Curvy Parke-Taylors

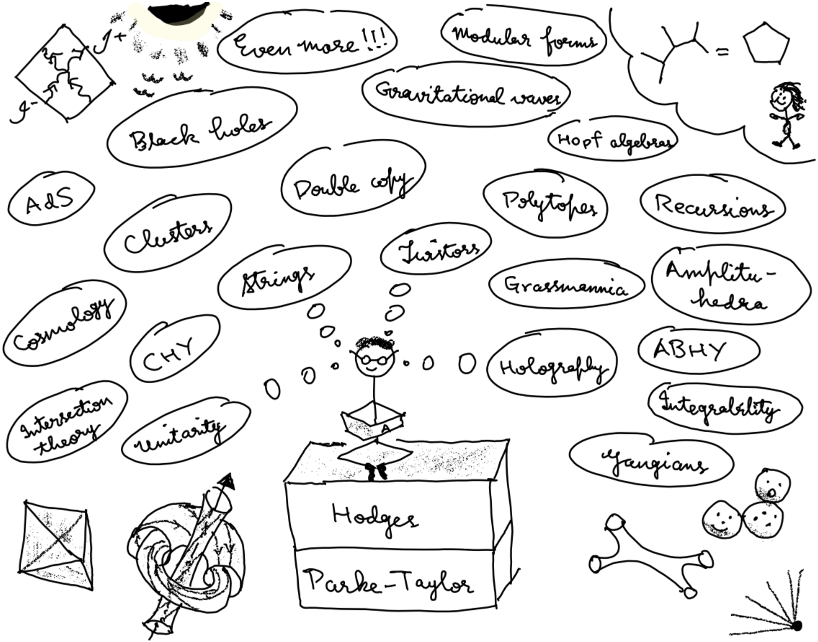
Atul Sharma

Mathematical Institute, Oxford

Dec 16, 2020

Young Theorists' Forum 2020

[Based on work with T. Adamo, L. Mason]


Even more!!!
Modular forms
Gravitational waves
Black holes
Hopf algebras
Double copy
AdS
Polytopes
Recursions
Clusters
Twistors
Strings
Grassmannia
Amplitu-
hedra
Cosmology
CHY
Holography
ABHY
Integrability
Intersection
theory
unitarity
Yangians
Hodges
Parke-Taylor

Motivations

Why study curved backgrounds?

- Test domain of validity of reformulations of perturbative QFT.
- Many future applications:
  - Strong field QED/QCD
  - Gravitational waves
  - AdS/CFT
  - Cosmology
  - Black hole physics
  - Instanton backgrounds

    ⋮

VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

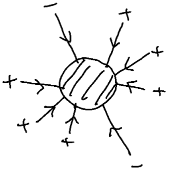
Amplitude for *n*-Gluon Scattering

Stephen J. Parke and T. R. Taylor
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 17 March 1986)

A nontrivial squared helicity amplitude is given for the scattering of an arbitrary number of gluons to lowest order in the coupling constant and to leading order in the number of colors.

Maximally helicity-violating (MHV) tree-level gluon amplitude

$$\mathcal{A}(1^+ \cdots r^- \cdots s^- \cdots n^+) = \frac{\langle r\, s\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle \cdots \langle n-1\; n\rangle\,\langle n\,1\rangle}\,\delta^4\!\left(\sum_{i=1}^{n} k_i\right)$$



VOLUME 56, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1986

Amplitude for *n*-Gluon Scattering

Stephen J. Parke and T. R. Taylor
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 17 March 1986)

A nontrivial squared helicity amplitude is given for the scattering of an arbitrary number of gluons to lowest order in the coupling constant and to leading order in the number of colors.

