Top FCNC interactions

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Disclaimer for the flavour-aware audience

Never underestimate the joy people derive
from hearing something they already know.
E. Fermi

A minimal hope for the flavour-indifferent audience

Before I came here I was confused about this subject.
Having listened to your lecture I am still confused.
But on a higher level.
E. Fermi

Outline

1. Introduction/Motivation
2. Top FNCN through example
   - Two Higgs doublets models
   - Vector-like quarks
3. Summary

Introduction/Motivation

Top FCNC interactions?

Flavour Changing Neutral Currents:

$t - q -$ Neutral boson interactions

- $q = u, c$
- Neutral boson $= \gamma, g$ (massless vectors, gauge symmetry)
- Neutral boson $= Z$
  (massive vector, spontaneously broken gauge symmetry)
- Neutral boson $=$ h (massive scalar)
  (SFCNC: Scalar Flavour Changing Neutral *Couplings*)
- (Neutral boson $= Z'$, H, ...)

Introduction/Motivation

SM couplings

- Fermion-Scalar (Yukawa couplings) ($\tilde{\phi} = i\sigma_2\phi$)

$$\mathscr{L}_{\rm Y} = -\mathbf{\bar{Q}^0_L}\,\tilde{\phi}\,\mathbf{Y_u}\,\mathbf{u^0_R} - \mathbf{\bar{Q}^0_L}\,\phi\,\mathbf{Y_d}\,\mathbf{d^0_R} \;+\text{H.c.}$$

  In flavour space, vectors $\mathbf{\bar{Q}^0_L}$, $\mathbf{u^0_R}$, $\mathbf{d^0_R}$, matrices $\mathbf{Y_u}$, $\mathbf{Y_d}$

- EW-SSB

$$\tilde{\phi} = \begin{pmatrix} \frac{v+\mathrm{h}-iG^0}{\sqrt{2}} \\ G^- \end{pmatrix}$$

- Mass terms & Higgs Yukawas

$$\mathscr{L}_{\rm Y} \supset -\left[\mathbf{\bar{u}^0_L}\,\frac{v}{\sqrt{2}}\mathbf{Y_u}\,\mathbf{u^0_R} + \mathbf{\bar{d}^0_L}\,\frac{v}{\sqrt{2}}\mathbf{Y_d}\,\mathbf{d^0_R}\right]\left(1+\frac{\mathrm{h}}{v}\right) + \text{H.c.}$$

Bidiagonalization of the mass matrices $\frac{v}{\sqrt{2}}\,\mathbf{Y_{u,d}}$ $\Rightarrow$ no tree SFCNC!

Introduction/Motivation

SM couplings

- Fermion-Gauge vector (from covariant derivative $\sum i\bar{\psi}\gamma^\mu D_\mu \psi$)

$$\mathscr{L}_Z = -\frac{g}{c_W} Z_\mu J_Z^\mu - e A_\mu J_{\mathrm{EM}}^\mu$$

where

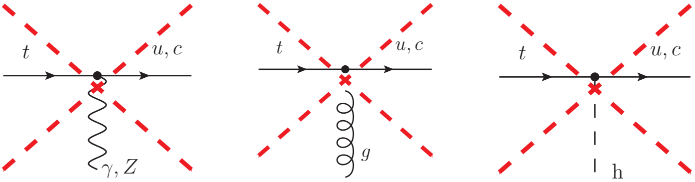
$$J_{\mathrm{EM}}^\mu = \frac{2}{3}\left[\bar{\mathbf{u}}_{\mathbf{L}}^{\mathbf{0}}\gamma^\mu \mathbf{u}_{\mathbf{L}}^{\mathbf{0}} + \bar{\mathbf{u}}_{\mathbf{R}}^{\mathbf{0}}\gamma^\mu \mathbf{u}_{\mathbf{R}}^{\mathbf{0}}\right] - \frac{1}{3}\left[\bar{\mathbf{d}}_{\mathbf{L}}^{\mathbf{0}}\gamma^\mu \mathbf{d}_{\mathbf{L}}^{\mathbf{0}} + \bar{\mathbf{d}}_{\mathbf{R}}^{\mathbf{0}}\gamma^\mu \mathbf{d}_{\mathbf{R}}^{\mathbf{0}}\right]$$

$$J_Z^\mu = -s_W^2\, J_{\mathrm{EM}}^\mu + \frac{1}{2}\left[\bar{\mathbf{u}}_{\mathbf{L}}^{\mathbf{0}}\gamma^\mu \mathbf{u}_{\mathbf{L}}^{\mathbf{0}} - \bar{\mathbf{d}}_{\mathbf{L}}^{\mathbf{0}}\gamma^\mu \mathbf{d}_{\mathbf{L}}^{\mathbf{0}}\right]$$

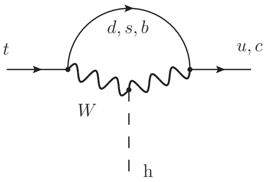
Again, bidiagonalization of the mass matrices ⇒ no tree FCNC!

