15/12/2020

Synthetic Flux Attachment

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Thanks to: A. Celi, L. Tarruell, B. Schroers, C. Hooley, I. Carusotto

Phys. Rev. Research 2, 033453 (2020)

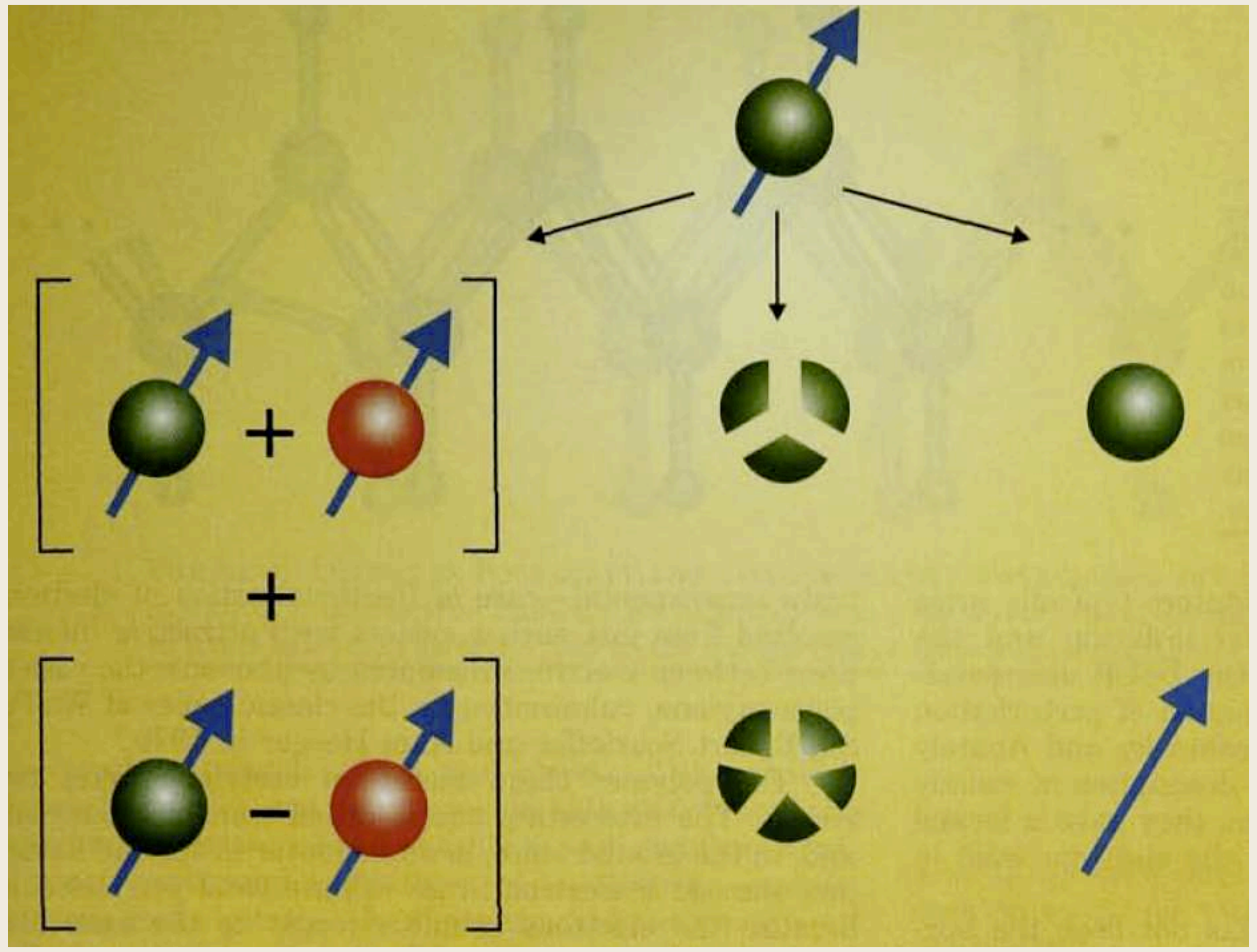
What are we after? Understanding fractionalisation

42 OCTOBER 1997 PHYSICS TODAY

WHEN THE ELECTRON FALLS APART

In condensed matter physics, some particles behave like fragments of an electron.

Philip W. Anderson



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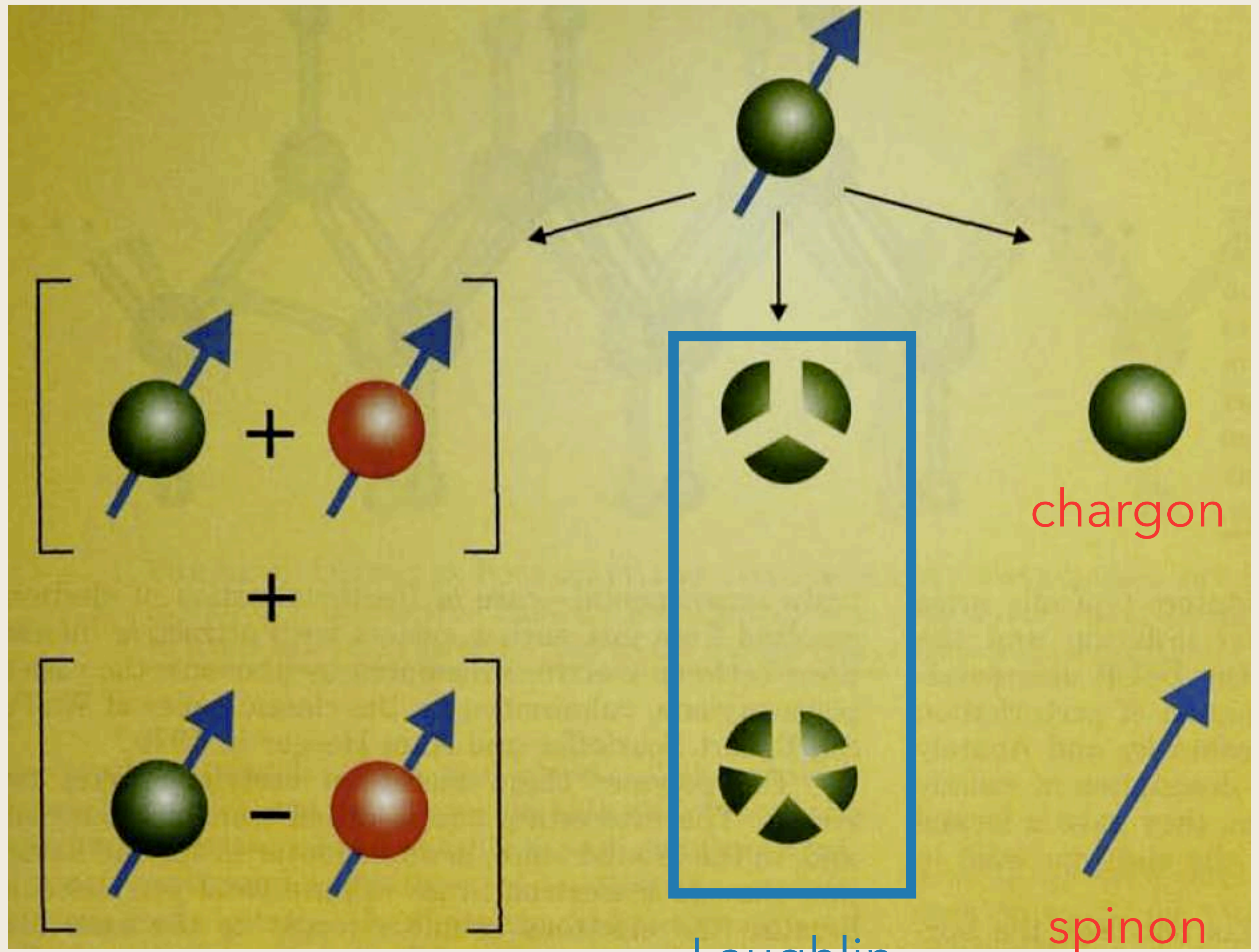


BCS quasiparticle

Laughlin quasiparticles

Fractionalisation

Quasi-examples: Laughlin quasiholes, spinons, chargons/holons, visons, Majorana zero modes

Topologically Ordered Phases:

Quantum spin liquids,
Fractional quantum Hall effects

No symmetry breaking order

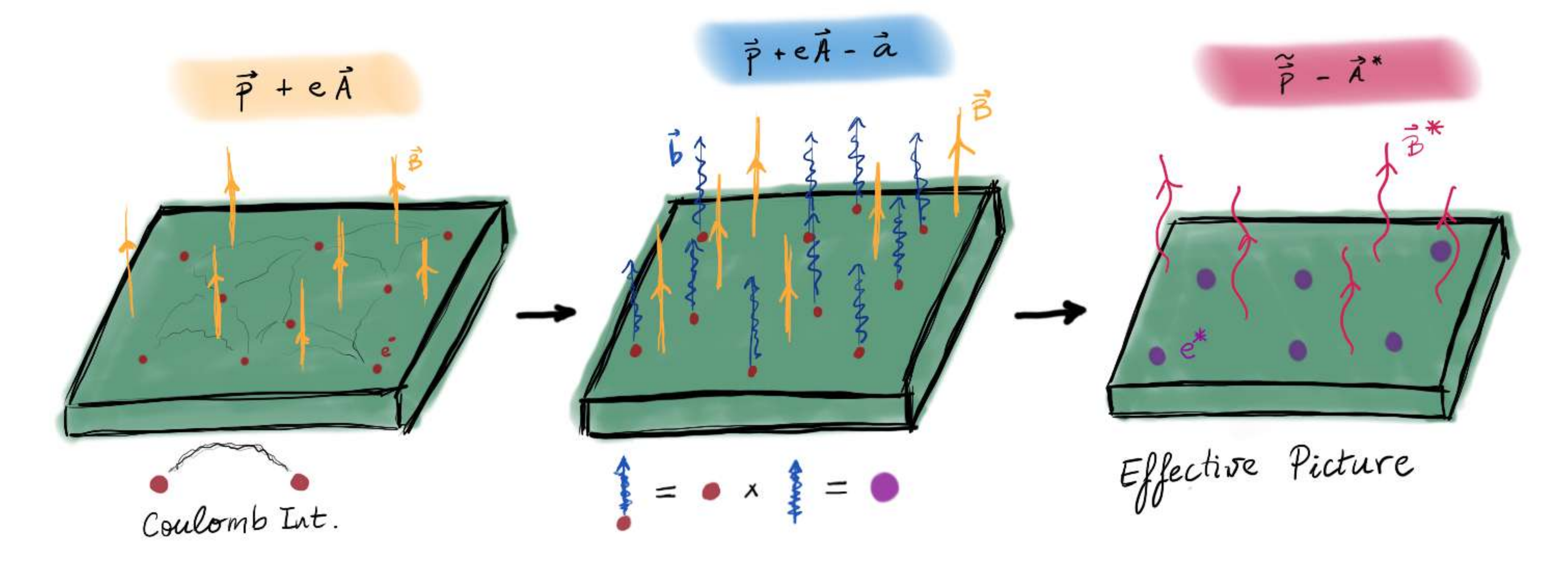
+ Emergent gauge structure
+ Topology
+ Interactions
+ Long-range entanglement
+ Groundstate degeneracy
+ Quantum anomalies

Chern-Simons Theory

Fractionalisation? Fractional Quantum Hall effect as an example

Take a 2+1D electron gas and put it in a strong (10-20T) magnetic field, spin is “frozen”

(i) (ii) (iii)



Emergent/self-generated statistical gauge field a

How does it work in practice ? The magic of a Chern-Simons term

<u>**Example in 2+1D: Flux Attachment / Chern-Simons Theory / Fractionalisation**</u>

Flux attachment is a mechanism by which charged particles capture magnetic flux quanta and form composite entities.
As a consequence of flux dressing, these composites may acquire fractional quantum numbers and statistics.

- Start from a Chern-Simons Term $$\mathcal{L}_{\text{CS}} = -\frac{\kappa}{4\pi}\,\epsilon^{\mu\nu\lambda} A_\mu\,\partial_\nu\,A_\lambda$$ Note this is similar to a Maxwell term but with one derivative less
- Equation of motion with a matter source $$j^{\,\mu} = \frac{\kappa}{2\pi}\,\epsilon^{\mu\nu\lambda}\,\partial_\nu\,A_\lambda$$ (Generic) matter content is found in the current
- Write in components and solve $$B \equiv \nabla \times \mathbf{A} = \frac{2\pi}{\kappa}\rho$$ **Flux Attachment**

Coulomb Gauge →

$$\mathbf{A}\,(\mathbf{r}) = \frac{1}{\kappa}\int d^2\mathbf{r}'\,\frac{\hat{\mathbf{e}}_z \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^2}\,\rho\,(\mathbf{r}')$$

