Towards an M5-Brane Model

with bits and pieces of Higher Gauge Theory and Lagrange multipliers and supersymmetry

Dominik Rist
Joint work with Christian Saemann and Miro van der Worp.

(2012.?????)

Heriot-Watt University, Edinburgh

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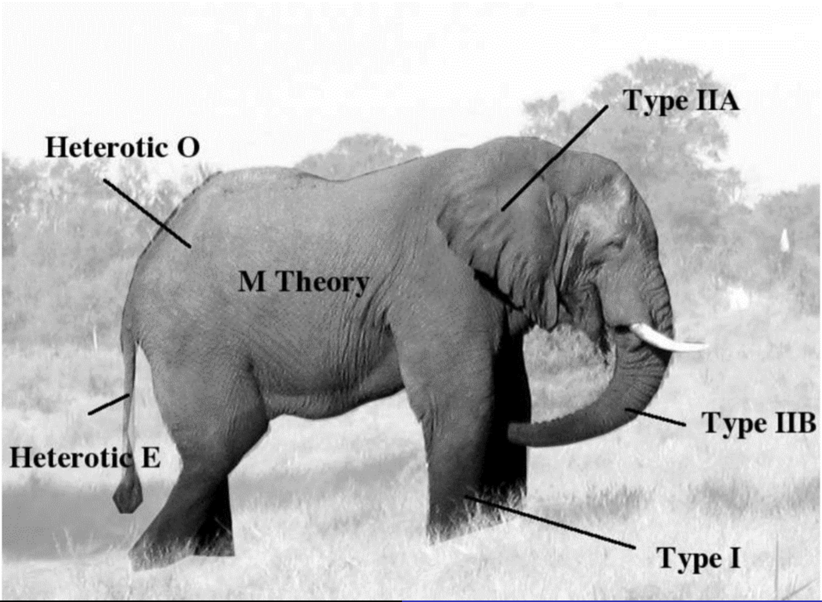
Outline

1. Motivation
2. Higher gauge theory
3. (1, 0) Lagrangian
4. Outlook

Motivation

What do we want to do?

- **Big goal:** Better understand M-theory


Type IIA
Heterotic O
M Theory
Type IIB
Heterotic E
Type I

Motivation

What do we want to do?

- **Big goal:** Better understand M-theory
- **Smaller goal:** understand (2,0)-Theory

(2, 0)-theory

- Representation theory and string theory predicts a mysterious superconformal field theory in six dimensions related to M-theory. (1995, Witten)

$(2, 0)$-theory

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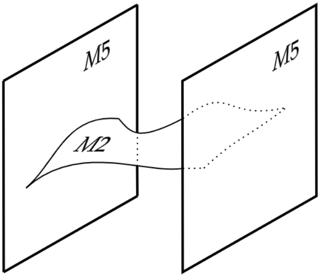
- Little is known.
- No Lagrangian exists – irreducibly quantum?

(2, 0)-theory



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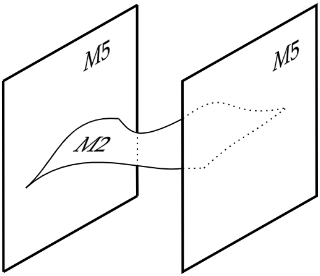

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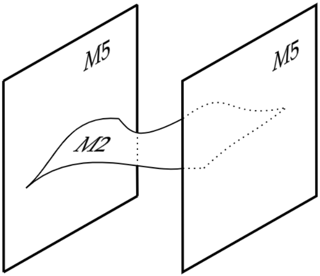

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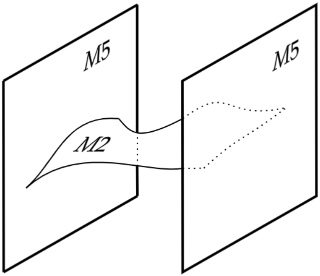

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Good news! There is such a framework – **higher gauge theory**.

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Yang-Mills theory specified by:

- principal G-bundle over M; Lie algebra $\mathfrak{g}$
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- Add SUSY: $S = -\frac{1}{2} \int_M (F, \star F) - \frac{1}{2} \int_M (\bar{\lambda}, \nabla\!\!\!/ \lambda)\mathrm{vol}$

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- elements $\in L_0$ parametrize generalized gauge transformations

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$$\mathcal{F} = F + \mu_1(B) = \mathrm{d}A + \tfrac{1}{2}\mu_2(A, A) + \mu_1(B)$$
$$H = \mathrm{d}B + \mu_2(A, B) - \tfrac{1}{3!}\mu_3(A, A, A) + \ldots$$
$$\vdots$$

Higher gauge theory: string structures

- **Problem:** consistency requires fake flatness
  $\mathcal{F} = \mathrm{d}A + \frac{1}{2}\mu_2(A, A) + \mu_1(B) = 0$
  trivial (only topological)
- **Solution:** modification = **string structures**
  - String Lie 2-algebra: $\mathfrak{string}(n) = \mathbb{R}[1] \to \mathfrak{g}(n)$

$$\mu_2(x_1, x_2) = [x_1, x_2] \ , \quad \mu_3(x_1, x_2, x_3) = (x_1, [x_2, x_3])$$