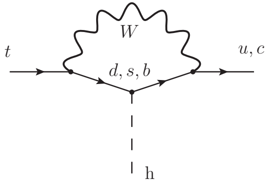
Introduction/Motivation

SM couplings



But





... and similarly for γ, g, Z

Introduction/Motivation

Beside charges (electric & colour)

the defining particularity of the top quark is its mass[1]:

$$m_t \simeq \frac{v}{\sqrt{2}}$$

$$m_t \gg m_f \quad f = \text{any other SM fermion}$$

$$m_t > m_Z, m_W, m_{\text{h}}$$

- Weak[2] decay before hadronization ($\tau \sim 5 \times 10^{-25}$s)

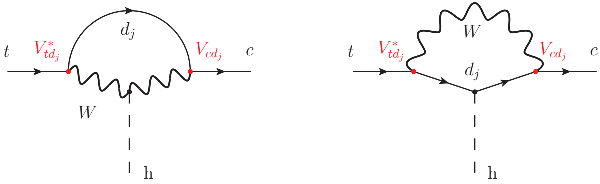
  $\Rightarrow$ No (pseudoscalar) meson oscillations (like D, K, B_d, B_s)
  e.g. B meson lifetimes $\sim 10^{-12}$s
- Only fermion with real h and Z in decays

[1] Flavour *is* mass

[2] Weak coupling $g = \frac{2M_W}{v} \sim \frac{2}{3}$

Introduction/Motivation

The SM picture, tch example



$$\text{Amplitude } \mathcal{M} = \# \sum_{d_j=d,s,b} V^*_{td_j} V_{cd_j} F(x_j), \qquad x_j = m^2_{d_j}/M^2_W$$

GIM suppression

- Masses, e.g. if $m_d = m_s = m_b$,
$$\mathcal{M} \to \# \left(V^*_{td} V_{cd} + V^*_{ts} V_{cs} + V^*_{tb} V_{cb} \right) = 0$$
- CKM, if $V = \mathbf{1}$, $\mathcal{M} \to 0$, $\quad |V| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$ with $\lambda \simeq 0.22$

Introduction/Motivation

The SM picture – Summary

- No tree level top FCNC interactions
- One loop top FCNC interactions
  - Full GIM suppression (mass and CKM)

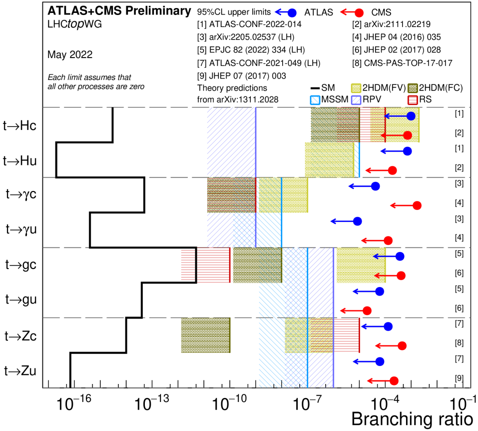
The SM picture – A New Physics lesson

- Any signal is New Physics!

An experiment disproving a prediction is discovery.

E. Fermi

Introduction/Motivation



LHC Top Working Group

Introduction/Motivation

Beyond the SM picture, different top FCNC interactions arise in

- MultiHiggs doublets models (2HDMs, 3HDMs, ...)
- MSSM
- R parity violating SUSY
- Warped extra-dimensions
- Composite Higgs
- Vector-like quarks (VLQs)
- ...

Top FCNC interactions in 2HDMs

- Up quark Yukawa couplings ($\tilde{\Phi}_j = i\sigma_2\Phi_j^*$)

$$\mathscr{L}_{\mathrm{Y}} = -\mathbf{\bar{Q}_L^0}\left(\tilde{\Phi}_1\mathbf{Y_{u1}} + \tilde{\Phi}_2\mathbf{Y_{u2}}\right)\mathbf{u_R^0} + \text{H.c.}$$

- Vacuum expansion for EWSSB

$$\Phi_j = e^{i\theta_j}\begin{pmatrix}\varphi_j^+ \\ \frac{v_j+\rho_j+i\eta_j}{\sqrt{2}}\end{pmatrix}, \qquad \langle\Phi_j\rangle = \frac{v_j e^{i\theta_j}}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}$$

