



Four top quark production in SMEFT

[2208.04962]

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Introduction to *t*ttt

- Rare processes with distinctive signatures
- Firstly computed at NLO in QCD in [1206.3064]
- Available in event generators [1405.0301] [1507.05640]
 [2110.15159]
- Complete NLO including EW were computed in [1711.02116] $\rightarrow \sigma_{NLO}^{SM} \sim 12$ fb $\pm 20\%$ at $\sqrt{s} = 13$ TeV
- Experiment; ATLAS: 24^{+7}_{-6} fb, CMS: $12.6^{+5.8}_{-5.2}$ fb, with evidence of 4.7σ by ATLAS

see also talks by Jan van der Linden and Melissa van Beekveld

Introduction to $t\bar{t}t\bar{t}$

[1602.01934][1711.02116][2008.11743][2010.05915]

In the SM, the EW scattering is important, however, there are large cancellations in interfering with purely-QCD ones



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In SMEFT, the EFT interference with the SM EW amplitudes can not be neglected, e.g.



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What did we do?

Considered **all** QCD and EW-induced, splitting to α and κ_t , e.g.

$$d\sigma_{\mathrm{int},gg,[4\mathsf{F}]} = \alpha_s^3 \, d\sigma_{\mathrm{int},gg}^{(3,0,0)} + \alpha_s^2 \left(\alpha \, d\sigma_{\mathrm{int},gg}^{(2,1,0)} + \kappa_t \, d\sigma_{\mathrm{int},gg}^{(2,0,1)} \right)$$
$$d\sigma_{\mathrm{int},gg,[4\mathsf{F}]} = \alpha_s^3 \cdots + \alpha_s^2 \cdots + \alpha_s^1 \cdots + \alpha_s^0 \ldots$$

the total interference cross-section reads

$$\sigma_{INCL} = \sigma_3 + \sigma_2 + \sigma_1 + \sigma_0$$

and then systematically ...

- obtain differential and inclusive predictions for all relevant SMEFT operators
- toy fit to illustrate potential bounds on the effective coefficients

An example: 4H LHC inclusive predictions



σ₂ is dominant in all 4-heavy → 'non-naive' Sub-leading terms dictate the sign of the interference

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The big picture

	4H	2L2H	2F	OF
$\alpha_{s} = 4$	×	×	C _{tG}	CG
$\alpha_{s} = 3$	-	$\begin{matrix} c_{Qq}^{83}, c_{Qu}^8, c_{tq}^8, c_{Qd}^8, c_{tu}^8, c_{dq}^8, c_{td}^8, c_{Qq}^{81} \\ c_{Qq}^{11}, c_{Qu}^1, c_{tq}^1, c_{Qd}^1, c_{tu}^1, c_{td}^1 \end{matrix}$	$c_{t\varphi}, c_{tZ}, c_{tW}$	-
$\alpha_{s} = 2$	$c_{QQ}^8, c_{QQ}^1, c_{Qt}^8, c_{Qt}^1, c_{tt}^1$	C ³¹ Qq	$c_{arphi t}, c_{arphi Q}^{(-)}$	-
$\alpha_{s} = 1$	-	-	-	$c_{arphi G}$
$\alpha_{s} = 0$	-	-	-	-

The set of **non-naive** operators are

all 4-heavy and
$$\{\mathcal{O}_{Qq}^{3,1}, \mathcal{O}_{t\varphi}, \mathcal{O}_{tG}, \mathcal{O}_{\varphi Q}^{(-)}, \mathcal{O}_{\varphi t}, \mathcal{O}_{\varphi G}\}$$

4H LHC differential predictions



Z/ γ contributions are dominant \rightarrow expected from the inclusive study

• Mild interference growth with $m_{tttt} \sim \sqrt{s}$

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Toy fit

 $\textbf{Goal} \rightarrow \text{provide}$ illustrative constraints on WCs

All fit results are shown in **two cases:** when including **only QCD contributions**, and when **including all terms**

LHC

- Available measurements are ATLAS: 24⁺⁷₋₆ fb, CMS: 12.6^{+5.8}₋₅ fb
- Theory prediction of 12 ±20% fb [1711.02116]
- HL-LHC
 - We assume the measurement sits on the SM rate, within an expected total experimental uncertainty of 28% [1902.04070]
 - Assume 20% total theoretical uncertainty
- FCC-hh
 - Same as HL-LHC but we keep 5% total experimental uncertainty

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We show three different fit results assessing different aspects of four-top production ...