  - Curvatures:

$$\begin{aligned} \mathcal{F} &= F + \mu_1(B) = \mathrm{d}A + \tfrac{1}{2}\mu_2(A, A) + \mu_1(B) \ , \\ H &= \mathrm{d}B + \mu_2(A, B) - \tfrac{1}{3!}\mu_3(A, A, A) + (A, \mathcal{F}) \\ &= \mathrm{d}B + \mathrm{cs}(A) \end{aligned}$$

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    - **Task:** find Lagrangian - gauge invariant, (1,0) SUSY, produce the right EOMs
    - **Methods:** HGT, Lagrange multipliers for self-duality

(1, 0)-theory

The string structure underlying (1, 0)-theory is the following.

$$\left(\begin{array}{ccccccc} \mathfrak{g}^*_v & \xrightarrow{\mu_1 = \mathrm{id}} & \mathfrak{g}^*_u & & \mathbb{R}^*_s & \xrightarrow{\mu_1 = \mathrm{id}} & \mathbb{R}^*_p \\ & & \oplus & & \oplus & & \oplus \\ & & \mathbb{R}_q & \xrightarrow{\mu_1 = \mathrm{id}} & \mathbb{R}_r & & \mathfrak{g}_t \end{array}\right)$$

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We also have the maps

$$\mu_2 : \mathfrak{g}_t \wedge \mathfrak{g}_t \to \mathfrak{g}_t \ , \qquad \mu_2(t_1, t_2) = [t_1, t_2] \ ,$$

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$$\mu_3 : \mathfrak{g}_t \wedge \mathfrak{g}_t \wedge \mathfrak{g}_t \to \mathbb{R}_r \ , \qquad \mu_3(t_1, t_2, t_3) = (t_1, [t_2, t_3]) \ ,$$

$$\mu_3 : \mathfrak{g}_t \wedge \mathfrak{g}_t \wedge \mathbb{R}_s^* \to \mathfrak{g}_u^* \ , \qquad \mu_3(t_1, t_2, s) = s\big((-, [t_1, t_2]) \big) \ ,$$

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as well as

$$\nu_2 : \mathfrak{g}_t \otimes \mathfrak{g}_t \to \mathbb{R}_r \ , \qquad \nu_2(t_1, t_2) = (t_1, t_2) \ ,$$
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The local description of a connection on this string structure then consists of gauge potential forms

$$A \in \Omega^1(\mathbb{R}^{1,5}) \otimes (\mathfrak{g}_t \oplus \mathbb{R}_p^*) \ , \qquad B \in \Omega^2(\mathbb{R}^{1,5}) \otimes (\mathbb{R}_r \oplus \mathbb{R}_s^*) \ ,$$
$$C \in \Omega^3(\mathbb{R}^{1,5}) \otimes (\mathfrak{g}_u^* \oplus \mathbb{R}_q) \ , \qquad D \in \Omega^4(\mathbb{R}^{1,5}) \otimes \mathfrak{g}_v^*$$

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and the corresponding curvatures are defined as

$$F = \mathrm{d}A + \tfrac{1}{2}\mu_2(A,A) + \mu_1(B) \qquad \in \ \Omega^2 \otimes (\mathfrak{g}_t \oplus \mathbb{R}_p^*) \ ,$$
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G &= \mathrm{d}C + \mu_2(A,C) + \tfrac{1}{2}\mu_3(A,A,B) + \mu_1(D) && \in \ \Omega^4 \otimes (\mathfrak{g}^*_u \oplus \mathbb{R}_q) ,\\
I &= \mathrm{d}D + \mu_2(A,D) + \nu_2(F,C) + \tfrac{1}{2}\nu_3(A,A,H) && \\
&\quad + \nu_3(F,A,B) && \in \ \Omega^5 \otimes \mathfrak{g}^*_v .
\end{aligned}$$

String structures

The curvatures satisfy Bianchi identities

$$\mathrm{d}F + \mu_2(A, F) - \mu_1(H) = 0\ , \qquad \mathrm{d}H - \nu_2(F, F) + \mu_1(G) = 0\ ,$$
$$\mathrm{d}G + \mu_2(A, G) - \mu_1(I) = 0\ , \qquad \mathrm{d}I + \mu_2(A, I) - \nu_2(F, G) = 0\ .$$

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Finally, we also have the evident pairings

$$\langle -, - \rangle : \mathfrak{g}_u^* \times \mathfrak{g}_t \to \mathbb{R} \;, \quad \mathbb{R}_s^* \times \mathbb{R}_r \to \mathbb{R} \;, \quad \mathbb{R}_p^* \times \mathbb{R}_q \to \mathbb{R} \;.$$