- Higgs basis, $c_\beta \equiv \cos\beta = \frac{v_1}{v}$, $s_\beta \equiv \sin\beta = \frac{v_2}{v}$, $t_\beta \equiv \tan\beta$, $\theta = \theta_2 - \theta_1$

$$\begin{pmatrix}H_1\\H_2\end{pmatrix} = \mathcal{R}_\beta\begin{pmatrix}e^{-i\theta_1}\Phi_1\\e^{-i\theta_2}\Phi_2\end{pmatrix}, \quad \text{with} \quad \mathcal{R}_\beta = \begin{pmatrix}c_\beta & s_\beta\\ -s_\beta & c_\beta\end{pmatrix}, \ \mathcal{R}_\beta^T = \mathcal{R}_\beta^{-1}$$

Top FCNC interactions in 2HDMs

- Higgs basis

$$\langle H_1\rangle = \frac{v}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}, \quad \langle H_2\rangle = \begin{pmatrix}0\\0\end{pmatrix}, \qquad v^2 = v_1^2 + v_2^2 = \frac{1}{\sqrt{2}G_F}$$

$$H_1 = \begin{pmatrix}G^+\\ \frac{v+H^0+iG^0}{\sqrt{2}}\end{pmatrix}, \quad H_2 = \begin{pmatrix}\mathrm{H}^+\\ \frac{R^0+iI^0}{\sqrt{2}}\end{pmatrix}$$

  - would-be Goldstone bosons G^0, $G^\pm$
  - physical charged scalar $H^\pm$
  - neutral scalars $\{H^0, R^0, I^0\}$, *not the mass eigenstates*

- Up quark Yukawa couplings again

$$\mathscr{L}_{\mathrm{Y}} \supset -\bar{\mathbf{Q}}_{\mathbf{L}}^{\mathbf{0}}\left(\mathrm{M}_{\mathbf{u}}^{\mathbf{0}}\left(1+\frac{H^0}{v}\right)+\mathrm{N}_{\mathbf{u}}^{\mathbf{0}}\left(\frac{R^0-iI^0}{v}\right)\right)\mathbf{u}_{\mathbf{R}}^{\mathbf{0}} + \mathrm{H.c.}$$

Two different structures

Top FCNC interactions in 2HDMs

- *Two* different structures

$$\mathrm{M}_{\mathbf{u}}^{\mathbf{0}} = \frac{e^{-i\theta_1} v}{\sqrt{2}} (c_\beta \mathbf{Y_{u1}} + e^{-i\theta} s_\beta \mathbf{Y_{u2}})$$

$$\mathrm{N}_{\mathbf{u}}^{\mathbf{0}} = \frac{e^{-i\theta_1} v}{\sqrt{2}} (-s_\beta \mathbf{Y_{u1}} + e^{-i\theta} c_\beta \mathbf{Y_{u2}})$$

- Usual bi-unitary diagonalization $\mathrm{M}_{\mathbf{u}}^{\mathbf{0}} \mapsto \mathrm{M}_u$, $\mathrm{N}_{\mathbf{u}}^{\mathbf{0}} \mapsto \mathrm{N}_u$ where
  - $\mathrm{M}_u = \mathrm{diag}(m_u, m_c, m_t)$ is the diagonal up quark mass matrix
  - N_u is the new flavour structure

    arbitrary in general, source of tree level SFCNC!
- N_u SFCNC *leak* into the physical Higgs h through scalar mixing

$$\begin{pmatrix} \mathrm{h} \\ \mathrm{H} \\ \mathrm{A} \end{pmatrix} = \mathcal{R}^T \begin{pmatrix} H^0 \\ R^0 \\ I^0 \end{pmatrix}$$

Top FCNC interactions in 2HDMs

- In general, SFCNC including $t c$h and $t u$h couplings!
- However, no tree level tcZ couplings
- Even if one imposes no tree level SFCNC,
  there are new one loop contributions with $W^{\pm} \to \mathrm{H}^{\pm}$
- Most general 2HDM (type III): one can have values within "present discovery range"... but one can just play with the couplings to satisfy the most sensitive bounds while remaining close to them ("discovery tomorrow")
- Interest of 2HDMs with additional properties (e.g. symmetries) which limit that possibility or force relations between different processes

Top FCNC interactions in 2HDMs

- Interest of 2HDMs with additional properties (e.g. symmetries) which limit that possibility or force relations between different processes
- An interesting example within a class of models due to Branco, Grimus, Lavoura (Minimal Flavour Violation *avant la lettre*) "Down BGL model type α" ($\alpha = d, s, b$)

$$[\mathrm{N}_u]_{jk} = \left(t_\beta \delta_{jk} - (t_\beta + t_\beta^{-1}) V_{j\alpha} V^*_{k\alpha}\right) [\mathrm{M}_u]_{kk}$$
$$[\mathrm{N}_d]_{jk} = \left(t_\beta - (t_\beta + t_\beta^{-1}) \delta_{j\alpha}\right) [\mathrm{M}_d]_{kk} \delta_{jk}$$

