Boostless bootstrap: amplitudes without Lorentz boosts

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Enrico Pajer, David Stefanyszyn, J. S.

Bootstrap in Minkowski: an overview

1. Constructing gauge invariant Lagrangians that are consistent with particle content, unitarity and locality is a nontrivial task.

$$F \coloneqq d\Psi, \quad S \overset{?}{\sim} \int F^{ab\cdots} F_{ab\ldots} + \cdots \quad \Rightarrow \quad \delta S_{diff} \neq 0 \text{ if spin} > 2$$

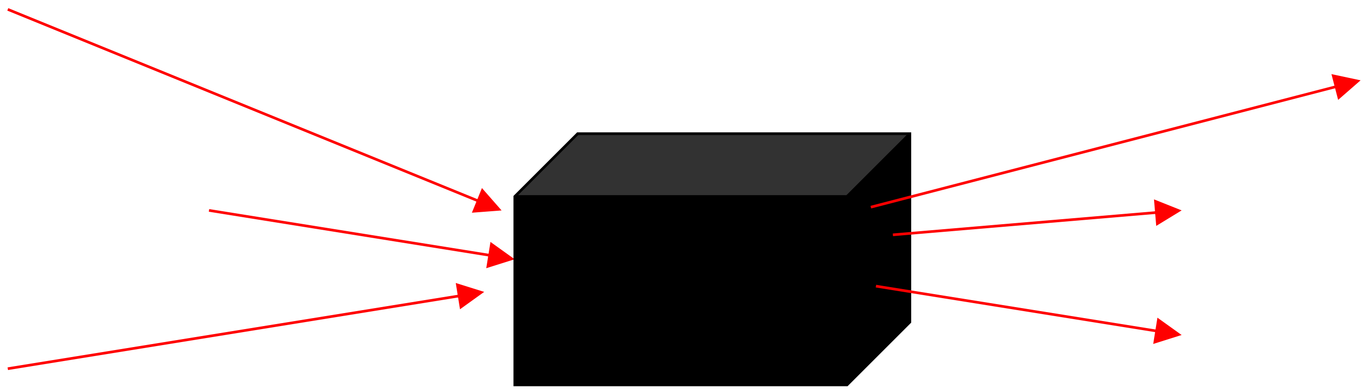
2. Computing amplitudes from Lagrangians is also hard: gg -> gggg (6-gluon) scattering (Parke & Taylor 1985) takes 100 pages of calculations at tree level. In fact, the result for helicities −−++++ is:

$$g^3 \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle}$$

Amplitudes bootstrap: it is easier to construct amplitudes directly by means of on-shell methods and check if they are consistent with locality.

Bootstrap rules

Rule 1. (On-shell principle) Amplitudes can only depend on coupling constants and on-shell quantities: momenta and helicities. (Throughout, we assume all particles to be massless.)



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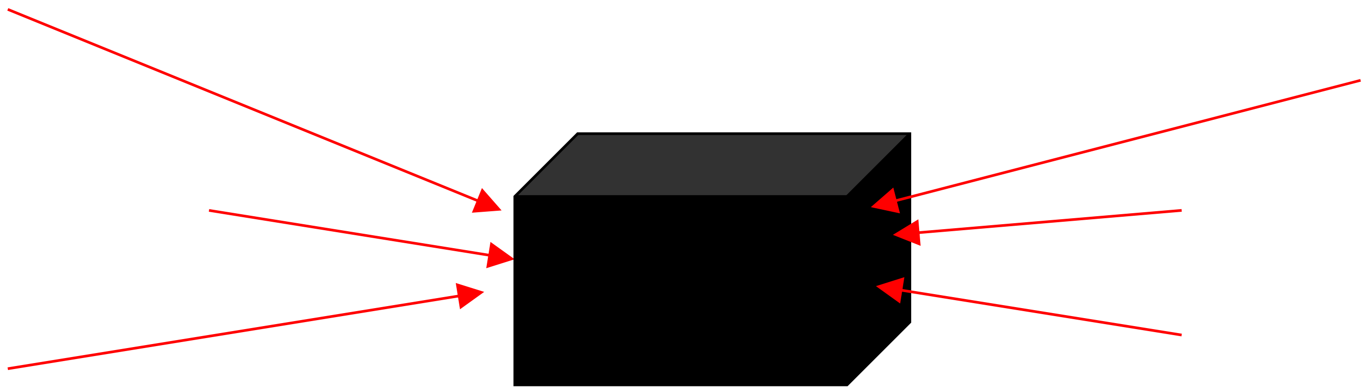


Without loss of generality, we can take all particles to be incoming (crossing symmetry).