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Toy fit: impact of subleading terms



At the linear interference level, the inclusion of subleading terms in 4H predictions is crucial

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Toy fit: impact of differential information

We add information from m_{tttt} in three bins ...



differential information improves sensitivity

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Toy fit: the big picture



■ The effect of subleading terms in 4H is diluted by including the quadratic contributions → 4H amplitudes are QCD-induced

• $\sqrt{s} = 100$ TeV will have a strong handle on all of the operators



- The EW scattering in four-top amplitudes is crucial element of its predictions
- Computed four-top SMEFT predictions considering all QCD and EW-induced amplitudes
- We defined a set of 'non-naive' operators for which formal subleading terms can not be neglected

all 4-heavy and
$$\{\mathcal{O}_{Qq}^{3,1}, \mathcal{O}_{t\varphi}, \mathcal{O}_{tG}, \mathcal{O}_{\varphi Q}^{(-)}, \mathcal{O}_{\varphi t}, \mathcal{O}_{\varphi G}\}$$

 Performed a toy fit to assess the impact of subleading terms and differential information in constraining relevant effective coefficients



Operators' definitions: 4F in Warsaw basis

$$\begin{aligned} \mathcal{Q}_{qq}^{1(ijkl)} &= (\bar{q}_i \gamma^{\mu} q_j) (\bar{q}_k \gamma_{\mu} q_l), \\ \mathcal{Q}_{qu}^{1(ijkl)} &= (\bar{q}_i \gamma^{\mu} q_j) (\bar{u}_k \gamma_{\mu} u_l), \\ \mathcal{Q}_{qd}^{1(ijkl)} &= (\bar{q}_i \gamma^{\mu} q_j) (\bar{d}_k \gamma_{\mu} d_l), \\ \mathcal{Q}_{ud}^{1(ijkl)} &= (\bar{u}_i \gamma^{\mu} u_j) (\bar{d}_k \gamma_{\mu} d_l), \\ \hat{\mathcal{Q}}_{ud}^{1(ijkl)} &= (\bar{q}_i u_j) \epsilon (\bar{q}_k d_l), \end{aligned}$$

$$\begin{aligned} \mathcal{Q}_{qq}^{3(ijkl)} &= (\bar{q}_i \gamma^{\mu} \tau^l q_j) (\bar{q}_k \gamma_{\mu} \tau^l q_l), \\ \mathcal{Q}_{qu}^{8(ijkl)} &= (\bar{q}_i \gamma^{\mu} T^A q_j) (\bar{u}_k \gamma_{\mu} T^A u_l), \\ \mathcal{Q}_{qd}^{8(ijkl)} &= (\bar{q}_i \gamma^{\mu} T^A q_j) (\bar{d}_k \gamma_{\mu} T^A d_l), \\ \mathcal{Q}_{ud}^{8(ijkl)} &= (\bar{u}_i \gamma^{\mu} T^A u_j) (\bar{d}_k \gamma_{\mu} T^A d_l), \\ ^{\dagger} \mathcal{Q}_{quqd}^{8(ijkl)} &= (\bar{q}_i T^A u_j) \epsilon (\bar{q}_k T^A d_l), \\ \mathcal{Q}_{uu}^{(ijkl)} &= (\bar{u}_i \gamma^{\mu} u_j) (\bar{u}_k \gamma_{\mu} u_l) \end{aligned}$$

Operators' definitions: 2F and 0F in top-basis

$$\begin{split} \mathcal{Q}_{tB} &= i \big(\bar{Q} \tau^{\mu\nu} t \big) \, \tilde{\varphi} \, \mathcal{B}_{\mu\nu} + \text{h.c.}, \\ \mathcal{O}_{tW} &= i \big(\bar{Q} \tau^{\mu\nu} \tau_l t \big) \, \tilde{\varphi} \, \mathcal{W}_{\mu\nu}^l + \text{h.c.}, \\ \mathcal{O}_{tZ} &= -\mathcal{Q}_{tB} / \sin \theta_{w}, \\ \mathcal{O}_{tG} &= i g_s \left(\bar{Q} \tau^{\mu\nu} \, T_A t \right) \, \tilde{\varphi} \, \mathcal{G}_{\mu\nu}^A + \text{h.c.}, \\ \mathcal{O}_{eq} &= i \big(\varphi^{\dagger} \stackrel{\leftrightarrow}{D}_{\mu} \varphi \big) \big(\bar{t} \, \gamma^{\mu} t \big), \\ \mathcal{O}_{eq} &= i \big(\varphi^{\dagger} \stackrel{\leftrightarrow}{D}_{\mu} \tau_l \varphi \big) \big(\bar{Q} \, \gamma^{\mu} \, \tau^l Q \big), \\ \mathcal{O}_{\varphi Q}^{(-)} &= i \big(\varphi^{\dagger} \stackrel{\leftrightarrow}{D}_{\mu} \varphi \big) \big(\bar{Q} \, \gamma^{\mu} \, Q \big) \end{split}$$

