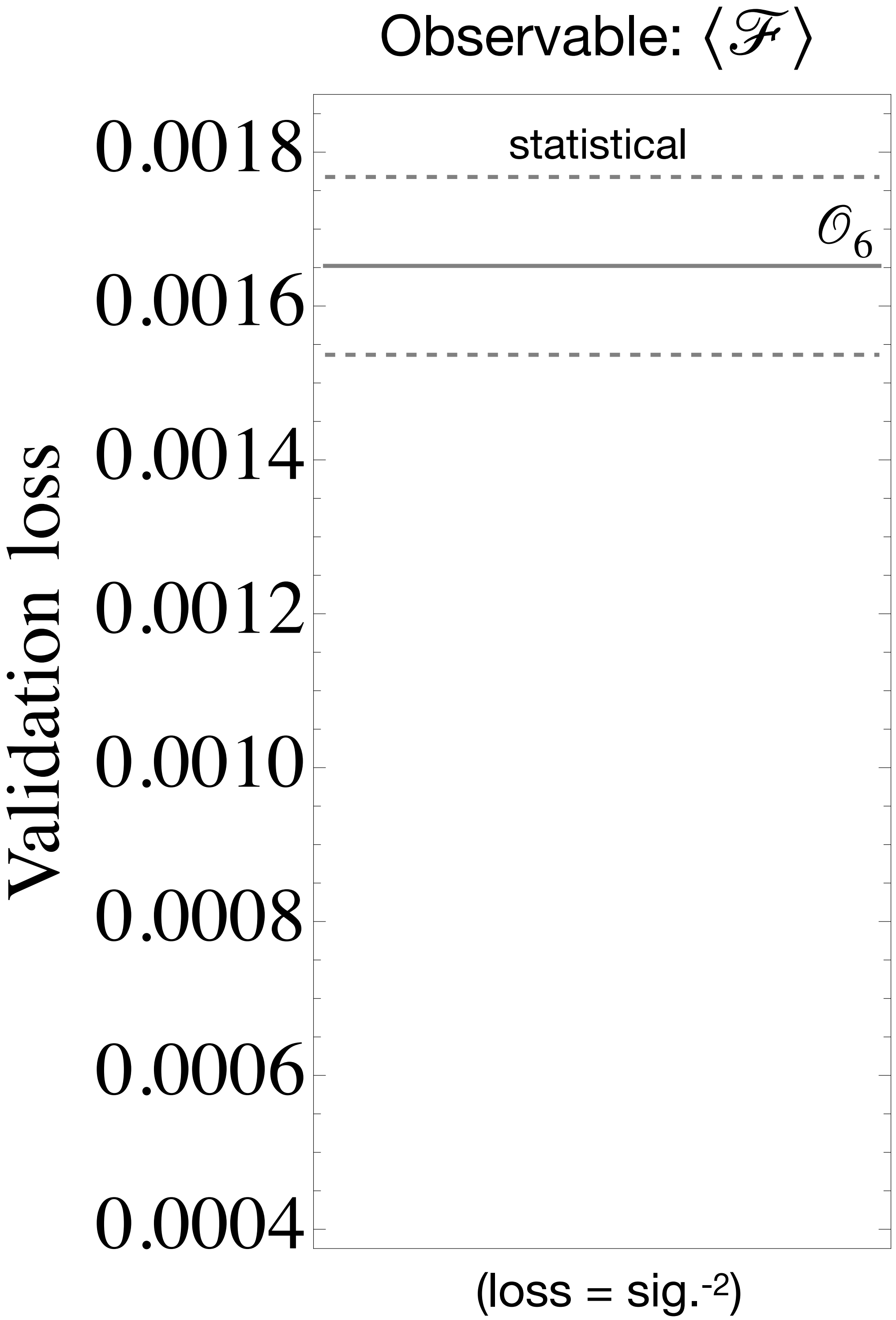
Optimisation goal**: maximize resulting observable significance**

$$\text{loss}(\boldsymbol{\alpha}) = \left(\frac{\text{mean}(\mathcal{F}(\boldsymbol{X};\boldsymbol{\alpha}))}{\text{std}(\mathcal{F}(\boldsymbol{X};\boldsymbol{\alpha}))/\sqrt{N}}\right)^{-2}$$

Optimisation procedure:

- simulate 10^7 events with fixed $\kappa = \tilde{\kappa} = 1$ using Madgraph
- train on a set of 7.5×10^6 events using Tensorflow
- validation sample of 2.5×10^6 events

Approaches to optimisation:



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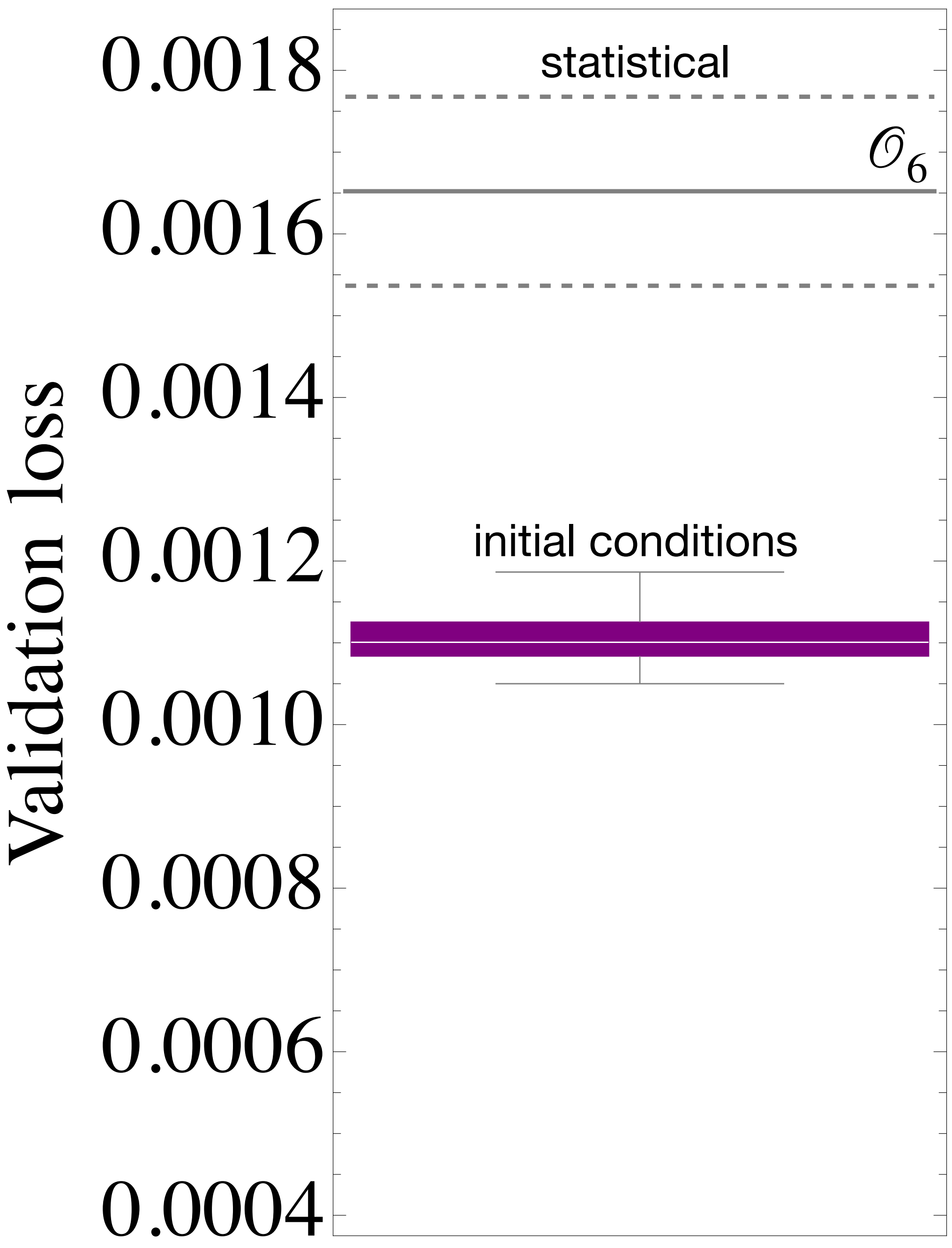
Approaches to optimisation:

1) Phase space optimisation of a single ω_i

$$\mathscr{F}\left(\boldsymbol{x}, \omega_i; \boldsymbol{\alpha}\right) = \underbrace{f(\boldsymbol{x}; \boldsymbol{\alpha})}\omega_i$$

NN=optimal weight function with $\boldsymbol{x} = \begin{pmatrix} (\boldsymbol{p}_{\ell^+} + \boldsymbol{p}_{\ell^-}) \cdot \boldsymbol{p}_h \\ (\boldsymbol{p}_{\ell^+} + \boldsymbol{p}_{\ell^-}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}}) \\ (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}}) \cdot \boldsymbol{p}_h \\ \boldsymbol{p}_{\ell^+} \cdot \boldsymbol{p}_{\ell^-} \\ \boldsymbol{p}_b \cdot \boldsymbol{p}_{\bar{b}} \end{pmatrix}$

Observable: $\langle \mathscr{F} \rangle$



(loss = sig.$^{-2}$)