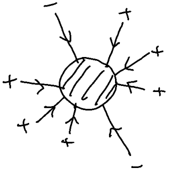
Maximally helicity-violating (MHV) tree-level gluon amplitude

$$\mathcal{A}(1^+\cdots r^-\cdots s^-\cdots n^+) = \frac{\langle r\, s\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\cdots\langle n-1\;n\rangle\,\langle n\,1\rangle}\,\delta^4\!\left(\sum_{i=1}^{n} k_i\right)$$

$$k_i^{\alpha\dot\alpha} := \frac{1}{\sqrt{2}}\sigma_\mu^{\alpha\dot\alpha}k_i^\mu\,, \qquad \sigma_\mu = (\mathbb{1}_2, \vec\sigma)\,, \qquad \det(k_i^{\alpha\dot\alpha}) = k_i^2 = 0\,,$$

$$k_i^{\alpha\dot\alpha} = \kappa_i^\alpha\bar\kappa_i^{\dot\alpha}\,, \qquad \langle i\,j\rangle := \kappa_i^\alpha\kappa_{j\alpha}\,, \qquad [i\,j] = \bar\kappa_i^{\dot\alpha}\bar\kappa_{j\dot\alpha}$$

$$\alpha,\beta = 0,1\,, \quad \dot\alpha,\dot\beta = \dot 0,\dot 1\,, \qquad \epsilon_{\alpha\beta} = \epsilon_{\dot\alpha\dot\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0\end{pmatrix}.$$



Cartan-valued SD radiative backgrounds

We study gauge field backgrounds $A \in \Omega^1(\mathbb{R}^4, \mathfrak{g}_{\mathbb{C}})$.

- **Self-dual** $\quad *F = \mathrm{i}\, F\,, \qquad F = \mathrm{d}A + A \wedge A$

Cartan-valued SD radiative backgrounds

We study gauge field backgrounds $A \in \Omega^1(\mathbb{R}^4, \mathfrak{g}_{\mathbb{C}})$.

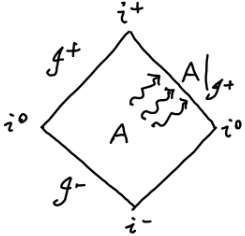
- **Self-dual** $\quad *F = \mathrm{i}\, F\,, \qquad F = \mathrm{d}A + A \wedge A$

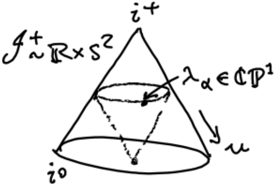
- **Radiative** $\quad A$ fixed by radiation data $A|_{\mathscr{I}^+}$.

$$A|_{\mathscr{I}^+} = \mathcal{A}(u,\lambda,\bar\lambda)\,\mathrm{D}\lambda + \tilde{\mathcal{A}}(u,\lambda,\bar\lambda)\,\mathrm{D}\bar\lambda\,.$$

$(u, \lambda_\alpha, \bar\lambda_{\dot\alpha})$ homog. coordinates on $\mathscr{I}^+ \simeq \mathbb{R} \times \mathbb{CP}^1$.

$\mathrm{D}\lambda = \langle \lambda\, \mathrm{d}\lambda \rangle$, $\mathrm{D}\bar\lambda = [\bar\lambda\, \mathrm{d}\bar\lambda]$ basis of $T^*\mathbb{CP}^1$.





Cartan-valued SD radiative backgrounds

We study gauge field backgrounds $A \in \Omega^1(\mathbb{R}^4, \mathfrak{g}_\mathbb{C})$.

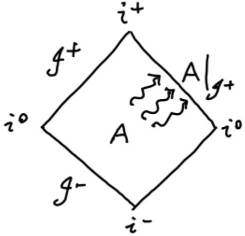
- **Self-dual** $\quad *F = \mathrm{i}\, F\,, \qquad F = \mathrm{d}A + A \wedge A$

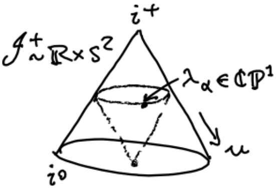
- **Radiative** $\quad A$ fixed by radiation data $A|_{\mathscr{I}^+}$.

