Charting the Fifth Force Landscape

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Based on 2009.12399 with Matthew McCullough

Outline

1. Dispersing a Fifth Force
2. The Experimental Landscape
3. Journeying Beyond QFT?

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3. Journeying Beyond QFT?

Motivation

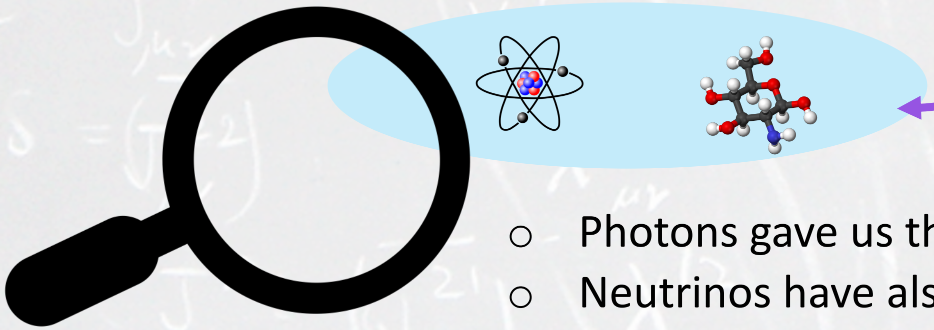
- Overwhelming evidence for Dark Matter:

e.g. CMB | Structure Formation | Galactic Rotation Curves | Bullet Cluster

- Accounts for ~ 85 % of matter in the Universe

But still no clues at the particle level…..

- The visible sector is staggeringly rich and complex!



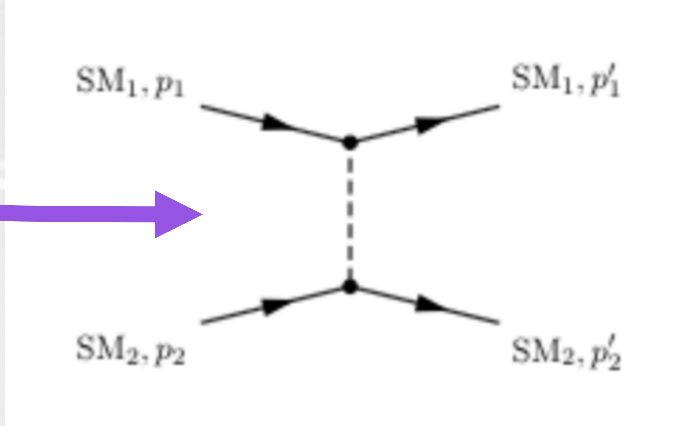
  - Photons gave us the first tools to explore it
  - Neutrinos have also had much to tell us…

- Could light neutral states from hidden sectors similarly open a window to the Dark Universe?

Motivation

- Exchange of new states between ordinary nucleons generates a "fifth" force

  e.g. The Yukawa Potential from tree level scalar exchange



- Host of experiments designed to search for new weakly coupled states

- Forces are "long-range" and therefore one cannot use EFT/decoupling to generalise effects on low-energy observables

  ⇒Experimental direction comes from considering "toy models" (mainly Yukawa)

Can we find a general framework which would allow us to move beyond case-by-case searches?

The Framework

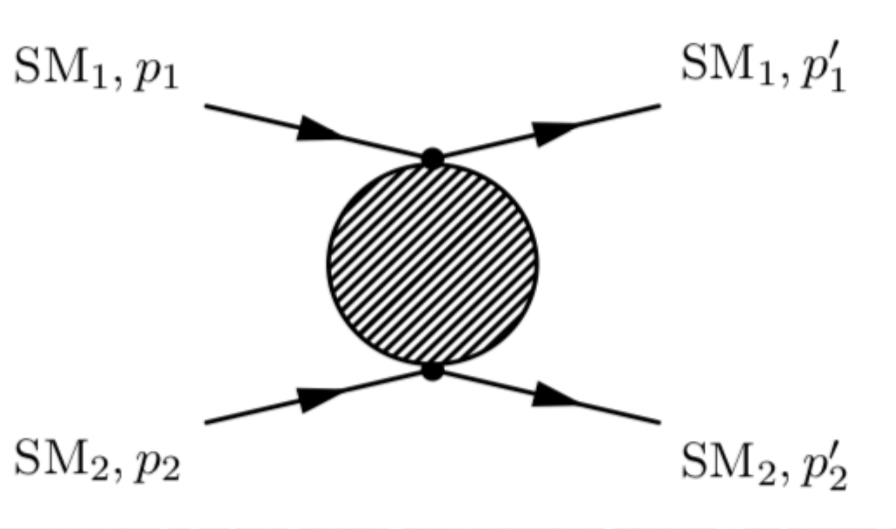
- Consider SM states of mass *M* coupled weakly to a scalar composite operator $\mathcal{O}_{DS}$:

$$\boxed{\mathcal{L}_{\text{int}} = \lambda \bar{\Psi}_{SM} \Psi_{SM} \mathcal{O}_{DS}}$$

- At $\mathcal{O}(\lambda^2)$ this generates a potential

$$\boxed{V(\boldsymbol{r}) = -\frac{1}{4M^2} \int d^3\boldsymbol{q}\, \frac{\mathcal{M}^{\text{NR}}}{(2\pi)^3} e^{i\boldsymbol{q}\cdot\boldsymbol{r}} \quad ,}$$

(Born Approx.)



where

$$\boxed{i\mathcal{M}^{\text{NR}} = -4i\lambda^2 M^2 \langle \mathcal{O}_{DS}(x) \mathcal{O}_{DS}(y) \rangle^{\text{NR}}}$$

- Nature of the force fully determined by the two-point function

$$\langle \mathcal{O}_{DS}(x)\mathcal{O}_{DS}(y)\rangle \equiv \Delta(q)$$

Unitarity,
Causality,
Locality

- Can express this using the Källén-Lehmann spectral representation:

$$\Delta(q) = 2\int_0^\infty \mu d\mu \; \frac{\rho(\mu^2)}{q^2-\mu^2+i\epsilon}$$

for $\rho(\mu^2)$ real and positive-definite.

