The Entropy of Black Holes from the Superconformal Index

Ziruo Zhang (SISSA)

Joint work with Francesco Benini, Edoardo Colombo, Saman Soltani, Alberto Zaffaroni (2005.12308)

YTF 20, December 16, 2020

General context

- Bekenstein and Hawking tell us that black holes radiate and carry entropy

$$S = \frac{c^3 A}{4 G_N \hbar} \,.$$

General context

- Bekenstein and Hawking tell us that black holes radiate and carry entropy
$$S = \frac{c^3 A}{4 G_N \hbar} \, .$$
- In statistical physics, entropy is the degeneracy of microstates. For this interpretation one needs quantum gravity.

General context

- Bekenstein and Hawking tell us that black holes radiate and carry entropy
$$S = \frac{c^3 A}{4 G_N \hbar} \, .$$
- In statistical physics, entropy is the degeneracy of microstates. For this interpretation one needs quantum gravity.
- Accounted for in string theory by eg. [Strominger and Vafa, 1996].

General context

- Bekenstein and Hawking tell us that black holes radiate and carry entropy
$$S = \frac{c^3 A}{4 G_N \hbar} \, .$$
- In statistical physics, entropy is the degeneracy of microstates. For this interpretation one needs quantum gravity.
- Accounted for in string theory by eg. [Strominger and Vafa, 1996].
- Want to examine this in AdS/CFT.

Summary

- In AdS/CFT, entropy of black holes
  $=\log(\text{degeneracy of some states in CFT})$.

Summary

- In AdS/CFT, entropy of black holes
  = log(degeneracy of some states in CFT).

- We look at the dualities

$$\text{IIB on AdS}_5 \times SE_5 \quad \leftrightarrow \quad 4d\ \mathcal{N} = 1 \text{ SCFT}$$

  and consider rotating, electrically charged supersymmetric black holes.

Summary

- In AdS/CFT, entropy of black holes
$=\log$(degeneracy of some states in CFT).

- We look at the dualities
$$\text{IIB on } \text{AdS}_5 \times SE_5 \quad \leftrightarrow \quad 4d \;\, \mathcal{N}=1 \text{ SCFT}$$
and consider rotating, electrically charged supersymmetric black holes.

- Gave predictions for the Bekenstein-Hawking entropy from the superconformal index using the Bethe ansatz formula

Summary

- In AdS/CFT, entropy of black holes
  = log(degeneracy of some states in CFT).

- We look at the dualities

$$\text{IIB on AdS}_5 \times SE_5 \;\; \leftrightarrow \;\; 4d \;\; \mathcal{N} = 1 \text{ SCFT}$$

  and consider rotating, electrically charged supersymmetric black holes.

- Gave predictions for the Bekenstein-Hawking entropy from the superconformal index using the Bethe ansatz formula

- Successfully compared it to the near-horizon geometry of black holes in $\text{AdS}_5 \times T^{1,1}$

Black hole entropy from field theory

- In $\mathrm{AdS}_5/\mathrm{CFT}_4$, supersymmetric black hole $\leftrightarrow$ ensemble of BPS states with $\{J_{1,2}, Q_{a=1,\cdots n}\}$. We study the superconformal index

$$\mathcal{I}(p,q,y) = \mathrm{Tr}(-1)^F p^{J_1+\frac{r}{2}} q^{J_2+\frac{r}{2}} \prod_{a=1}^{n-1} y_a^{Q_a} \,.$$

Black hole entropy from field theory

- In AdS_5/CFT_4, supersymmetric black hole $\leftrightarrow$ ensemble of BPS states with $\{J_{1,2}, Q_{a=1,\cdots n}\}$. We study the superconformal index

$$\mathcal{I}(p,q,y) = \mathrm{Tr}(-1)^F p^{J_1+\frac{r}{2}} q^{J_2+\frac{r}{2}} \prod_{a=1}^{n-1} y_a^{Q_a} \ .$$

- Degeneracy of states is obtained via the Fourier transform

$$e^{S(J,Q)} = d(J,Q) = \int d\tau d\sigma \prod_{a=1}^{n-1} \left(d\Delta_a \ y_a^{-Q_a}\right) p^{-J_1} q^{-J_2} \mathcal{I}(p,q,y) \ ,$$

$$2\pi i\tau = \log p,\ 2\pi i\sigma = \log q,\ 2\pi i\Delta_a = \log y_a \ .$$

Black hole entropy from field theory

- In AdS_5/CFT_4, supersymmetric black hole $\leftrightarrow$ ensemble of BPS states with $\{J_{1,2}, Q_{a=1,\cdots n}\}$. We study the superconformal index

$$\mathcal{I}(p,q,y) = \mathrm{Tr}(-1)^F p^{J_1+\frac{r}{2}} q^{J_2+\frac{r}{2}} \prod_{a=1}^{n-1} y_a^{Q_a} \ .$$