Density-dependent Gauge Field

How does it work in practice ? The magic of a Chern-Simons term

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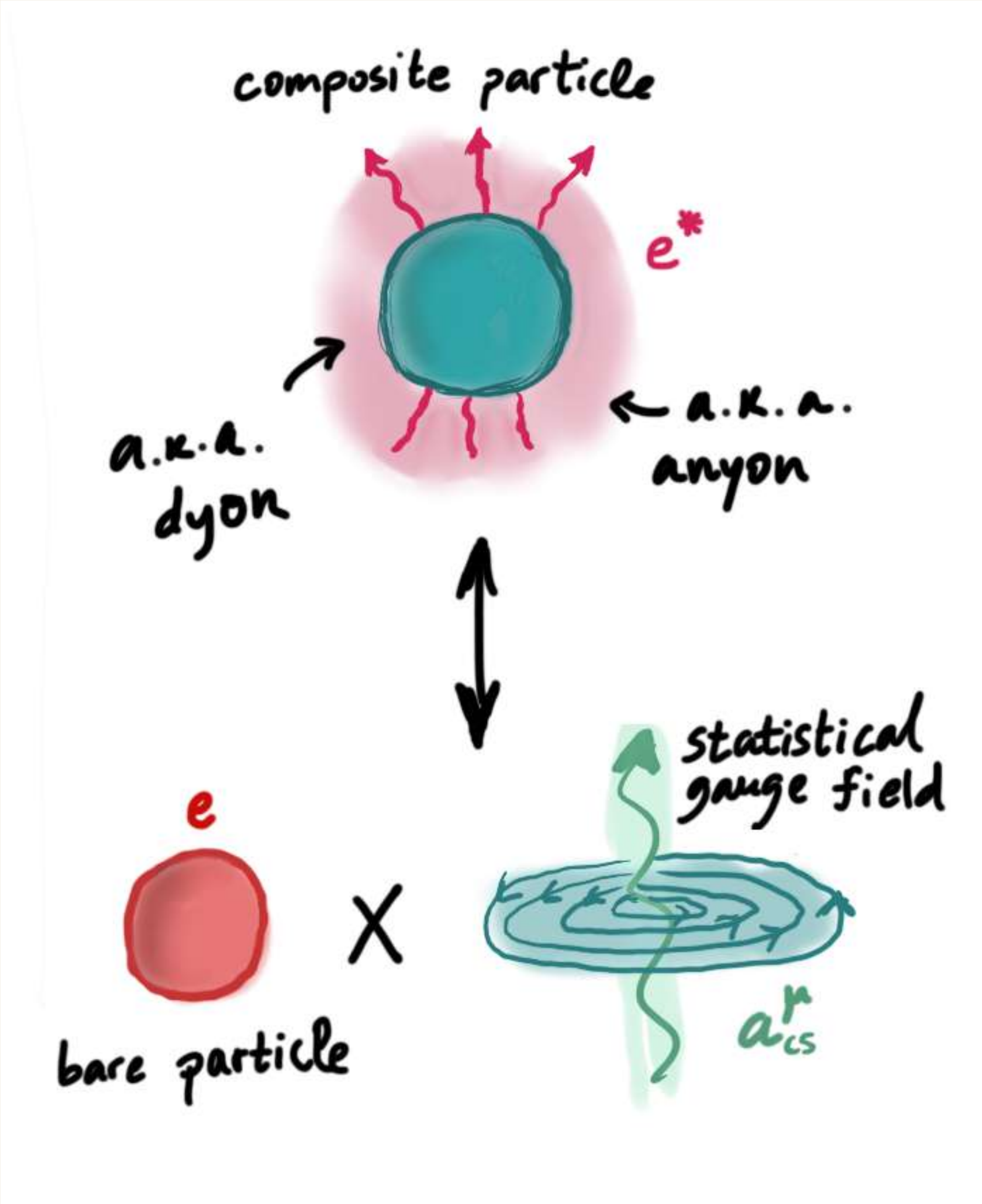
Flux Attachment — **Density-dependent Gauge Field**



Hamiltonian Duality $\hat{\mathcal{H}}_{\rm B} = \frac{\left[\hat{\mathbf{p}} - \hat{\mathbf{A}}\right]^2}{2m} \iff \hat{\tilde{\mathcal{H}}}_{\rm C} = \frac{\hat{\tilde{\mathbf{p}}}^{\,2}}{2m}$

or

Operator Duality $\hat{\Psi}_B(\mathbf{r}) = \hat{U}_{\rm CS}(\mathbf{r})\,\hat{\varphi}_C(\mathbf{r})$

U (r) contains a topological soliton

How does it work in practice ? The magic of a Chern-Simons term

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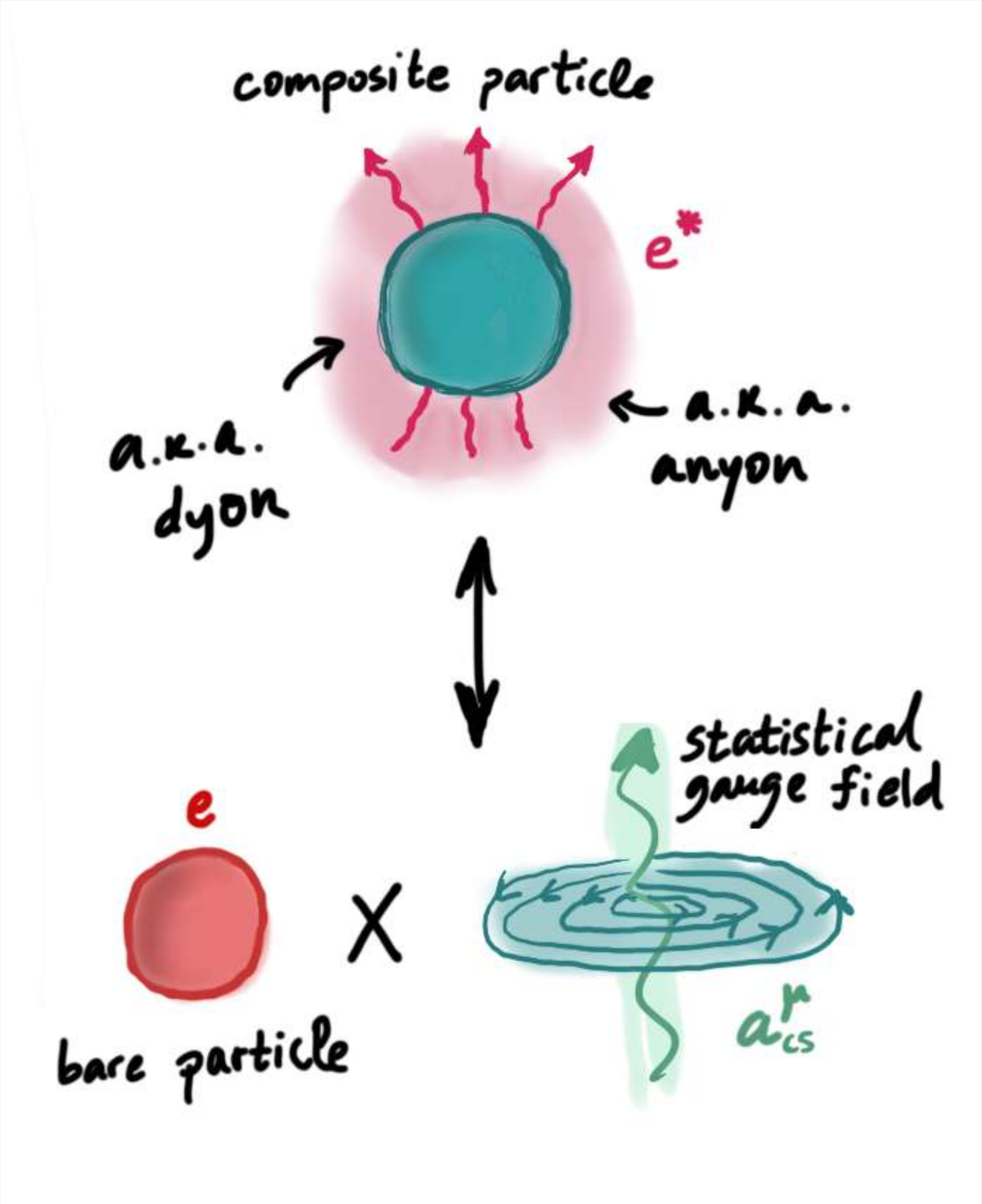
Flux Attachment **Density-dependent Gauge Field**



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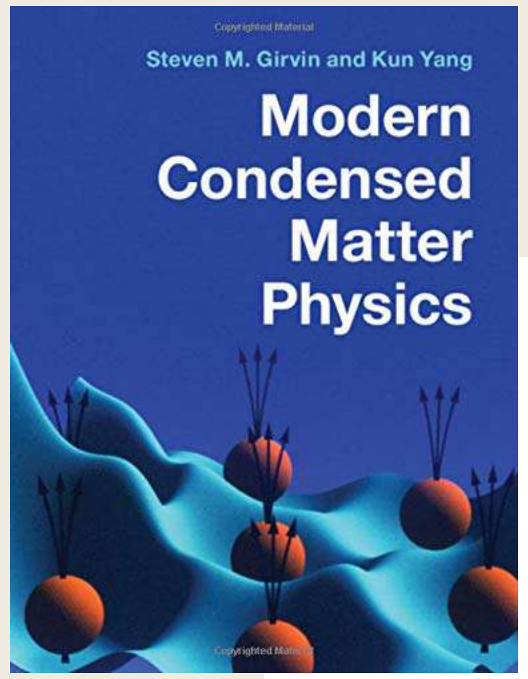
or

Operator Duality $\hat{\Psi}_B(\mathbf{r}) = \hat{U}_{\mathrm{CS}}(\mathbf{r})\,\hat{\varphi}_C(\mathbf{r})$ **U (r) contains a topological soliton**

$e^* = \frac{e}{m}$ (e, Ψ_B) (e^*, φ_C)

$\Psi_B\,(\mathbf{r}_1, \mathbf{r}_2) = (\pm)\Psi_B\,(\mathbf{r}_2, \mathbf{r}_1)$ $\varphi_C\,(\mathbf{r}_1, \mathbf{r}_2) = (\pm\, e^{\mp i\frac{\pi}{m}})\varphi_C\,(\mathbf{r}_2, \mathbf{r}_1)$

The Conundrum: Where does the statistical gauge field come form in the first place ?



S. M. Girving and K. Yang, "Modern Condensed Matter Physics", Cambridge (2019)

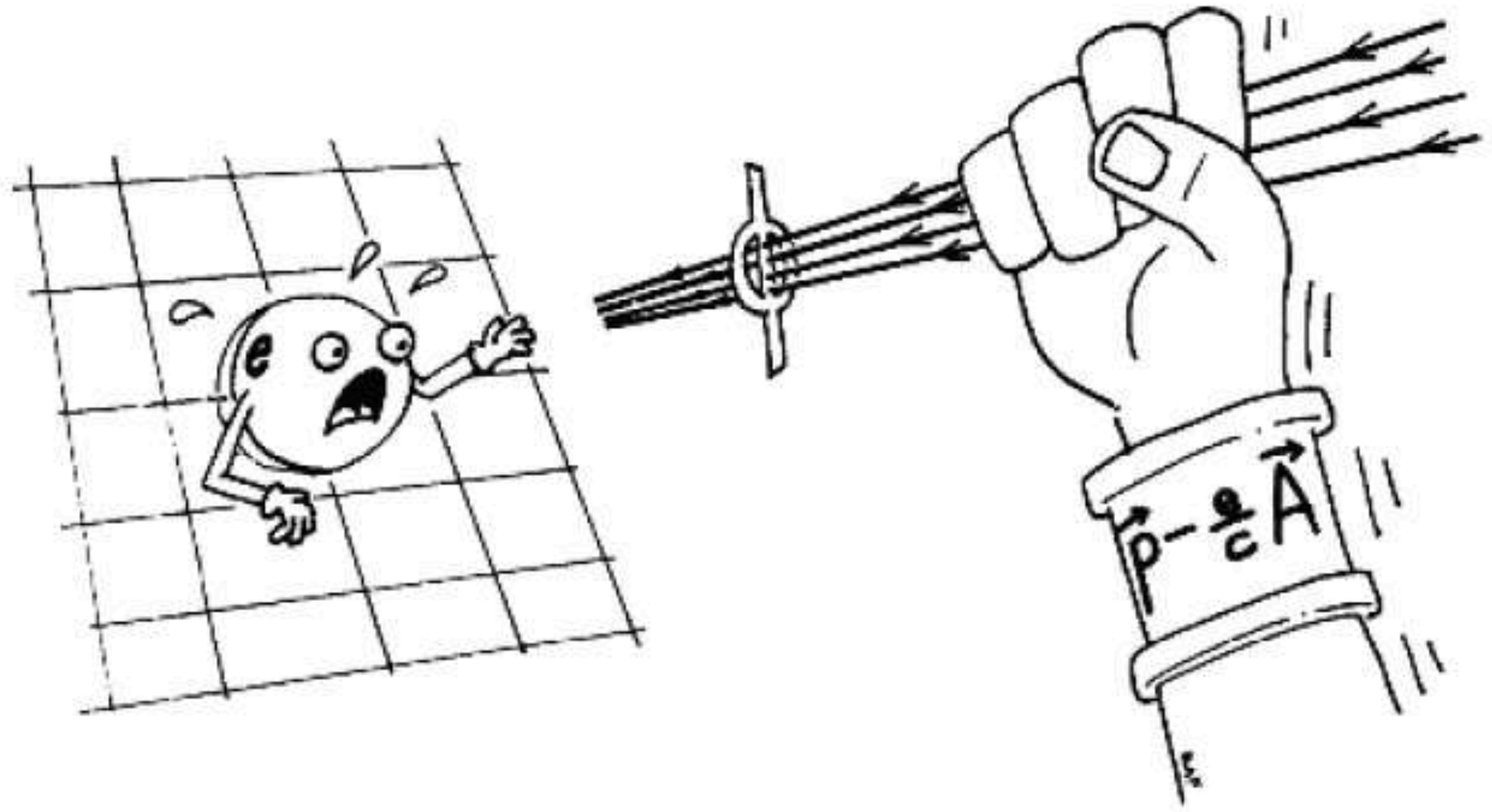


Figure 16.9 The piercing of a charge (which is confined to 2D) with a flux tube. The resulting composite object can have fractional statistics. To date, experimentalists have not succeeded in performing this operation; however, nature has been (as always) more clever. Figure reprinted with permission from [90]. Copyright 1989 World Scientific Publishing Co. Pte Ltd.