- Inserting into the Born Approximation yields:

$$V(r) = -\frac{\lambda^2}{2\pi r}\int_0^\infty \mu d\mu \; \rho(\mu^2) e^{-\mu r} \quad .$$

Most general form of potential from scalar operator exchange within QFT!

$$V(r) = -\frac{\lambda^2}{2\pi r}\int_0^\infty \mu d\mu \ \rho(\mu^2) e^{-\mu r} \quad .$$

Most general form of the potential generated from scalar operator exchange within the axioms of QFT

Valid regardless of the form of the hidden sector: perturbative, strongly coupled, minimal, complex ….

Positivity of $\boldsymbol{\rho}$ implies:

1. Attractive force
2. Force is a monotonic function of distance : no turning points

- In this language observables from standard fifth force searches can be re-cast in completely general terms:

Experiment	Observable
Molecular Spectroscopy	$\Delta E_\psi = -\frac{\lambda^2}{2\pi}\int d^3\boldsymbol{r}\ \psi^*(r)\frac{1}{r}\left(\int_0^\infty d\mu\ \mu\ \rho(\mu^2)e^{-\mu r}\right)\psi(r)$
Bouncing Neutrons	$\delta V(z) = \frac{-\lambda^2\rho_{\rm glass}}{m_n}\int_0^\infty d\mu\ \frac{\rho(\mu^2)e^{-\mu z}}{\mu}$
Planar Experiments	$F(s) = -\frac{2\pi R\lambda^2}{m_n^2}\int_0^\infty d\mu\ \frac{\rho(\mu^2)e^{-\mu s}}{\mu^2}\left(\rho_{\rm Au} + \left(\rho_{\rm sap} - \rho_{\rm Au}\right)e^{-\mu\Delta}\right)\left(\rho_{\rm Au} + \left(\rho_{\rm pol} - \rho_{\rm Au}\right)e^{-\mu\Delta}\right)$
Cold Neutron Scattering	$l_{\rm BSM}(\mathbf{q}) = 2m_N V(\mathbf{q}) = -2m_N\lambda^2\int_0^\infty d\mu^2\,\frac{\rho(\mu^2)}{\lvert\mathbf{q}\rvert^2+\mu^2}$
Lunar Perihelion Precession	$\delta\theta = \frac{\lambda^2}{Gm_n^2(1-\epsilon^2)}\int_0^\infty d\mu\ \mu\ \rho(\mu^2)e^{-\mu a}\left[1+\mu a+\frac{(\mu a)^2}{2}\right]$

Straightforward extraction of limits to any possible model!

Extracting ρ

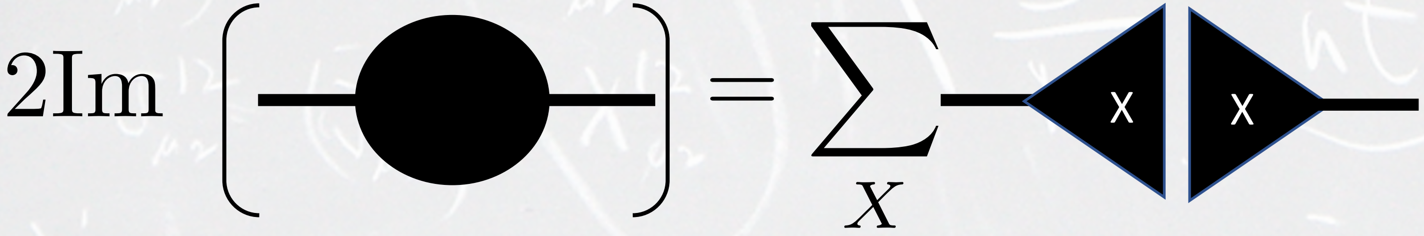
- Note that

$$\rho(q^2) = -\frac{1}{\pi}\mathrm{Im}\{\Delta(q)\} \quad .$$

$\Rightarrow$ just need to compute the **imaginary** part of the propagator

- For loops – can exploit the optical theorem for forward scattering:

$$2\mathrm{Im}\{\mathcal{M}(A \to A)\} = \sum_X \int d\Pi_X (2\pi)^4 \delta^4(p_A - p_X)|\mathcal{M}(A \to X)|^2 \quad .$$

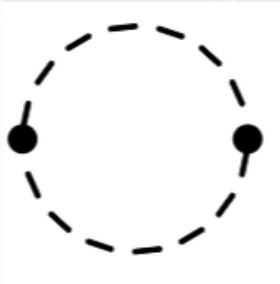


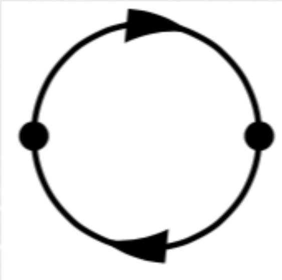
i.e. For single loops just need to compute the tree-level cut!

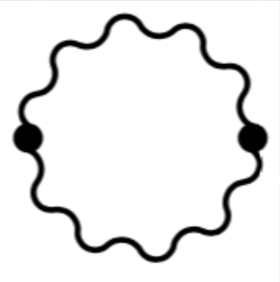
1-loop Examples:

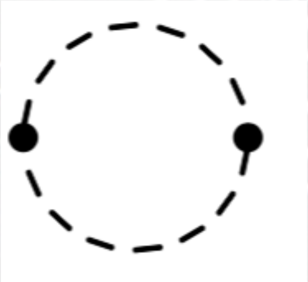
(A) $\frac{1}{\Lambda}\mathcal{O}_{SM}|\phi|^2$ (B) $\frac{1}{\Lambda^2}\mathcal{O}_{SM}\bar{\psi}\psi$ (C) $\frac{m^2}{\Lambda^3}\mathcal{O}_{SM}|V|^2$ (D) $\frac{1}{\Lambda^3}\mathcal{O}_{SM}\partial_\mu\phi^*\partial^\mu\phi$

$\Delta(q)$:

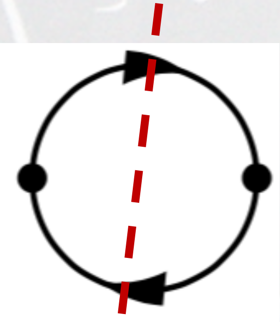


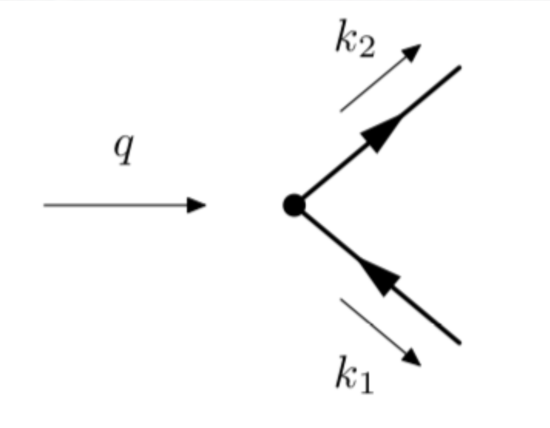






Tree Level Cut:





Analogous to the decay of an external source of momentum q to a pair of equal mass particles

(A) $\frac{1}{\Lambda}\mathcal{O}_{SM}|\phi|^2$ (B) $\frac{1}{\Lambda^2}\mathcal{O}_{SM}\bar{\psi}\psi$ (C) $\frac{m^2}{\Lambda^3}\mathcal{O}_{SM}|V|^2$ (D) $\frac{1}{\Lambda^3}\mathcal{O}_{SM}\partial_\mu\phi^*\partial^\mu\phi$

Operator	$\rho(\mu^2)$	$V(r)$
(A)	$\frac{\eta}{8\pi^2}\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{1}{2}}\Theta(\mu^2-4m^2)$	$-\frac{\eta m}{8\Lambda^2\pi^3 r^2}K_1(2mr)$
(B)	$\frac{\mu^2\eta}{4\pi^2}\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{3}{2}}\Theta(\mu^2-4m^2)$	$-\frac{3\eta m^2}{2\Lambda^4\pi^3 r^3}K_2(2mr)$
(C)	$\frac{\mu^4\eta}{32m^4\pi^2}\left(1+\frac{12m^4}{\mu^4}-\frac{4m^2}{\mu^2}\right)\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{1}{2}}\Theta(\mu^2-4m^2)$	$-\frac{3m^3\eta(5+m^2r^2)}{8\Lambda^6\pi^3 r^4}K_3(2mr)$
(D)	$\frac{\mu^4\eta}{32\pi^2}\left(1-\frac{4m^2}{\mu^2}+\frac{4m^4}{\mu^4}\right)\left(1-\frac{4m^2}{\mu^2}\right)^{\frac{1}{2}}\Theta(\mu^2-4m^2)$	$-\frac{\eta}{8\Lambda^6\pi^3}\left(\frac{15m^3}{r^4}+\frac{m^5}{r^2}\right)K_1(2mr)$ - $\frac{\eta}{4\Lambda^6\pi^3}\left(\frac{15m^2}{r^5}+\frac{3m^4}{r^3}\right)K_2(2mr)$

K_n is the n^{th} modified Bessel function of the second kind, Θ is the Heaviside step function, and η takes a value of 1 if the field is self conjugate and 1/2 if not.

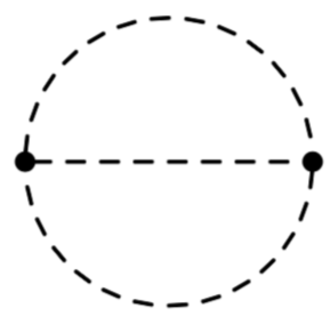
Consistent with 1710.00850

Short distance behaviour: $r^{-3}, r^{-5}, r^{-7}, r^{-7}$

Beyond 1-loop:

- The spectral densities corresponding to higher order loop exchange can also be straightforwardly calculated via the optical theorem, without needing to perform loop calculations:

e.g. $$\frac{1}{\Lambda^2}\mathcal{O}_{SM}\phi^3 \qquad \Delta(q):$$



$$\boxed{\rho_3(\mu^2) = \frac{3\sqrt{(\mu-m)(\mu+3m)}}{128\mu^2\pi^4}\left(\frac{(\mu-m)(\mu^2+3m^2)}{2}E(\tilde{k}) - 4m^2\mu K(\tilde{k})\right)\Theta(\mu^2-9m^2)}$$

K, E are the complete elliptical integrals of the first and second kind and $\tilde{k} = \sqrt{\frac{(\mu+m)^3(\mu-3m)}{(\mu-m)^3(\mu+3m)}}$.