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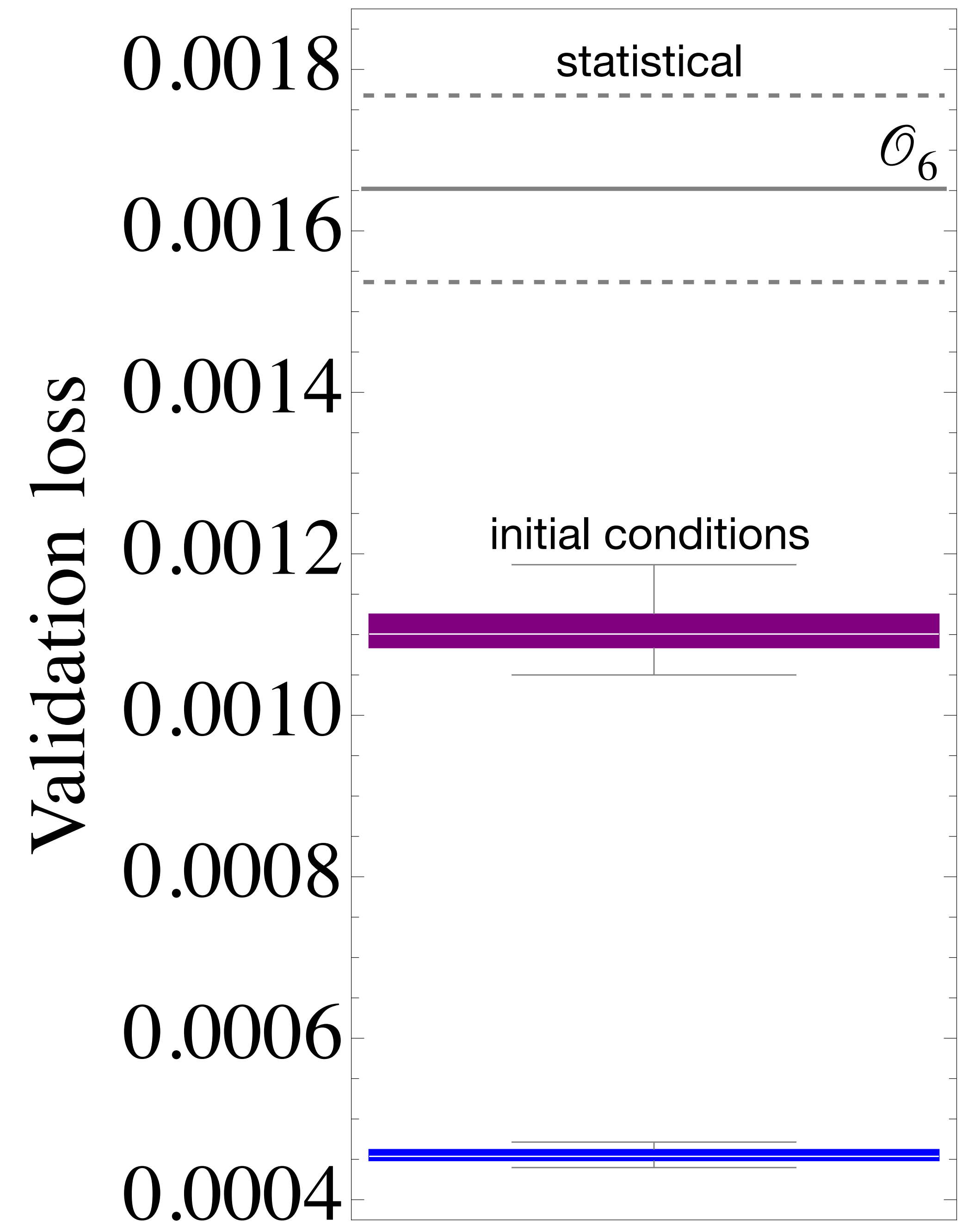
2) NN as a CP-odd observable

$$\mathscr{F}(\boldsymbol{X}; \boldsymbol{\alpha}) = F(\boldsymbol{\omega}; \boldsymbol{\alpha}) \text{ with } F(\boldsymbol{\omega}; \boldsymbol{\alpha}) = -F(-\boldsymbol{\omega}; \boldsymbol{\alpha})$$

Observable: $\langle \mathscr{F} \rangle$



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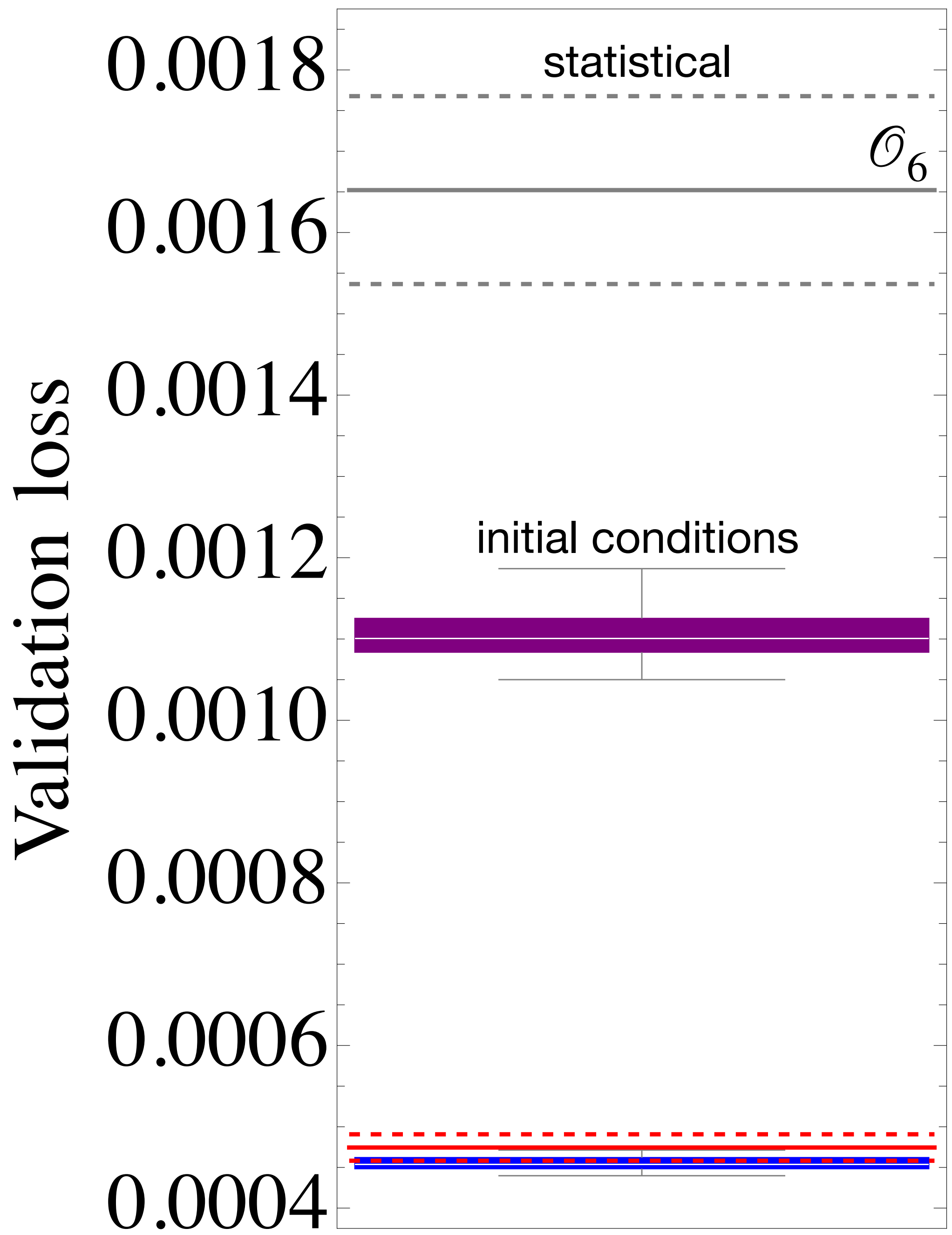
2) NN as a CP-odd observable