$$A|_{\mathscr{I}^+} = \mathcal{A}(u,\lambda,\bar\lambda)\,\mathrm{D}\lambda + \tilde{\mathcal{A}}(u,\lambda,\bar\lambda)\,\mathrm{D}\bar\lambda\,.$$

$(u, \lambda_\alpha, \bar\lambda_{\dot\alpha})$ homog. coordinates on $\mathscr{I}^+ \simeq \mathbb{R} \times \mathbb{CP}^1$.
$\mathrm{D}\lambda = \langle \lambda\, \mathrm{d}\lambda \rangle$, $\mathrm{D}\bar\lambda = [\bar\lambda\, \mathrm{d}\bar\lambda]$ basis of $T^*\mathbb{CP}^1$.

- **SD Radiative** $\qquad \mathcal{A}(u,\lambda,\bar\lambda) = 0$





Cartan-valued SD radiative backgrounds

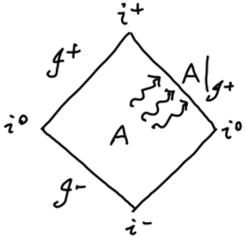
We study gauge field backgrounds $A \in \Omega^1(\mathbb{R}^4, \mathfrak{g}_{\mathbb{C}})$.

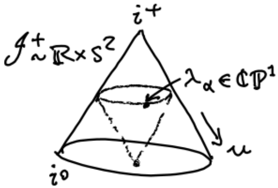
- **Self-dual** $\quad *F = \mathrm{i}\, F\,, \qquad F = \mathrm{d}A + A \wedge A$

- **Radiative** $\quad A$ fixed by radiation data $A|_{\mathscr{I}^+}$.

$$A|_{\mathscr{I}^+} = \mathcal{A}(u, \lambda, \bar{\lambda})\, \mathrm{D}\lambda + \tilde{\mathcal{A}}(u, \lambda, \bar{\lambda})\, \mathrm{D}\bar{\lambda}\,.$$

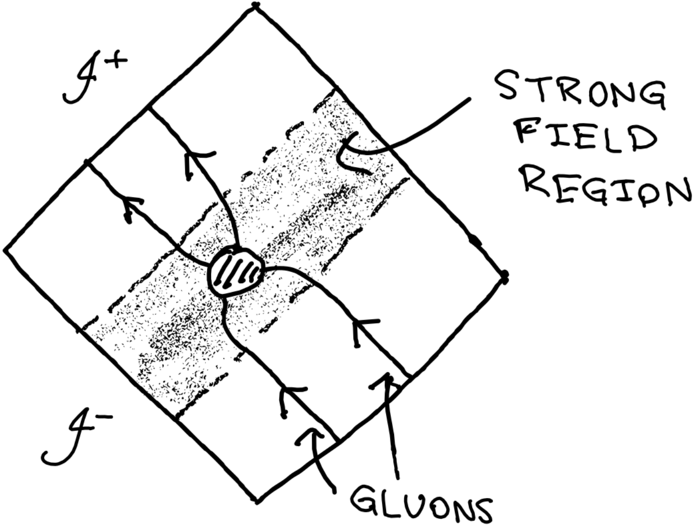
$(u, \lambda_\alpha, \bar{\lambda}_{\dot{\alpha}})$ homog. coordinates on $\mathscr{I}^+ \simeq \mathbb{R} \times \mathbb{CP}^1$.
$\mathrm{D}\lambda = \langle \lambda\, \mathrm{d}\lambda \rangle$, $\mathrm{D}\bar{\lambda} = [\bar{\lambda}\, \mathrm{d}\bar{\lambda}]$ basis of $T^*\mathbb{CP}^1$.