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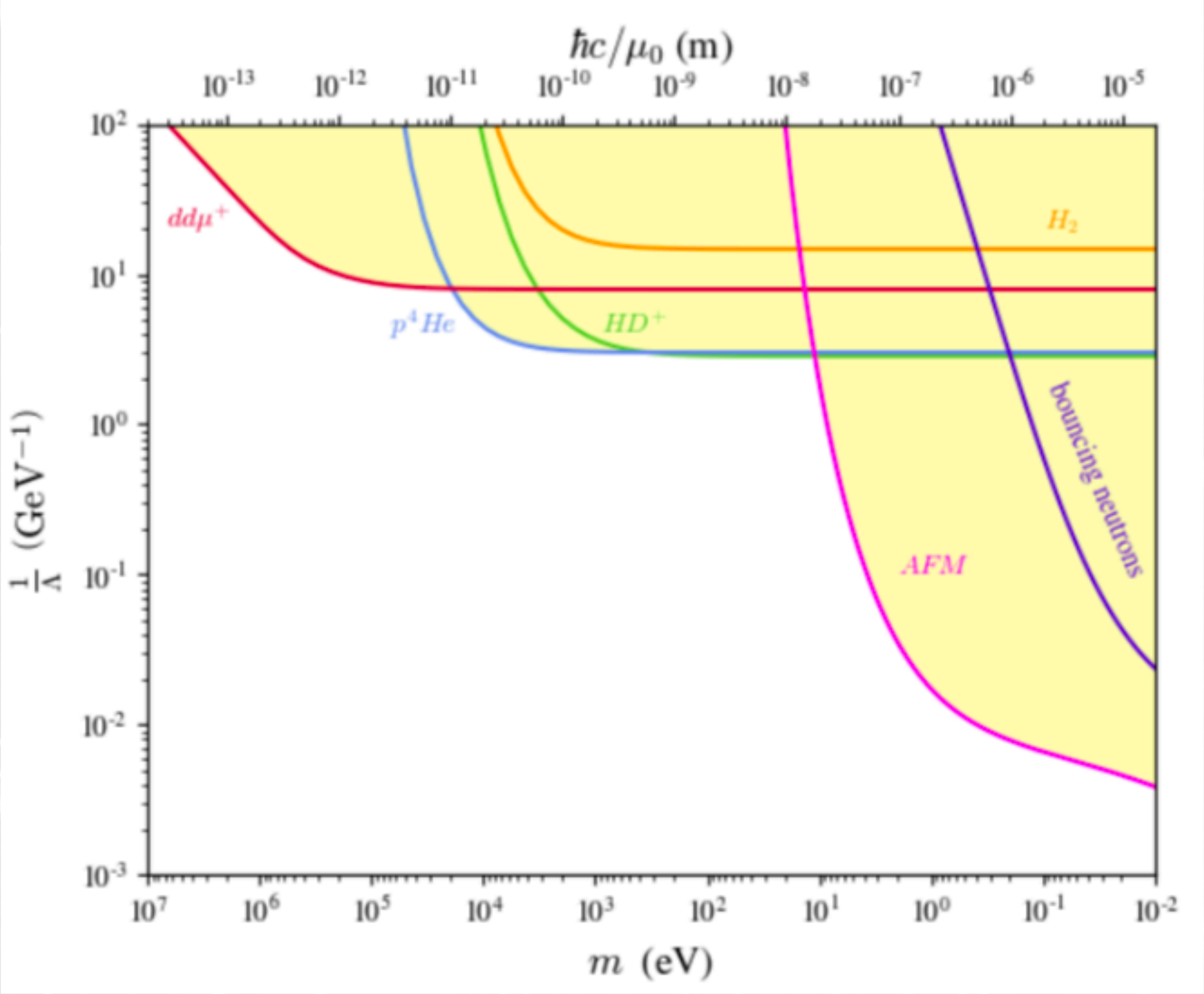
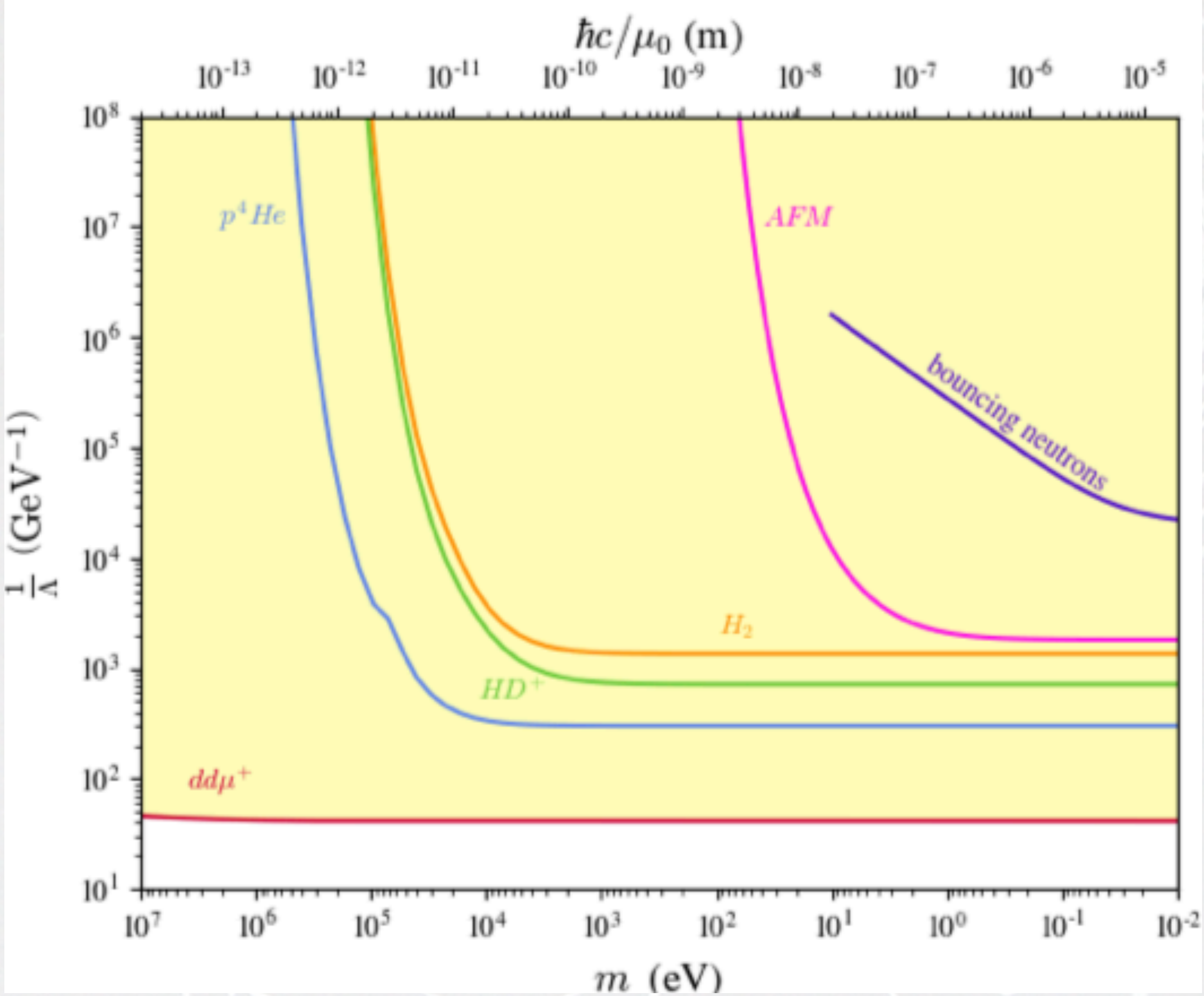
2 **The Experimental Landscape**