$$\mathscr{F}(X; \boldsymbol{\alpha}) = F(\boldsymbol{\omega}; \boldsymbol{\alpha}) \text{ with } F(\boldsymbol{\omega}; \boldsymbol{\alpha}) = -F(-\boldsymbol{\omega}; \boldsymbol{\alpha})$$

3) Linear approximation of 2)

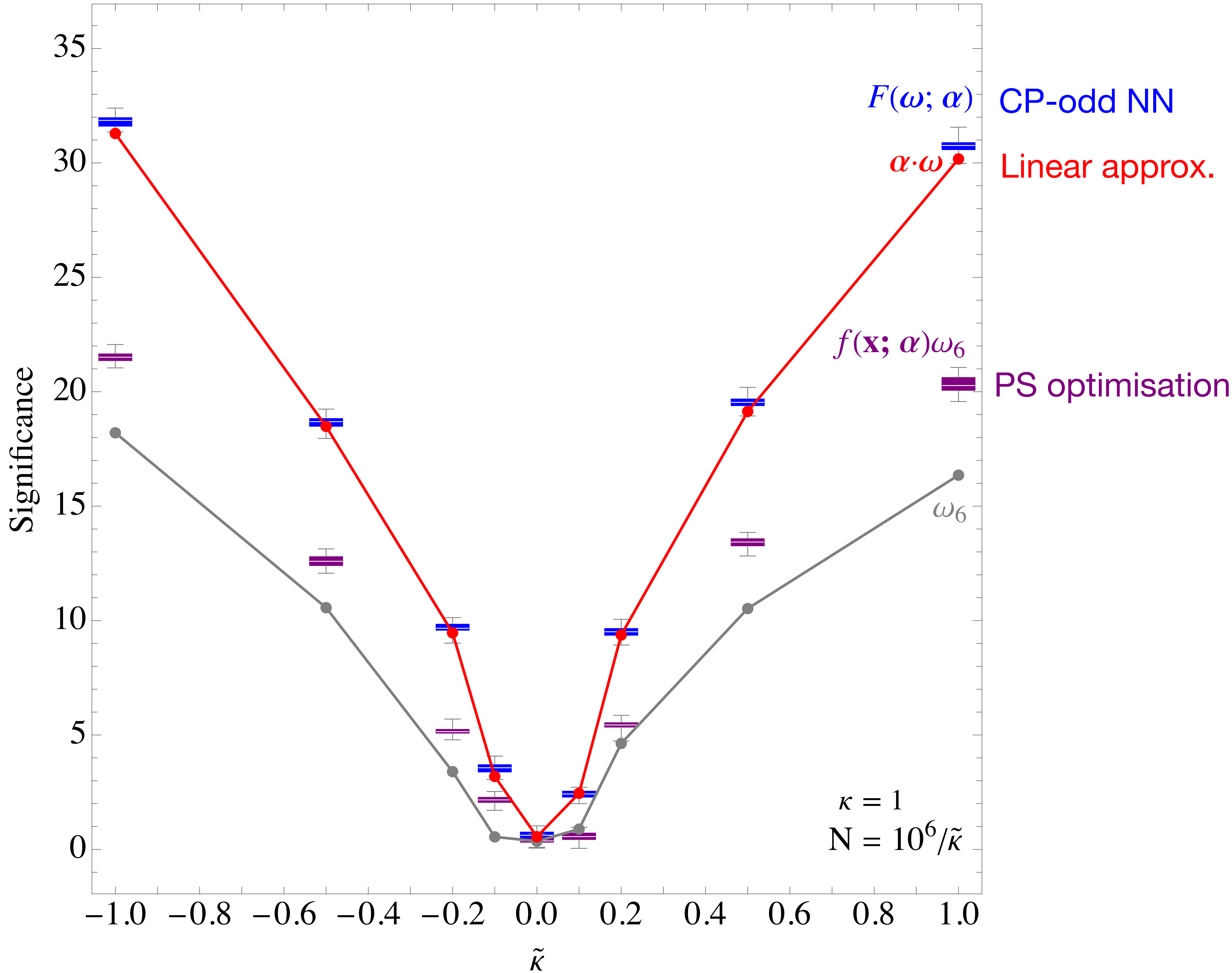
$$F(\boldsymbol{\omega}; \boldsymbol{\alpha}) = \sum_{j=1}^{22} \alpha_j \omega_j + \mathscr{O}\left(\omega^3\right)$$

