Soft Anomalous Dimension in QCD

Niamh Maher

Supervisor: Einan Gardi

THE UNIVERSITY
of EDINBURGH

CENTRE FOR THEORETICAL PHYSICS · Higgs

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Outline

- Motivation for studying the Soft Anomalous Dimension
- Introduction:Set-up and Definition
- Wilson Lines and Lie Algebra
- Color and Kinematics in diagrams

Properties of the Soft Anomalous Dimension

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- Only fully connected diagrams present (Gardi, Smilie, White 2013)
- Bootstrapped at 3 loops for massless partons (Almelid, Duhr,Gardi Mcleod, White 2017)
- High energy (Regge) limit of the SAD=soft limit of the BFKL resummation (Del Duca, Duhr, Gardi, Magnea, White 2011)
- Important for resummation of large logs in n-jet process to N^3LL accuracy, possible check for higgs production calculations

Current Status of Soft Anomalous Dimension

Massless Partons

- Ansatz at four loops in QCD (Becher, Neubert 2019)
- Exact calculation and bootstrapping at three loops (Almelid, Duhr Gardi 2016, + McLeod, White 2017))

Other

- Production of single top quark at three loops (Kidonakis 2019)
- Massive particle 2 loops (Mitov 2009)
- Exact Soft Anomalous Dimension in Super Yang-Mills (Giombi,Komatsu 2020)

Introduction: Set-up 1/2

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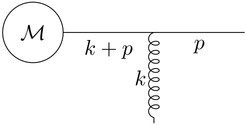
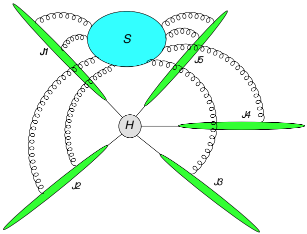
  $2p \cdot k = 2p_0 k_0(1 - \cos\theta_{pk})$

  Soft: $k_0 = 0$
  Collinear: $\cos\theta_{pk} = 1$



Introduction: Set-up 2/2

- Hard/Soft/Jet: Amplitude factorisation



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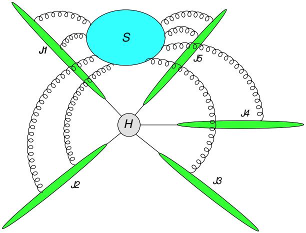
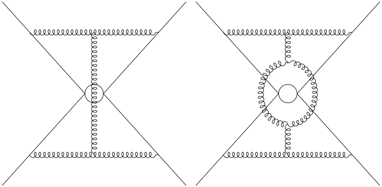


- Loops



Soft Anomalous dimension(SAD) Definition

Amplitude Factorises with μ_f as the scale

$$\mathcal{A}_n = Z_n \mathcal{H}_n \tag{1}$$

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Renormalisation group equation for the scale μ_f

$$\frac{d}{d \ln \mu_f} Z_n \left(\{p_i\}, \{\mathbf{T}_i\}, \epsilon, \alpha_s(\mu_f)\right) = -Z_n \, \Gamma_n \left(\{p_i\}, \{\mathbf{T}_i\}, \mu_f, \alpha_s(\mu_f)\right) \tag{2}$$

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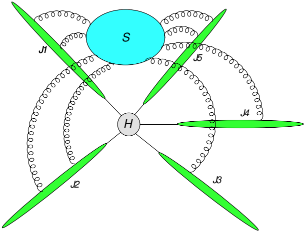
$$Z_n = \mathcal{P}\exp\left\{-\frac{1}{2}\int_0^{\mu_f^2}\frac{d\lambda^2}{\lambda^2}\Gamma_n\left(\{p_i\},\{\mathbf{T}_i\},\lambda,\alpha_s(\lambda^2)\right)\right\}, \tag{3}$$

Wilson Lines

Non-abelian exponentiation: Gardi, Smilie, White 1304.7040

Eikonal Approximation



$$\frac{i(\not{p} + \not{k})}{(p+k)^2 + i0}\mathcal{M} \xrightarrow[k \ll p]{} \frac{p^\mu}{p \cdot k + i0}\mathcal{M} = \frac{\beta^\mu}{\beta \cdot k + i0} \qquad (4)$$

Wilson Line representation

$$\Phi_\beta(0,\infty) = \mathcal{P}\left(\exp\left[i\int_C dx^\mu \beta_i A^a_\mu(\beta_i x) T^a\right]\right) \qquad (5)$$

$SU(N_c)$Lie Algebra 1/2

1. Commutator

$$\boldsymbol{T}_1^a \boldsymbol{T}_1^b - \boldsymbol{T}_1^b \boldsymbol{T}_1^a = [\boldsymbol{T}_1^a, \boldsymbol{T}_1^b] = i\, f^{abc}\, T_1^c \tag{7}$$

- f^{abc} is **antisymmetric** under any swap of indices
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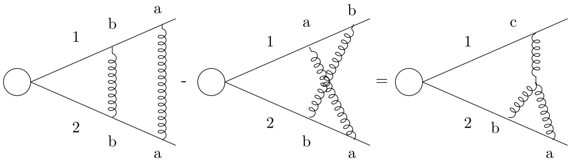

$$\boldsymbol{T}_1^a \boldsymbol{T}_1^b \boldsymbol{T}_2^a \boldsymbol{T}_2^b - \boldsymbol{T}_1^b \boldsymbol{T}_1^a \boldsymbol{T}_2^a \boldsymbol{T}_2^b = i\, f^{abc}\, T_1^c \boldsymbol{T}_2^a \boldsymbol{T}_2^b \tag{8}$$