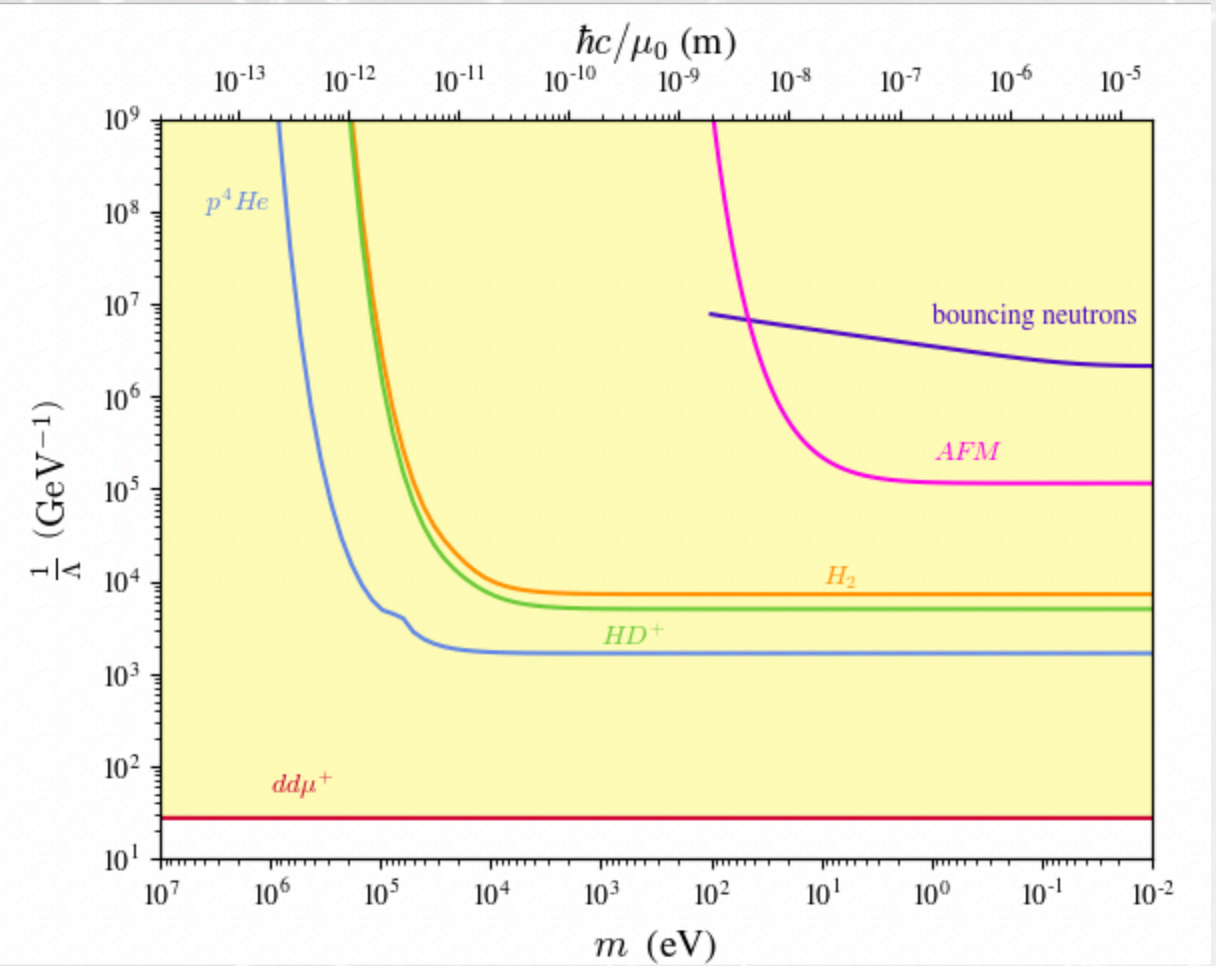
3 Journeying Beyond QFT?



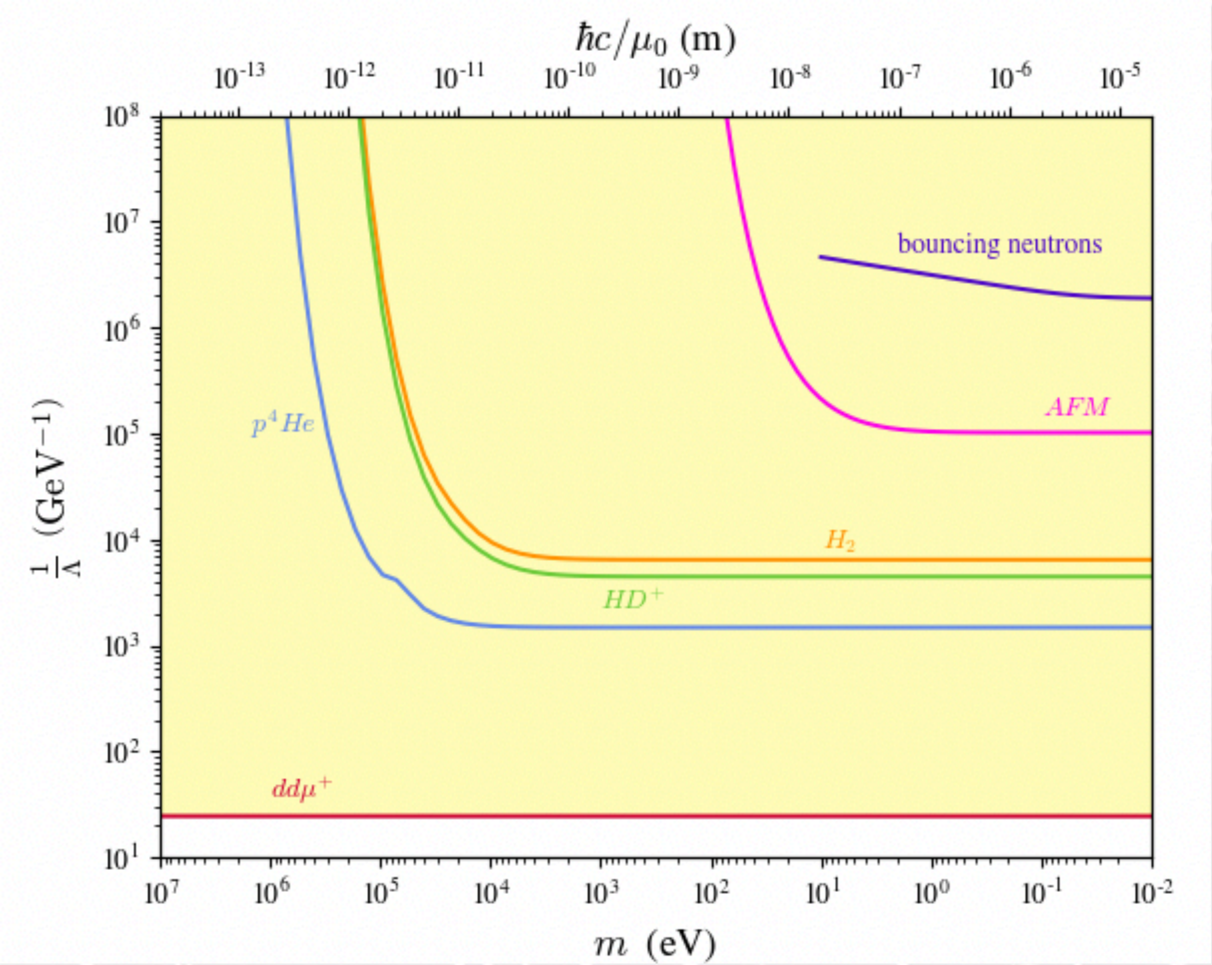
(A) $\frac{1}{\Lambda}\mathcal{O}_{SM}|\phi|^2$