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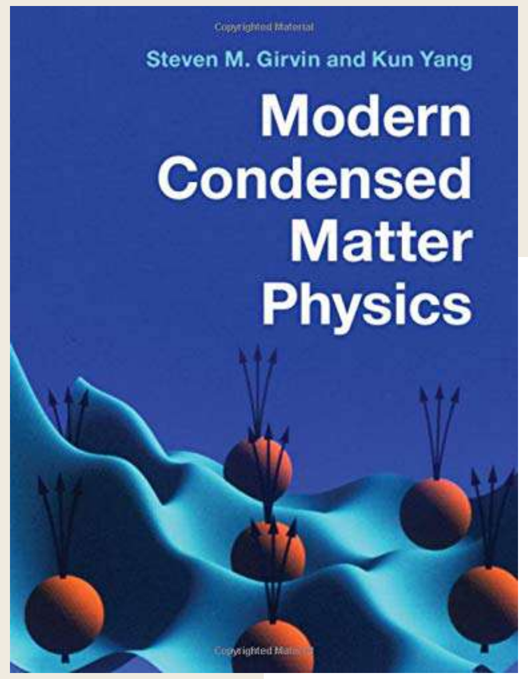

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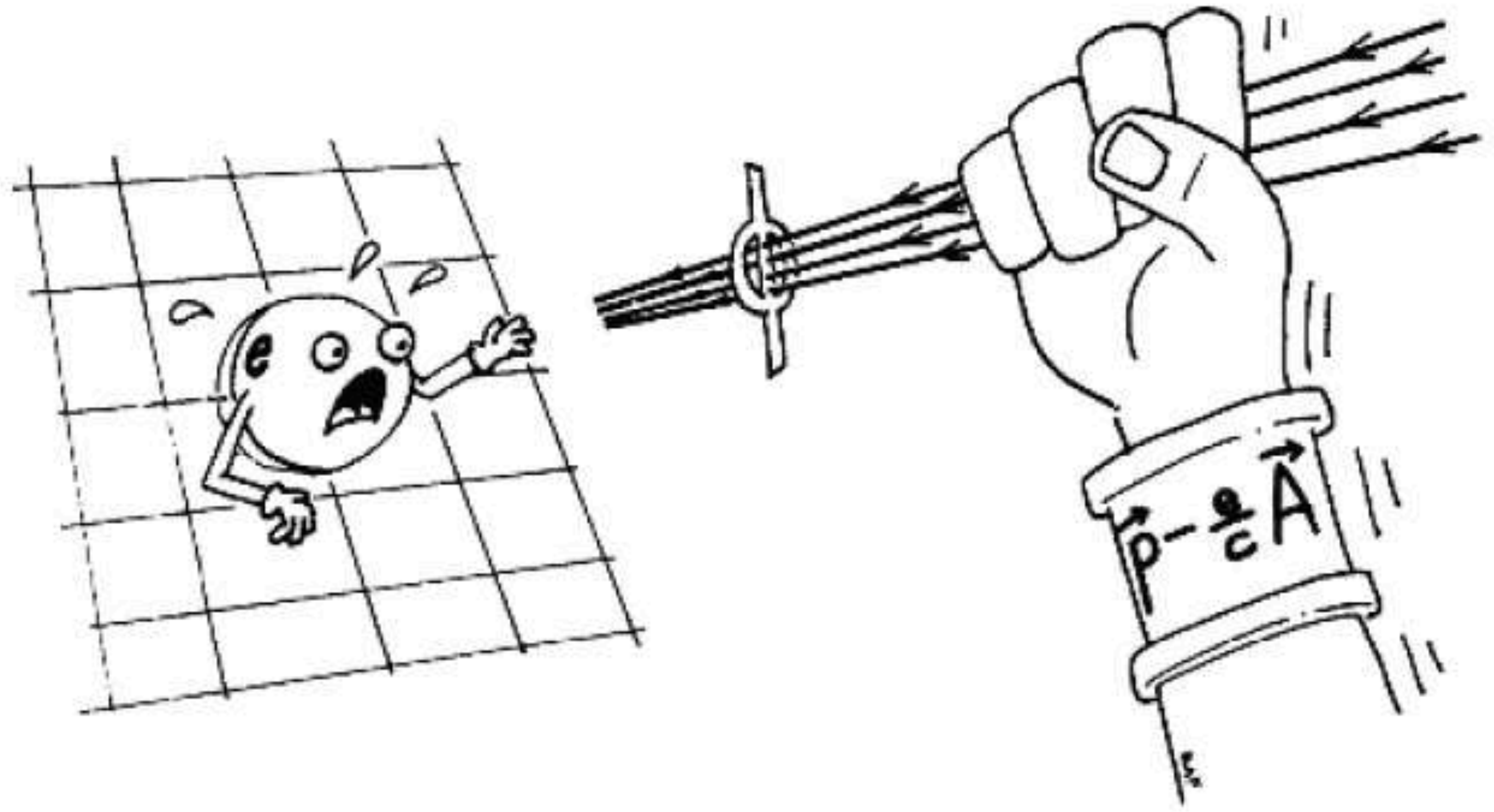

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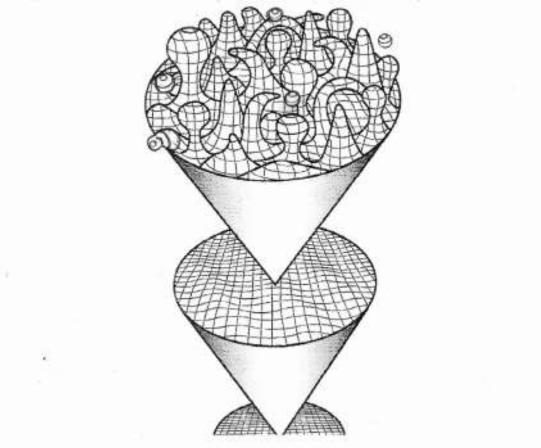
The problem with all this is the microscopic "origin" or the "emergence" of this Chern-Simons Term

A Bottom-Up Approach

<u>GOAL:</u> From a **microscopic** interacting quantum-many body system, derive the "self-generation" of a Chern-Simons term so that it performs flux attachment at an effective level.

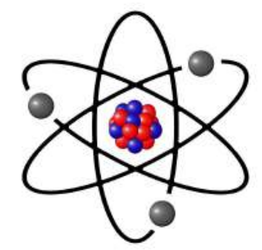
Challenges



Flux Attachment is Emergent

$$H_{CS} = 0$$

It is a Topological Field Theory



$$\mathbf{p} \not\to \mathbf{p} - q\mathbf{A}$$

Ultracold Atoms: Dilute & Charge Neutral

A Bottom-Up Approach

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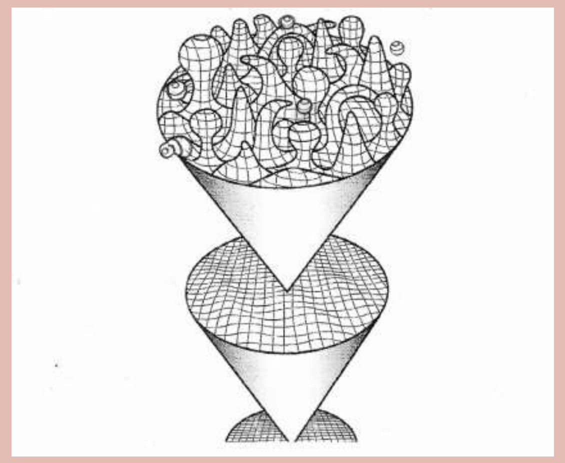

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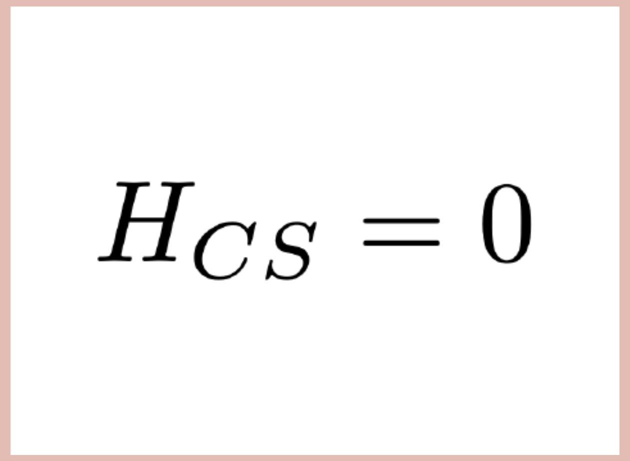

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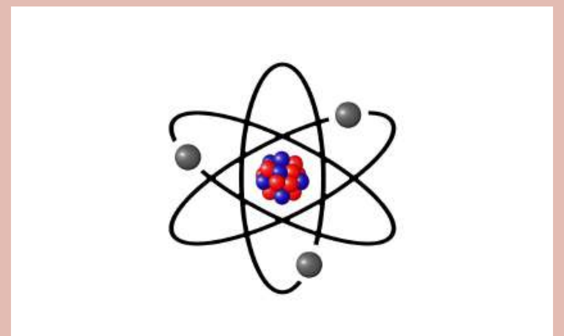

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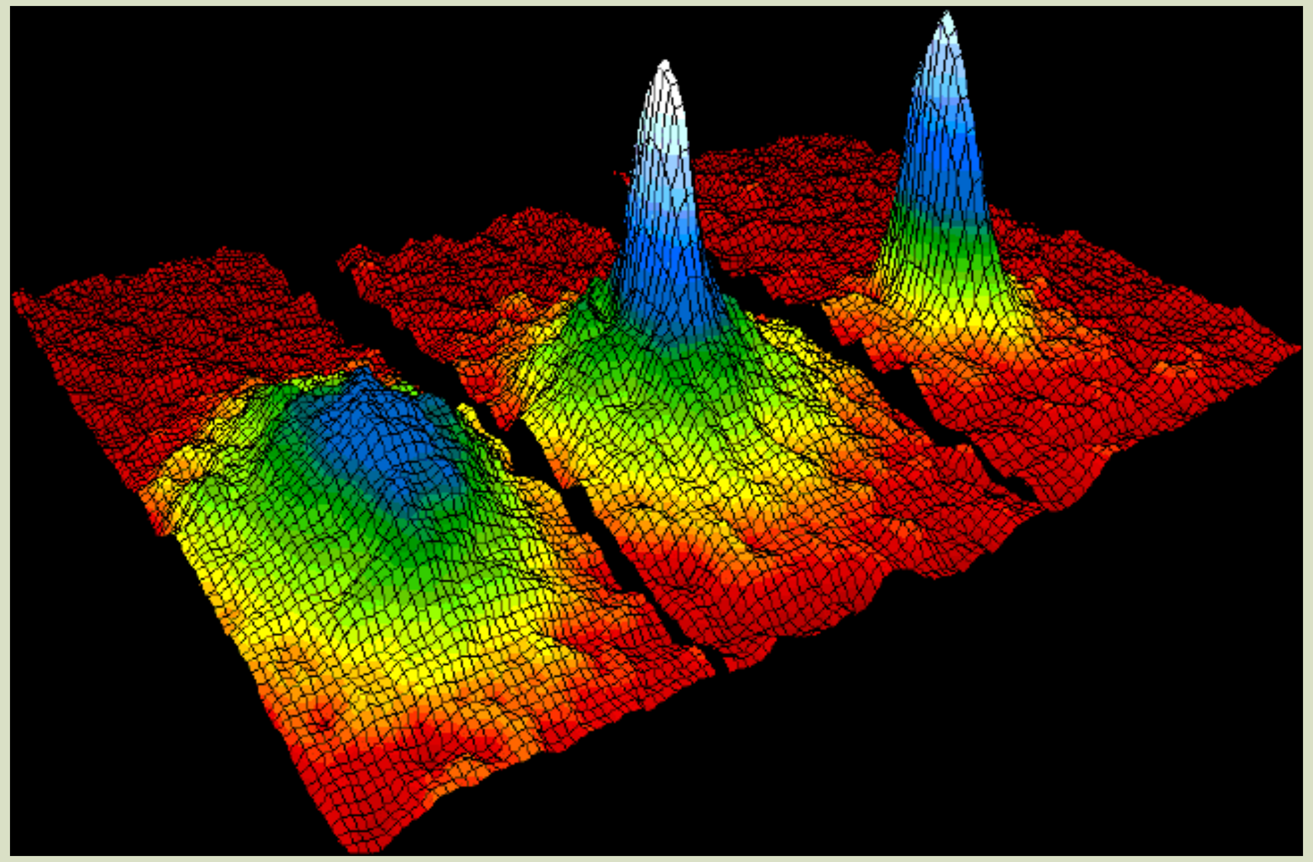
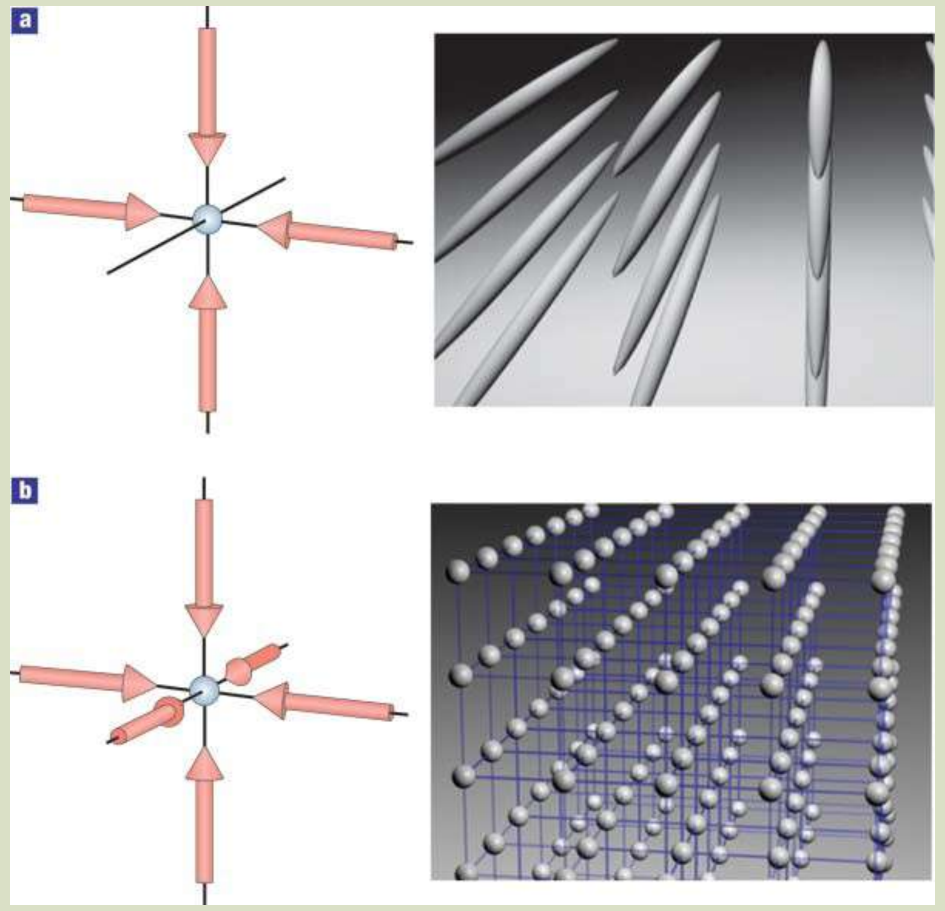
Ultracold Atoms: Dilute & Charge Neutral

Quantum Fluids



Engineered Systems: Quantum Simulation

Quantum "Solids"



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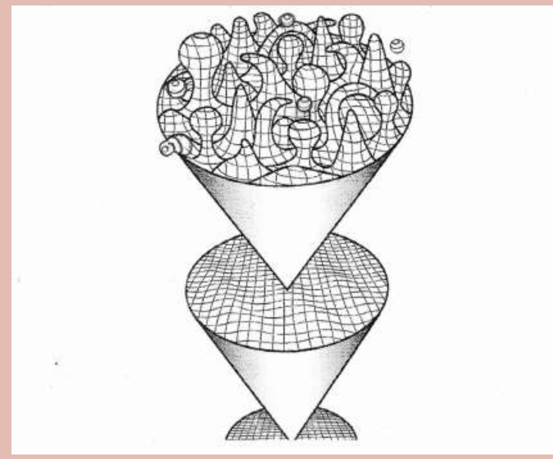