 Q_{tB} introduced in the Warsaw basis to show the SMEFTatNLO rotation of the O_{tZ} operator; the coefficients of O_{tW} and O_{tZ} reads

$$c_{tW} = C_{tW}, \ c_{tZ} = -\sin\theta_{w}C_{tB} + \cos\theta_{w}C_{tW}$$

 $c_i \rightarrow$, $C_i \rightarrow$ Warsaw basis. Purely bosonic operators are defined as follows:

$$\mathcal{O}_{\varphi G} = \left(\varphi^{\dagger}\varphi - \frac{v^{2}}{2}\right) G_{A}^{\mu\nu} G_{\mu\nu}^{A}, \qquad \qquad \mathcal{O}_{G} = g_{s} f_{ABC} G_{\mu\nu}^{A} G_{\rho}^{B,\nu\rho} G_{\rho}^{C,\mu}$$

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Decomposition of the SM cross-section

\sqrt{s}	$\mathscr{O}(\alpha_{s}^{4})$	$\mathscr{O}(\alpha_s^3 \alpha)$	$\mathscr{O}(\alpha_s^3 \alpha_t)$	$\sum_{n} \mathscr{O}(\alpha_{s}^{2} \alpha_{w}^{n})$	$\sum_{n} \mathscr{O}(\alpha_{s} \alpha_{w}^{n})$	$\sum_{n} \mathscr{O}(\alpha_{\mathrm{w}}^{n})$	Inclusive
13 TeV	6.15	-1.44	-0.58	2.33	×	×	6.46
100 TeV	2570	-313	-197	753	×	×	2812

In orders of α_s and $\alpha_w \equiv \alpha + \alpha_t$

- EW contributions, i.e. $\mathscr{O}(\alpha_s^2 \alpha_w^2)$, are significant, yet the interference of the corresponding amplitudes with QCD ones, i.e. $\mathscr{O}(\alpha_s^3 \alpha_w)$, dilutes their effect
- Same cancellations, yet even larger, are observed at NLO in [1711.02116]

tttt in SMEFT: operators's insertions



Four-fermion (4F); 4-heavy (4H) and 2-heavy 2-light (2H2L)



Relevant two-fermion (2F) and purely-bosonic (0F);

$$\{\mathcal{O}_{t\varphi}, \mathcal{O}_{tZ}, \mathcal{O}_{tW}, \mathcal{O}_{tG}, \mathcal{O}_{\varphi Q}^{(-)}, \mathcal{O}_{\varphi t}, \mathcal{O}_{G}, \mathcal{O}_{\varphi G}\}$$

LO expansion

In the presence of dim-6 SMEFT operators, the scattering amplitude reads

$$\mathcal{A} = \mathcal{A}_{\mathrm{SM}} + \frac{1}{\Lambda^2} \mathcal{A}_{(\mathrm{d6})} + \frac{1}{\Lambda^4} \big(\mathcal{A}_{(\mathrm{d6})^2} + \mathcal{A}_{(\mathrm{d8})} \big),$$

leading to the partonic differential cross-section

$$d\sigma = d\sigma_{\rm SM} + rac{1}{\Lambda^2} d\sigma_{\rm int} + rac{1}{\Lambda^4} (d\sigma_{
m quad} + d\sigma_{
m dbl} + d\sigma_{
m d8}).$$

Lets just focus on the linear interference,

$$egin{aligned} & d\sigma_{ ext{int},gg} + d\sigma_{ ext{int},qq} \ & \sim & 2 \mathfrak{R} \left(\mathcal{A}_{ ext{SM},gg} \, \mathcal{A}_{ ext{EFT},gg}^{\dagger}
ight) + 2 \mathfrak{R} \left(\mathcal{A}_{ ext{SM},qq} \, \mathcal{A}_{ ext{EFT},qq}^{\dagger}
ight) \end{aligned}$$

LO expansion

Write down the SM amplitudes (skipping $q\bar{q}$ -mode for simplicity),

$$\mathcal{A}_{\mathrm{SM},gg}^{(ij,k)} = \alpha_s^2 \, \mathcal{A}_{\mathrm{SM},gg}^{(2,0,0)} + \alpha_s \left(\alpha \, \mathcal{A}_{\mathrm{SM},gg}^{(1,1,0)} + \alpha_t \, \mathcal{A}_{\mathrm{SM},gg}^{(1,0,1)} \right),$$

and the EFT ones,

$$\mathcal{A}_{\mathrm{EFT},gg,[4\mathsf{F}]}^{(i,j,k)} = \alpha_{\mathsf{s}} \mathcal{A}_{\mathrm{EFT},gg\,[4\mathsf{F}]}^{(1,0,0)}.$$

