The Holographic Swampland

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Based on: 2006.01021 [Joseph Conlon,FR]
+ other works in progress
(Also in collaboration with Joseph Conlon and Sirui Ning)

Introduction

Swampland program: criteria to distinguish low energy Lagrangians admitting a UV completion in ST (QG)

Topic of this talk: AdS/CFT

Swampland constraints on 4d EFTS ⟷ ? ⟷ CFT inconsistencies or reformulation

Why?

- Develop independent framework to test debated constructions
- AdS/CFT universal tool for QG, not just ST
- Bootstrap very successful in mapping space of allowed theories

Plan of the talk

Intro: Holographic CFTs

CFT positivity bounds and how to apply them

LVS dual as a motivating scenario

A conjecture on mixed anomalous dimensions

Other examples (E.g. KKLT)

Relation to distance conjecture

A new inequality on mixed anomalous dimensions

DBI "at the edge of the Swamp"

Conclusions and Outlook

Holographic CFTs

Gravity dual is <u>weakly coupled</u> and amenable to perturbative analysis

$$\lambda = g^2 N \longrightarrow (R/\ell_S)^d >> 1$$

Expansion in large N parametr or equivalent

Large gap in the spectrum $\Delta_{gap} >> 1$

Single trace primaries $\mathcal{O}_1, \mathcal{O}_2$ Dimension Δ_1, Δ_2

OPE: $\mathcal{O}_1 \times \mathcal{O}_2 \supset \mathbb{1}, \mathcal{O}_1, \ \mathcal{O}_2, \ [\mathcal{O}_1\mathcal{O}_2]_{n,l}$

Double trace operators: $[\mathcal{O}_1\mathcal{O}_2]_{n,l} \sim \mathcal{O}_1 \Box^n \partial_{\mu_1}\partial_{\mu_2}...\partial_{\mu_l}\mathcal{O}_2$

$\Delta = \Delta_1 + \Delta_2 + 2n + \ell + \gamma(n,l)$ ⟶ anomalous dimension

The bootstrap

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_1(x_2)\,\mathcal{O}_2(x_3)\,\mathcal{O}_2(x_4)\rangle = \sum_{\mathcal{O}} C_{11\mathcal{O}} C_{22\mathcal{O}} \frac{G_{\Delta,\ell}(u,v)}{|x_{12}|^{2\Delta_1}\,|x_{34}|^{2\Delta_2}}$$

conformal blocks

OPE coefficients

Bootstrap Equation:

$$u^{\Delta}\left(1+\sum_{\Delta,\ell} c^2_{\Delta,\ell} G_{\Delta,\ell}(v,u)\right) = v^{\Delta}\left(1+\sum_{\Delta,\ell} c^2_{\Delta,\ell} G_{\Delta,\ell}(u,v)\right)$$

No constraints from crossing symmetry alone

Solution to bootstrap equations

[Heemskerk,Penedones, Polchinski,Sully '05]

Quartic (derivative) vertices

CFT positivity bounds

Minimal twist operators dominate Lorentzian OPE on the light cone

$\tau = \Delta - \ell$

$$\frac{\tau^*_{\ell_3} - \tau^*_{\ell_1}}{\ell_3 - \ell_1} \leq \frac{\tau^*_{\ell_2} - \tau^*_{\ell_1}}{\ell_2 - \ell_1}$$

→ $\gamma(0,\ell)$ for identical operators

convex & negative for $\ell \geq \ell_c$

$$\ell_c \geq 2$$

Analytical bootstrap [Komargodski, Zhiboedov '12]

Inversion formula [Caron-Huot '17] + [Costa, Hansen, Penedones '17]

Causality arguments in the CFT:

$$\gamma(0,2) \leq 0$$

[Hartman, Jain, Kundu '16]

→

$$\mathcal{L} = \frac{g}{\Lambda^4}(\nabla\varphi)^4 \quad g > 0$$

on AdS

Generalization of flat space S-matrix bounds [Adams,Arkani-Hamed,Dubovsky, Nicolis,Rattazzi '06]

Plan of the talk

Intro: Holographic CFTs ✓

CFT positivity bounds and how to apply them ✓

LVS dual as a motivating scenario

↓

A conjecture on mixed anomalous dimensions

↓

Other examples (E.g. KKLT)

↓

Relation to distance conjecture

A new inequality on mixed anomalous dimensions

↓

DBI "at the edge of the Swamp"

↓

Conclusions and Outlook

The Large Volume Scenario

Motivating scenario: Low energy dynamics of moduli in ST

LVS: Type IIB flux compactification, with all moduli stabilised at an exponentially large volume

$$V = V_0 e^{-\lambda\Phi/M_P}\left(-\left(\frac{\Phi}{M_P}\right)^{3/2} + A\right) \qquad \Phi = \sqrt{\frac{2}{3}}\ln\mathcal{V}$$

canonically normalised volume modulus

AdS vacuum:

$$V_{\rm min} = -3M_P^2 R_{AdS}^{-2} \qquad m_\Phi = \sqrt{\frac{3}{2}}\frac{\lambda}{R_{AdS}}$$

[Balasubramanian,Berglund,Conlon,Quevedo ’05]

[Conlon,Quevedo,Suruliz ’05]

Holographic reformulation of LVS

AdS/CFT

4d moduli EFT in AdS

Crucial property of LVS:

$m_\phi << m_{3/2}$ as $\mathcal{V} \to \infty$

+

Putative 3d CFT dual

$$m^2 R^2_{AdS} = \Delta(\Delta - 3)$$

Small number of low lying operators:

Mode	Spin	Parity	Conformal dimension
$T_{\mu\nu}$	2	+	3
a	0	-	3
Φ	0	+	$8.038 = \frac{3}{2}\left(1+\sqrt{19}\right)$

[Conlon,Quevedo '18]

LVS Effective Lagrangian

All interactions fixed in term of R_{AdS} only-
uniquely determined theory in large $\mathcal{V}$ limit

$$\mathcal{L}_{(\delta\Phi)^n} = (-\lambda)^n \frac{3M_P^2}{R_{AdS}^2}\frac{n-1}{n!}\left(\frac{\delta\Phi}{M_P}\right)^n\left(1+\mathcal{O}\left(\frac{1}{\lambda\langle\Phi\rangle}\right)\right)$$

$$\mathcal{L}_{(\delta\Phi)^{n-2}aa} = \left(+\sqrt{\frac{8}{3}}\right)^{(n-2)}\frac{1}{2(n-2)!}\left(\frac{\delta\Phi}{M_P}\right)^{n-2}\partial_\mu a\partial^\mu a$$

Some modifications certainly in the Swampland

Sign flip in axion kinetic term → Divergent f_a as $\mathcal{V}\to\infty$

$$\mathcal{L}\supset\frac{3}{4}e^{+\sqrt{\frac{8}{3}}\frac{\Phi}{M_P}}\partial_\mu a\partial^\mu a$$

What happens to the CFT?