Spinor-helicity formalism

We can choose a convenient parametrization of the kinematics, defining:

$$p^\mu = \sigma^\mu_{\alpha\dot{\alpha}} \lambda^\alpha \tilde{\lambda}^{\dot{\alpha}}$$

$$\langle ij \rangle = \lambda_1^{(i)} \lambda_2^{(j)} - \lambda_2^{(i)} \lambda_1^{(j)}$$

$$[ij] = \tilde{\lambda}_1^{(i)} \tilde{\lambda}_2^{(j)} - \tilde{\lambda}_2^{(i)} \tilde{\lambda}_1^{(j)}$$

Useful identities for 4 particles with conserved energy and momentum:

$$\langle 12 \rangle [12] = 2\, p_1 . p_2$$

$$\langle 12 \rangle [24] = -\langle 13 \rangle [34]$$

$$A_h\left(1^{h_1} 2^{h_2} \cdots\right) = \varepsilon_1^{\alpha_1 \alpha_2 \dots} \varepsilon_2^{\beta_1 \cdots} \cdots A_{\alpha_1 \alpha_2 \cdots \beta_1 \cdots}$$

Bootstrap rules

Rule 2. (Symmetry) Amplitudes must be consistent with the underlying symmetries.

We can have contractions $p_{1,\mu}p_2^\mu$ or $p_{1,\mu}\varepsilon_2^\mu$, but nothing like $a_i p_{1,i}$, because of space(time) rotation symmetry.

Bose/Fermi statistics when identical particles are present.

The amplitude must also have the correct scaling under helicity transformations:

$$(\lambda_i, \tilde{\lambda}_i) \to (t\lambda_i, t^{-1}\tilde{\lambda}_i):\ \ A \to t^{-2h_i}A$$

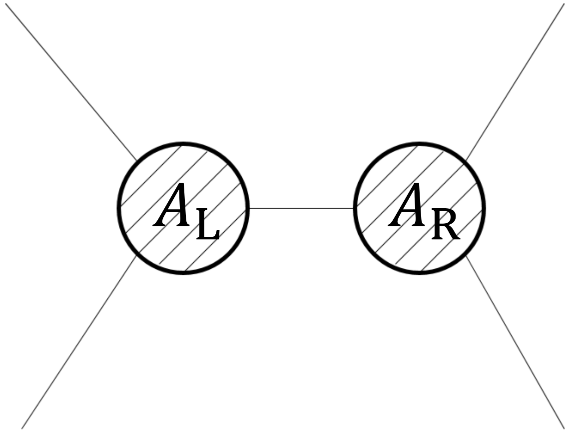
Bootstrap rules

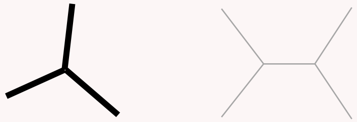
Rule 3. (Unitarity + locality)

Factorization Theorem (4p):

- **The only possible poles are simple poles in the Mandelstam variables s,t,u.**
- **Each residue must factorize into a product of 3p amplitudes.**

$$A_4 \sim \frac{1}{p_I^2} A_{\mathrm{L}}\, A_{\mathrm{R}}$$





Three-particle amplitudes (Lorentz invariant)

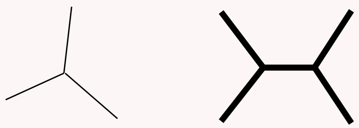
On-shell principle and symmetry imply that Lorentz invariant 3p amplitudes for particles of helicity h_1, h_2, h_3 take the following form:

$$A_3 = k\,\langle 12\rangle^{h_3-h_1-h_2}\,\langle 23\rangle^{h_1-h_2-h_3}\,\langle 31\rangle^{h_2-h_3-h_1} \text{ if } \sum h_i < 0$$

$$A_3 = k'[12]^{h_1+h_2-h_3}\,[23]^{h_2+h_3-h_1}\,[31]^{h_3+h_1-h_2} \text{ if } \sum h_i > 0$$

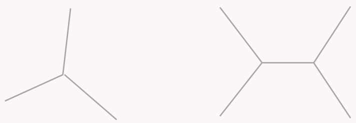
Introducing the amplitudes bootstrap

Lorentz invariant bootstrap



Why study boost-violating theories?