Observable: $\langle \mathscr{F} \rangle$

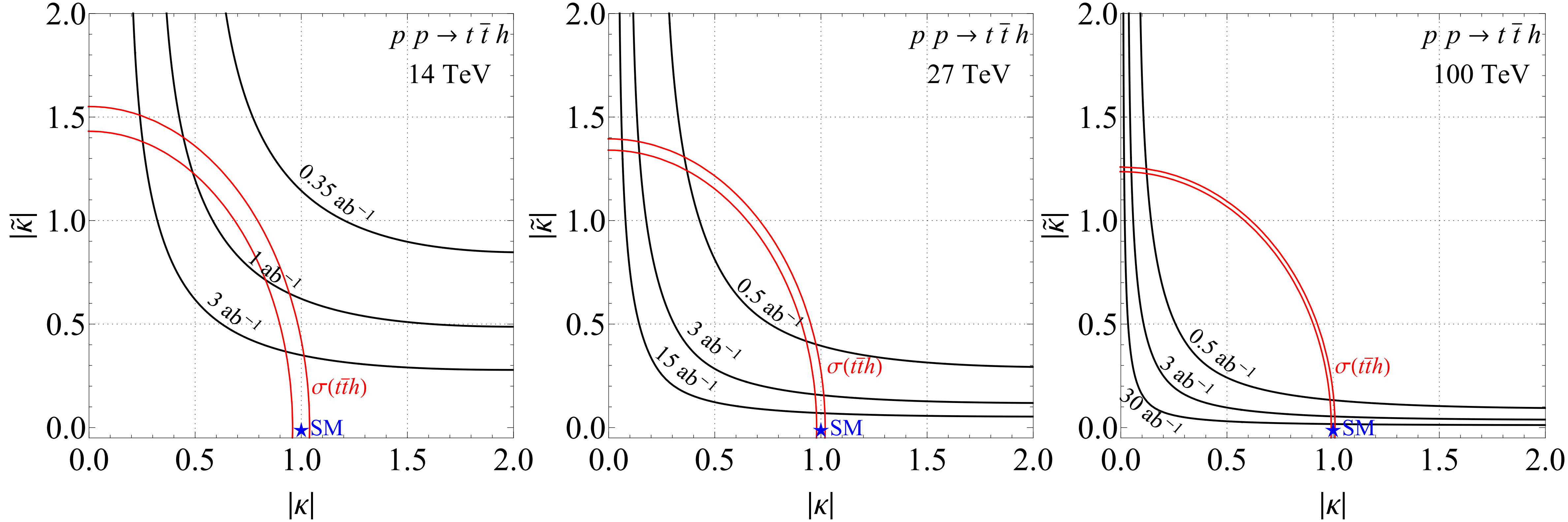


(loss = sig.$^{-2}$)

Test of generalization: independent datasets with various $\tilde{\kappa}$:



Example 2σ exclusion bounds, $h \to b\bar{b}$ channel assumed, using $\alpha \cdot \omega$:



Complementary probe to cross-section measurements

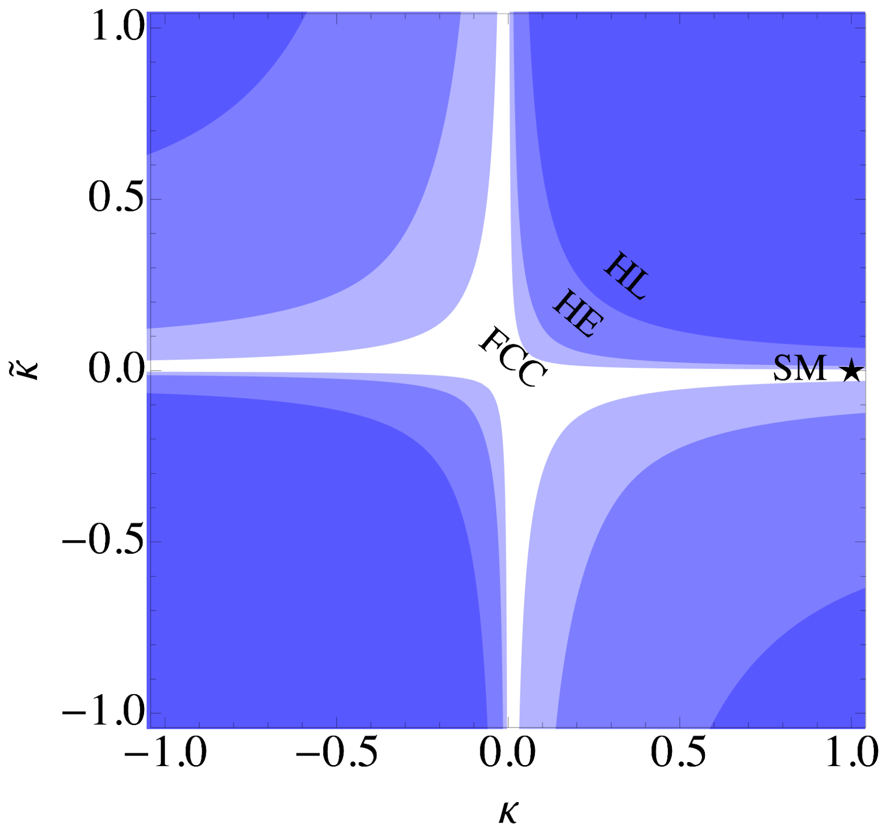
14 TeV: $|\tilde{\kappa}| < 1.1 @ 350\text{fb}^{-1}$