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2.Jacobi Relation

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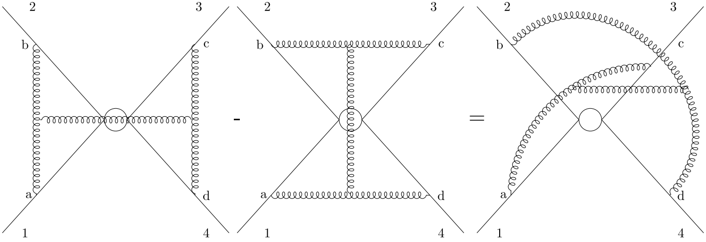

$$f^{abx}f^{cdx}\boldsymbol{T}_1^a\boldsymbol{T}_2^b\boldsymbol{T}_3^c\boldsymbol{T}_4^d - f^{axd}f^{bcx}\boldsymbol{T}_1^a\boldsymbol{T}_2^b\boldsymbol{T}_3^c\boldsymbol{T}_4^d = f^{acx}f^{bdx}\boldsymbol{T}_1^a\boldsymbol{T}_2^b\boldsymbol{T}_3^c\boldsymbol{T}_4^d \tag{10}$$

Kinematics and Weight

- weight: a Multiple PolyLogarithm of weight n is

$$G(a_1, a_2, \cdots, a_n; z) = \int_0^z \frac{dt}{t - a_1} G(a_2, \cdots, a_n; t), \qquad (11)$$

  where G is a MPL and a_i are indices that make up a length n vector.
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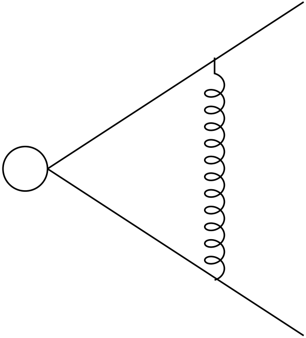
- For 4 loops, Γ_n contains weight 7 functions.

Dipole



$$\boldsymbol{T}_i \cdot \boldsymbol{T}_j = (\boldsymbol{T}_i^a)_{km}(\boldsymbol{T}_j^a)_{pq}$$

$\{i, j\}$ are the leg indices, the color index a is summed over.

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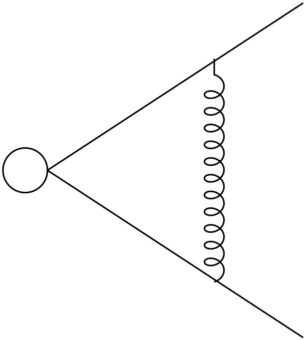

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Amplitude: weight $2L$
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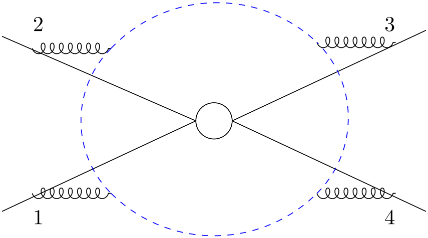
$\log(\frac{-s_{ij}}{\mu^2})$

New at four loops

- $\frac{1}{6}\sum_\sigma \text{Tr}(\boldsymbol{T}_R^a \boldsymbol{T}_R^{\sigma(b)} \boldsymbol{T}_R^{\sigma(c)} \boldsymbol{T}_R^{\sigma(d)}) \boldsymbol{T}_1^a \boldsymbol{T}_2^b \boldsymbol{T}_3^c \boldsymbol{T}_4^d$



- Kinematics: Completely symmetric
- Two independent cross ratios
- $G^R(\beta_{1243}, \beta_{1342})$

Conclusion and outlook

- Provided motivation for studying the soft anomalous dimension
- Looked at diagrammatic representation of the color structures
- Commented on some symmetries and the kinematics
- Find the form of the kinematic functions at four loops using the limits

References

- 0901.1091
- 1008.0098
- 1304.7040
- 1908.11379
- 1911.10174
- https://wmresources.org/august-2019-the-long-and-winding-road/

Kinematic Functions: Single-Valued Harmonic Polylogarithms

- Harmonic Polylogarithm

$$H(a_1, a_2, \cdots, a_n; z) = \int_0^z \frac{dt}{t - a_1} H(a_2, \cdots, a_n; t), \qquad (12)$$

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where $a_i \in \{0, -1, 1\}$.

- Single-valued means there are no branch cuts e.g

$$\begin{aligned}\mathcal{L}_0 &= \ln(z) + \ln(\bar{z}) = H(0, z) + H(0, \bar{z}) = G(0, z) + G(0, \bar{z}) \\ \mathcal{L}_1 &= \ln(1 - z) + \ln(1 - \bar{z}) = H(1, z) + H(1, \bar{z}) \\ &= -(G(1, z) + G(1, \bar{z}))\end{aligned} \qquad (13)$$

- The kinematic functions of Γ_s are made up weight 5 SVHPLs

Casimirs

Quadratic Casimir: $C_2(i)\mathbb{I} = (\boldsymbol{T}_i^a)_{km}(\boldsymbol{T}_i^a)_{mq}$