Boostless bootstrap (new results)



Future steps

Lorentz-invariant bootstrap: 4p test

Rules 1 and 2 entail that four-particle amplitude should be a function of the following variables only:

$$\langle ij\rangle, [ij], s, t, u$$

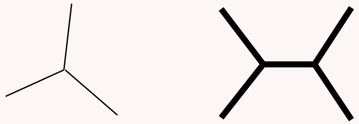
For example, the 4p amplitude for elastic scatterring of 2 opposite-helicity gravitons (+−+−) is:

4 square half-brackets for each (+2)-helicity particle

$$A(+,-,+,-) = g^2 \frac{[13]^4\langle 24\rangle^4}{stu}$$

4 angle half-brackets for each (-2)-helicity particle

Mandelstam variables



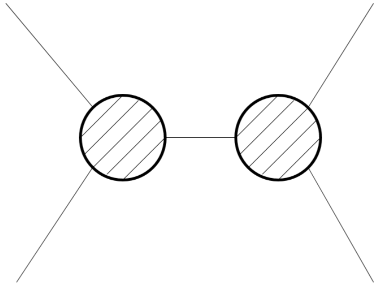
Lorentz-invariant bootstrap: 4p test

Factorization Theorem (tree-level):

- There are no singularities other than simple poles in the Mandelstam variables s,t,u.
- Each residue must factorize into a product of 3p amplitudes.

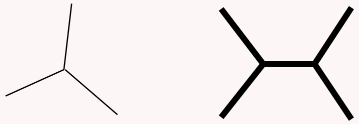
For 4 scalars:

$$A(0,0,0,0) = k^2\left(\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right)$$



Can we do it for spin 4? No!

$$A(+,-,+,-) = k^2\frac{[13]^8\langle 24\rangle^8}{s^a t^b u^c}, \qquad a+b+c=5.$$



Equivalence principle

I. Minimal coupling of any particle with spin S ≤ 2 to gravitons must be equal to the graviton GR self-coupling (or must be zero).

II. Fundamental particles with spin S > 2 cannot minimally couple to gravity.

For example, for scalars,

$$A(1,2,3^{+2}) = g\,\frac{[23]^2[31]^2}{[12]^2}$$

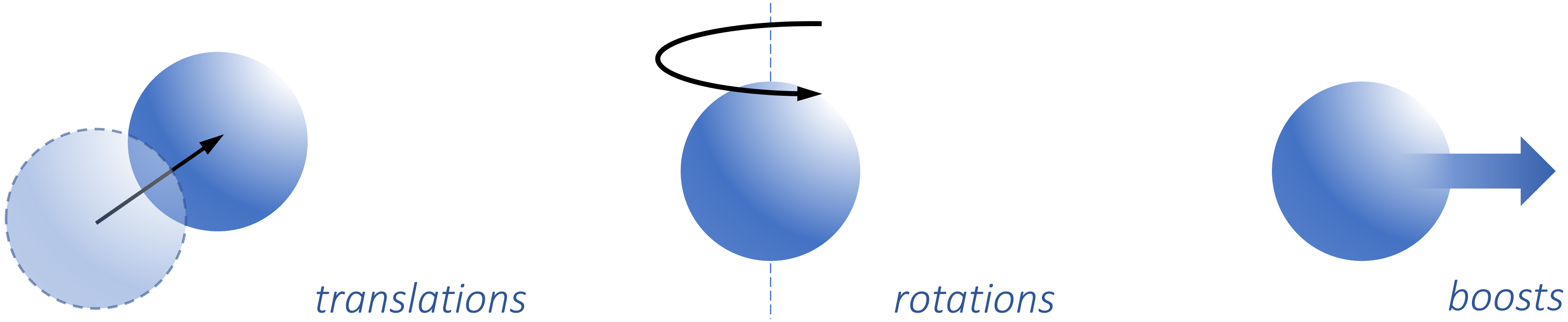
$$A(1^{+2},2^{+2},3^{-2}) = g\,\frac{[12]^6}{[23]^2[31]^2}$$

(same $g\sim\frac{1}{M_{pl}}$)

Lorentz symmetry

All confirmed theories with Poincaré-invariant vacuum are invariant under the Poincaré group:

$$x^{\mu} \rightarrow \Lambda^{\mu}{}_{\nu} x^{\nu} + a^{\mu}, \text{ for } \Lambda_{\mu}{}^{\sigma} \Lambda^{\mu}{}_{\nu} = \Lambda^{\sigma}{}_{\mu} \Lambda_{\nu}{}^{\mu} = \delta^{\sigma}{}_{\nu}$$

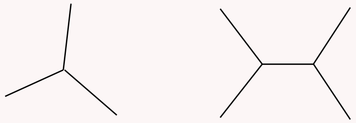


But Lorentz violations should be taken into account in several contexts:

- EFTs of condensed matter
- Lorentz-violating extensions of the Standard Model (Kostelecky 2002)

