QCD Instantons at Colliders

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Outline

- Motivation
- QCD Instanton
- Cross section calculation
- Collider phenomenology

Motivation

- Prediction of the Standard Model.
- Better understanding of non-abelian gauge theories.
- Could be important in the early universe.
- Related to the vacuum structure of the Standard Model.
- Also important in BSM theories such as SUSY.

The QCD vacuum

Instantons are solutions of the classical equations of motion which tunnel between degenerate vacua.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - [A_\mu, A_\nu] = 0$$

This is satisfied when A_μ is a pure gauge.

$$A_\mu(x) = iU^\dagger \partial_\mu U \quad U \in SU(2)$$

At large x, the field strength tensor must vanish so this defines a map

$$f : S^3 \to SU(2)$$

Borrowing results from topology such maps are categorised by $\Pi_3\left(S^3\right) \sim \mathbb{Z}$ [Belavin et al., 1975].

The explicit form of the instanton

$$D_\mu \tilde{F}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} D_\mu F_{\rho\sigma}$$

$$= \frac{1}{6}\varepsilon_{\mu\nu\rho\sigma}\left(D_\mu F_{\rho\sigma} + D_\rho F_{\sigma\mu} + D_\sigma F_{\mu\rho}\right) \equiv 0.$$

Hence the explicit form of the instanton is [Belavin et al., 1975]

$$A^a_\mu = \frac{2}{g}\eta_{a\mu\nu}\frac{(x-x_0)_\nu}{(x-x_0)^2+\rho^2}$$

where

$$\eta_{a\mu\nu} = \begin{cases} \varepsilon_{a\mu\nu} & 1 \le \mu,\nu \le 3 \\ -\delta_{a\nu} & \mu = 4, 1 \le \nu \le 3 \\ \delta_{a\mu} & \nu = 4, 1 \le \mu \le 3 \\ 0 & \mu,\nu = 4 \end{cases}.$$

Expansion around the saddle point

Topological charge is given by

$$\int d^4x Tr\left(F_{\mu\nu}\tilde{F}_{\mu\nu}\right) = \frac{32\pi^2 Q}{g^2}$$

and simply performing this integral, the instanton has topological charge 1.

To calculate the cross section for instanton processes we expand around the instanton solution:

$$\int DA\, e^{-S} = e^{-S_0}\int DA\, e^{-A\Box A} = e^{-S_0}\frac{1}{\det^{\frac{1}{2}}(\Box)}$$

where $S_0 = \frac{2\pi}{\alpha}$ and $\Box = \frac{\delta^2 S}{\delta A^2}$. If $\Box$ has zero modes then

$$\frac{1}{\det^{\frac{1}{2}}(\Box)} \rightarrow \frac{d\tau}{\det'^{\frac{1}{2}}(\Box)}.$$

The Instanton process

The main instanton process is

$$g + g \to \sum_{i=1}^{N_f} \left(q_R^0 + \bar{q}_L^0 \right) + n_g \, g.$$

The number of fermions is determined by both the number of zero modes (coming from $\left(-i\not{D} - im\right)\psi = 0$) and the Adler-Bell-Jackiw anomaly.

$$\partial_\mu j^\mu = \frac{N_f g^2}{16\pi^2} Tr\left(F_{\mu\nu} \tilde{F}_{\mu\nu} \right) \implies$$

$$\Delta Q_A = 2 N_f \Delta Q_T$$

where

$$j^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$$

The master integral

The instanton cross section can be calculated through the use of the optical theorem

$$\sigma^{I}_{TOT} = \frac{1}{s'} \mathrm{Im}\mathcal{M}\,(p_1, p_2, -p_1, -p_2)$$

This is given by [Khoze et al., 2020]

$$\hat{\sigma}^{\rm inst}_{\rm tot} \simeq \frac{1}{E^2}\,\mathrm{Im}\,\frac{\kappa^2\pi^4}{36\cdot 4}\int\frac{d\rho}{\rho^5}\int\frac{d\bar{\rho}}{\bar{\rho}^5}\int d^4R\int d\Omega\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{14}$$

$$(\rho^2 E)^2(\bar{\rho}^2 E)^2\mathcal{K}_{\rm ferm}(z)(\rho\mu_r)^{b_0}(\bar{\rho}\mu_r)^{b_0}$$

$$\exp\left(R_0 E - \frac{4\pi}{\alpha_s(\mu_r)}\mathcal{S}(z) - \frac{\alpha_s(\mu_r)}{16\pi}(\rho^2+\bar{\rho}^2)\,E^2\,\log\frac{E^2}{\mu_r^2}\right)$$