- No tree SFCNC for down quarks
- Tree SFCNC for up quarks, controlled by t_β and CKM elements

Top FCNC interactions in 2HDMs

- Interest of 2HDMs with additional properties (e.g. symmetries) which limit that possibility or force relations between different processes
- Example: type s BGL model

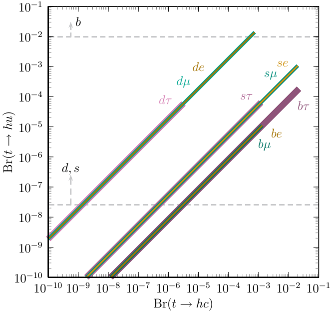
tq couplings

$$[\mathrm{N}_u]_{tc} = -(t_\beta + t_\beta^{-1}) V_{ts} V_{cs}^* m_c$$

$$[\mathrm{N}_u]_{ct} = -(t_\beta + t_\beta^{-1}) V_{cs} V_{ts}^* m_t$$

$$[\mathrm{N}_u]_{tu} = -(t_\beta + t_\beta^{-1}) V_{ts} V_{us}^* m_u$$

$$[\mathrm{N}_u]_{ut} = -(t_\beta + t_\beta^{-1}) V_{us} V_{ts}^* m_t$$



$t \to \mathrm{h}u$ vs $t \to \mathrm{h}c$

Top FCNC interactions from Vector-like Quarks

- Another BSM scenario with top FCNC interactions
- Instead of an extended scalar sector, an extended fermion sector
- One simple possibility, one extra $SU(2)_L$ singlet generation:

$$\text{Up} \quad p_L^0,\ p_R^0, \qquad \text{Down} \quad n_L^0,\ n_R^0$$

in addition to SM content

- Yukawa + mass terms

$$\mathscr{L}_{YM} = -\,\bar{\mathbf{Q}}_{\mathbf{L}}^{\mathbf{0}}\,\tilde{\phi}\,\mathbf{Y_u}\,\mathbf{u}_{\mathbf{R}}^{\mathbf{0}} - \bar{\mathbf{Q}}_{\mathbf{L}}^{\mathbf{0}}\,\phi\,\mathbf{Y_d}\,\mathbf{d}_{\mathbf{R}}^{\mathbf{0}} - \bar{\mathbf{Q}}_{\mathbf{L}}^{\mathbf{0}}\,\tilde{\phi}\,\mathbf{y_u}\,p_R^0 - \bar{\mathbf{Q}}_{\mathbf{L}}^{\mathbf{0}}\,\phi\,\mathbf{y_d}\,n_R^0$$
$$-\bar{p}_L^0\,\mathbf{X_u}\,p_R^0 - \bar{n}_L^0\,\mathbf{X_d}\,n_R^0 - \bar{p}_L^0\,\mu_u\,p_R^0 - \bar{n}_L^0\,\mu_d\,n_R^0 \ + \text{H.c.}$$

Some bookkeeping necessary to track FCNC and SFCNC

Top FCNC interactions from Vector-like Quarks

- Mass matrices

$$\mathscr{L}_{YM} \supset -\begin{pmatrix}\bar{\mathbf{u}}^{\mathbf{0}}_{\mathbf{L}} & \bar{p}^0_L\end{pmatrix} \mathbf{M}^{\mathbf{0}}_{\mathbf{u}} \begin{pmatrix}\mathbf{u}^{\mathbf{0}}_{\mathbf{R}} \\ p^0_R\end{pmatrix} - \begin{pmatrix}\bar{\mathbf{d}}^{\mathbf{0}}_{\mathbf{L}} & \bar{n}^0_L\end{pmatrix} \mathbf{M}^{\mathbf{0}}_{\mathbf{d}} \begin{pmatrix}\mathbf{d}^{\mathbf{0}}_{\mathbf{R}} \\ n^0_R\end{pmatrix}$$

with

$$\mathbf{M}^{\mathbf{0}}_{\mathbf{f}} = \begin{pmatrix} \left[\frac{v}{\sqrt{2}}\mathbf{Y_f}\right]_{3\times 3} & \left[\frac{v}{\sqrt{2}}\mathbf{y_f}\right]_{3\times 1} \\ [\mathbf{X_f}]_{1\times 3} & \mu_f \end{pmatrix} \qquad f = u, d$$