Can we apply the positivity bounds?

Contact diagrams:

Finite support in the spin

$$\ell \leq \frac{\#\partial}{2}$$

[Heemskerk,Penedones, Polchinski,Sully '05]

[Fitzpatrick,Katz,Poland, Simmons-Duffin '12]

VS

Exchange diagrams:

Arbitrary large spin

BUT

Manifestly negative

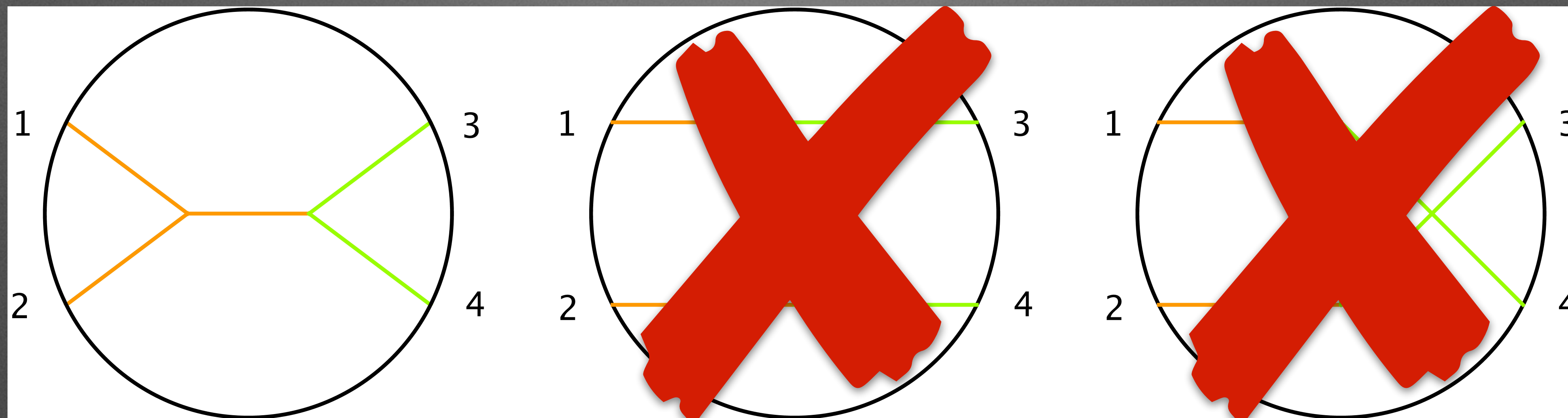
$$\gamma(0,\ell) \propto -\lambda^2$$

For identical operators, not quite...

Claim: $\gamma^{\varphi a}(0,\ell)$ correlates with swampland conditions

Mixed anomalous dimensions

$$\mathcal{L}_{LVS} = g\varphi^3 - \mu\varphi\partial_\mu a\partial^\mu a$$



$$\mathcal{A}(u,v) \longleftrightarrow M(s,t)$$

Mandelstam-like variables

$$\gamma(0,\ell) \underset{\ell>>1}{\simeq} -\int_{-i\infty}^{+i\infty} \frac{ds}{2\pi i} M(s,0)\,\Gamma\left(\frac{s}{2}\right)^2 \frac{\Gamma(\Delta_1)\Gamma(\Delta_2)}{\ell^s}$$

[Costa,Goncalves,Penedones '12]

Lowest lying pole in s-channel dominates the integral

$$M(s,t) \simeq \sum_{m=0}^{\infty} \frac{Q_{\ell,m}(t)}{s-\Delta+\ell-2m} + P_{\ell-1}(t) + s \leftrightarrow t + s \leftrightarrow u$$

Consequences for LVS

$$\gamma^{\varphi a}(0,\ell) \propto -\frac{g\mu(\Delta_\varphi - 6)}{\Gamma(\frac{6-\Delta_\varphi}{2})}\frac{1}{M_P^2 R_{AdS}^2}\frac{1}{\ell^{\Delta_\varphi}}$$