Do some work and then write the cross-section,

$$d\sigma_{\mathrm{int},gg,[4\mathrm{F}]} = \alpha_{\mathrm{s}}^{3} d\sigma_{\mathrm{int},gg}^{(3,0,0)} + \alpha_{\mathrm{s}}^{2} \left(\alpha \, d\sigma_{\mathrm{int},gg}^{(2,1,0)} + \kappa_{t} \, d\sigma_{\mathrm{int},gg}^{(2,0,1)} \right),$$

The total interference (including $q\bar{q}$ -mode) cross-section reads

$$\sigma_{INCL} = \sigma_3 + \sigma_2 + \sigma_1 + \sigma_0,$$

where σ_3 is the cross-section induced from all terms with α_s^3 , etc.

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$q\bar{q}$ -initiated σ_{int} in 4F

$$d\sigma_{\text{int},qq,[4F]} = \alpha_s^3 \, d\sigma_{\text{int},qq}^{(3,0,0)} + \alpha_s^2 \left(\alpha \, d\sigma_{\text{int},qq}^{(2,1,0)} + \alpha_t \, d\sigma_{\text{int},qq}^{(2,0,1)} \right) + \alpha_s \left(\alpha^2 \, d\sigma_{\text{int},qq}^{(1,2,0)} + \alpha^{3/2} \, \alpha_t^{1/2} \, d\sigma_{\text{int},qq}^{(1,3/2,1/2)} + \alpha \alpha_t \, d\sigma_{\text{int},qq}^{(1,1,1)} + \alpha_t^2 \, d\sigma_{\text{int},qq}^{(1,0,2)} \right) + (\alpha^3) \, d\sigma_{\text{int},qq}^{(0,3,0)} + (\alpha^{5/2} \, \alpha_t^{1/2}) \, d\sigma_{\text{int},qq}^{(0,5/2,1/2)} + (\alpha^2 \, \alpha_t) \, d\sigma_{\text{int},qq}^{(0,2,1)} + (\alpha^{3/2} \, \alpha_t^{3/2}) \, d\sigma_{\text{int},qq}^{(0,3/2,3/2)} + (\alpha \, \alpha_t^2) \, d\sigma_{\text{int},qq}^{(0,1,2)}$$

2H2L at $\sqrt{s} = 13$ TeV



dominant $\sigma_3 \rightarrow$ almost all 2H2L are 'naive' operators

■ All enter in *qq*-induced production → EW scattering effects are less critical in interference with *qq*-initiated amplitudes?

Relevant 2F and 0F at $\sqrt{s} = 13$ TeV



Non-four-fermion operators can also be 'non-naive'

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Rest of 4-heavy at $\sqrt{s} = 13$ TeV



2F and 0F differential predictions $\sqrt{s} = 13$ TeV



Different EFT structure than contact-term insertions (4F) \rightarrow can be inferred from the amplitudes scaling with $\sim E$

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2F and 0F differential predictions $\sqrt{s} = 13$ TeV

coefficients approximate values extracted from [2105.00006]



FCC-hh 4H



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FCC-hh 2H2L

	$\sigma_{SM}^{tttt}(LO) =$	2.8pb @ √	<u>s</u> = 100 <i>Te</i>	v	2-heavy 2-light $\sigma_{int.}[fb]$										
INCL -	0.48	4.37	3.19	5.38	2.27	2.49	1.87	1.72	-1.24	0.14	0.15	-0.16	-0.35	-0.94	• 5
σ_3 ·	0.59	3.39	2.52	4.27	1.79	2.01	1.42	-0.28	-1.57	-0.12	-0.25	-0.14	-0.93	-0.67	• 3
σ2 -	-0.19	0.41	0.29	0.54	0.25	0.18	0.23	1.56	0.24	0.18	0.32	0.01	0.44	-0.22	• 2
σ_1 ·	0.08	0.57	0.34	0.58	0.23	0.31	0.23	0.26	0.06	0.04	0.05	-0.02	0.08	-0.03	• 1
σ_0 -	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	0.04	0.03	0.03	-0.01	0.05	-0.02	· -1
	cQq83	cQq81	cQu8	ctq8	cQd8	ctu8	ctd8	cQq13	cQq11	cQu1	ctq1	cQd1	ctu1	ctd1	

FCC-hh relevant 2F and 0F



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FCC 4H differential predictions



FCC 2F and 0F differential predictions