- Condense right-handed singlets, Yukawas and mass terms

$$\begin{pmatrix}\mathbf{u}^{\mathbf{0}}_{\mathbf{R}} \\ p^0_R\end{pmatrix} = \mathbf{u}^{\mathbf{0}\prime}_{\mathbf{R}}, \quad \begin{pmatrix}\mathbf{d}^{\mathbf{0}}_{\mathbf{R}} \\ n^0_R\end{pmatrix} = \mathbf{d}^{\mathbf{0}\prime}_{\mathbf{R}}, \quad \tilde{\mathbf{Y}}_{\mathbf{f}} = \begin{pmatrix}\mathbf{Y_f} & \mathbf{y_f}\end{pmatrix}, \quad \tilde{\mathbf{X}}_{\mathbf{f}} = \begin{pmatrix}\mathbf{X_f} & \mu_f\end{pmatrix}$$

$$\mathbf{M}^{\mathbf{0}}_{\mathbf{f}} = \begin{pmatrix} \left[\frac{v}{\sqrt{2}}\tilde{\mathbf{Y}}_{\mathbf{f}}\right]_{3\times 4} \\ [\tilde{\mathbf{X}}_{\mathbf{f}}]_{1\times 4} \end{pmatrix} \qquad f = u, d$$

Top FCNC interactions from Vector-like Quarks

- Bidiagonalization $\mathbf{M_u^0} \mapsto \mathbf{M_u}$, $\mathbf{M_d^0} \mapsto \mathbf{M_d}$

$$\mathbf{M_u} = \text{Diag}(m_u, m_c, m_t, m_T), \qquad \mathbf{M_d} = \text{Diag}(m_d, m_s, m_b, m_B)$$

- The critical ingredient: diagonalization "mixes" fields from different representations $\mathbf{u_L^0}$ and p_L^0, $\mathbf{d_L^0}$ and n_L^0
- Implications for Yukawa couplings to h and Z couplings, which only involve $\mathbf{u_L^0}$, $\mathbf{d_L^0}$
- Higgs Yukawa couplings with up quarks

$$\mathscr{L}_{YM} \supset -\frac{\text{h}}{v} \begin{pmatrix} \bar{\mathbf{u}}_\mathbf{L}^\mathbf{0} & \bar{p}_L^0 \end{pmatrix} \begin{pmatrix} \left[\frac{v}{\sqrt{2}} \tilde{\mathbf{Y}}_\mathbf{u}\right]_{3\times 4} \\ [\mathbf{0}]_{1\times 4} \end{pmatrix} \mathbf{u_R^0}'$$

$\Rightarrow$ non-diagonal, i.e. SFCNC!

Top FCNC interactions from Vector-like Quarks

Neutral currents – EM

$$\mathscr{L}_A = -\, eA_\mu J^\mu_{\mathrm{EM}}$$

$$J^\mu_{\mathrm{EM}} = \frac{2}{3}\left[\bar{\mathbf{u}}^{\mathbf{0}}_{\mathbf{L}}\gamma^\mu \mathbf{u}^{\mathbf{0}}_{\mathbf{L}} + \bar{\mathbf{u}}^{\mathbf{0}}_{\mathbf{R}}\gamma^\mu \mathbf{u}^{\mathbf{0}}_{\mathbf{R}} + \bar{p}_L\gamma^\mu p_L + \bar{p}_R\gamma^\mu p_R\right] - \frac{1}{3}\left[\bar{\mathbf{d}}^{\mathbf{0}}_{\mathbf{L}}\gamma^\mu \mathbf{d}^{\mathbf{0}}_{\mathbf{L}} + \bar{\mathbf{d}}^{\mathbf{0}}_{\mathbf{R}}\gamma^\mu \mathbf{d}^{\mathbf{0}}_{\mathbf{R}} + \bar{n}_L\gamma^\mu n_L + \bar{n}_R\gamma^\mu n_R\right]$$

Into the mass bases (as expected)

$$J^\mu_{\mathrm{EM}} = \frac{2}{3}\bar{\mathbf{u}}_{\mathbf{L}}\gamma^\mu \mathbf{u}_{\mathbf{L}} - \frac{1}{3}\bar{\mathbf{d}}_{\mathbf{L}}\gamma^\mu \mathbf{d}_{\mathbf{L}}$$

Top FCNC interactions from Vector-like Quarks

Neutral currents – Z

$$\mathscr{L}_Z = -\frac{g}{c_W} Z_\mu J_Z^\mu$$

$$J_Z^\mu = -s_W^2 J_{\text{EM}}^\mu + \frac{1}{2}\left[\bar{\mathbf{u}}_{\mathbf{L}}^{\mathbf{0}}\gamma^\mu \mathbf{u}_{\mathbf{L}}^{\mathbf{0}} - \bar{\mathbf{d}}_{\mathbf{L}}^{\mathbf{0}}\gamma^\mu \mathbf{d}_{\mathbf{L}}^{\mathbf{0}}\right]$$

Into the mass bases

- EM piece is fine
- FCNC from the second piece!