$|\tilde{\kappa}| < 0.3 @ 3\text{ab}^{-1}$

100 TeV: $|\tilde{\kappa}| < 0.01 @ 30\text{ab}^{-1}$

Many further improvements possible:
including multiple Higgs decay channels,
simple cut-based analysis could be optimised

Sensitivity to the sign of $\tilde{\kappa}$:

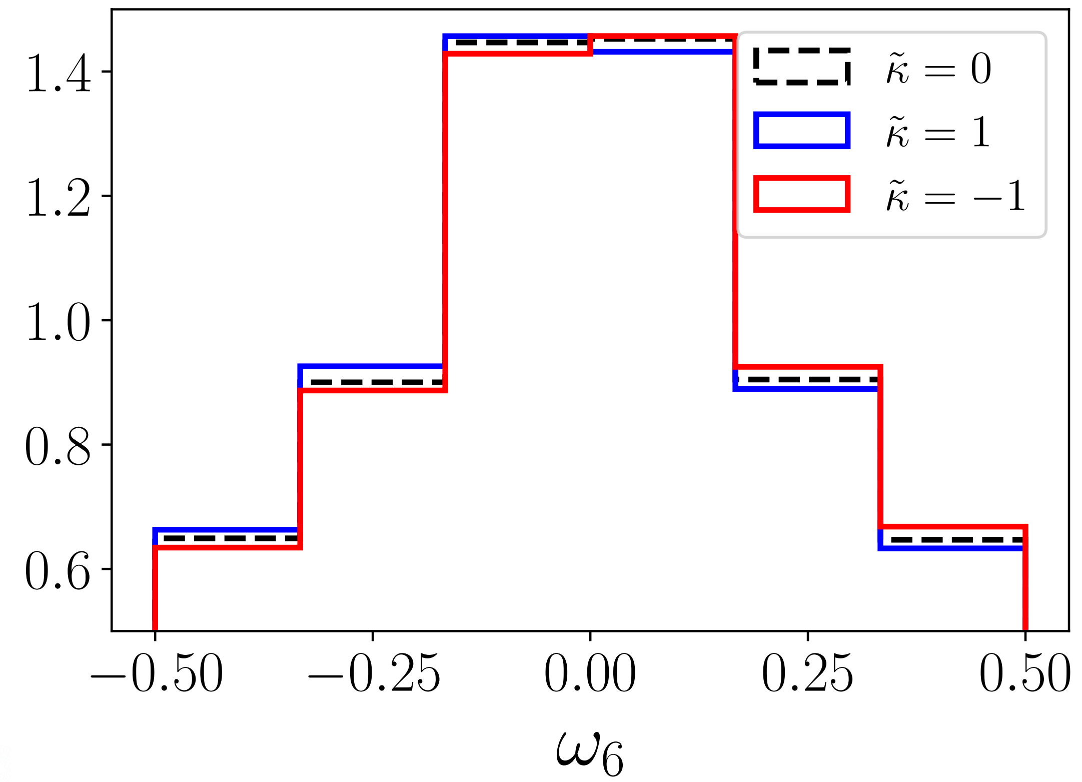


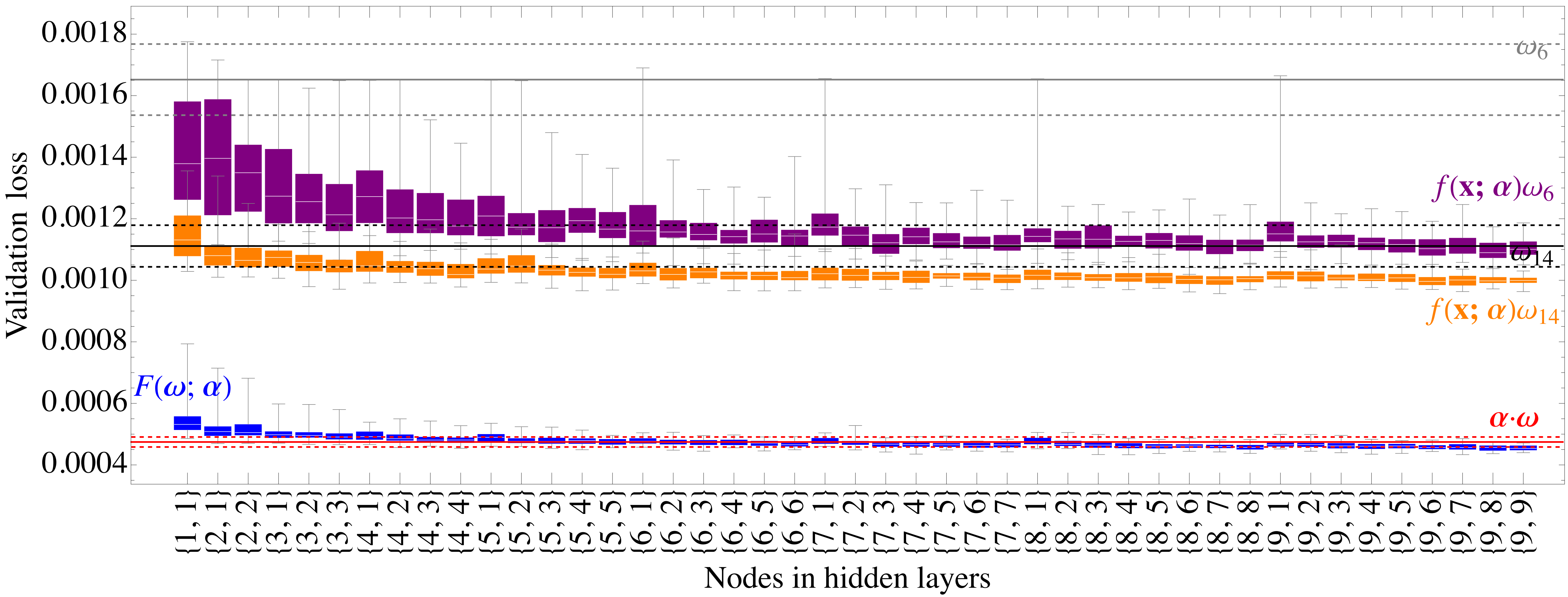
Conclusions

- CPV $\tilde{\kappa} \neq 0$ clear sign of physics beyond the Standard Model
- Ideally it should be measured directly
- Many avenues not explored in this talk: top polarization observables, $pp \to tqh, pp \to tWh, \ldots$
- Proposed manifestly CP-odd lab. frame variables ω_i offer direct access to $\tilde{\kappa}$
- Constructed ω_i for $pp \to t\bar{t}h$ with dileptonic $t\bar{t}$, but any Higgs decay channel allowed (results shown with $h \to b\bar{b}$ only)
- The information contained in various ω_i could be combined to further optimise sensitivity to $\tilde{\kappa}$