LSZ procedure

$$A^a_\mu(p) = \frac{4i\pi^2\rho^2}{g}\frac{\eta^a_{\mu\nu}p_\nu}{p^2}e^{ip.x_0}$$

$$A^a_{LSZ}(p) = \lim_{p^2\to 0} p^2\epsilon^\mu(\lambda)A^a_\mu(p) = \epsilon^\mu(\lambda)\bar{\eta}^a_{\mu\nu}p_\nu\frac{4i\pi^2\rho^2}{g}e^{ip.x_0}$$

$$\sum_{spins,polarisations} A_\mu(p,\lambda)A_\mu(-p,\lambda) = \frac{1}{6}\left(\frac{2\pi^2}{g}\rho^2\sqrt{s}\right)^2 e^{iR.p}$$

The instanton–anti-instanton action

We want to solve

$$\epsilon^2(\tau)\, w(x,\tau)\, \frac{\partial A_\tau}{\partial \tau} = \left.\frac{\delta S}{\delta A}\right|_{A=A_\tau} .$$

Upon the ansatz:

$$A_\mu = \frac{-i}{g} \frac{\bar{\sigma}_\mu \sigma_\nu - \delta_{\mu\nu}}{(x-x_0)^2} (x-x_0)_\nu\, s\left((x-x_0)^2\right)$$

the action reduces to a quantum mechanical double-well

$$S_{YM} = \frac{48\pi^2}{g^2} \int dt \left(\left(\frac{ds}{dt}\right)^2 + \frac{1}{2}\left[\left(s-\frac{1}{2}\right)^2 - \frac{1}{4}\right]^2 \right) .$$

It turns out that the sum of an instanton and an anti-instanton is numerically close to a numerical solution over the whole range of separations.

After substituting back into the gauge theory we get [Yung, 1988, Khoze and Ringwald, 1991]

$$S = \frac{48\pi^2}{g^2}\left[\frac{6z^2-14}{\left(z-\frac{1}{z}\right)^2} - \frac{17}{3} - \ln z\left(\frac{\left(z-\frac{5}{z}\right)\left(z+\frac{1}{z}\right)^2}{\left(z-\frac{1}{z}\right)^3} - 1\right)\right]$$

where

$$z = \frac{R^2+\rho_1^2+\rho_2^2+\sqrt{\left(R^2+\rho_1^2+\rho_2^2\right)^2-4\rho_1\rho_2}}{2\rho_1\rho_2}$$

The instanton density

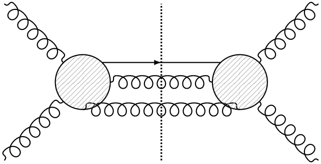
$$\frac{\partial g}{\partial \log \mu} = -b_0 \frac{g^3}{16\pi^2} \implies$$

$$\frac{8\pi^2}{g(\rho)^2} = \frac{8\pi^2}{g(\mu)^2} - b_0 \log(\mu\rho)$$

We also have the renormalisation constant ['t Hooft, 1976]

$$\kappa = \frac{2\, e^{5/6-1.511374 N_c}}{\pi^2 (N_c-1)!(N_c-2)!}\, e^{0.291746 N_f} \simeq 0.0025\, e^{0.291746 N_f},$$

Fermionic interactions



Assuming that the instanton and anti-instanton are well separated we have the fermionic overlap integral [Ringwald and Schrempp, 1998, Shuryak and Verbaarschot, 1992]:

$$\omega_{ferm} = \int d^4x \psi_0^{\bar{I}}(x)\, i\not{D} \psi_0^I(x) = \frac{3\pi}{8} \frac{1}{z^{3/2}} {}_2F_1\left(\frac{3}{2}, \frac{3}{2}; 4; 1 - \frac{1}{z^2}\right)$$