Field content of (1,0)-theory

multiplet	symbol	field type	values in	SUSY transformation $\delta_{\mathrm{SUSY},0}$
vector	A	1-form	$\mathfrak{g}_t \oplus \mathbb{R}^*_p$	$-\bar{\varepsilon}\gamma_{(1)}\lambda$
	λ^i	MW spinors	$\mathfrak{g}_t \oplus \mathbb{R}^*_p$	$\frac{1}{4}\not{F}\varepsilon^i - \frac{1}{2}Y^{ij}\varepsilon_j + \frac{1}{4}\mu_1(\phi)\varepsilon^i$
	$Y^{(ij)}$	aux. scalars	$\mathfrak{g}_t \oplus \mathbb{R}^*_p$	$-\bar{\varepsilon}^{(i}\gamma^\mu\nabla_\mu\lambda^{j)} + 2\mu_1(\bar{\varepsilon}^{(i}\chi^{j)})$
tensor	B	2-form	$\mathbb{R}_r \oplus \mathbb{R}^*_s$	$-\bar{\varepsilon}\gamma_{(2)}\chi - \nu_2(\delta_{\mathrm{SUSY}}A, A)$
	χ^i	MW spinors	$\mathbb{R}_r \oplus \mathbb{R}^*_s$	$\frac{1}{8}\not{H}\varepsilon^i + \frac{1}{4}\not{\partial}\phi\varepsilon^i - \frac{1}{2}\nu_2(\gamma^\mu\lambda^i, \bar{\varepsilon}\gamma_\mu\lambda)$
	ϕ	scalar field	$\mathbb{R}_r \oplus \mathbb{R}^*_s$	$\bar{\varepsilon}\chi$
none	C	3-form	$\mathfrak{g}^*_u \oplus \mathbb{R}_q$	$\nu_2(\bar{\varepsilon}\gamma_{(3)}\lambda, \phi) - \nu_2(\delta_{\mathrm{SUSY}}A, B)$
	D	4-form	$\mathfrak{g}^*_v$	$-\nu_2(\delta_{\mathrm{SUSY}}A, C)$

Equations of motion

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Higher curvatures vanishing

$$\mathscr{G} := G - \nu_2(\star F, \phi) + 2\nu_2(\bar{\lambda}, \star\gamma_{(2)}\chi) = 0$$
$$\mathscr{I} := I + 2\nu_3(\bar{\lambda}, \star\gamma_{(1)}\lambda, \phi) = 0$$

(1,0) Lagrangian

$$\begin{aligned}\mathcal{L}^{(1,0)} = & -\langle \mathrm{d}\phi, \star\mathrm{d}\phi\rangle - 4\mathrm{vol}\langle\bar{\chi}, \not{\partial}\chi\rangle \\ & + \big\langle \phi\,,\, \nu_2(F, \star F) - 2\mathrm{vol}\;\nu_2(Y_{ij}, Y^{ij}) + 4\mathrm{vol}\;\nu_2(\bar{\lambda}, \not{\nabla}\lambda)\big\rangle \\ & + 4\big\langle\bar{\chi}, \nu_2(\not{F}, \lambda)\big\rangle - 8\mathrm{vol}\big\langle\bar{\chi}^j, \nu_2(Y_{ij}, \lambda^i)\big\rangle \\ & - H_s \wedge \star\mathscr{H}_r + H_s \wedge C_q\end{aligned}$$

- supplies other eoms

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- Use Sen's Lagrange multiplier approach

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- Find stationary points of $f(x)$ subject to the constraint $g(x) = 0$

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Consider a self-dual 3-form $\daleth_s = \star\daleth_s \in \Omega^3(\mathbb{R}^{1,5}) \otimes \mathbb{R}^*_s$ and form

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- Variation w.r.t. B_s gives $\mathscr{G}_q = 0$.

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Obtain $\mathscr{G}_u = 0$ by introducing an auxiliary 2-form $\daleth_t \in \Omega^2(\mathbb{R}^{1,5}) \otimes \mathfrak{g}_t$ and form

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- Produces $\mathscr{G}_u = 0$ and $\daleth_t = 0$.
- Extra terms required for SUSY.

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- $\delta_{\mathrm{SUSY},1}\, C_q = 2\nu_2(\delta A_t, \daleth_t)$,
- Also $\delta_{\mathrm{SUSY}} \beth_s = 0$, $\delta_{\mathrm{SUSY}} \daleth_t = 0$.
- SUSY algebra closes on-shell thanks to $\beth_s = 0$ and $\daleth_t = 0$.

The full Lagrangian

We present the fully supersymmetric, and non-trivially interacting higher gauge theory which produces $\mathscr{H} = \mathscr{G} = \mathscr{I} = 0$ as equations of motion.

$$
\begin{aligned}
\mathcal{L}^{(1,0)} = & -\langle \mathrm{d}\phi, \star \mathrm{d}\phi \rangle - 4\mathrm{vol}\langle \bar{\chi}, \not{\partial}\chi \rangle \\
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\end{aligned}
$$

Outlook

- Study this model further
- Show conformality at the quantum level
- Quantum field theory

The End

Thank you for your attention!