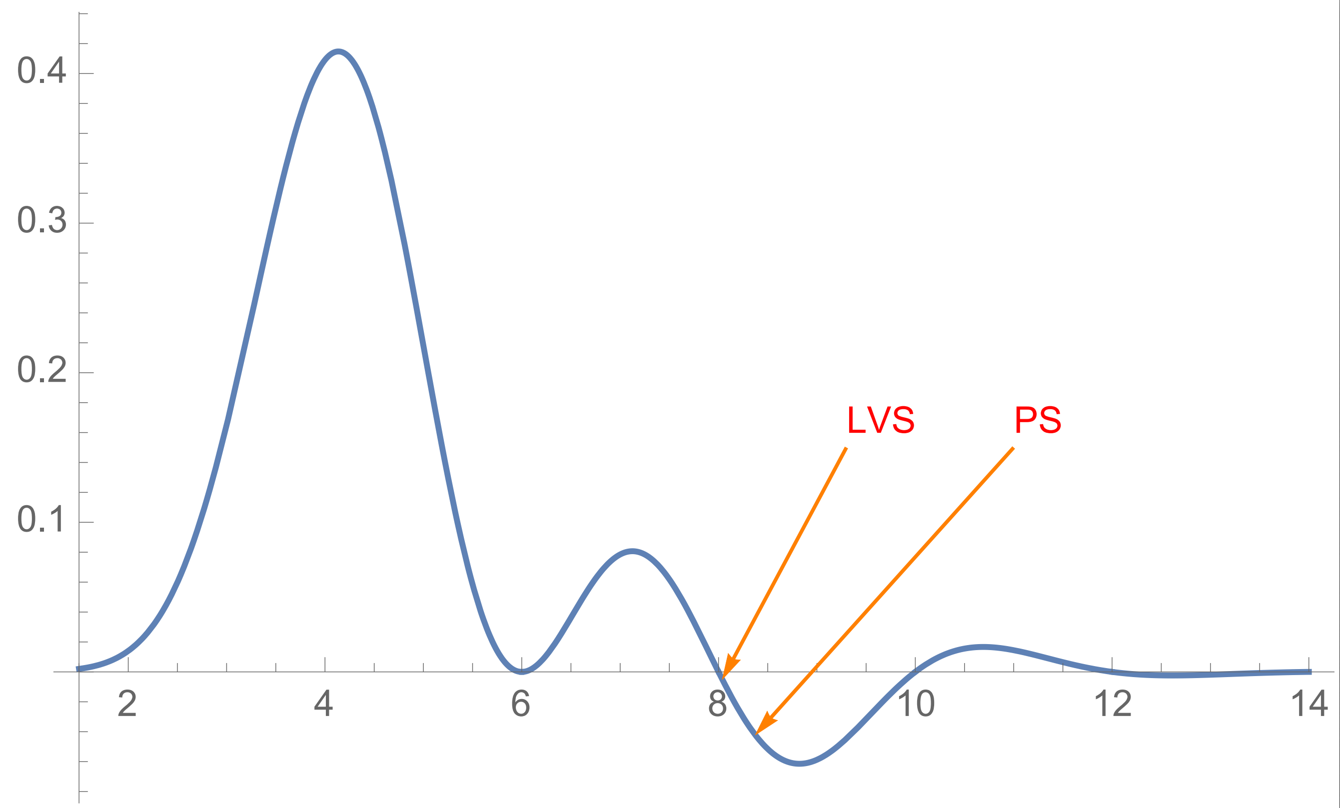
decouples higher order contributions at large volume

With LVS couplings, $\gamma^{\varphi a}(0,\ell) < 0$

Swampland

Sign flip in $g\mu$ ⟷ $f_a \sim M_P \mathcal{V}^{\frac{2}{3}}$

$\Delta_\varphi \leq 6$ ⟷ $V \gtrsim M_S^4$



LVS very close to boundary of the allowed region!

Other examples

Perturbative Stabilisation:
[Berg, Haack,Kors '06,
von Gersdorff, Hebecker '05]

$$V_{eff} = Ae^{-\lambda_1\varphi} - Be^{-\lambda_2\varphi}$$

LVS-like: Sign flip has same effect

$$\gamma(0,\ell)_{\varphi a} < 0 \text{ for } \lambda_1 = \frac{10}{3}\sqrt{\frac{3}{2}}, \lambda_2 = 3\sqrt{\frac{3}{2}}$$

KKLT (before uplifting): [Kachru,Kallosh,Linde,Trivedi '03]

$$\mathcal{K} = -3\log\left[-i(T-\bar{T})\right] \quad \mathcal{W} = W_0 + Ae^{-\alpha T}$$

Racetreack Stabilisation: [Krasnikov '87, Taylor '90,
De Carlos, Casas, Munoz '93]

$$\mathcal{K} = -3\log[-i(T-\bar{T})] \quad \mathcal{W} = Ae^{-\alpha T} - Be^{-\beta T}$$

For both $\gamma(0,\ell)_{\varphi a} < 0$ in their regime of validity

$$\sigma_c >> 1, \sigma_c\alpha >> 1, (\sigma_c\beta >> 1)$$

Qualitatively different from LVS

Potential for a → $\Delta_a > 3$

$$\mathcal{L} \supset \sigma^3, \sigma a^2, \sigma\partial_\mu a\partial^\mu a$$

Connection to the distance conjecture

Interactions with heavy modes:

$$\mathcal{L}_{\psi\bar{\psi}} = K_{\psi\bar{\psi}}\partial_\mu\psi\partial^\mu\bar{\psi} + \frac{1}{\mathcal{V}^2}K^{\psi\bar{\psi}}\psi\bar{\psi}$$

Since superpotential W does depend on T

E.g. KK mode: $K_{\psi\bar{\psi}} \sim \mathcal{V}^{-1/3}$

$$f_{\varphi\psi\psi} = \frac{\Gamma\left(\frac{2\Delta_\psi+\Delta_\varphi-3}{2}\right)}{2\Gamma\left(\Delta_\varphi\right)\Gamma\left(\Delta_\psi\right)^2}\left(\sqrt{\frac{1}{6}}\left(\Delta_\varphi+2\Delta_\psi-\Delta_\psi^2-3\right)+\frac{5}{\sqrt{6}}\Delta_\psi\left(\Delta_\psi-3\right)\right)$$

$$f_{\varphi\psi\psi} > 0 \equiv \frac{\partial m^2(\psi)}{\partial\varphi} < 0$$

$$f_{\varphi\psi\psi} < 0 \equiv \frac{\partial m^2(\psi)}{\partial\varphi} > 0$$

Negative $\gamma(0,\ell)$ requires heavy states to become light in LV limit

↓

Distance Conjecture

Caveats

LVS with a fibred direction $\mathcal{V} = \alpha\left(\tau_1\sqrt{\tau_2} - \tau_3^{3/2}\right)$

Flat direction - lifted by loop effects but still $m\,R_{AdS} \xrightarrow{\mathcal{V}\to+\infty} 0$

$$\varphi_1 = \sqrt{\frac{2}{3}}\log\left(\mathcal{V}\right) \qquad \varphi_2 = \frac{\sqrt{3}}{2}\log\left(\tau_1\mathcal{V}^{-2/3}\right)$$

$$\mathcal{L}_{kin} \supset \frac{1}{4}e^{-2\sqrt{\frac{2}{3}}(\varphi_1+2\varphi_2)}\partial_u a_1\partial^u a_1 + \frac{\alpha^2}{2}e^{\left(\sqrt{\frac{2}{3}}-2\sqrt{\frac{3}{2}}\right)\varphi_1+2\sqrt{\frac{2}{3}}\varphi_2}\partial_u a_2\partial^u a_2$$

$$\gamma^{a_i\varphi_1}(0,\ell) < 0 \qquad \gamma^{a_1a_2}(0,\ell) > 0$$

Positivity only for volume modulus?