Introducing the amplitudes bootstrap

Lorentz invariant bootstrap



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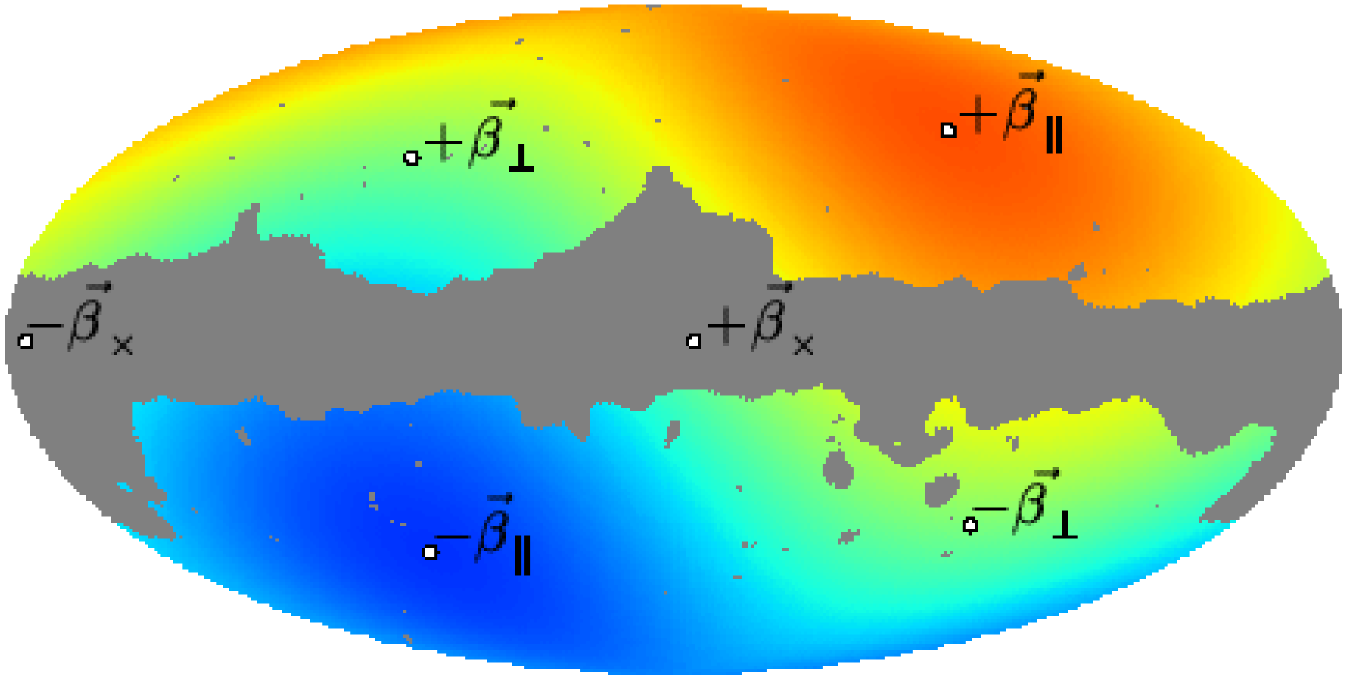
Boostless bootstrap (new results)

Future steps

Cosmology and the breaking of boosts

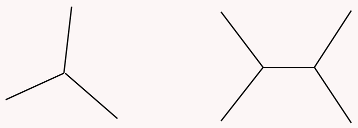


Planck 2013 results. XXVII. Doppler boosting of the CMB: Eppur si muove

Example of the breaking of boosts: there is only one reference frame in which the CMB looks isotropic.

Introducing the amplitudes bootstrap

Lorentz invariant bootstrap



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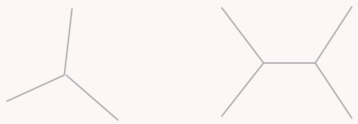
Boostless bootstrap (new results)



Future steps

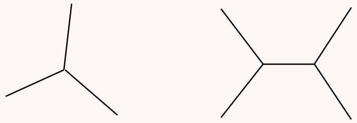
The breaking of time translations is suppressed

1. Time translations are approximately restored during inflation, as indicated by the scale invariance of the power spectrum:

$$n_s - 1 \approx -0.04$$

2. We can make the breaking of time translations arbitrarily small by going to high energies, while keeping the breaking of boosts large.