Top FCNC interactions from Vector-like Quarks

- We have argued that h-SFCNC and Z-FCNC are present, "how?"
- One last ingredient is necessary to bring light into the picture…
- Charged currents

$$\mathscr{L}_W = -\frac{g}{\sqrt{2}} W_\mu^+ J_W^\mu + \text{H.c.}, \qquad J_W^\mu = \bar{\mathbf{u}}_{\mathbf{L}}^{\mathbf{0}} \, \gamma^\mu \, \bar{\mathbf{d}}_{\mathbf{L}}^{\mathbf{0}}$$

Into the mass bases

$$J_W^\mu = \bar{\mathbf{u}}_{\mathbf{L}} \, V \, \gamma^\mu \, \bar{\mathbf{d}}_{\mathbf{L}}$$

where V is the 4×4 CKM matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{uB} \\ V_{cd} & V_{cs} & V_{cb} & V_{cB} \\ V_{td} & V_{ts} & V_{tb} & V_{tB} \\ V_{Td} & V_{Ts} & V_{Tb} & V_{TB} \end{pmatrix}$$

Top FCNC interactions from Vector-like Quarks

- But in this scenario V is not 4×4 unitary!
- It is embedded in a 5×5 unitary matrix

$$\hat{V} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{uB} & V_{u5} \\ V_{cd} & V_{cs} & V_{cb} & V_{cB} & V_{c5} \\ V_{td} & V_{ts} & V_{tb} & V_{tB} & V_{t5} \\ V_{Td} & V_{Ts} & V_{Tb} & V_{TB} & V_{T5} \\ V_{5d} & V_{5s} & V_{5b} & V_{5B} & 0 \end{pmatrix}$$

- The deviations from 4×4 unitarity

$$VV^\dagger = H_{U_L}, \qquad V^\dagger V = H_{D_L}$$

are the ones that control FCNC and SFCNC!

Vector-like Quarks

- Back to tree level Z-FCNC

$$\mathscr{L}_Z = -\frac{g}{c_W} Z_\mu J_Z^\mu$$

$$J_Z^\mu = -s_W^2 J_{\text{EM}}^\mu + \frac{1}{2}\left[\bar{\mathbf{u}}_{\mathbf{L}} H_{U_L} \gamma^\mu \mathbf{u}_{\mathbf{L}} - \bar{\mathbf{d}}_{\mathbf{L}} H_{D_L} \gamma^\mu \mathbf{d}_{\mathbf{L}}\right]$$

- Back to tree level h-SFCNC

$$\mathscr{L}_{YM} \supset -\frac{\text{h}}{v} \bar{\mathbf{u}}_{\mathbf{L}} H_{U_L} \mathbf{M}_{\mathbf{u}} \mathbf{u}_{\mathbf{R}} + \text{H.c.}$$

- both Z-FCNC and h-SFCNC are tied to non-unitarity of CKM!

Summary

- Top FCNC interactions are *New Physics hunting ground*
- Impressive experimental progress!
- Top FCNC arise easily in BSM scenarios (tree or loop level)
  within sensitivity reach (but not necessarily)
- However, if they do appear, patterns should come along
  (correlations, new particles)
- For illustration 2HDMs, and models with VLQ,
  but many more in the market

Thank you!

Backup

Yukawa couplings

Neutral scalars

$$\mathscr{L}_{\mathrm{S}\bar{f}f} = -\frac{\mathrm{S}}{v}\,\bar{f}\,\left[\mathcal{R}_{1s}\mathrm{M}_f + \mathcal{R}_{2s}\frac{\mathrm{N}_f + \mathrm{N}_f^\dagger}{2} + i\epsilon_{(f)}\mathcal{R}_{3s}\frac{\mathrm{N}_f - \mathrm{N}_f^\dagger}{2}\right] f$$
$$- \frac{\mathrm{S}}{v}\,\bar{f}\,\gamma_5\,\left(\mathcal{R}_{2s}\frac{\mathrm{N}_f - \mathrm{N}_f^\dagger}{2} + i\epsilon_{(f)}\mathcal{R}_{3s}\frac{\mathrm{N}_f + \mathrm{N}_f^\dagger}{2}\right) f$$

where $s = 1, 2, 3$ in correspondence with $\mathrm{S} = \mathrm{h}, \mathrm{H}, \mathrm{A}$; $f = u, d, \ell$; in terms proportional to $\mathcal{R}_{3s}$, $\epsilon_{(d)} = \epsilon_{(\ell)} = -\epsilon_{(u)} = 1$