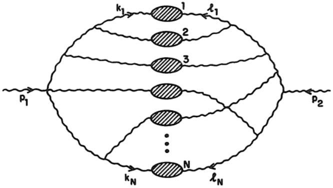
Then

$$\mathcal{K}_{ferm} = \omega_{ferm}^{2N_f}$$

Initial state interactions

The gluon propagator in the instanton background is [Mueller, 1991a]

$$G^{ab}_{\mu\nu}(p_1, p_2) = -\frac{g^2\rho^2}{64\pi^2} s \log(s) A^a_\mu(p_1) A^b_\nu(p_2)$$



Insert this propogator in all possible ways and we find it exponentiates [Mueller, 1991b]

$$e^{-\frac{\alpha\rho^2}{16\pi} s \log(s)} A^a_\mu(p_1) A^b_\nu(p_2)$$

Orientation integral and Dimensionless coordinates

The integral over the orientation is given by [Balitsky and Braun, 1993]

$$\int d\Omega\, e^{-\frac{4\pi}{\alpha_s(\mu_r)}\mathcal{S}(z,\Omega)} = \frac{1}{9\sqrt{\pi}}\left(\frac{3}{U_{\rm int}(z)}\right)^{7/2} e^{-\frac{4\pi}{\alpha_s(\mu_r)}\mathcal{S}(z)} =$$

$$\frac{1}{9\sqrt{\pi}}\left(\frac{3\,\alpha_s(\mu_r)}{4\pi(1-\mathcal{S}(z))}\right)^{7/2} e^{-\frac{4\pi}{\alpha_s(\mu_r)}\mathcal{S}(z)}$$

We define dimensionless integration variables

$$r_0 = R_0 E, \quad r = |\vec{R}|E$$

$$y = \rho\bar{\rho}E^2, \quad x = \frac{\rho}{\bar{\rho}}$$

Final integral

$$\hat{\sigma}^{\text{inst}}_{\text{tot}}(E) = \frac{1}{E^2}\,\mathrm{Im}\int_{-\infty}^{+\infty} dr_0\, e^{r_0}\, G(r_0, E)\,,$$

where

$$G(r_0, E) = \frac{\kappa^2\pi^4}{2^{17}}\sqrt{\frac{\pi}{3}}\int_0^\infty r^2\, dr \int_0^\infty \frac{dx}{x}\int_0^\infty \frac{dy}{y}\left(\frac{4\pi}{\alpha_s}\right)^{21/2}$$

$$\left(\frac{1}{1-\mathcal{S}(z)}\right)^{7/2}\mathcal{K}_{\text{ferm}}(z)\exp\left(-\frac{4\pi}{\alpha_s}\mathcal{S}(z) - \frac{\alpha_s}{4\pi}\frac{x+1/x}{4}\,y\log y\right)$$

where

$$\frac{4\pi}{\alpha_s}(y; E) = \frac{4\pi}{0.416} + 2b_0\log\frac{E}{1\text{GeV}} - b_0\log y\,.$$

Numerical results

$$\sigma_{pp\to I}(\hat{s} > \hat{s}_{\rm min}) = \int_{\hat{s}_{min}}^{s_{pp}} dx_1\, dx_2\, f(x_1, Q^2) f(x_2, Q^2) \hat{\sigma}(\hat{s} = x_1 x_2 s_{pp})$$

$\sqrt{\hat{s}}$	$\langle n_g \rangle$	$\hat{\sigma}^{\rm inst}_{\rm tot}$ [pb]
50	9.43	207.33×10^{3}
100	11.2	1.29×10^{3}
150	12.22	53.1
200	12.94	5.21
300	13.96	165.73×10^{-3}
400	14.68	13.65×10^{-3}
500	15.23	1.89×10^{-3}

$E_{\rm min}$	$\sigma_{pp\to I}$
50	58.19 μb
100	129.70 nb
150	2.769 nb
200	270.61 pb
300	3.04 pb
400	114.04 fb
500	8.293 fb

Shape variables

Instanton events produce many jets and so will not stand out from the QCD background. We want to exploit the lack of angular dependence to search for the instanton signal. We use shape variables; define the sphericity tensor:

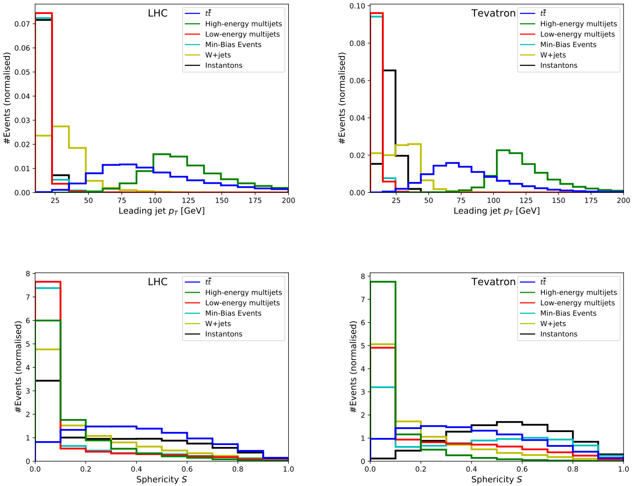
$$S^{\alpha\beta} = \frac{\sum_i p_i^\alpha p_i^\beta}{\sum_i |\mathbf{p}_i|^2}.$$

$$S = \frac{3}{2}(\lambda_2 + \lambda_3)$$

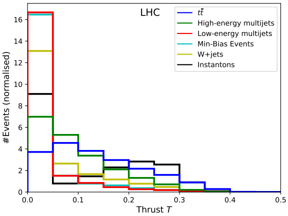
Thrust:

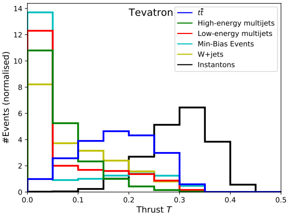
$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{p}_i \cdot \vec{n}|}{\sum_i |\vec{p}_i|},$$

Distributions



Distributions cont.





Recoil from a jet

- There are two possibilities, the jet originates from the initial state or from the final state.
- The emission from the final state is suppressed by the phase space; instanton events are a priori spherically symmetric.
- To have a jet from the initial state, one of the inital state gluons would have to have some virtuality.
- This introduces the factor $e^{-Q\rho}$ into our previous calculation

$\sqrt{\hat{s}}$ [GeV]	$\hat{\sigma}_{tot}^{inst}$ [pb]
310	3.42×10^{-23}
350	1.35×10^{-18}
375	1.06×10^{-17}
400	1.13×10^{-16}
450	9.23×10^{-16}
500	3.10×10^{-15}

$\sqrt{\hat{s}}$ [GeV]	$\hat{\sigma}_{tot}^{inst}$ [pb]
100	1.68×10^{-7}
150	1.20×10^{-9}
200	3.24×10^{-11}
300	1.84×10^{-13}
400	4.38×10^{-15}
500	2.38×10^{-16}

Tevatron

- We were not able to set any constraints on instanton production from the Tevatron
- However we found that triggers at the Tevatron would have recorded many instanton events
- Later selection criteria rendered the analyses insensitive to instantons.
- Mainly there was a trigger which required six jets each with $p_T > 15\ \mathrm{GeV}$
- Need to reanalyse old data with new selection criteria

The LHC

- Triggers in ATLAS are insensitive to instantons
- Lowest unprescaled single jet trigger requires $p_T > 360$ GeV, while lower p_T triggers, e.g. $p_T > 20$ GeV can be prescaled by 10^6.
- Multijet triggers are also not sensitive e.g. six jets each with $p_T > 45$ GeV.
- However there was a low luminosity run, collecting $335 pb^{-1}$ of data with a minimum-bias trigger.
- This data was only used for luminosity determination but if we require six jets each with $p_T > 10$ GeV and $\tau > 0.2$.
- Then we obtain $\frac{s}{\sqrt{b}} = 50.1$ CoM energies greater than 100 GeVand $\frac{s}{\sqrt{b}} = 7.1$ for energies greater than 200 GeV.

Summary

- Instanton processes are a good further test of the standard model.
- Instantons cannot be constrained by existing Tevatron analyses.
- However Tevatron triggers were sensitive to instantons so a reanalysing the existing data could prove fruitful.
- Current triggers are not sensitive to QCD instantons so are unlikely to be discovered in Run 1 or 2 LHC data.
- Special low luminosity runs of the LHC potentially recorded many instanton events which could be separated from background using shape variables.