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LVS dual as a motivating scenario

A conjecture on mixed anomalous dimensions

Other examples (E.g. KKLT)

Relation to distance conjecture

A new inequality on mixed anomalous dimensions

DBI "at the edge of the Swamp"

Conclusions and Outlook

CFT positivity bounds again

For even spins $\ell \geq 2$

$$\gamma_{12}(0,\ell) \geq \frac{1}{2}\Big[\gamma_{11}(0,\ell) + \gamma_{22}(0,\ell)\Big]$$

[Kundu '20]

Trivially satisfied at large spin, where it becomes equivalent to

$$\sum_{\mathcal{O}} \left(C_{11\mathcal{O}} - C_{22\mathcal{O}}\right)^2 \geq 0$$

Can however be used to constrain multi-derivative quartic vertices

$$\sum_{\ell=0}^{\infty} \gamma_{ij}(0,\ell)\left(f^{i,j}_{[ij]_{0,\ell}}\right)^2 Q_{\ell,0}\left(-s+\Delta_1-\Delta_3\right)\alpha\left(\Delta_i,\ell\right) = -4M_{ii\to jj}\left(s,\Delta_1+\Delta_3\right)$$

"Forward" Mellin amplitude $\sim s^k$ contributes up to spin k

The constraints

$$\mathcal{L} \supset \frac{c_{11}}{\Lambda^4}\left(\partial\varphi_1\right)^4 + \frac{c_{12}}{\Lambda^4}\left(\partial\varphi_1\right)^2\left(\partial\varphi_2\right)^2 + \frac{c_{22}}{\Lambda^4}\left(\partial\varphi_2\right)^4 + \frac{\tilde{c}_{12}}{\Lambda^4}\left(\partial\varphi_1\partial\varphi_2\right)^2$$

(Assuming no cubic terms)

$$c_{11} f_d\left(\Delta_1, \Delta_1\right) - \left(c_{12} + \frac{\tilde{c}_{12}}{2}\right) f_d\left(\Delta_1, \Delta_2\right) + c_{22} f_d\left(\Delta_2, \Delta_2\right) \geq 0$$

$$f_d(x, y) = \frac{\Gamma(x+2)\Gamma(y+2)\Gamma\left(x+y+2-\frac{d}{2}\right)}{\Gamma\left(x+\frac{d}{2}-1\right)\Gamma\left(y+\frac{d}{2}-1\right)\Gamma(x+y+4)}$$

Correct flat spacetime limit for $m(R)R \to 0$

Application: multi-field DBI

Probe D3-brane moving along warped throat

$$ds^2 = f^{-1/2}\left(y^K\right) g_{\mu\nu} dx^\mu dx^\nu + f^{1/2}\left(y^K\right) G_{IJ}\left(y^K\right) dy^I dy^J$$

Compact coordinates give rise to scalars in 4d -
inherently stringy and "universal" EFT

Many applications, E.g. DBI Inflation
[Silverstein, Tong '04]
[Alishahiha, Silverstein, Tong '04]

Full multifield action:

[Langlois, Renaux-Petel, Steer, Tanaka '08]

$$S = -\int \mathrm{d}^4x \sqrt{-g} \frac{1}{f\left(\varphi^I\right)} \left(\sqrt{\mathcal{D}_S} - 1\right)$$

$$\mathcal{D}_S = 1 - 2 f G_{IJ} X^{IJ} + 4 f^2 X^{[I}_I X^{J]}_J + \mathcal{O}\left(X^3\right) \qquad X^{IJ} = -\frac{1}{2}\partial_\mu \varphi^I \partial^\mu \varphi^J$$

DBI at the "edge of the Swamp"