Yukawa couplings

Charged scalars

$$\mathscr{L}_{\mathrm{H}^{\pm}ud} = \frac{\mathrm{H}^-}{\sqrt{2}v}\bar{d}\left[V^\dagger \mathrm{N}_u - \mathrm{N}_d^\dagger V^\dagger + \gamma_5\left(V^\dagger \mathrm{N}_u + \mathrm{N}_d^\dagger V^\dagger\right)\right] u$$
$$+ \frac{\mathrm{H}^+}{\sqrt{2}v}\bar{u}\left[\mathrm{N}_u^\dagger V - V\mathrm{N}_d + \gamma_5\left(\mathrm{N}_u^\dagger V + V\mathrm{N}_d\right)\right] d$$

and

$$\mathscr{L}_{\mathrm{H}^{\pm}\ell\nu} = -\frac{\sqrt{2}}{v}\mathrm{H}^+\,\bar{\nu}_L\,U^\dagger \mathrm{N}_\ell\,\ell_R - \frac{\sqrt{2}}{v}\mathrm{H}^-\,\bar{\ell}_R\,\mathrm{N}_\ell^\dagger U\,\nu_L$$

V and U are, respectively, the CKM and PMNS mixing matrices (massless neutrinos assumed, one can set $U \to \mathbf{1}$)

Vector-like Quarks – Detailed bookkeeping

$$\begin{pmatrix}\mathbf{u_R^0}\\ p_R^0\end{pmatrix}=\mathcal{U}_{u_R}\,\mathbf{u_R}\qquad \begin{pmatrix}\mathbf{u_L^0}\\ p_L^0\end{pmatrix}=\mathcal{U}_{u_L}\,\mathbf{u_L}=\begin{pmatrix}[A_{u_L}]_{3\times4}\\ [B_{u_L}]_{1\times4}\end{pmatrix}\mathbf{u_L}$$

Unitary $\mathcal{U}_{u_R}, \mathcal{U}_{u_L}$

$$\mathcal{U}_{u_R}\mathcal{U}_{u_R}^\dagger=\mathcal{U}_{u_R}^\dagger\mathcal{U}_{u_R}=\mathbf{1}_{4\times4}$$

$$\mathcal{U}_{u_L}\mathcal{U}_{u_L}^\dagger=\begin{pmatrix}A_{u_L}\\ B_{u_L}\end{pmatrix}\begin{pmatrix}A_{u_L}^\dagger & B_{u_L}^\dagger\end{pmatrix}=\begin{pmatrix}A_{u_L}A_{u_L}^\dagger & A_{u_L}B_{u_L}^\dagger\\ B_{u_L}A_{u_L}^\dagger & B_{u_L}B_{u_L}^\dagger\end{pmatrix}=\begin{pmatrix}\mathbf{1}_{3\times3} & \mathbf{0}_{3\times1}\\ \mathbf{0}_{1\times3} & 1_{1\times1}\end{pmatrix}$$

$$\mathcal{U}_{u_L}^\dagger\mathcal{U}_{u_L}=\begin{pmatrix}A_{u_L}^\dagger & B_{u_L}^\dagger\end{pmatrix}\begin{pmatrix}A_{u_L}\\ B_{u_L}\end{pmatrix}=A_{u_L}^\dagger A_{u_L}+B_{u_L}^\dagger B_{u_L}=\mathbf{1}_{4\times4}$$

$$H_{U_L}=A_{u_L}^\dagger A_{u_L},\quad H_{U_L}A_{u_L}^\dagger=A_{u_L}^\dagger,\quad H_{U_L}B_{u_L}^\dagger=0$$

Vector-like Quarks – Detailed bookkeeping

Charged currents

$$\mathscr{L}_W = -\frac{g}{\sqrt{2}} W^+_\mu J^\mu_W + \text{H.c.}, \qquad J^\mu_W = \bar{\mathbf{u}}^{\mathbf{0}}_{\mathbf{L}}\, \gamma^\mu\, \bar{\mathbf{d}}^{\mathbf{0}}_{\mathbf{L}}$$

Into the mass bases

$$J^\mu_W = \bar{\mathbf{u}}_{\mathbf{L}}\, A^\dagger_{u_L}\, \gamma^\mu\, A_{d_L}\, \bar{\mathbf{d}}_{\mathbf{L}} = \bar{\mathbf{u}}_{\mathbf{L}}\, V\, \gamma^\mu\, \bar{\mathbf{d}}_{\mathbf{L}}$$

$V = A^\dagger_{u_L} A_{d_L}$ is the CKM matrix

- 4×4 but not unitary

$$V V^\dagger = A^\dagger_{u_L} A_{u_L} = H_{U_L}, \qquad V^\dagger V = A^\dagger_{d_L} A_{d_L} = H_{D_L}$$