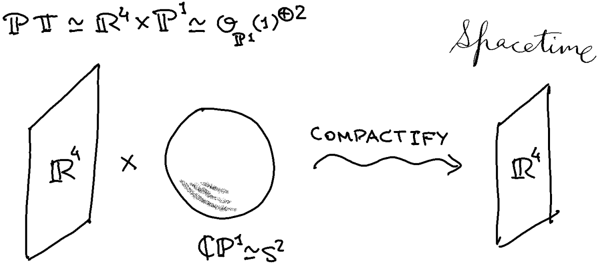
- **SD Radiative** $\qquad \mathcal{A}(u, \lambda, \bar{\lambda}) = 0$

- **Cartan-valued** $\quad A \in$ Cartan $\subset \mathfrak{g}_{\mathbb{C}}$.



$\mathcal{I}^+$
STRONG
FIELD
REGION
$\mathcal{I}^-$
GLUONS

A weak-weak duality [Mason '05] [Boels, Mason, Skinner '07]

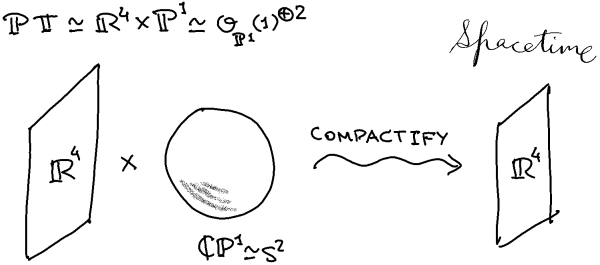


Twistor action → YM + θ-term

$\int_{\mathbb{PT}} b\bar{\partial}a + \sum_{n=2}^{\infty} \int_{\mathbb{R}^4} \int_{(\mathbb{P}^1)^n} b^2 a^{n-2}$

$\int F \wedge *F - \frac{1}{4} F \wedge F$

A weak-weak duality [Mason '05] [Boels, Mason, Skinner '07]



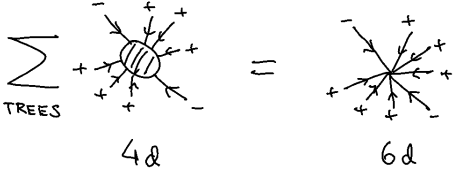
Twistor action

$\int_{\mathbb{PT}} b\bar{\partial}a + \sum_{n=2}^{\infty} \int_{\mathbb{R}^4} \int_{(\mathbb{P}^1)^n} b^2 a^{n-2}$

YM + θ-term

$\int F \wedge *F - \frac{1}{4} F \wedge F$

A MAGIC TRICK :



MHV gluon amplitudes on SD radiative $A = A_\mu \, \mathrm{d}x^\mu \in$ Cartan with gluons r, s negative helicity : [Adamo, Mason, AS '19]

$$\frac{\langle r\, s\rangle^4}{\langle 1\, 2\rangle \, \langle 2\, 3\rangle \cdots \langle n-1\; n\rangle \, \langle n\, 1\rangle} \int \mathrm{d}^4 x \; \exp \sum_{i=1}^{n} \big(\mathrm{i} k_i \cdot x + e_i \, g(x, \kappa_i)\big) \, .$$

- e_i gluon charge w.r.t. Cartan, $\sum_i e_i = 0$; $k_i^{\alpha\dot\alpha} = \kappa_i^\alpha \bar\kappa_i^{\dot\alpha}$.

MHV gluon amplitudes on SD radiative $A = A_\mu \, \mathrm{d}x^\mu \in$ Cartan with gluons r, s negative helicity : [Adamo, Mason, AS '19]

$$\frac{\langle r\, s\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle \cdots \langle n-1\; n\rangle\,\langle n\,1\rangle} \int \mathrm{d}^4x \;\, \exp \sum_{i=1}^{n} \big(\mathrm{i}k_i \cdot x + e_i\, g(x,\kappa_i)\big)\,.$$

- e_i gluon charge w.r.t. Cartan, $\sum_i e_i = 0$; $k_i^{\alpha\dot\alpha} = \kappa_i^\alpha \bar\kappa_i^{\dot\alpha}$.
- $g(x,\kappa)$ gives holographic imprint of A: $A|_{\mathscr{I}^+} = \tilde{\mathcal{A}}(u,\lambda,\bar\lambda)\,\mathrm{D}\bar\lambda$.