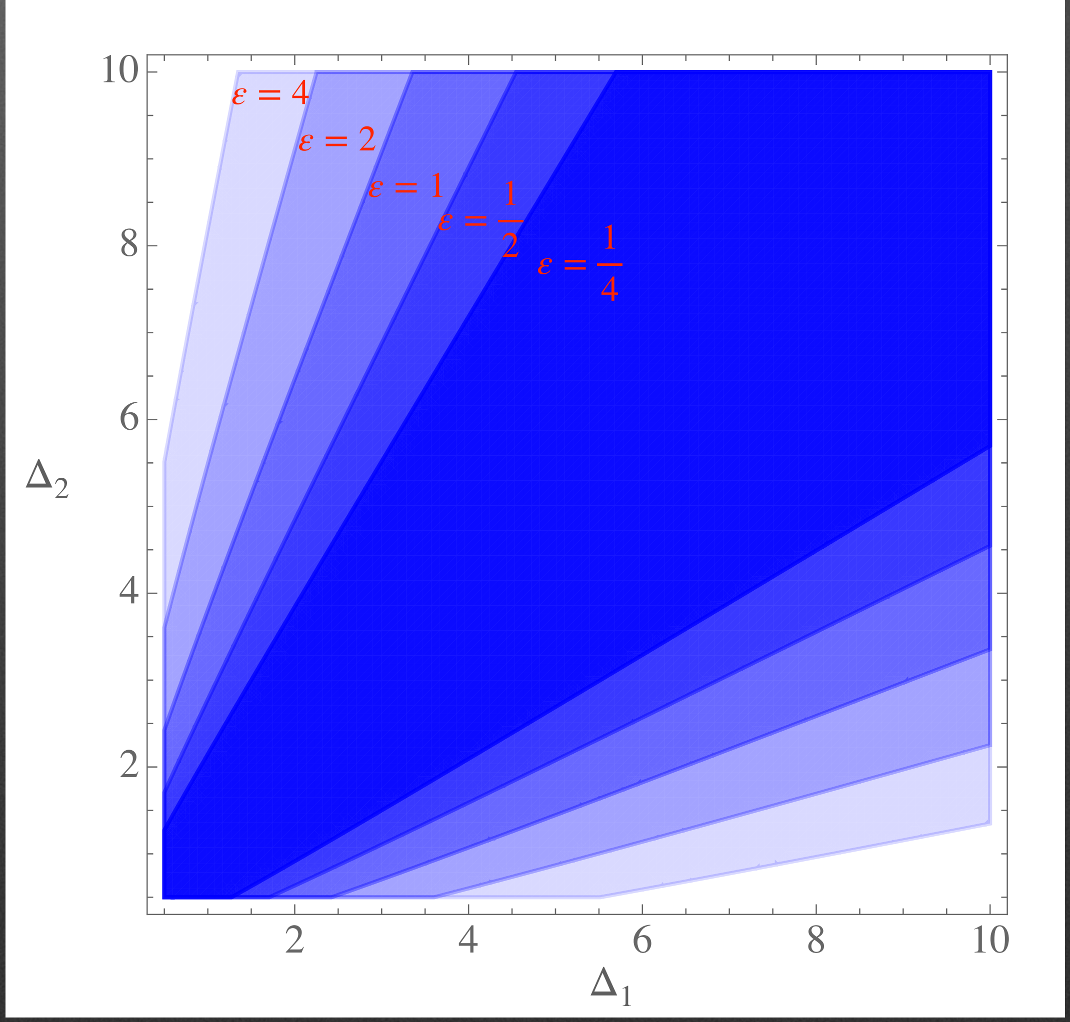
$$\mathcal{L} \supset \frac{1}{2f(\varphi^I)} \sum_I (\partial\varphi_I)^4 + \frac{1}{f(\varphi^I)} \sum_{I<J} 4(\partial\varphi_I \partial\varphi_J)^2 - 3(\partial\varphi_I)^2(\partial\varphi_J)^2$$

$c_{11} = c_{22} = 1$ $\qquad \tilde{c_{12}} = 8$ $\qquad c_{12} = -6$

Bounds satisfied for any value of Δ_1, Δ_2

Not true if we add small perturbation

$$c_{12} + \frac{\tilde{c_{12}}}{2} = -2 - \varepsilon$$



Boundary theories special! (E.g. Ising)

Conclusions and outlook

CFT techniques promising tool to address AdS stringy EFTs

Not proven, but very similar to known positivity bounds

- Sign of $\gamma^{\text{mix}}(0,\ell)$ correlates with Swampland conditions
- Applications: LVS, KKLT, Racetrack, Perturbative stabilisation
- Connection to the distance conjecture
- Just a feature of volume modulus?

Work in progress on more examples: Fibred models, Type IIA…

- New CFT inequality on mixed & identical anomalous dimensions gives non-trivial constraints on higher derivative theories
- Immediate application: DBI at the edge of the Swamp

Applications to Cosmology?

Long term goal: invert the relationship to "navigate" the Swampland

Thank you for your attention!

Back-up slides

The Large Volume Scenario

"Big" and "small" Kähler moduli T_b, T_s

$$\mathcal{V} = \frac{1}{\kappa}\left(\tau_b^{3/2} - \tau_s^{3/2}\right)$$

$$K = -2\ln\left(\frac{1}{\kappa}\left(\left(\frac{T_b + \bar{T}_b}{2}\right)^{3/2} - \left(\frac{T_s + \bar{T}_s}{2}\right)^{3/2}\right) + \frac{\xi}{g_s^{3/2}}\right)$$

$$W = W_0 + A_s e^{-a_s T_s}$$

α'^3 correction

Minimum for an exponentially large volume $\langle \mathcal{V} \rangle \sim e^{a_s \langle \tau_s \rangle}$

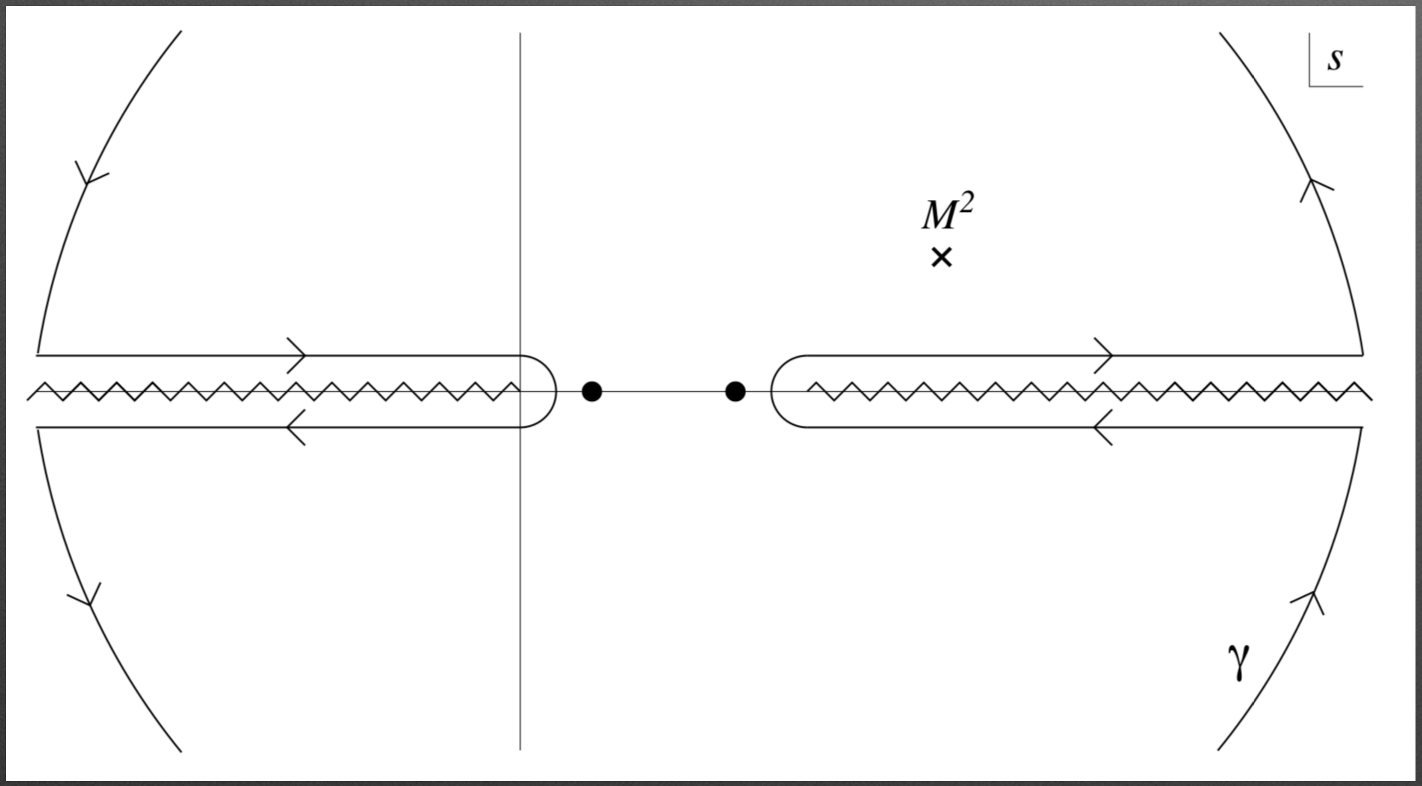
$$V_{eff} = \frac{1}{\mathcal{V}^3}\left(-A(\ln \mathcal{V})^{3/2} + \frac{B}{g_s^{3/2}}\right) \qquad \langle \tau_s \rangle \sim \frac{\zeta^{2/3}}{g_s}$$