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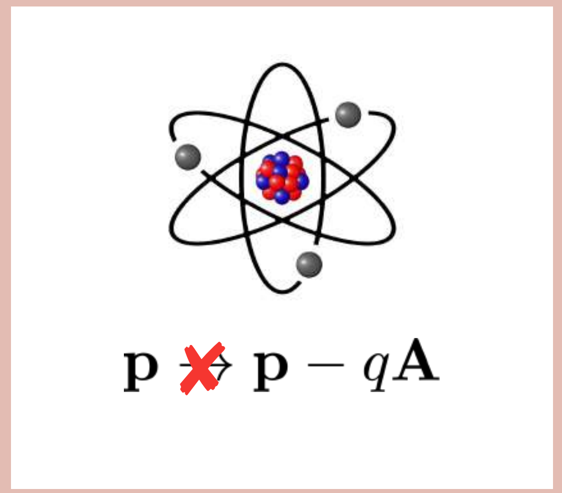

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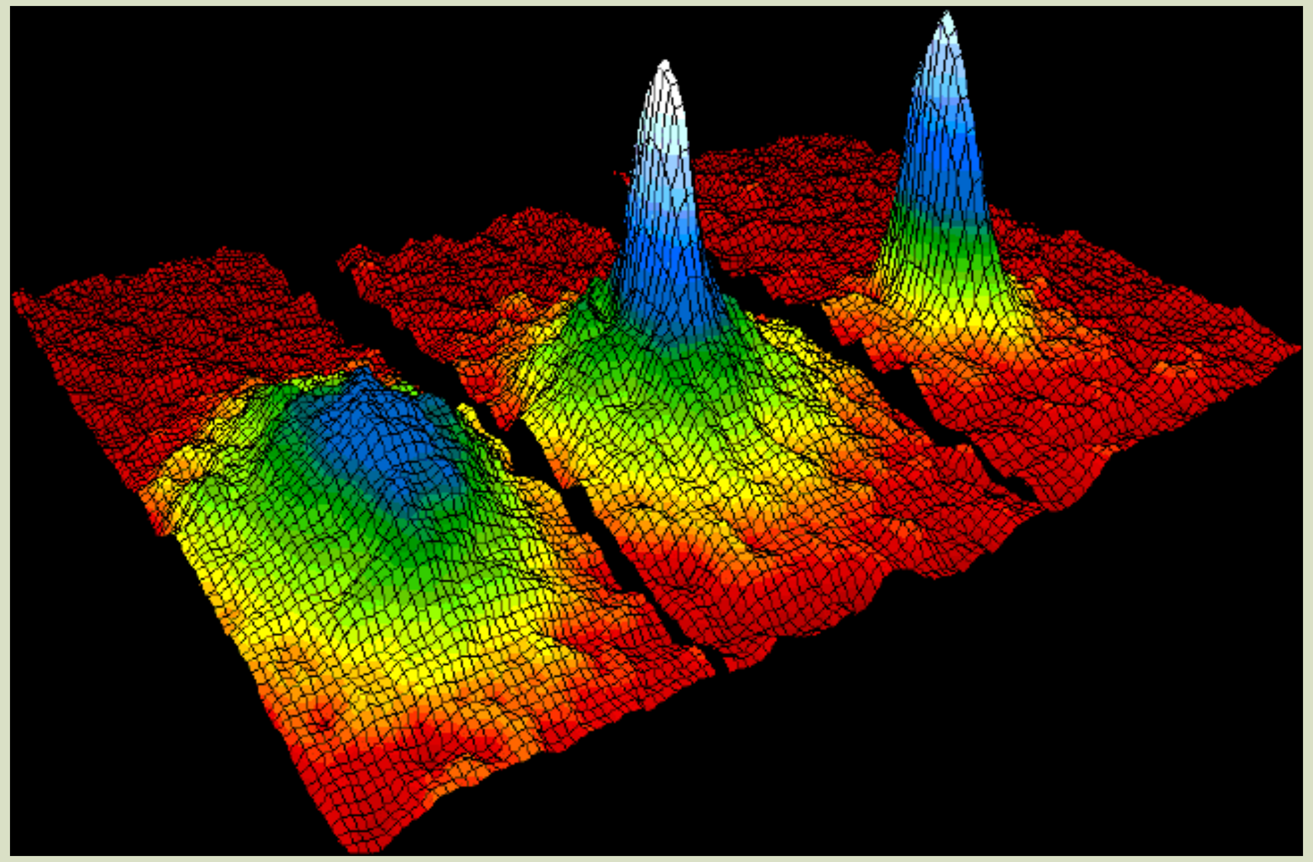

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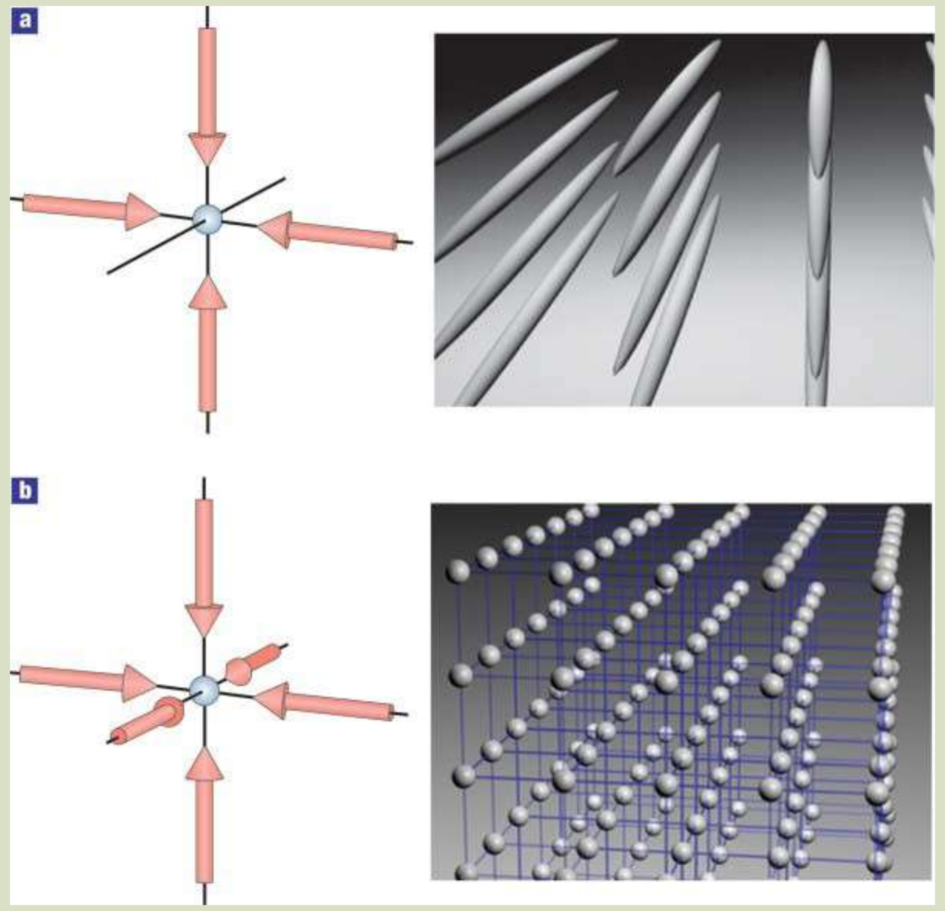


How to go from a theory of background gauge fields to a gauge theory ? Supplement with some constraint!

F. Görg et al., Nature Phys. 15, 1161 (2019)
V. Lienhard et al., Phys. Rev. X 10, 021031 (2020)
C. Schweizer et al., Nature Phys. 15, 1168 (2019)

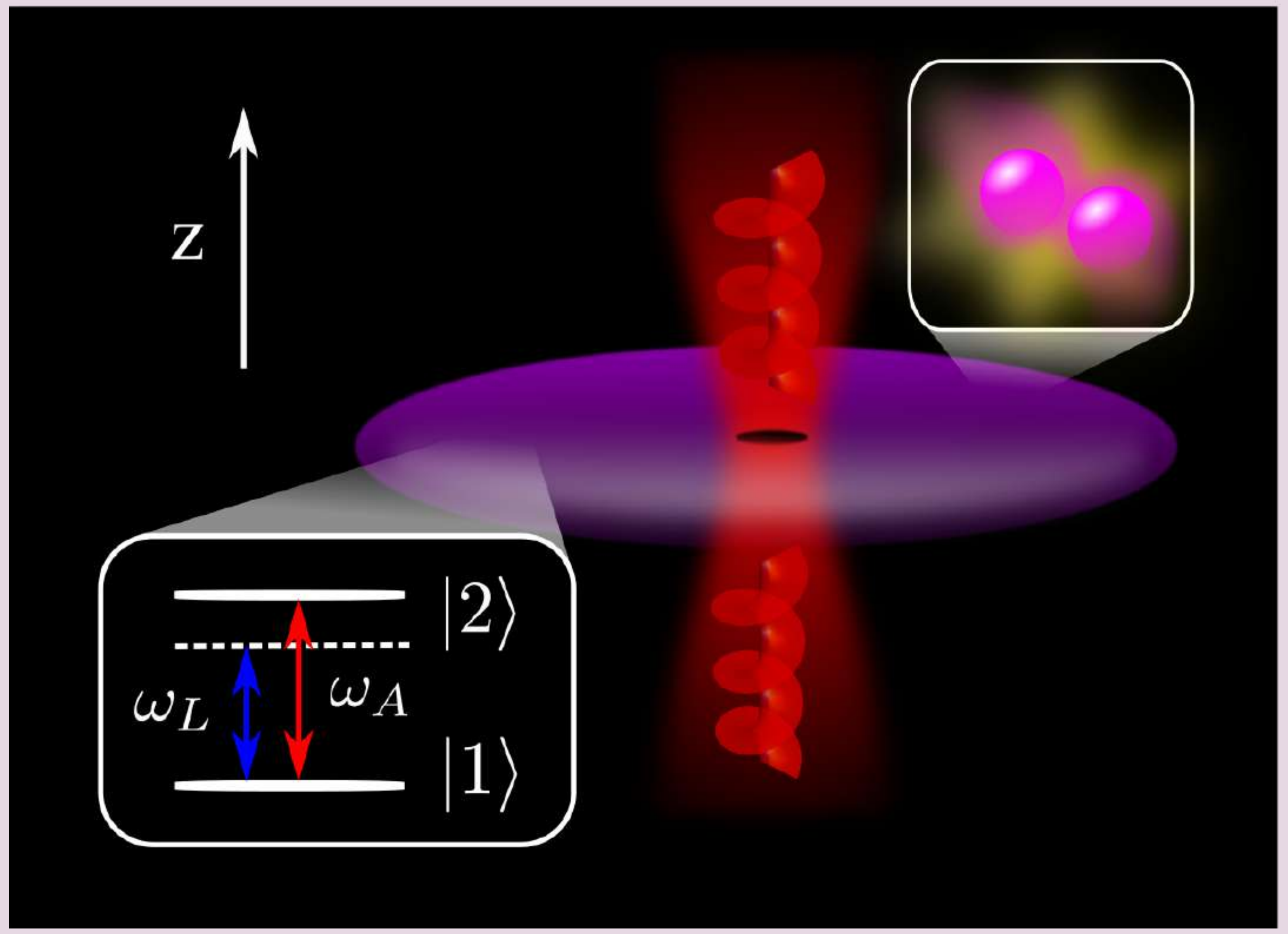
$$i\hbar\frac{\partial}{\partial t}\Psi(t,\mathbf{r}) = -\frac{\hbar^2}{2m}\left(\nabla - i\frac{e}{\hbar}\mathbf{A}\right)^2\Psi(t,\mathbf{r})$$

+

$$A_i\left(t,\mathbf{r}\right) = f\left[n\left(t,\mathbf{r}\right)\right]\hat{e}_i \quad \text{where} \quad n\left(t,\mathbf{r}\right) = |\Psi|^2$$

Gauge field is some function of matter density

Microscopic Scheme



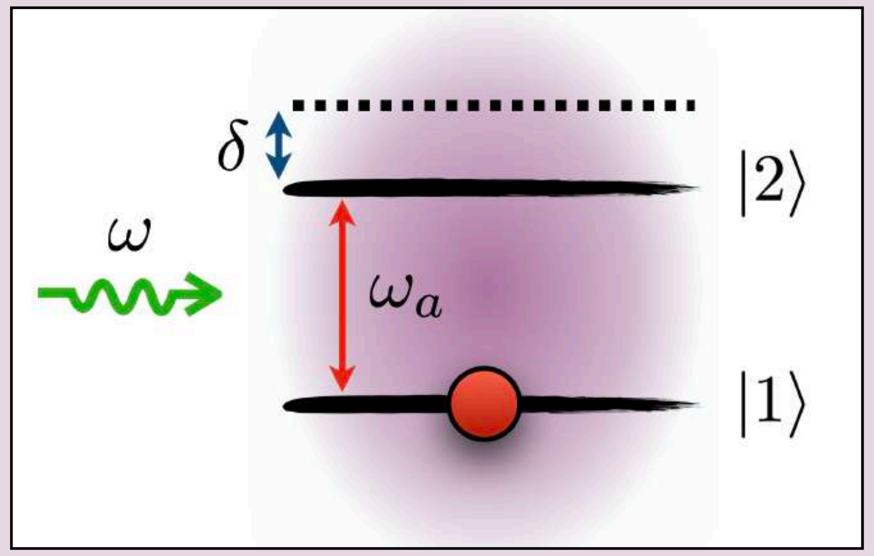
BEC of atoms with 2 internal levels coupled by a laser beam

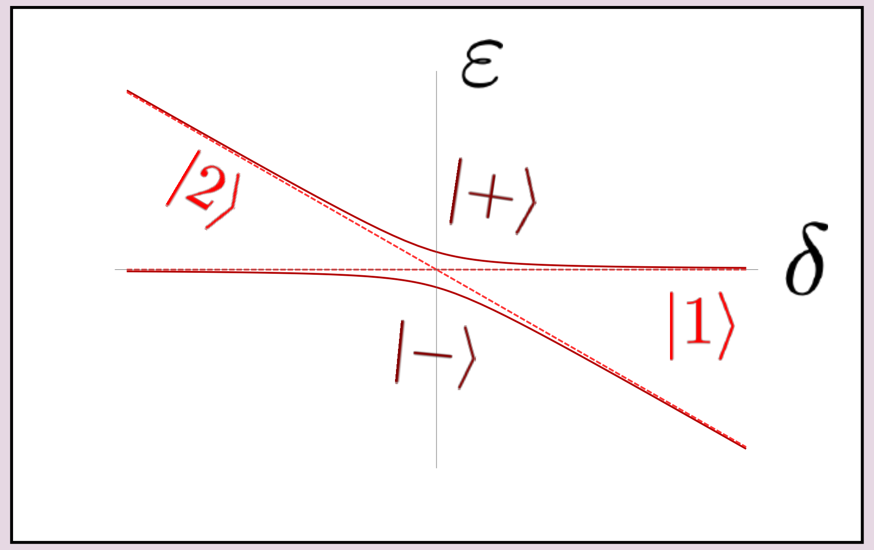
$$H = \sum_i \left(\frac{\mathbf{p}_i^2}{2m} + V_{\text{ext}}(\mathbf{r}_i) + U(\mathbf{r}_i) \right) + \sum_{\sigma,\sigma'=1}^{2} \sum_{i<j} g_{\sigma\sigma'}\, \delta\left(\mathbf{r}_i - \mathbf{r}_j\right)$$