$$g(x,\kappa) = \frac{1}{2\pi \mathrm{i}} \int_{\mathbb{CP}^1} \frac{\langle \kappa\, \xi\rangle}{\langle \kappa\, \lambda\rangle\,\langle \lambda\, \xi\rangle}\, \mathrm{D}\lambda \wedge \mathrm{D}\bar\lambda\; \tilde{\mathcal{A}}\big|_{u = x^{\alpha\dot\alpha}\lambda_\alpha \bar\lambda_{\dot\alpha}}\,.$$

$\lambda_\alpha \in \mathbb{CP}^1$ (celestial sphere), ξ_α a reference spinor, $\mathrm{D}\lambda = \langle \lambda\, \mathrm{d}\lambda\rangle$.

Features of our formula

$$\frac{\langle r\, s\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle \cdots \langle n-1\; n\rangle\,\langle n\,1\rangle} \int \mathrm{d}^4x\; \exp \sum_{i=1}^{n} \big(\mathrm{i}k_i \cdot x + e_i\, g(x, \kappa_i)\big)\,.$$

- Proven using twistor action.

Features of our formula

$$\frac{\langle r\, s\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\cdots\langle n-1\;n\rangle\,\langle n\,1\rangle}\int \mathrm{d}^4x\;\exp\sum_{i=1}^{n}\big(\mathrm{i}k_i\cdot x + e_i\, g(x,\kappa_i)\big)\,.$$

- Proven using twistor action.
- Clear flat space limit: $A = 0 \implies g = 0$.

Features of our formula

$$\frac{\langle r\, s\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\cdots\langle n-1\;n\rangle\,\langle n\,1\rangle}\int \mathrm{d}^4x\;\exp\sum_{i=1}^{n}\big(\mathrm{i}k_i\cdot x + e_i\, g(x,\kappa_i)\big)\,.$$

- Proven using twistor action.
- Clear flat space limit: $A = 0 \implies g = 0$.
- Feynman diagram checks at 3 and 4 points.

Features of our formula

$$\frac{\langle r\, s\rangle^4}{\langle 1\, 2\rangle\, \langle 2\, 3\rangle \cdots \langle n-1\ n\rangle\, \langle n\, 1\rangle} \int \mathrm{d}^4x\ \exp \sum_{i=1}^{n} \big(\mathrm{i}k_i \cdot x + e_i\, g(x, \kappa_i)\big)\,.$$

- Proven using twistor action.
- Clear flat space limit: $A = 0 \implies g = 0$.
- Feynman diagram checks at 3 and 4 points.
- Simplicity without a shred of momentum conservation!

Features of our formula

$$\frac{\langle r\, s\rangle^4}{\langle 1\, 2\rangle\, \langle 2\, 3\rangle \cdots \langle n-1\ n\rangle\, \langle n\, 1\rangle} \int \mathrm{d}^4x\ \exp \sum_{i=1}^{n} \big(\mathrm{i}k_i \cdot x + e_i\, g(x, \kappa_i)\big)\,.$$

- Proven using twistor action.
- Clear flat space limit: $A = 0 \implies g = 0$.
- Feynman diagram checks at 3 and 4 points.
- Simplicity without a shred of momentum conservation!
- Lot less integrals in comparison to Feynman rules.

  For n-gluon scattering, $(n-2)$ cubic vertices should give $(n-2)$ d^4x integrals, but we found only one.

  Behind the scenes: remarkable IBP relations collapse most integrals.

Conclusions

- A similar story holds for graviton scattering in SD radiative spacetimes (to appear "soon").

  See also [Adamo, Mason, AS '19].

- Many important future directions:
  - SD backgrounds with colour/gravitational sources and black holes.
  - Double copy of amplitudes on curved backgrounds.
  - Non-self-dual backgrounds using ambitwistor strings.
  - Generalisation of other modern methods like unitarity, on-shell recursion, polytopes...