Intermezzo: S-Matrix positivity

unitarity + analiticity + weak coupling expansion

$$\mathcal{A}(s)_{tree} = g(c_2 s^2 + c_4 s^4 \ldots)$$

$$g\, c_i > 0$$



E.g.

$$\mathcal{L} = \frac{g}{\Lambda^4}(\partial\varphi)^4 \qquad g > 0$$

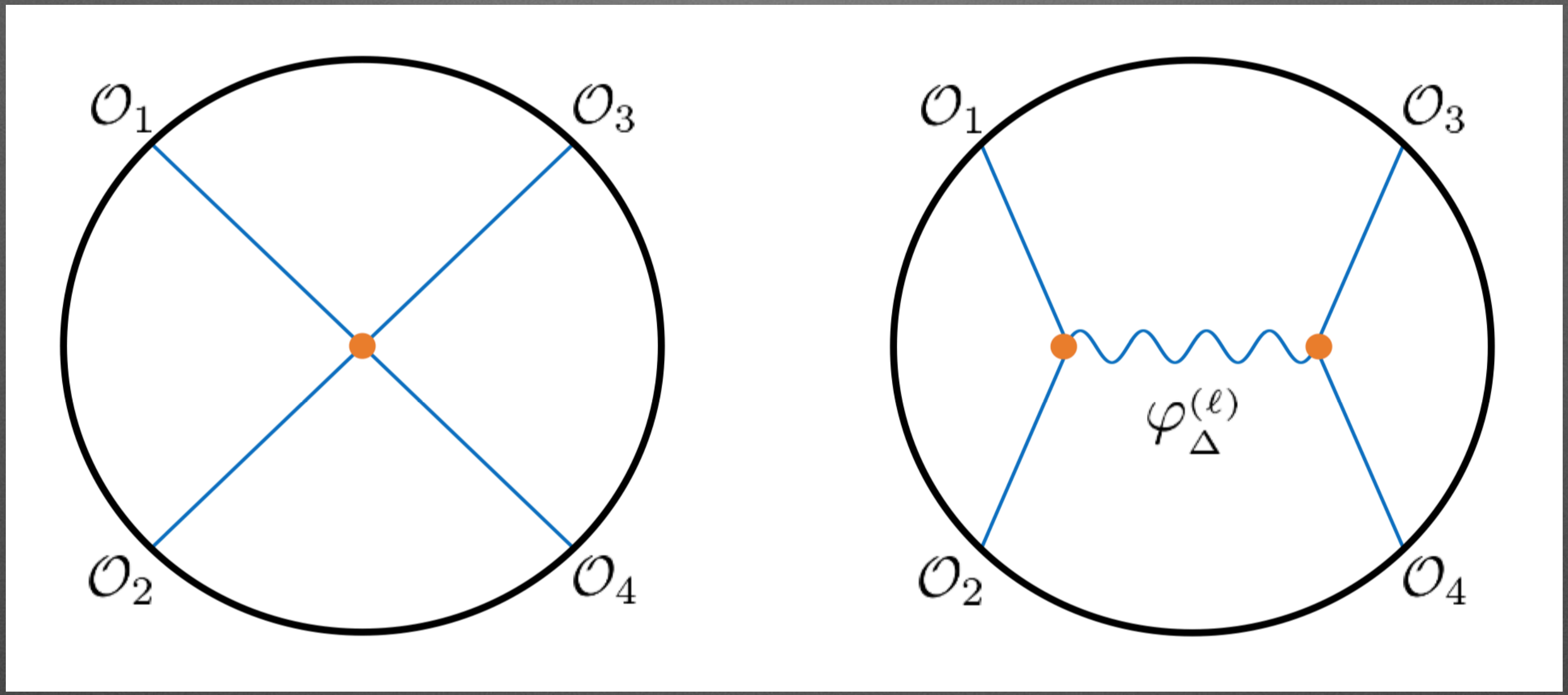
" QFT Swampland"

CAVEAT: $A_{grav}(s,t) \supset \frac{1}{M_{PL}^2}\frac{s^2}{t}$ → $\Lambda << M_{PL}$

[Adams,Arkani-Hamed,Dubovsky,Nicolis,Rattazzi '06]

How do we compute ?