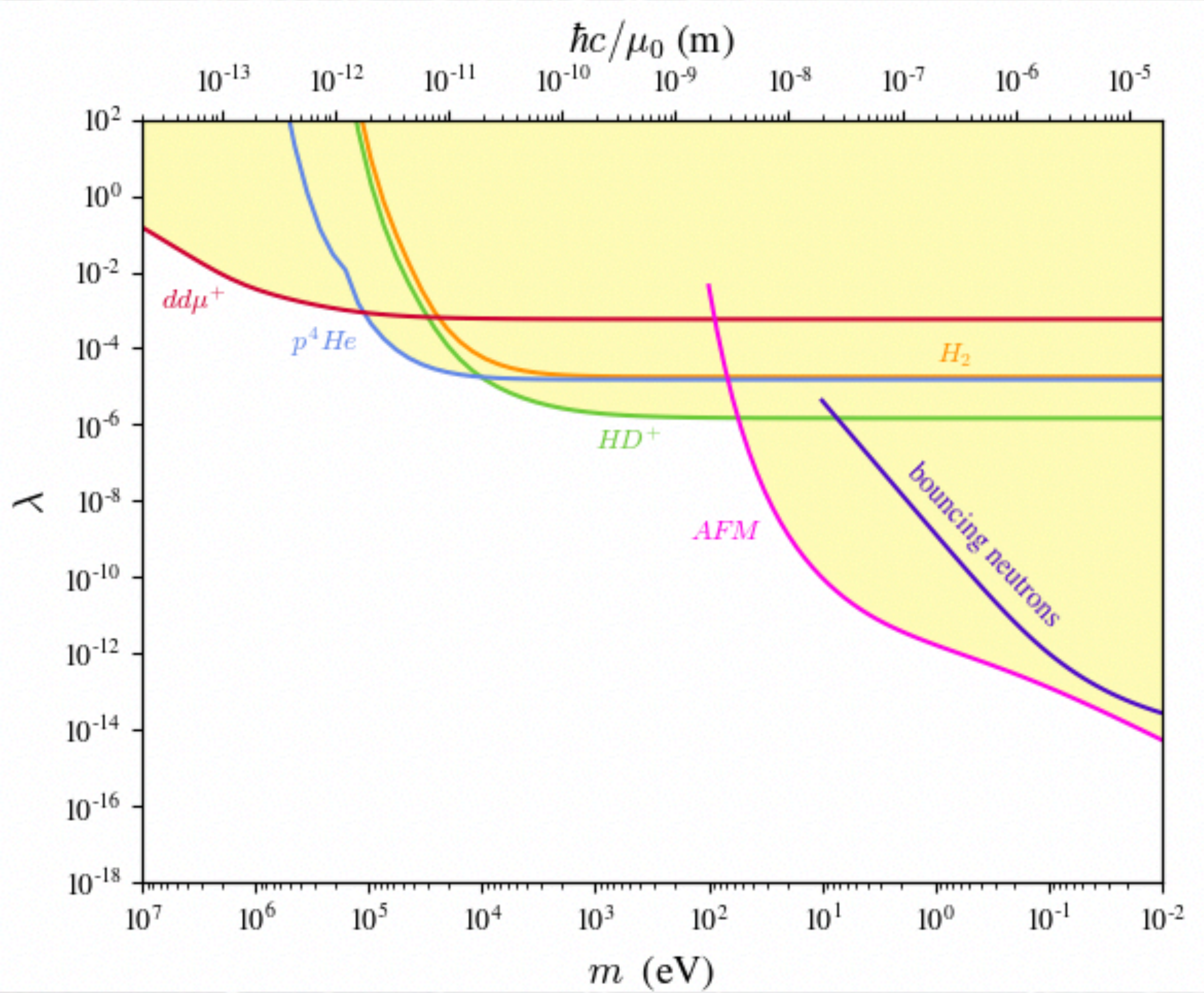
(B) $\frac{1}{\Lambda}\mathcal{O}_{SM}\bar{\psi}\psi$