Interparticle contact pairwise Interaction $\mathcal{V}_{ij}$

External Potential e.g. trapping potential

Light-Matter Interaction $U(\mathbf{r}_i) = \hbar\Omega(\mathbf{r}_i)\left(\mathbf{n}(\mathbf{r}_i)\cdot\vec{\sigma}\right)$





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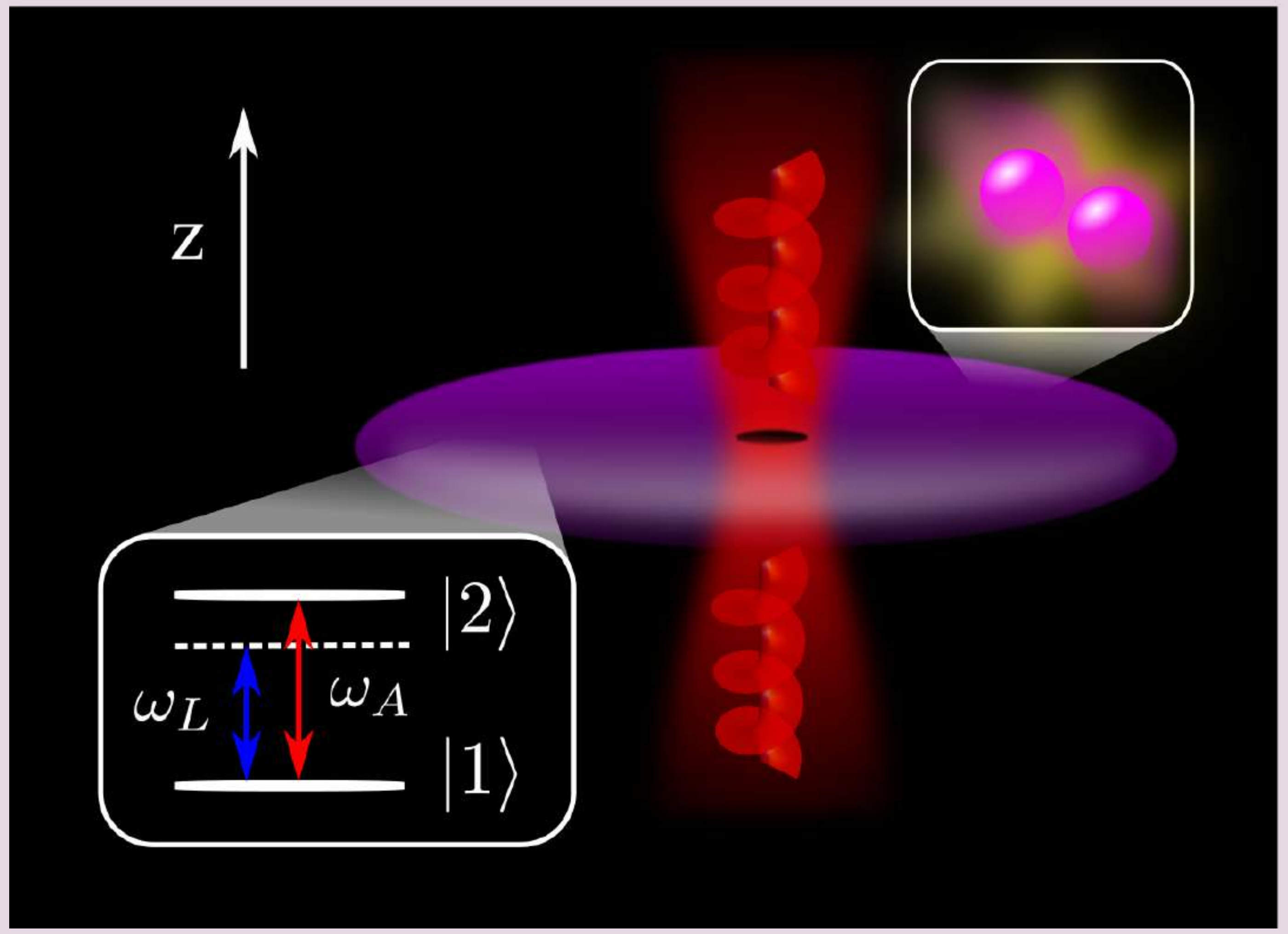

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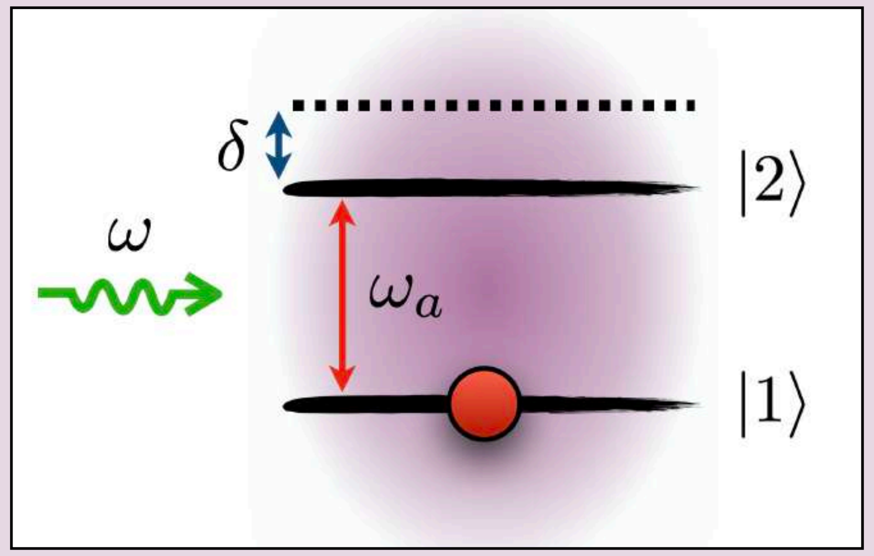
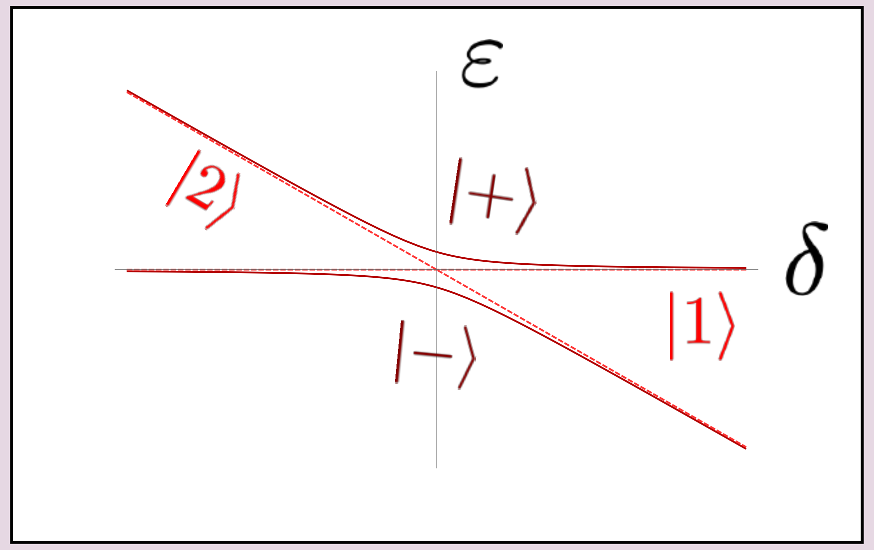
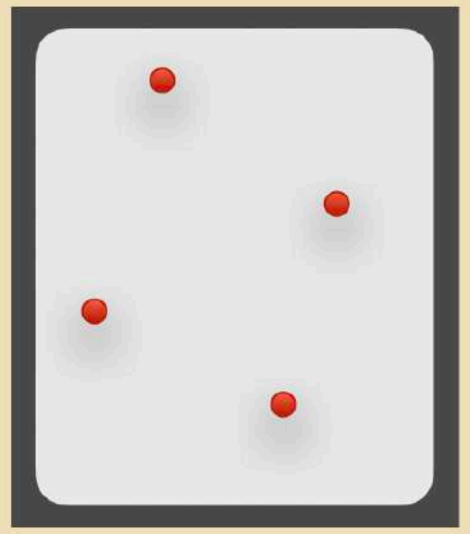


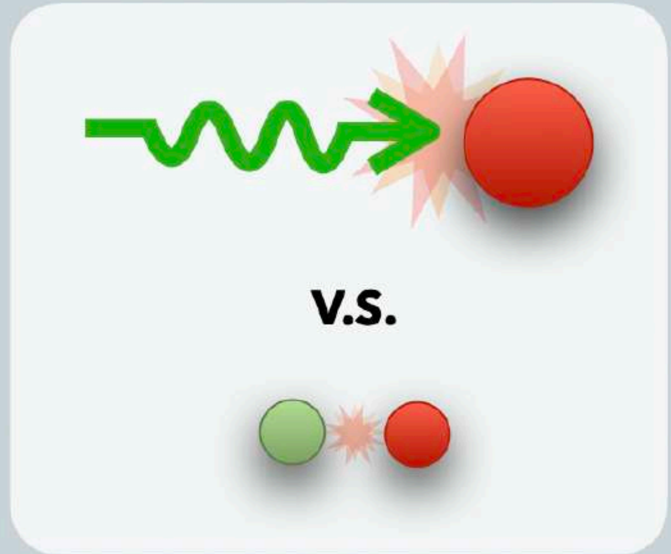


Approximations: "Deriving" emergence



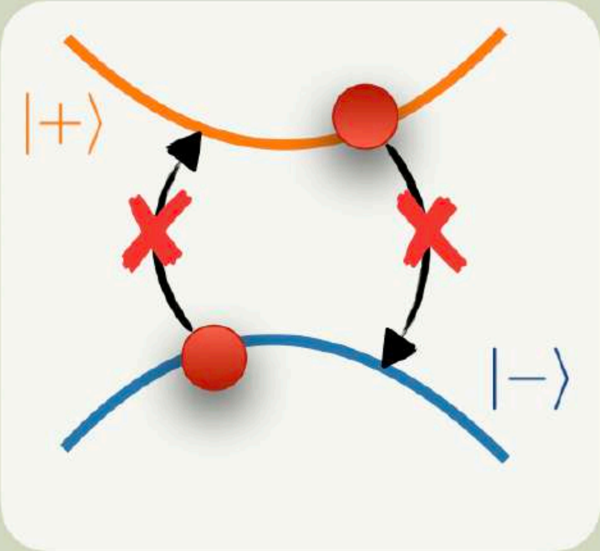
$\Psi(\mathbf{r}_1,\dots,\mathbf{r}_N) \approx \tilde{\Psi}(\mathbf{r}_1)\dots\tilde{\Psi}(\mathbf{r}_N)$

Mean-Field



$\Omega \gg g$

Perturbation



$|\tilde{\Psi}(t,\mathbf{r})\rangle \approx \Phi_\pm(t,\mathbf{r})\, |\pm(\mathbf{r})\rangle$

Adiabatic

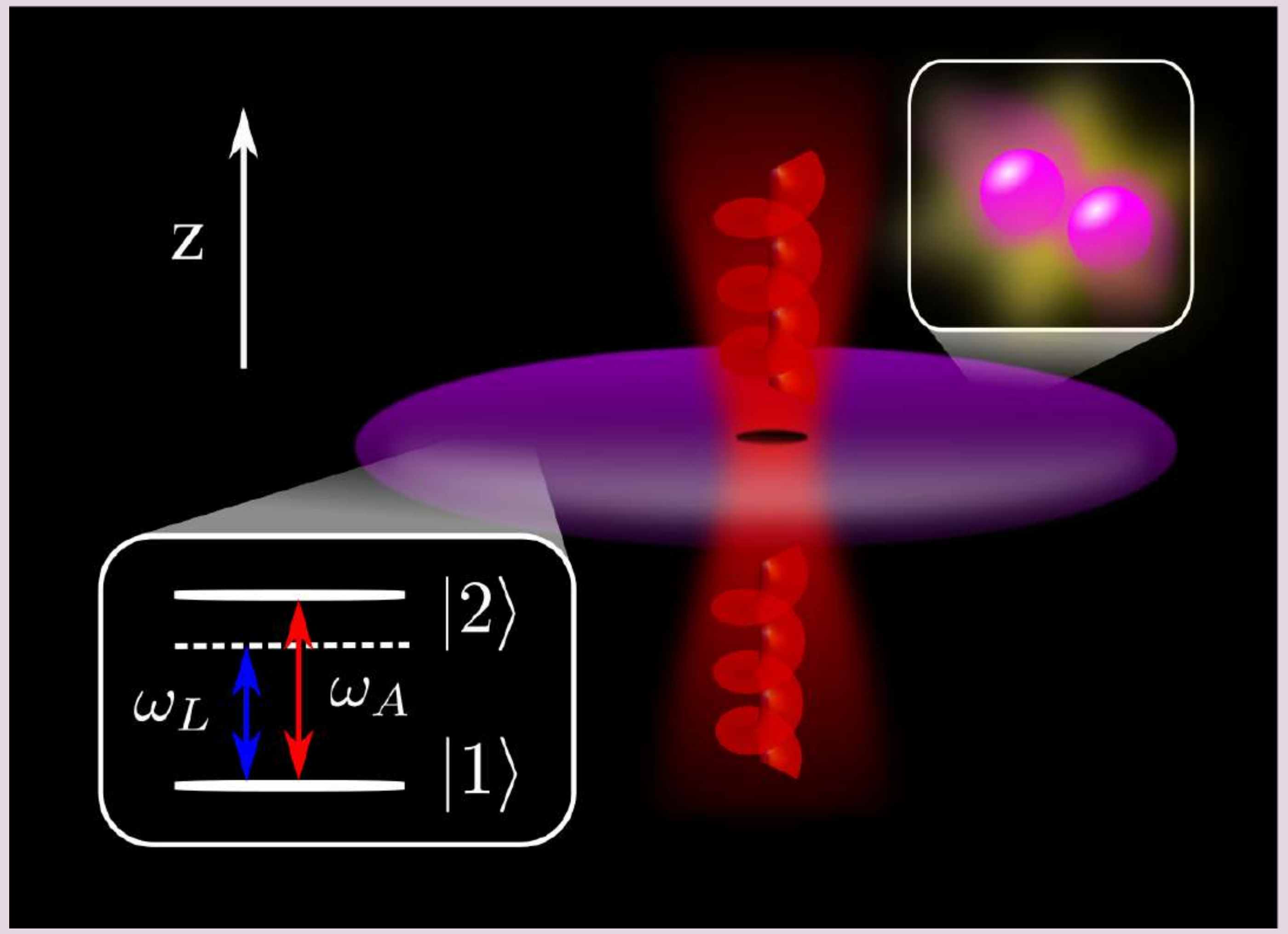
$$H_{\text{MF}} = \frac{\mathbf{p}^2}{2m} + V + U + \mathcal{V}_{\text{MF}}$$

$$|\pm\rangle \approx |\pm^{(0)}\rangle + |\pm^{(1)}\rangle$$

$$i\hbar\,\partial_t\,\Phi_\pm = H_{\text{eff}}^{\pm}\,\Phi_\pm$$

Mean-field Hamiltonian is projected onto the eigenstate in which the system is prepared