Correlation functions → Witten Diagrams in AdS



Other techniques:

- Bootstrapping correlators (good for exchanges)
  [Alday,Bissi,Perlmutter '17 + others]
- Eigenvalues of dilatation operator : $\gamma(n,\ell) = \langle n,\ell|\Delta H|n,\ell\rangle$
  [Fitzpatrick,Katz,Poland,Simmons-Duffin '12]
- Mellin amplitude formula for $\gamma(0,\ell)$
  [Costa,Goncalves,Penedones '12]

Perturbative stabilisation

$$V_{eff} = Ae^{-\lambda_1 \varphi} - Be^{-\lambda_2 \varphi} \qquad \varphi_{min} = \frac{1}{\lambda_1 - \lambda_2} \log\left(\frac{A\lambda_1}{B\lambda_2}\right)$$

string loops $\sim \mathcal{V}^{-\frac{10}{3}}$

α'^3 correction $\sim \mathcal{V}^{-3}$

typical example

$$\Delta_\varphi = \frac{3}{2}\left(1 \pm \sqrt{1 + \frac{4}{3}\lambda_1\lambda_2}\right) \qquad \Delta_a = 3$$

$$\gamma^{\varphi a}(0,\ell) \propto -\lambda_1\lambda_2(\lambda_1+\lambda_2)\frac{g\mu(\Delta_\varphi - 6)}{\Gamma\left(\frac{6-\Delta_\varphi}{2}\right)}\frac{1}{M_P^2 R_{AdS}^2}$$

$\lambda_1\lambda_2 \geq \frac{40}{3}$?

De Sitter conjecture → $\lambda_i > C$

[Obied, Ooguri, Spodyneiko, Vafa '18]

KKLT

Well known example of dS vacuum - we consider it before uplifting

$$\mathcal{K} = -3\log(-i(T-\bar{T})) \qquad \mathcal{W} = W_0 + Ae^{-\alpha T}$$

[Kachru,Kallosh, Linde,Trivedi '03]

SUSY vacuum

$$\frac{W_0}{A} = -e^{-a\sigma_c}(1+\frac{2}{3}a\sigma_c)$$

$\sigma_c a >> 1$

$\sigma >> 1$

Potential for a ⟶ $\Delta_a > 3$

$\mathcal{L} \supset \sigma^3, \sigma a^2, \sigma\partial_\mu a \partial^\mu a$

} qualitatively different

Negative anomalous dimensions in its regime of validity

$$\gamma^{\sigma a}(0,\ell) \;\; \propto -\Big[a\sigma_c(a\sigma_c+2)(2a\sigma_c+3)$$
$$+2\Delta_\varphi(a\sigma_c) - 2\Delta_a^2(a\sigma_c) + 4\Delta_a(a\sigma_c) - 6\Big]$$

Racetrack

$$\mathcal{K} = -3\log -i(T-\bar{T}) \qquad \mathcal{W} = Ae^{-aT} - Be^{-bT}$$

[Krasnikov '87, Taylor '90, De Carlos, Casas, Munoz '93]

Potential for a → $\Delta_a > 3$

$\mathcal{L} \supset \sigma^3, \sigma a^2, \sigma\partial_\mu a \partial^\mu a$

} similar to KKLT

Negative anomalous dimensions for $\sigma_c \gtrsim \frac{1}{a}, \frac{1}{b}$

$\Delta_a < \Delta_\varphi < 2\Delta_a$ → no need to decouple higher order contributions

Mellin Amplitudes (1)

Convenient representation of correlators

$$A\left(x_i\right) \supset \langle \mathcal{O}_1\left(x_1\right) \ldots \ldots \mathcal{O}_n\left(x_4\right)\rangle_c = \prod_{1\leq i<j\leq n} \int_{-i\infty}^{+i\infty} \frac{\mathrm{d}\delta_{ij}}{2\pi i} M(\delta_{ij})\Gamma\left(\delta_{ij}\right)\left(x_{ij}^2\right)^{-\delta_{ij}}$$

$$e^{-s_i s_j P_{ij}} = \int_{c-i\infty}^{c+i\infty} \frac{d\gamma_{ij}}{2\pi i}\Gamma\left(\gamma_{ij}\right)\left(s_i s_j P_{ij}\right)^{-\gamma_{ij}}$$

→ arises naturally due to Euler star formula

with $\delta_{ii} = -\Delta_i \quad \sum_i \delta_{ij} = 0$

→ n(n-3)/2 independent variables - analogue to n-particle scattering

Can introduce fictions moments s.t.

$$p_i \cdot p_j = \delta_{ij} \quad \sum_{i=1}^{n} p_i = 0 \qquad s_{ij} = -(p_i + p_j)^2 = \Delta_i + \Delta_j - 2\delta_{ij}$$

Mellin Amplitudes (2)

$$\mathcal{A}(u,v) = \int_{-i\infty}^{+i\infty} \frac{dtds}{(4\pi i)^2} M(s,t) u^{s/2} v^{-(s+t)/2} \Gamma\left(\frac{\Delta_1+\Delta_2-s}{2}\right)\Gamma\left(\frac{\Delta_3+\Delta_4-s}{2}\right) \Gamma\left(\frac{\Delta_{34}-t}{2}\right)\Gamma\left(\frac{-\Delta_{12}-t}{2}\right)\Gamma\left(\frac{t+s}{2}\right)\Gamma\left(\frac{t+s+\Delta_{12}-\Delta_{34}}{2}\right)$$

Contact interactions:

$$M(s,t) = \frac{g\pi^{\frac{d}{2}}}{2}\Gamma\Big(\frac{\sum \Delta_i - d}{2}\Big)\prod_{i=1}^{n}\frac{1}{\Gamma(\Delta_i)}$$

scalar vertex

$$M(s,t) = \frac{g\pi^{\frac{d}{2}}}{2}\Gamma\Big(\frac{\sum \Delta_i - d}{2}\Big)\prod_{i=1}^{n}\frac{1}{\Gamma(\Delta_i+\beta_i)}\prod_{i<j}^{n}(-2\delta_{ij})^{\alpha_{ij}}$$

derivative vertex

Closest analogue to scattering amplitudes in AdS!