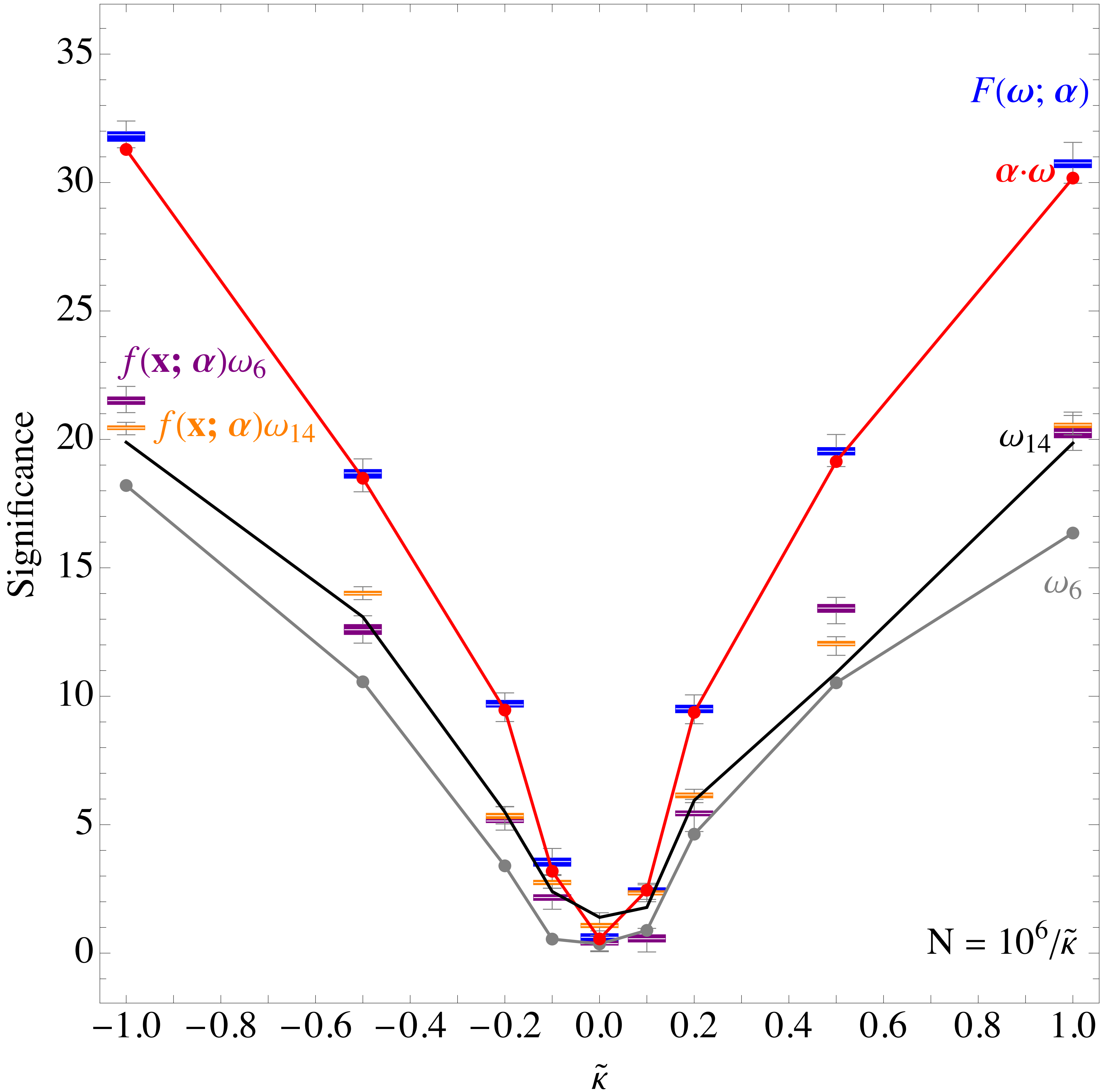
Additional slides

$$\omega_1 \sim [(\boldsymbol{p}_{\ell^-} \times \boldsymbol{p}_{\ell^+}) \cdot \boldsymbol{p}_h]\,[(\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot \boldsymbol{p}_h]\,,$$
$$\omega_2 \sim [(\boldsymbol{p}_{\ell^-} \times \boldsymbol{p}_{\ell^+}) \cdot \boldsymbol{p}_h]\,[(\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+})]\,,$$
$$\omega_3 \sim [(\boldsymbol{p}_{\ell^-} \times \boldsymbol{p}_{\ell^+}) \cdot \boldsymbol{p}_h]\,[(\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})]\,,$$
$$\omega_4 \sim [(\boldsymbol{p}_{\ell^-} \times \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})]\,[(\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot \boldsymbol{p}_h]\,,$$
$$\omega_5 \sim [(\boldsymbol{p}_{\ell^-} \times \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})]\,[(\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+})]\,,$$
$$\omega_6 \sim [(\boldsymbol{p}_{\ell^-} \times \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})]\,[(\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})]\,.$$
$$\omega_7 \sim [(\boldsymbol{p}_b \times \boldsymbol{p}_{\bar{b}}) \cdot \boldsymbol{p}_h]\,[(\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}) \cdot \boldsymbol{p}_h]\,,$$
$$\omega_8 \sim [(\boldsymbol{p}_b \times \boldsymbol{p}_{\bar{b}}) \cdot \boldsymbol{p}_h]\,[(\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+})]\,,$$
$$\omega_9 \sim [(\boldsymbol{p}_b \times \boldsymbol{p}_{\bar{b}}) \cdot \boldsymbol{p}_h]\,[(\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})]\,,$$
$$\omega_{10} \sim [(\boldsymbol{p}_b \times \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+})]\,[(\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}) \cdot \boldsymbol{p}_h]\,,$$
$$\omega_{11} \sim [(\boldsymbol{p}_b \times \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+})]\,[(\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+})]\,,$$
$$\omega_{12} \sim [(\boldsymbol{p}_b \times \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+})]\,[(\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})]\,,$$
$$\omega_{13} \sim [(\boldsymbol{p}_b \times \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+})]\,[(\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+})]\,,$$