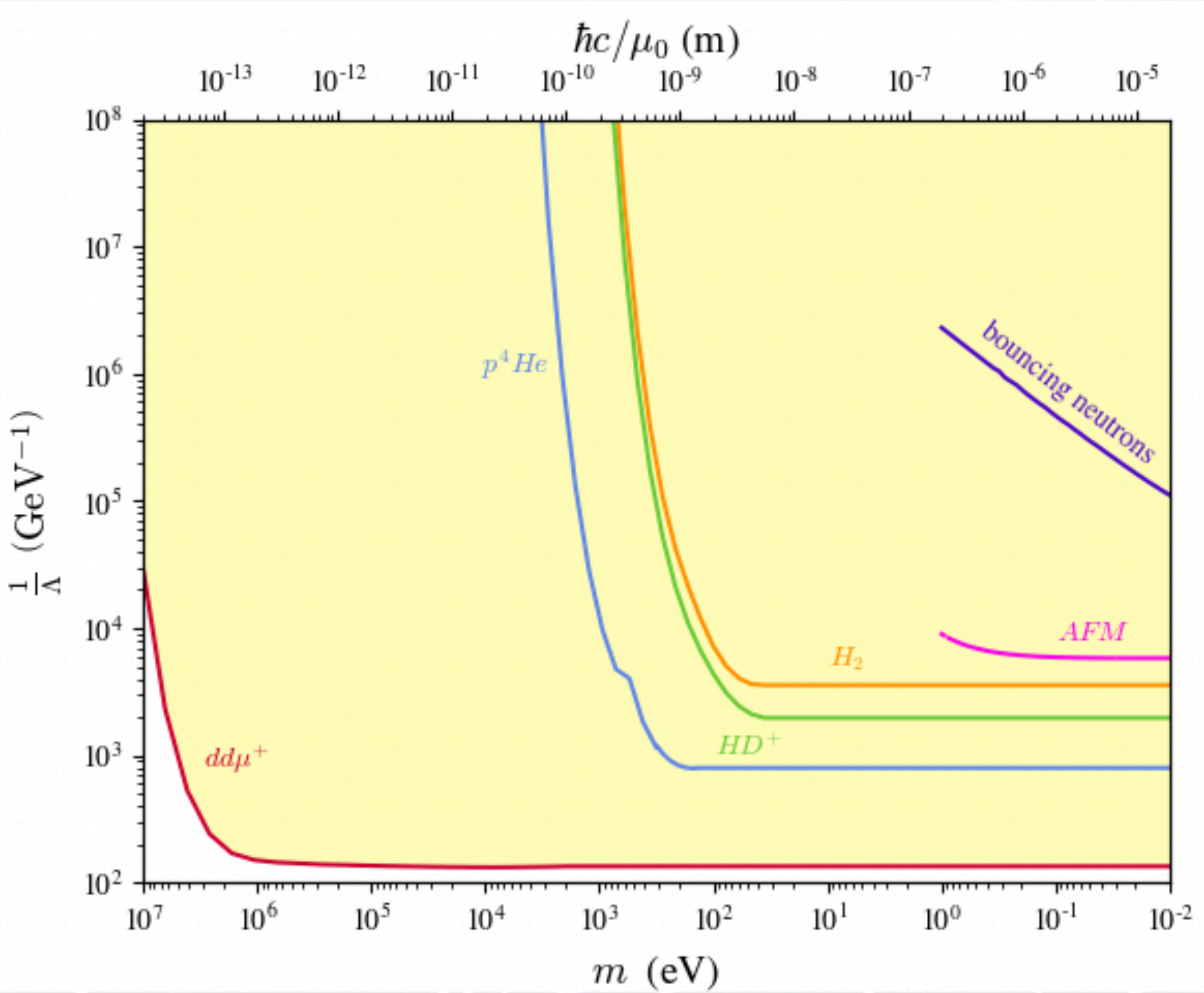
(C) $\frac{m^2}{\Lambda^3}\mathcal{O}_{SM}|V|^2$



(D) $\frac{1}{\Lambda^3}\mathcal{O}_{SM}\partial_\mu\phi^*\partial^\mu\phi$



Yukawa



$$\frac{1}{\Lambda^2}\mathcal{O}_{SM}\phi^3$$

Comments

Constraint hierarchy largely dominated by short-distance behaviour:

- For Yukawa (r^{-1}) and Interaction A (r^{-3}), experimental reach improves for lighter states

- For B-D and 2-loop interactions (r^{-5} / r^{-7}) strongest bounds come from shortest distance experiments

- Landscape of limits can vary significantly from the Yukawa picture – motivating a more systematic study of possible fifth forces

A two-pronged approach:

01

Take your desired interaction

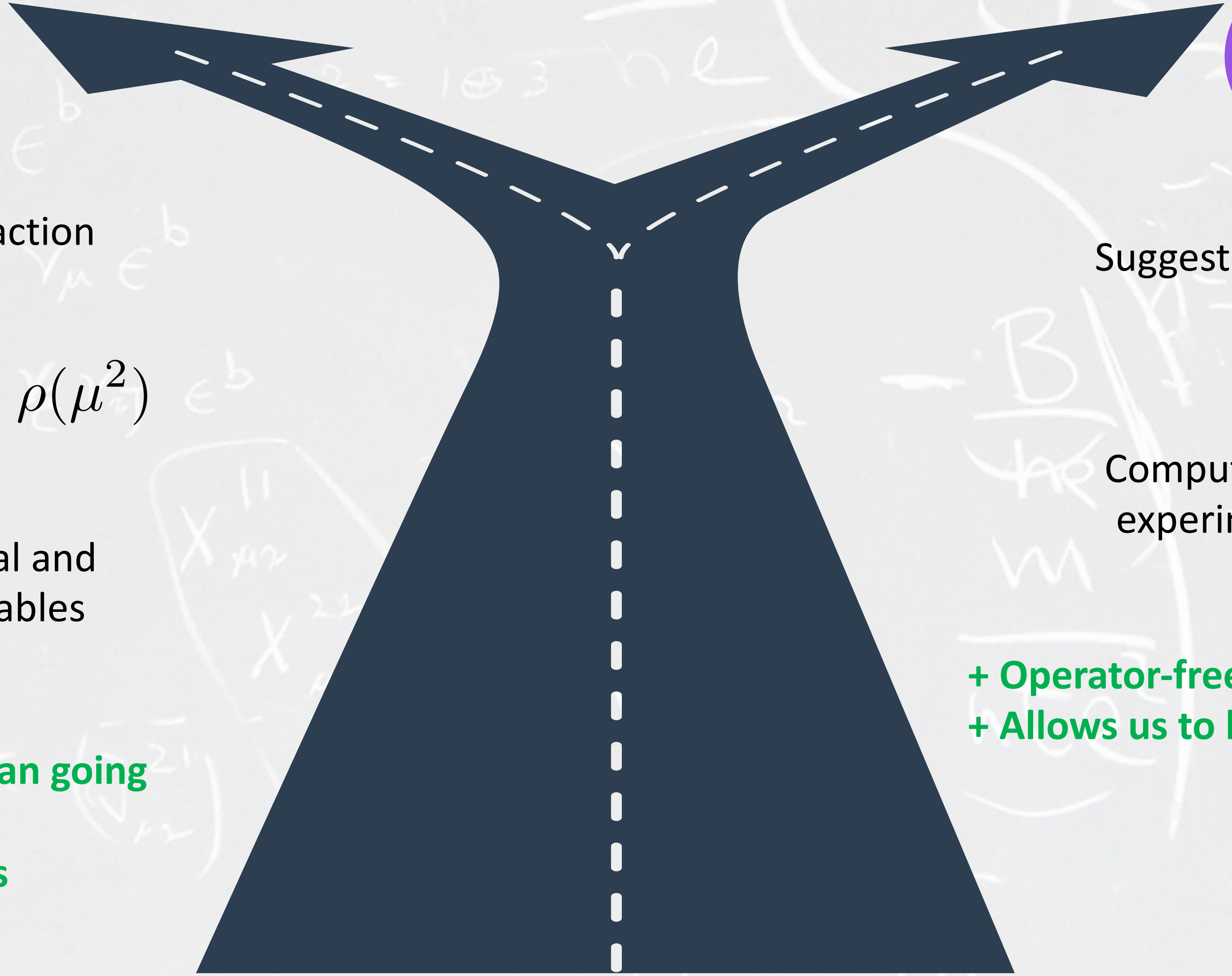
↓

Find the corresponding $\rho(\mu^2)$

↓

Compute the potential and experimental observables

+ Computationally simpler than going via full matrix element
+ No explicit loop calculations



02

Suggest a form for $\rho(\mu^2)$

↓

Compute the potential and experimental observables

+ Operator-free approach
+ Allows us to look beyond QFT?