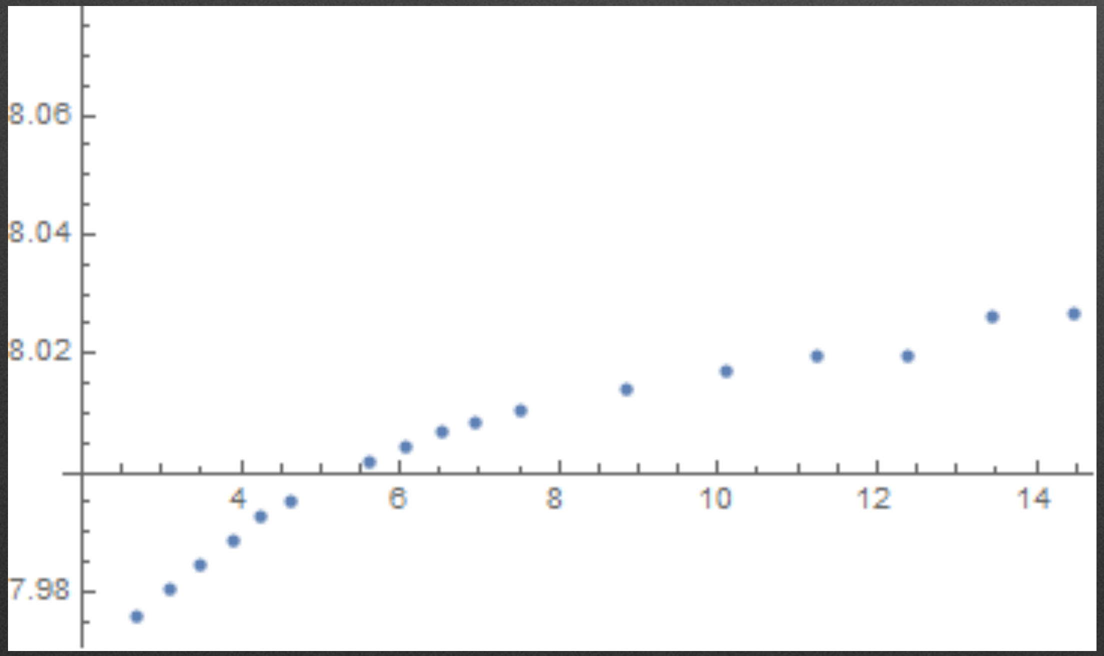
Finite Volume

At small volumes, Δ_φ can in principle drop below 8

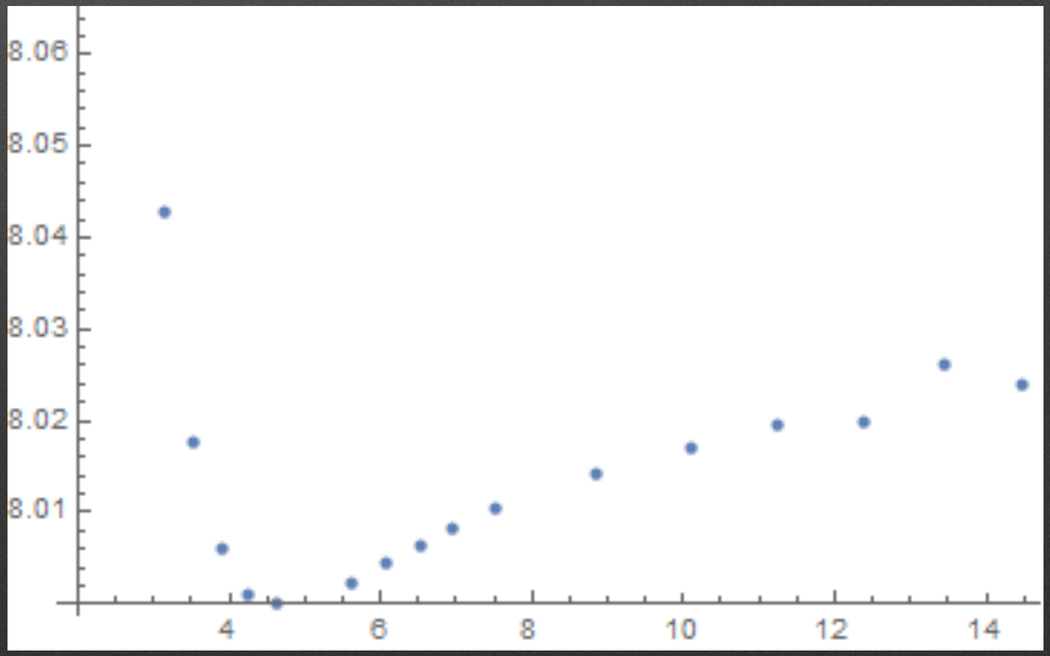
$$\Delta = \frac{3(1+\sqrt{19})}{2}\left(1 - \sqrt{\frac{2}{27}}\frac{1}{\langle\Phi\rangle} + \mathcal{O}\left(\frac{1}{\langle\Phi\rangle}\right)^2\right)$$

However, subheading volume corrections become important

$$K = -2\ln(\mathcal{V}+\xi) \to K = -2\ln\left(\mathcal{V}+\xi+\frac{\xi^2}{\mathcal{V}}\right)$$



Pure LVS



With corrections

Flat spacetime limit

Flat space bounds

$$\begin{cases} \alpha^2 c_{11} + \alpha\beta\left(c_{12} + \frac{\tilde{c}_{12}}{2}\right) + \beta^2 c_{22} \geq 0 \qquad \forall \quad \alpha, \beta \\ \tilde{c}_{12} \geq 0 \\ \Downarrow \\ -2\sqrt{c_{11}c_{22}} - \tilde{c}_{12} \leq c_{12} \leq 2\sqrt{c_{11}c_{22}} \end{cases}$$

$$\gamma(0,2)_{R\to+\infty} \sim \frac{1}{R^{3d/2}}\left[m^{4-3d/2} + \frac{m^{3-3d/2}}{R} + \frac{m^{2-3d/2}}{R^2} + \dots\right]$$

Only for $m(R)R \to 0$ as $R \to +\infty$ the AdS bounds reduce to flat space

AdS Distance Conjecture? [Lust,Palti,Vafa '19]