- Degeneracy of states is obtained via the Fourier transform

$$e^{S(J,Q)} = d(J,Q) = \int d\tau d\sigma \prod_{a=1}^{n-1} \left(d\Delta_a \ y_a^{-Q_a}\right) p^{-J_1} q^{-J_2} \mathcal{I}(p,q,y) \ ,$$

$$2\pi i\tau = \log p,\ 2\pi i\sigma = \log q,\ 2\pi i\Delta_a = \log y_a \ .$$

- $S(J,Q)$ at $\mathcal{O}(N^2)$ is the integrand at the saddle point, i.e. the extremal value wrt. τ, σ, Δ_a of the entropy function

$$\mathcal{S} = \log \mathcal{I}(\tau,\sigma,\Delta) - 2\pi i \left(\sum_{a=1}^{n} \Delta_a Q_a + \tau J_1 + \sigma J_2 \right) \ .$$

Bethe ansatz formula

- For angular fugacities $p = e^{2\pi i a\omega}$, $q = e^{2\pi i b\omega}$, $\gcd\{a,b\} = 1$, the superconformal index can be expressed as [Benini and Milan, 2018]

$$\mathcal{I}(p,q,y) = \kappa \oint_{\mathbb{T}^{\mathrm{rk}(G)}} \mathcal{Z}(u;p,q,y)$$
$$= \kappa \sum_{\hat{u}\in\mathrm{BAEs}} \sum_{\{m_i\}=1}^{ab} \mathcal{Z}(\hat{u}+m\omega;p,q,y) H^{-1} \, .$$

Bethe ansatz formula

- For angular fugacities $p = e^{2\pi i a\omega}$, $q = e^{2\pi i b\omega}$, $\gcd\{a, b\} = 1$, the superconformal index can be expressed as [Benini and Milan, 2018]

$$\begin{aligned}\mathcal{I}(p, q, y) &= \kappa \oint_{\mathbb{T}^{\mathrm{rk}(G)}} \mathcal{Z}(u; p, q, y) \\ &= \kappa \sum_{\hat{u} \in \mathrm{BAEs}} \sum_{\{m_i\}=1}^{ab} \mathcal{Z}(\hat{u} + m\omega; p, q, y) H^{-1} \,.\end{aligned}$$

- The Bethe ansatz equations are defined on a torus with modulus ω and have the form

$$Q_i(u; \Delta, \omega) = 1, \quad i = 1, \cdots, \mathrm{rk}(G) \,.$$

Prediction for the entropy

- Conjecture that one term in the sum is dual to the contribution of BHs: $\hat{u}$ evenly distributed on the B-cycle, and $m_i = i \mod ab$.

Prediction for the entropy

- Conjecture that one term in the sum is dual to the contribution of BHs: $\hat{u}$ evenly distributed on the B-cycle, and $m_i = i \mod ab$.
- For this contribution, we computed $\log \mathcal{Z}$ at leading order for toric quivers and found

$$\log \mathcal{Z} = -i\pi N^2 \sum_{a,b,c=1}^{n} \frac{C_{abc}\Delta_a\Delta_b\Delta_c}{6\tau\sigma}$$

C_{abc} are 't Hooft anomaly coefficients.

Prediction for the entropy

- Conjecture that one term in the sum is dual to the contribution of BHs: $\hat{u}$ evenly distributed on the B-cycle, and $m_i = i \mod ab$.
- For this contribution, we computed $\log \mathcal{Z}$ at leading order for toric quivers and found

$$\log \mathcal{Z} = -i\pi N^2 \sum_{a,b,c=1}^{n} \frac{C_{abc}\Delta_a\Delta_b\Delta_c}{6\tau\sigma}$$

  C_{abc} are 't Hooft anomaly coefficients.
- Reproduces the entropy function for known black holes [Gutowski and Reall, 2004] in $AdS_5 \times S^5$ [Hosseini et al., 2017] but does this work in other cases?

Supergravity checks

For $SE_5 \neq S^5$:

- Supergravity duals contain hypermultiplets and full supersymmetric black hole solutions are not known.

Supergravity checks

For $SE_5 \neq S^5$:

- Supergravity duals contain hypermultiplets and full supersymmetric black hole solutions are not known.
- Simple check: truncating to minimal gauged supergravity, entropy of Gutowski-Reall black hole with single R-charge is reproduced by the entropy function.

Supergravity checks

For $SE_5 \neq S^5$:

- Supergravity duals contain hypermultiplets and full supersymmetric black hole solutions are not known.
- Simple check: truncating to minimal gauged supergravity, entropy of Gutowski-Reall black hole with single R-charge is reproduced by the entropy function.
- Otherwise, try to apply the strategy [Hosseini et al., 2017] .

The strategy

- Rotating charged black holes are topologically AdS_2 fibered over S^3.

The strategy

- Rotating charged black holes are topologically AdS_2 fibered over S^3.
- Reducing along the Hopf fiber of S^3, generically get rotating charged black hole in 4d with the same entropy.