- Embedded in a 5×5 unitary matrix

$$\hat{V} = \begin{pmatrix} V & B^\dagger_{u_L} \\ B_{d_L} & 0 \end{pmatrix}$$

Vector-like Quarks – Detailed bookkeeping

Mass Diagonalization

$$-\frac{\mathrm{h}}{v}\begin{pmatrix}\bar{\mathbf{u}}_{\mathbf{L}}^{\mathbf{0}} & \bar{p}_L^0\end{pmatrix}\begin{pmatrix}\frac{v}{\sqrt{2}}\tilde{\mathbf{Y}}_{\mathbf{u}}\\ \tilde{\mathbf{X}}_{\mathbf{u}}\end{pmatrix}\mathbf{u}_{\mathbf{R}}^{\mathbf{0}\prime} = -\frac{\mathrm{h}}{v}\bar{\mathbf{u}}_{\mathbf{L}}\mathcal{U}_{u_L}^{\dagger}\begin{pmatrix}\frac{v}{\sqrt{2}}\tilde{\mathbf{Y}}_{\mathbf{u}}\\ \tilde{\mathbf{X}}_{\mathbf{u}}\end{pmatrix}\mathcal{U}_{u_R}\mathbf{u}_{\mathbf{R}}$$

$$A_{u_L}^{\dagger}\frac{v}{\sqrt{2}}\tilde{\mathbf{Y}}_{\mathbf{u}}\mathcal{U}_{u_R} + B_{u_L}^{\dagger}\tilde{\mathbf{X}}_{\mathbf{u}}\mathcal{U}_{u_R} = \mathbf{M}_{\mathbf{u}} = \mathrm{Diag}(m_u, m_c, m_t, m_T)$$

Higgs Yukawa couplings with up quarks

$$\begin{pmatrix}\bar{\mathbf{u}}_{\mathbf{L}}^{\mathbf{0}} & \bar{p}_L^0\end{pmatrix}\begin{pmatrix}\left[\frac{v}{\sqrt{2}}\tilde{\mathbf{Y}}_{\mathbf{u}}\right]_{3\times 4}\\ [\mathbf{0}]_{1\times 4}\end{pmatrix}\mathbf{u}_{\mathbf{R}}^{\mathbf{0}\prime} = \bar{\mathbf{u}}_{\mathbf{L}}\mathcal{U}_{u_L}^{\dagger}\begin{pmatrix}\left[\frac{v}{\sqrt{2}}\tilde{\mathbf{Y}}_{\mathbf{u}}\right]_{3\times 4}\\ [\mathbf{0}]_{1\times 4}\end{pmatrix}\mathcal{U}_{u_R}\mathbf{u}_{\mathbf{R}}$$

$$A_{u_L}^{\dagger}\frac{v}{\sqrt{2}}\tilde{\mathbf{Y}}_{\mathbf{u}}\mathcal{U}_{u_R} = \mathbf{M}_{\mathbf{u}} - B_{u_L}^{\dagger}\tilde{\mathbf{X}}_{\mathbf{u}}\mathcal{U}_{u_R} = ?$$

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Higgs Yukawa couplings with up quarks

$$A_{u_L}^{\dagger}\frac{v}{\sqrt{2}}\tilde{\mathbf{Y}}_{\mathbf{u}}\mathcal{U}_{u_R}=\mathbf{M}_{\mathbf{u}}-B_{u_L}^{\dagger}\tilde{\mathbf{X}}_{\mathbf{u}}\mathcal{U}_{u_R}=?$$

Left H_{U_L} multiplication, $H_{U_L}A_{u_L}^{\dagger}=A_{u_L}^{\dagger}$, $H_{U_L}B_{u_L}^{\dagger}=0$

$$H_{U_L}A_{u_L}^{\dagger}\frac{v}{\sqrt{2}}\tilde{\mathbf{Y}}_{\mathbf{u}}\mathcal{U}_{u_R}=A_{u_L}^{\dagger}\frac{v}{\sqrt{2}}\tilde{\mathbf{Y}}_{\mathbf{u}}\mathcal{U}_{u_R}=H_{U_L}\mathbf{M}_{\mathbf{u}}$$

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Neutral currents – Z

$$\mathscr{L}_Z = -\frac{g}{c_W} Z_\mu J_Z^\mu$$

$$J_Z^\mu = -s_W^2 J_{\mathrm{EM}}^\mu + \frac{1}{2}\left[\bar{\mathbf{u}}_{\mathbf{L}}^{\mathbf{0}}\gamma^\mu \mathbf{u}_{\mathbf{L}}^{\mathbf{0}} - \bar{\mathbf{d}}_{\mathbf{L}}^{\mathbf{0}}\gamma^\mu \mathbf{d}_{\mathbf{L}}^{\mathbf{0}}\right]$$

Into the mass bases

$$J_Z^\mu = -s_W^2 J_{\mathrm{EM}}^\mu + \frac{1}{2}\left[\bar{\mathbf{u}}_{\mathbf{L}} H_{U_L}\gamma^\mu \mathbf{u}_{\mathbf{L}} - \bar{\mathbf{d}}_{\mathbf{L}} H_{D_L}\gamma^\mu \mathbf{d}_{\mathbf{L}}\right]$$