Microscopic Scheme



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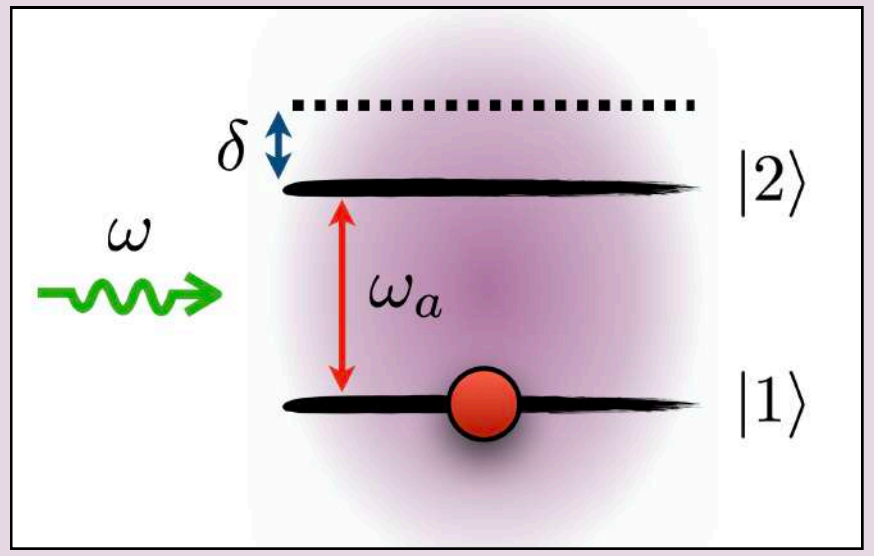

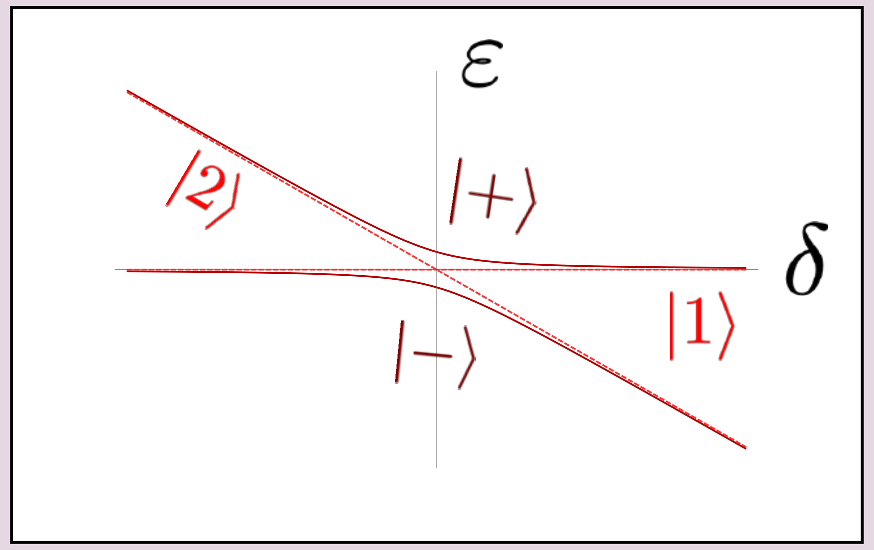

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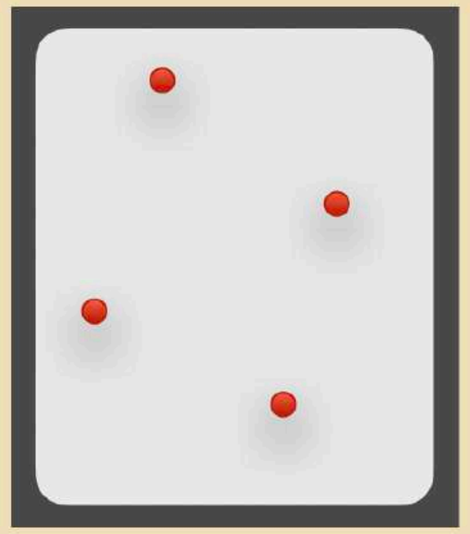

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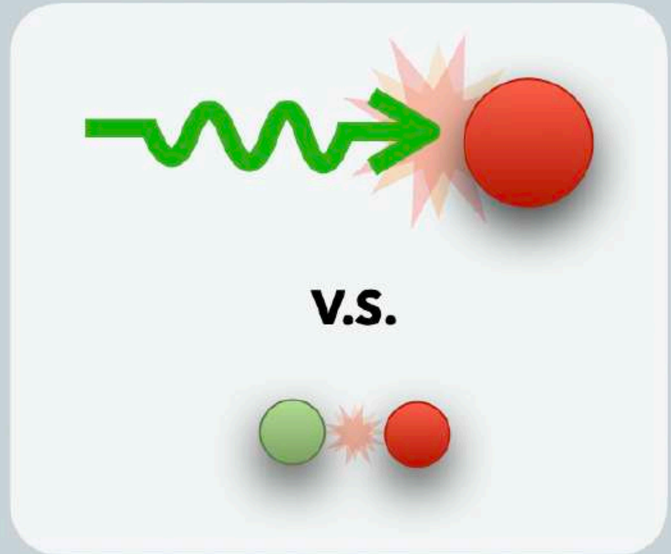

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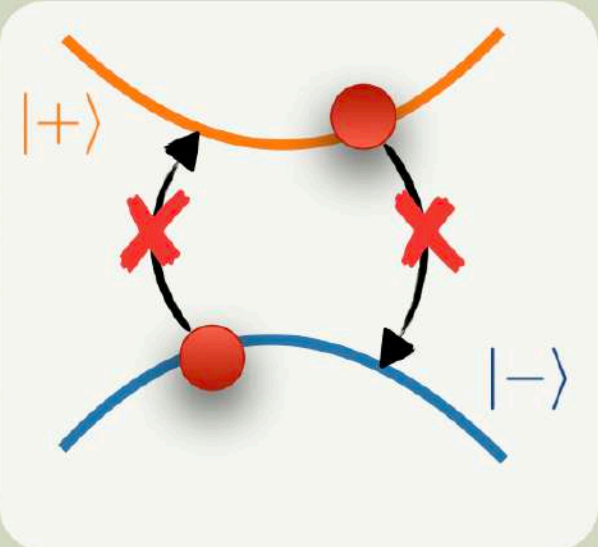

$|\tilde{\Psi}(t,\mathbf{r})\rangle \approx \Phi_\pm(t,\mathbf{r})\,|\pm(\mathbf{r})\rangle$

Adiabatic

$i\hbar\partial_t\Phi_\pm = H_{\mathrm{eff}}^{\pm}\Phi_\pm$

Mean-field Hamiltonian is projected onto the eigenstate in which the system is prepared

Effective model: A topological gauge theory in 2+1D

$$\mathcal{L}_{\mathrm{eff}} \approx -\frac{\kappa}{4\pi\hbar}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda + i\hbar\,\Phi^* D_t\Phi - \frac{\hbar^2}{2m}|\mathbf{D}\Phi|^2 - \frac{g}{2}|\Phi|^4 - \tilde{W}|\Phi|^2$$

$$D_\mu = \partial_\mu - \frac{i}{\hbar}(A_\mu + a_\mu) \qquad \text{where} \qquad \mu = t, x, y$$

Berry Connection

$$i\hbar\langle +|\vec{\nabla}|+\rangle \approx \mathbf{A}^{(0)} + \mathbf{A}^{(1)} = \vec{A} + \vec{a}$$

Perturbative Expansion

Background gauge field
Single-Particle contribution

(Dynamical) Chern-Simons gauge field
Interaction (two-body) contribution

Phenomenology

Effective Model Corresponds to

Macroscopic or Composite Boson description of a FQH fluid

VOLUME 62, NUMBER 1 — PHYSICAL REVIEW LETTERS — 2 JANUARY 1989

Effective-Field-Theory Model for the Fractional Quantum Hall Effect

S. C. Zhang
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106

T. H. Hansson and S. Kivelson
Physics Department, State University of New York at Stony Brook, Stony Brook, New York 11794
(Received 26 July 1988)

Super Nice Review !

International Journal of Modern Physics B, Vol. 6, No. 1 (1992) 25–58
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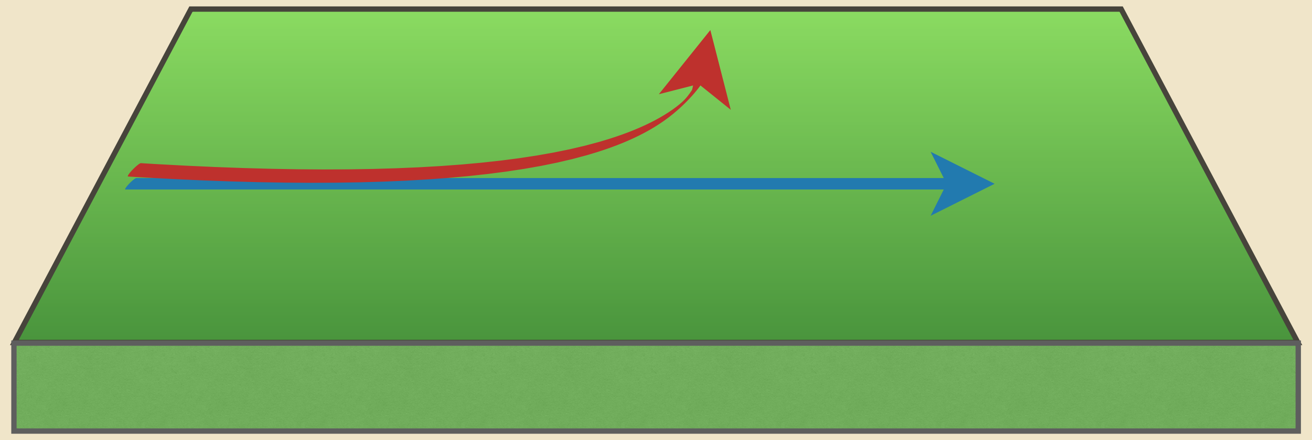
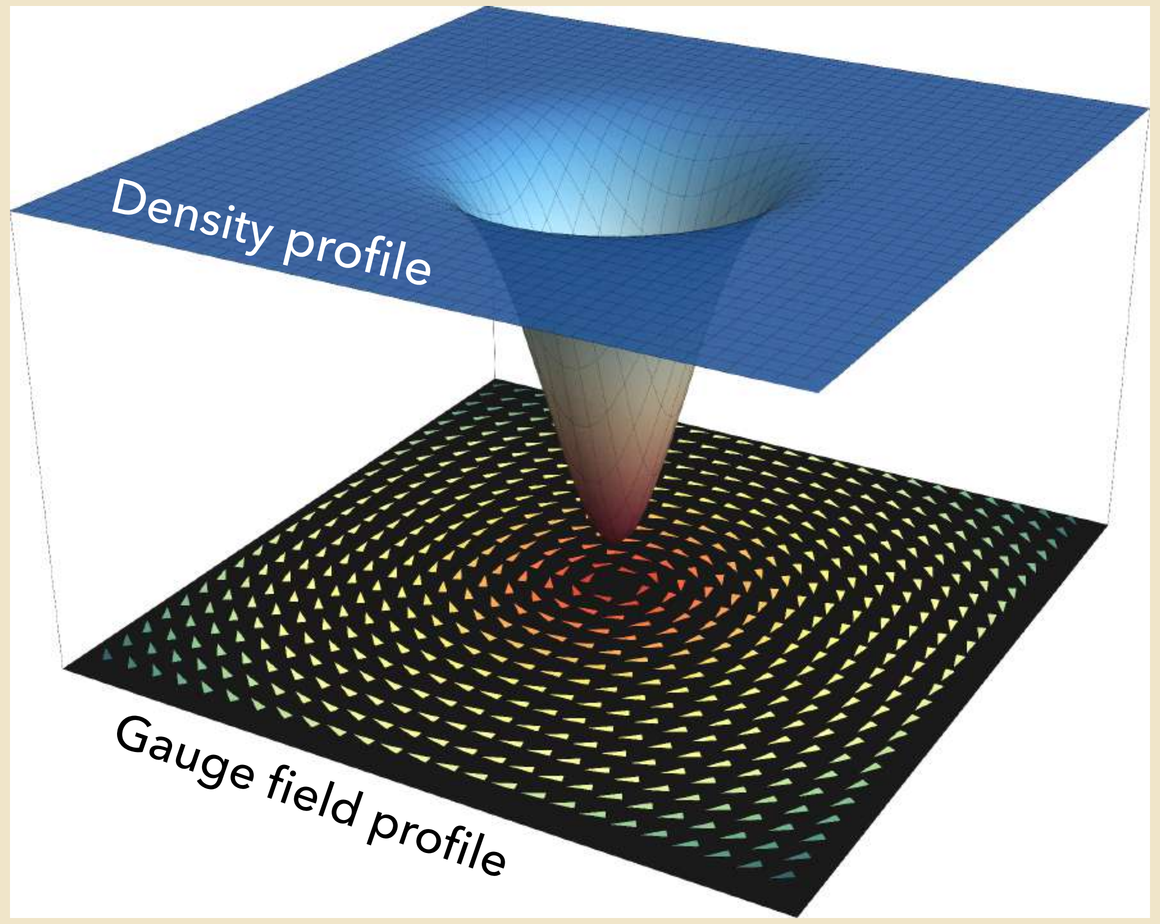
THE CHERN–SIMONS–LANDAU–GINZBURG THEORY OF THE FRACTIONAL QUANTUM HALL EFFECT*