$$\omega_{14} \sim [(\boldsymbol{p}_{\ell^-} \times \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}})]\,[(\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+})]\,.$$
$$\omega_{15} \sim [\boldsymbol{p}_h \times (\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}})]\,[(\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}) \cdot \boldsymbol{p}_h]\,,$$
$$\omega_{16} \sim [\boldsymbol{p}_h \times (\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}})]\,[(\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+})]\,,$$
$$\omega_{17} \sim [\boldsymbol{p}_h \times (\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}})]\,[(\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})]\,,$$
$$\omega_{18} \sim [\boldsymbol{p}_h \times (\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})]\,[(\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot \boldsymbol{p}_h]\,,$$
$$\omega_{19} \sim [\boldsymbol{p}_h \times (\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})]\,[(\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_{\ell^-} + \boldsymbol{p}_{\ell^+})]\,,$$
$$\omega_{20} \sim [\boldsymbol{p}_h \times (\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})]\,[(\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b + \boldsymbol{p}_{\bar{b}})]\,,$$
$$\omega_{21} \sim [\boldsymbol{p}_h \times (\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}})]\,[(\boldsymbol{p}_{\ell^-} - \boldsymbol{p}_{\ell^+}) \cdot (\boldsymbol{p}_b - \boldsymbol{p}_{\bar{b}})]\,.$$





0.0018
0.0016
0.0014
0.0012
0.0010
0.0008
0.0006
0.0004
Validation loss
ω_6
$f(\mathbf{x};\,\alpha)\omega_6$
ω_{14}
$f(\mathbf{x};\,\alpha)\omega_{14}$
$F(\omega;\,\alpha)$
$\alpha\cdot\omega$
{1, 1} {2, 1} {2, 2} {3, 1} {3, 2} {3, 3} {4, 1} {4, 2} {4, 3} {4, 4} {5, 1} {5, 2} {5, 3} {5, 4} {5, 5} {6, 1} {6, 2} {6, 3} {6, 4} {6, 5} {6, 6} {7, 1} {7, 2} {7, 3} {7, 4} {7, 5} {7, 6} {7, 7} {8, 1} {8, 2} {8, 3} {8, 4} {8, 5} {8, 6} {8, 7} {8, 8} {9, 1} {9, 2} {9, 3} {9, 4} {9, 5} {9, 6} {9, 7} {9, 8} {9, 9}
Nodes in hidden layers



Significance
$F(\omega;\alpha)$
$\alpha\cdot\omega$
$f(\mathbf{x};\alpha)\omega_6$
$f(\mathbf{x};\alpha)\omega_{14}$
ω_{14}
ω_6
$N = 10^6/\tilde{\kappa}$
$\tilde{\kappa}$