The strategy

- Rotating charged black holes are topologically AdS_2 fibered over S^3.
- Reducing along the Hopf fiber of S^3, generically get rotating charged black hole in 4d with the same entropy.
- $J_1 - J_2$ is the 4d angular momentum $\implies$ black holes with $J_1 = J_2$ become spherically symmetric in 4d.

The strategy

- Rotating charged black holes are topologically AdS_2 fibered over S^3.
- Reducing along the Hopf fiber of S^3, generically get rotating charged black hole in 4d with the same entropy.
- $J_1 - J_2$ is the 4d angular momentum $\implies$ black holes with $J_1 = J_2$ become spherically symmetric in 4d.
- Restricting to this case, the entropy is determined by horizon data, via the attractor mechanism, which is also an extremization problem.

Dimensional reduction from 5d to 4d

Start with a generic 5d $\mathcal{N} = 2$ abelian gauged supergravity with n_V vector multiplets and n_H hypermultiplets

Dimensional reduction from 5d to 4d

Start with a generic 5d $\mathcal{N} = 2$ abelian gauged supergravity with n_V vector multiplets and n_H hypermultiplets

We reduce along S^1 that is the Hopf fiber of S^3.

Dimensional reduction from 5d to 4d

Start with a generic 5d $\mathcal{N} = 2$ abelian gauged supergravity with n_V vector multiplets and n_H hypermultiplets

We reduce along S^1 that is the Hopf fiber of S^3. Schematically,

- The metric gives an extra gauge field A^0.

Dimensional reduction from 5d to 4d

Start with a generic 5d $\mathcal{N} = 2$ abelian gauged supergravity with n_V vector multiplets and n_H hypermultiplets

We reduce along S^1 that is the Hopf fiber of S^3. Schematically,

- The metric gives an extra gauge field A^0.
- Vector scalars become complex, extra real scalar from component along S^1. Also include flat connections ξ^I for A^I along S^1.

Dimensional reduction from 5d to 4d

Start with a generic 5d $\mathcal{N} = 2$ abelian gauged supergravity with n_V vector multiplets and n_H hypermultiplets

We reduce along S^1 that is the Hopf fiber of S^3. Schematically,

- The metric gives an extra gauge field A^0.
- Vector scalars become complex, extra real scalar from component along S^1. Also include flat connections ξ^I for A^I along S^1.
- Hyperscalars do not vary along S^1.

Dimensional reduction from 5d to 4d

Start with a generic 5d $\mathcal{N} = 2$ abelian gauged supergravity with n_V vector multiplets and n_H hypermultiplets

We reduce along S^1 that is the Hopf fiber of S^3. Schematically,

- The metric gives an extra gauge field A^0.
- Vector scalars become complex, extra real scalar from component along S^1. Also include flat connections ξ^I for A^I along S^1.
- Hyperscalars do not vary along S^1.

End up with

A 4d $\mathcal{N} = 2$ abelian gauged supergravity with $n_V + 1$ vector multiplets and n_H hypermultiplets, determined by 5d data and ξ^I.

The conifold

- List of truncations of IIB on SE_5 is limited, the simplest example is a truncation on $T^{1,1}$ given in [Cassani and Faedo, 2011].

The conifold

- List of truncations of IIB on SE_5 is limited, the simplest example is a truncation on $T^{1,1}$ given in [Cassani and Faedo, 2011].
- Only $n_V = 2$ vector multiplets dual to the field theory R charge and baryonic charge were kept. $n_H = 2$.

The conifold

- List of truncations of IIB on SE_5 is limited, the simplest example is a truncation on $T^{1,1}$ given in [Cassani and Faedo, 2011].
- Only $n_V = 2$ vector multiplets dual to the field theory R charge and baryonic charge were kept. $n_H = 2$.
- Restricting to black holes carrying these two charges and performing the dimensional reduction, we found that indeed

$$\text{entropy function} \quad \equiv \quad \text{attractor mechanism}$$

The conifold

- List of truncations of IIB on SE_5 is limited, the simplest example is a truncation on $T^{1,1}$ given in [Cassani and Faedo, 2011].
- Only $n_V = 2$ vector multiplets dual to the field theory R charge and baryonic charge were kept. $n_H = 2$.
- Restricting to black holes carrying these two charges and performing the dimensional reduction, we found that indeed

$$\text{entropy function} \;\equiv\; \text{attractor mechanism}$$

- Charges were matched using the $\text{AdS}_5/\text{CFT}_4$ dictionary and then reduced.

Outlook

- Construction of rotating black holes with generic electric charges in truncations on SE_5.
- Is there an attractor mechanism in 5d? Or a rotating attractor mechanism in 4d?
- Can we compute subleading (in N) corrections to the Bekenstein-Hawking entropy?
- Can we go beyond extremality and SUSY?