SHOU CHENG ZHANG
IBM Research Division, Almaden Research Center, 650 Harry Road, San Jose, CA 95120-6099, USA



Fractionally quantised (atomic) Hall conductance and transverse flow



Flux attached vortices

- Fractional (synthetic) charge
- Anyonic statistics

They act as Laughlin quasiparticles

Summary & Conclusions

- We derive emergent topological gauge theory in **2+1D** in **continuum**. Chern-Simons gauge field is understood as a **density-dependent** Berry connection (synthetic gauge field)
- We introduce a proof-of-concept scheme for a potential quantum simulation using a BEC. Only **one species needed** as compared to two species used in conventional LGTs
- We obtain an effective (strongly correlated) FQH fluid with fractionalised excitations (vortices) out of a dilute weakly interacting system. We "induce" flux attachment
- Systems with density-dependent gauge fields can be understood as gauge theories with (certain) topological structure
- Discretisation of the model for a lattice realisation is straightforward. Extensions as coupling to fermions or higher-spin structures are a subject for further work

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Thank You

G. V-R, N. Westerberg, P. Öhberg, Phys. Rev. Research 2, 033453 (2020)

OUTLOOK

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More Slides

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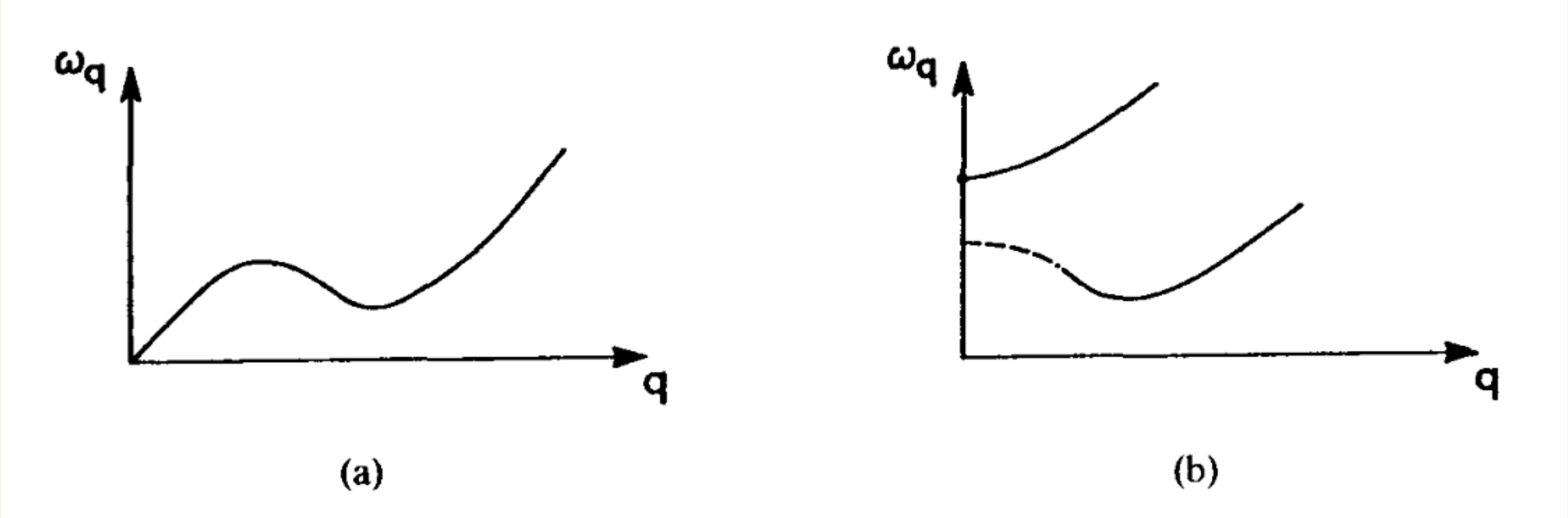
Other properties of the ZHK model



Phonon-Roton Spectrum (Superfluid)

Magneto-Phonon-Roton Spectrum (FQH fluid)