Approximate symmetry	Time translations	Boosts
Order of symmetry breaking effects	$\sim (E t_b)^{-1}$	$\sim v/c$

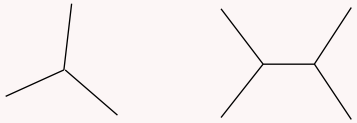


Our goal and assumptions

Goal: find all consistent, 3p scattering amplitudes in Minkowski allowing for violations of boost invariance, but keeping all other symmetries intact.

Our (well-motivated) assumptions:

- Analyticity of the S-matrix
- Unitarity and locality
- (3+1)-dim, flat space
- Weak coupling (we work with tree-level 4p amplitudes)
- Linearly realized rotations and spacetime translations (but not boosts)
- <u>Luminal propagation</u>: $E^2 - p^2 = 0$

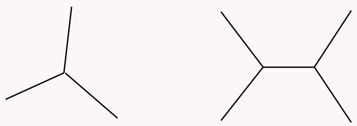


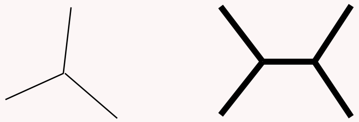
Three-particle amplitudes (boost-violating)

We found that the modification of 3p amplitudes is simple - they are now allowed to depend on energies too:

$$A_3 = A_3^{LI} \times F(E_1, E_2, E_3)$$

But are these amplitudes consistent with Rule 3 (factorization theorem)?



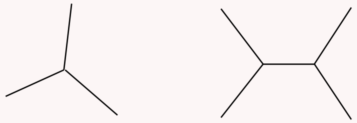


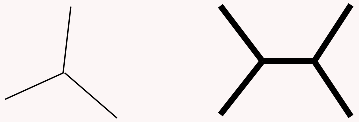
Overview of new results: spin 1

We assume the on-shell condition is $E^2 - p^2 = 0$.

LI – Lorentz-invariant only
BV – boost violations permitted

	Boost-invariant	Non-boost invariant
Spin 1, ++-/+--	✗	✗
Spin 1, +++/---	✗	✓ BV
Multiple spin 1, ++-/+--	✓ Yang-Mills structure	✓ LI Yang-Mills structure
Couplings of spin S to spin 1 (+1, +S, -S)	✓ S < 1, ✗ S > 1	✓ BV S < 1, ✗ S > 1





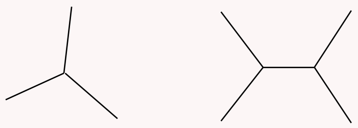
Overview of new results: spin 2

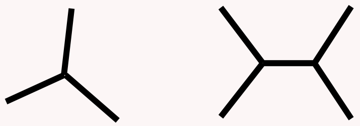
We assume the on-shell condition is $E^2 - p^2 = 0$.

LI – Lorentz-invariant only
BV – boost violations permitted

	Boost-invariant	Non-boost invariant
Spin 2, ++-/+-- and +++/---	✓	✓ **LI**
Couplings of spin S to spin 2 (+2, +S, -S)	✓ S < 2, ✗ S > 2	✓ **LI** S < 2, ✗ S > 2

AND: any particle minimally coupled to gravity must have Lorentz-invariant self-interactions!





The highlight of our analysis

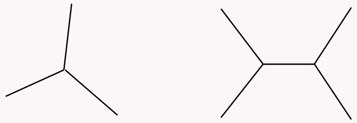
Our finding: <u>in flat space</u>, all boost-breaking interactions at cubic level (and probably for higher orders as well) are prohibited, provided that:

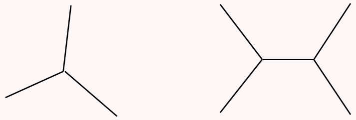
- the field has a minimal coupling to gravity,
- the speeds of the scalar and the graviton are the same (but this assumption is probably not necessary – results on that will be published soon)

Including $\dot{\phi}^3$ and $\dot{\phi}(\partial_i\phi)^2$

inconsistent

(These are the simplest couplings in the EFT of inflation, but that's not flat space.)





Summary & words of caution

- We drop the assumption of boost-invariance to make the bootstrap program more general (formally applicable to condensed matter physics and flat-space backgrounds where boosts are broken).
- We retain the Lorentz-invariant propagator: $E^2 – p^2 = 0$.

- <u>The constraints are sensitive to IR modifications</u>.
  - Previous and new results rely on infrared behaviour of 4p amplitudes, so they are known not to work in e.g. AdS, no matter how large the radius is. Seemingly „bad" theories can be saved by introducing an IR cutoff or an arbitrarily small mass.
  - Thus, the applicability to cosmology (as well as realistic condensed matter systems, which are finite in size) is still limited.
  - However, it is possible to apply similar methods in the context of cosmological correlators (*in progress*).