Gapped (Incompressible) spectrum

Anderson-Higgs Mechanism

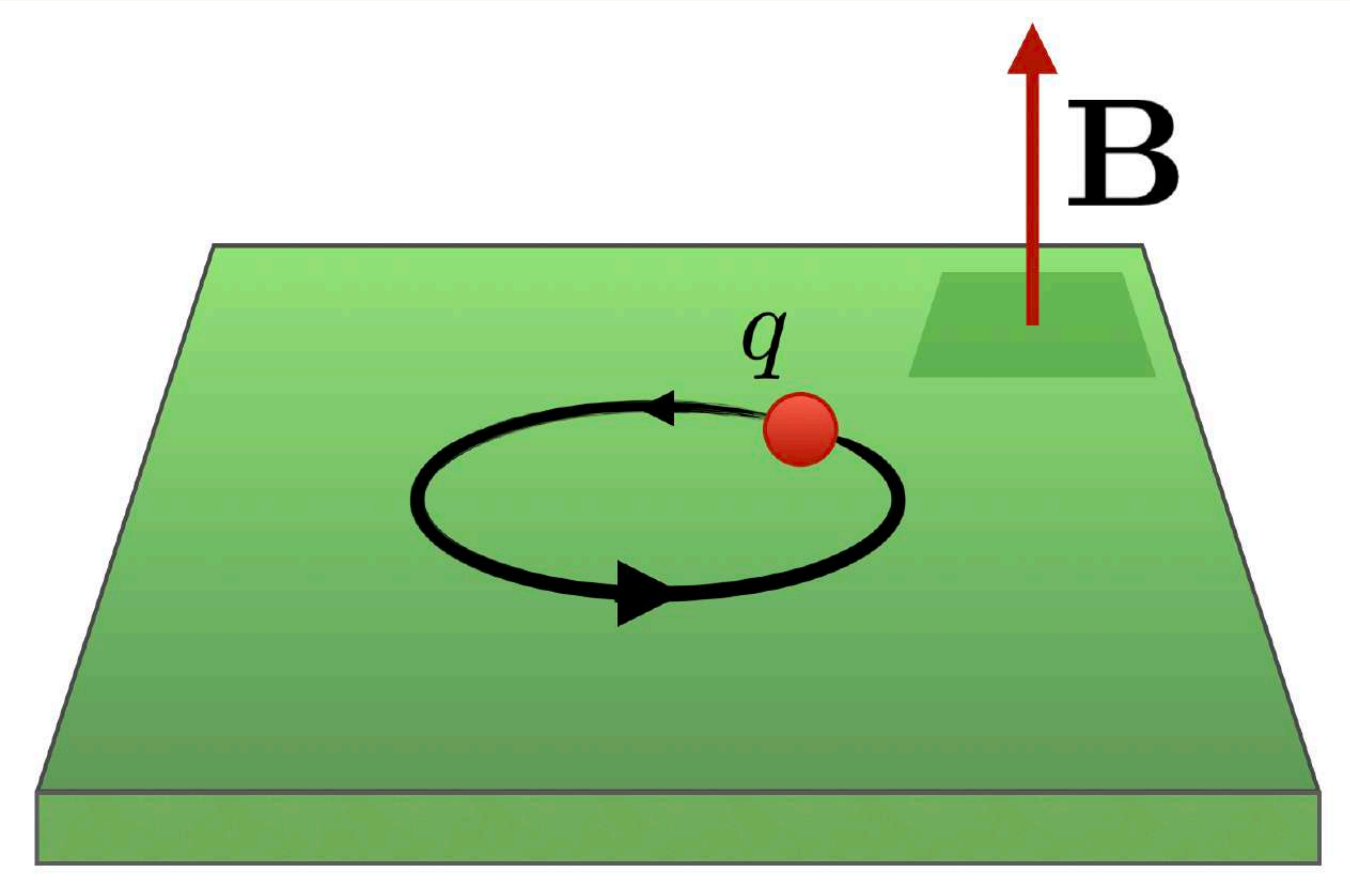
Nonlocal order parameter

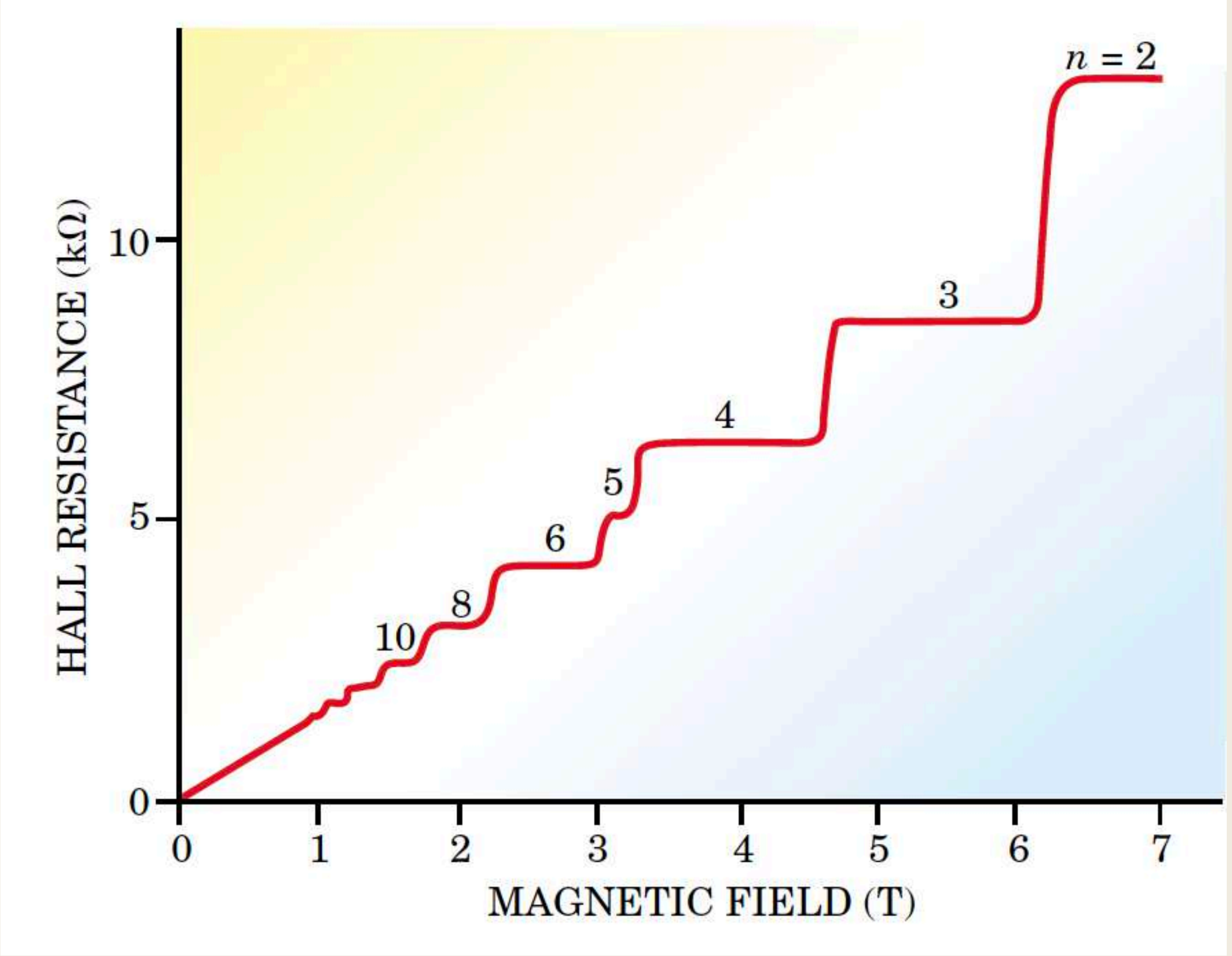
Off-diagonal long-range order

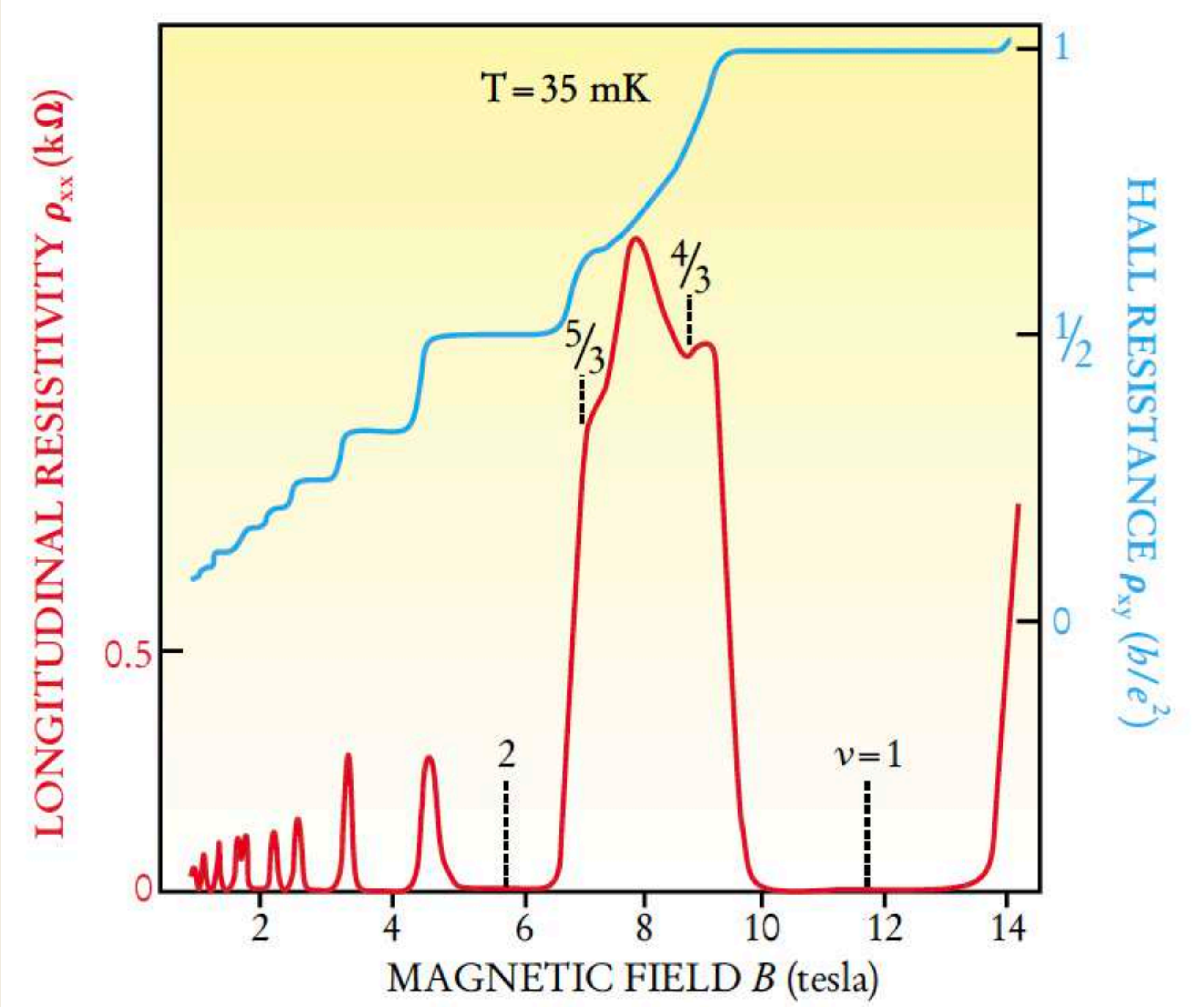
Laughlin wavefunction for vortices

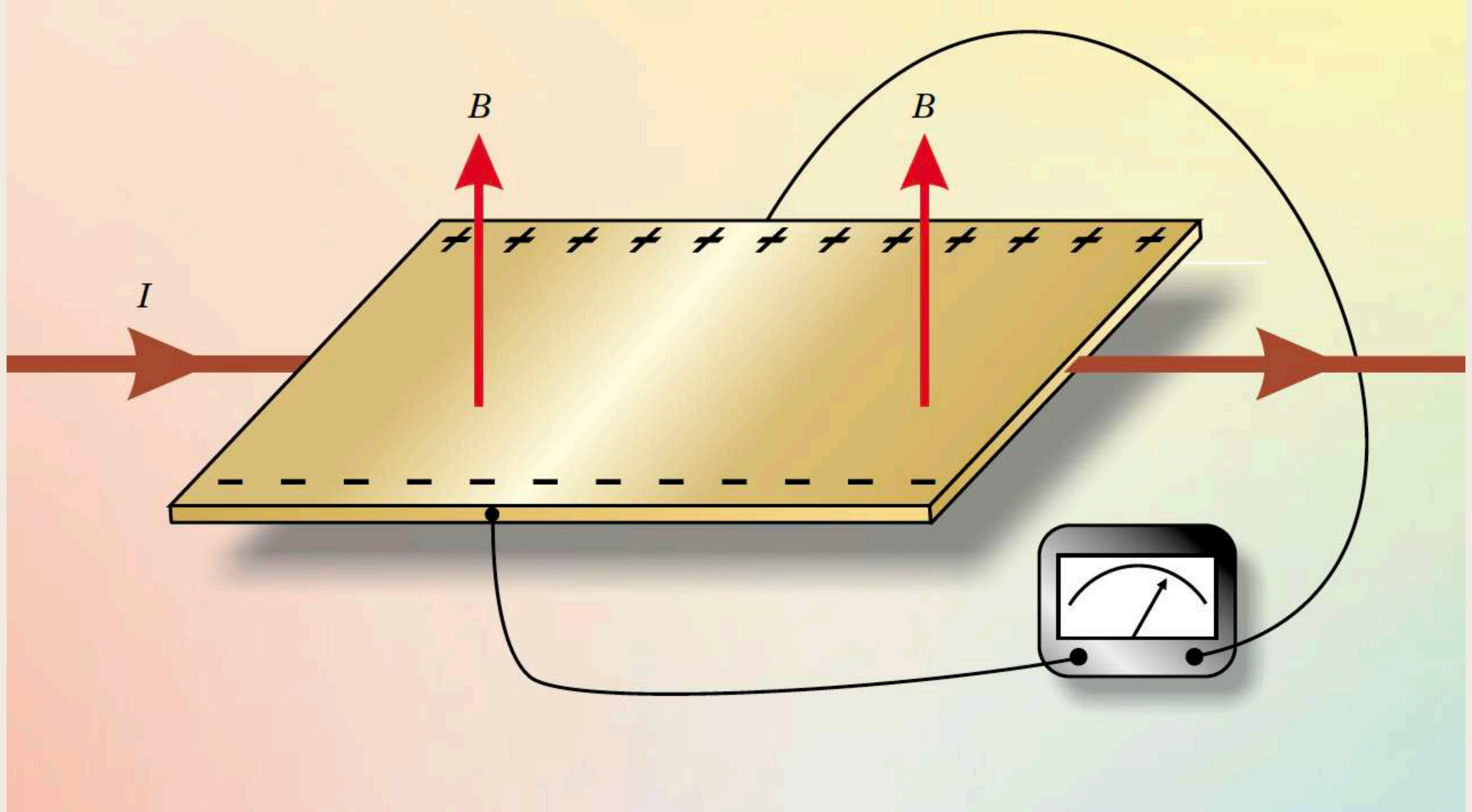
Vortices cost finite energy

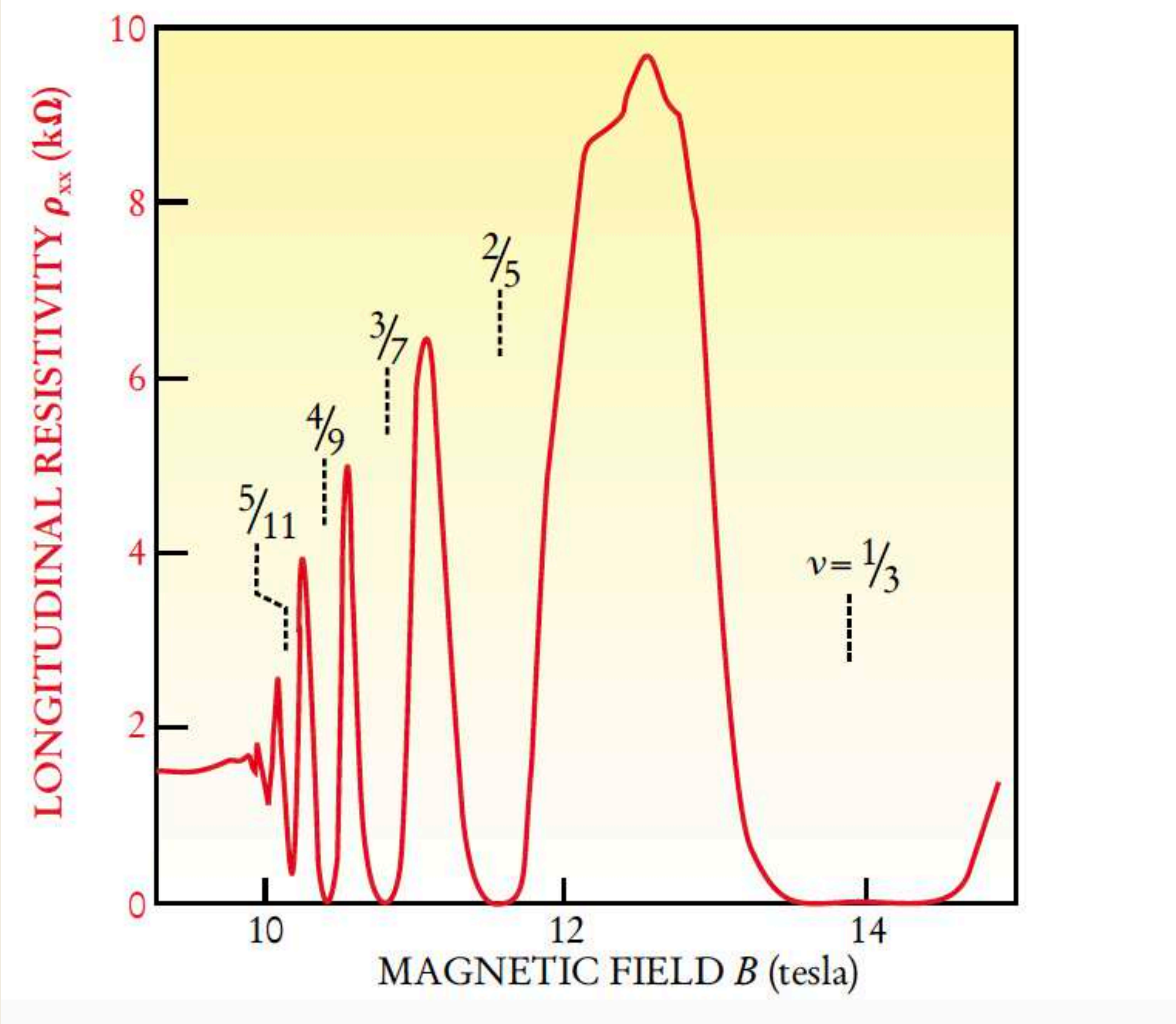
The Quantum Hall effect: Integer vs Fractional











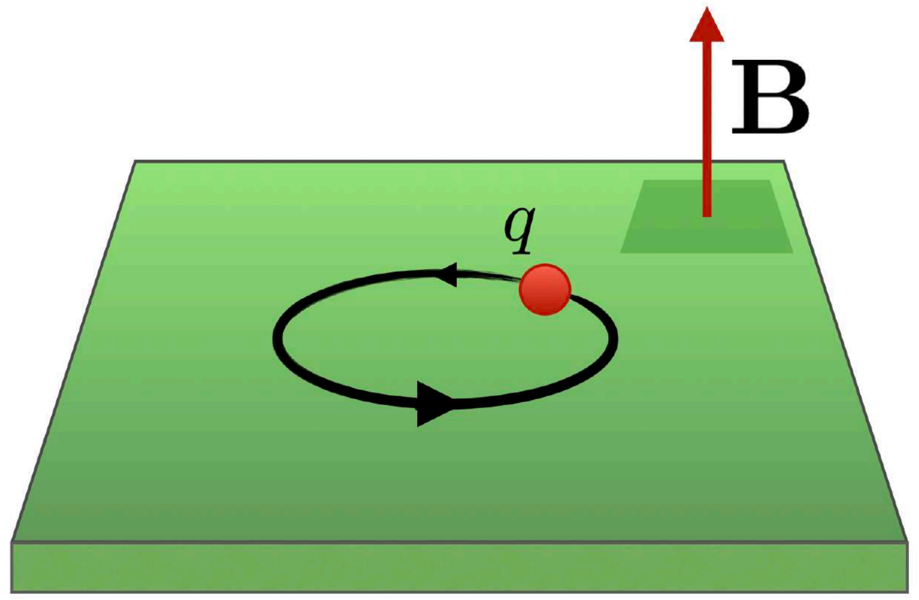
Background vs Dynamical gauge fields

(1)

Is this a gauge theory ?

NO! Gauge field is Background (non-dynamical)

Charged particle in a magnetic field

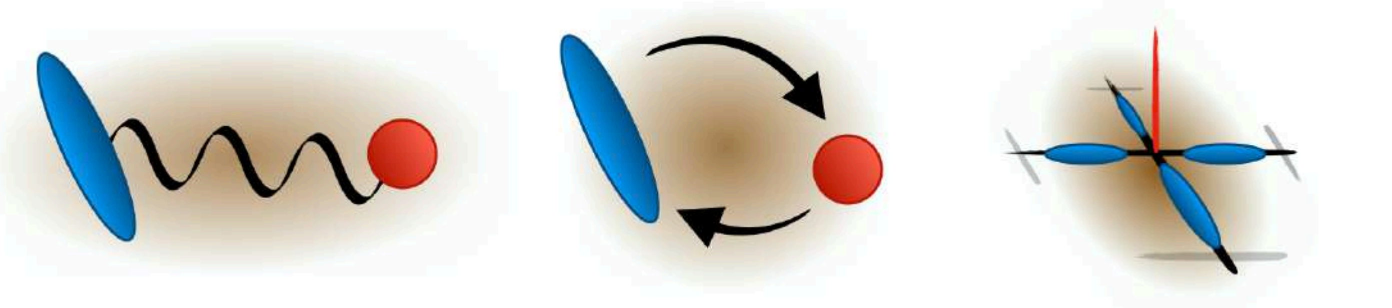


$$H = \frac{(\mathbf{p} - q\mathbf{A})^2}{2m}$$

(2)

Recipe for a gauge theory

A backaction mechanism is needed between gauge and matter sectors

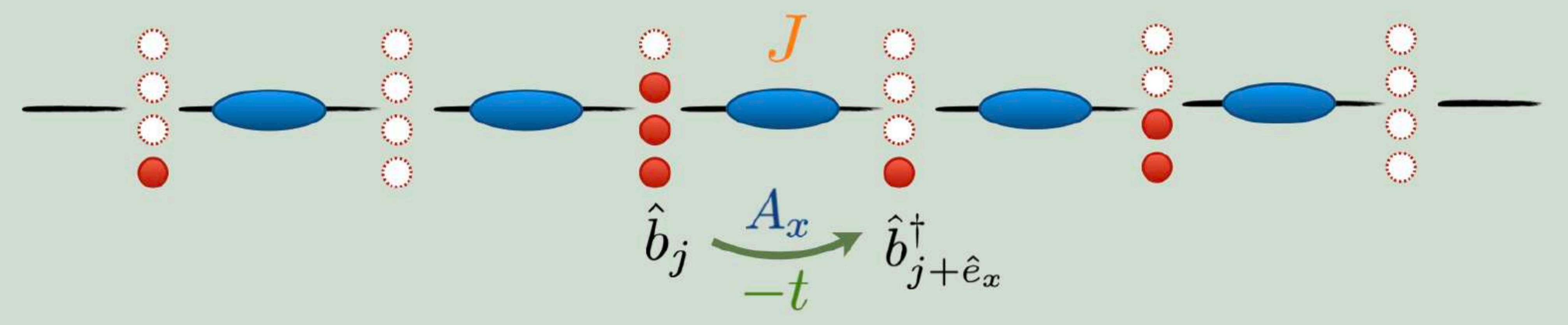


(i) Gauge-Matter Coupling

(ii) Dynamical Gauge fields

(iii) Local Gauge Invariance

Some random example: Lattice "Scalar Electrodynamics" in 1+1D



Impose Local Constraint

$E_{j+1} - E_j = (N\,●)_j$ → To continuum → $\vec{\nabla} \cdot \mathbf{E} = \rho$

$$H = -t \sum_j (\hat{b}^\dagger_{j+\hat{e}_x}\, e^{iA_x}\, \hat{b}_j + \mathrm{H.c.}) + J \sum_j E_j^2$$

where $E_j = -\partial_t A_x(j)$