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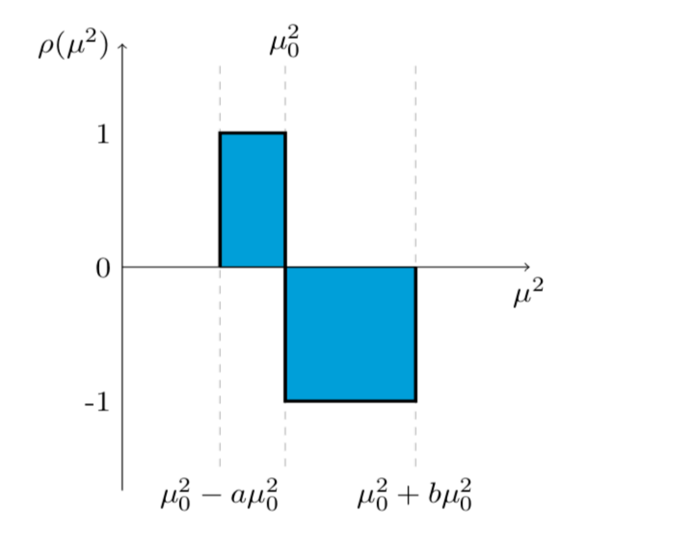
What happens if positivity of $\rho(\mu^2)$ relaxed? e.g.

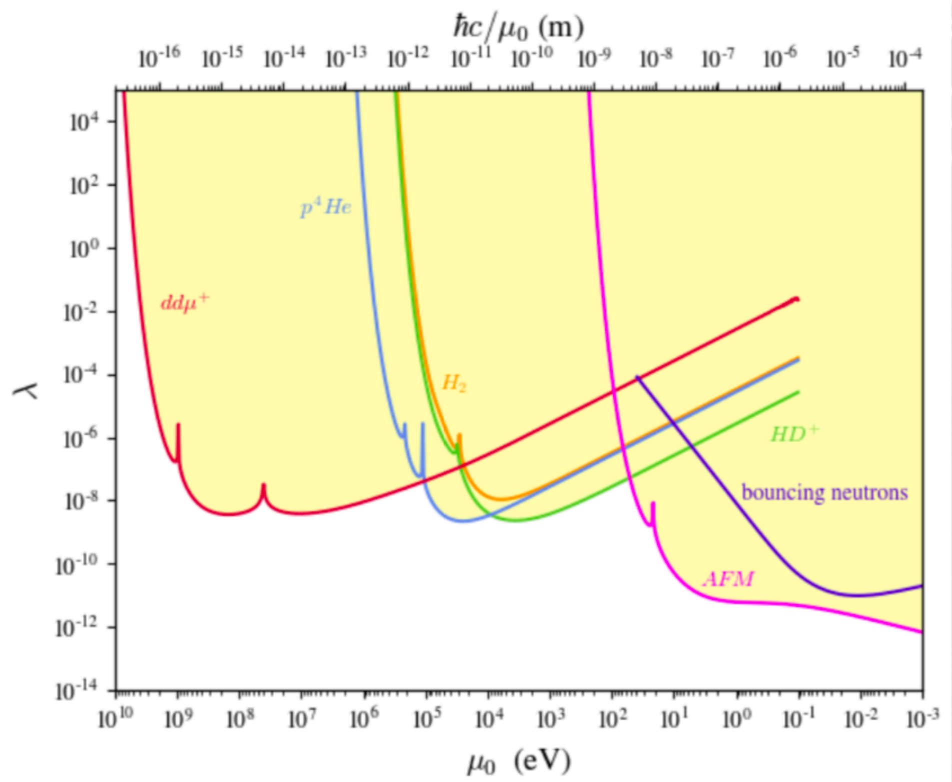


$$V(r) = -\lambda^2 \frac{f(a) + f(-b) - 2e^{-\mu_0 r}(1 + \mu_0 r)}{2\pi r^3},$$

where

$$f(x) = e^{-\mu_0 r\sqrt{(1-x)}}\left(1 + \mu_0 r\sqrt{(1-x)}\right)$$

Corresponds to violations of causality, unitarity…..



Interesting Cases:

- IFF a = b, potential is "screened" with finite value in short distance limit
- If b > a potential develops a turning point in r

What happens if positivity of $\rho(\mu^2)$relaxed? e.g.

$$V(r) = -\lambda^2 \frac{f(a) + f(-b) - 2e^{-\mu_0 r}(1 + \mu_0 r)}{2\pi r^3},$$

where

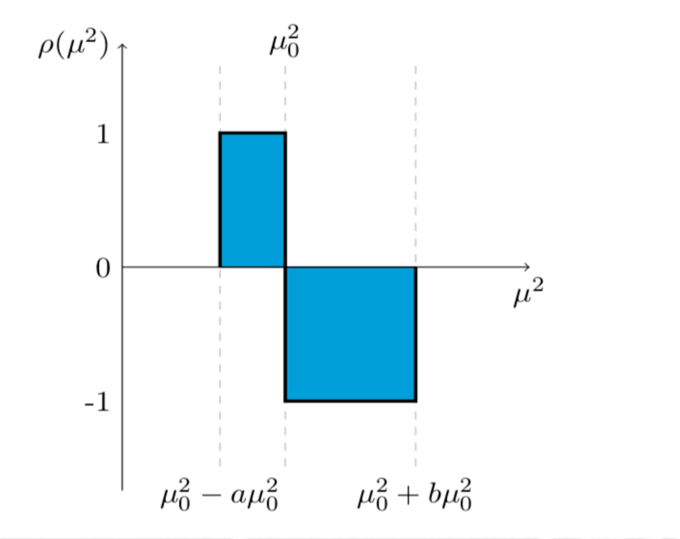
$f($

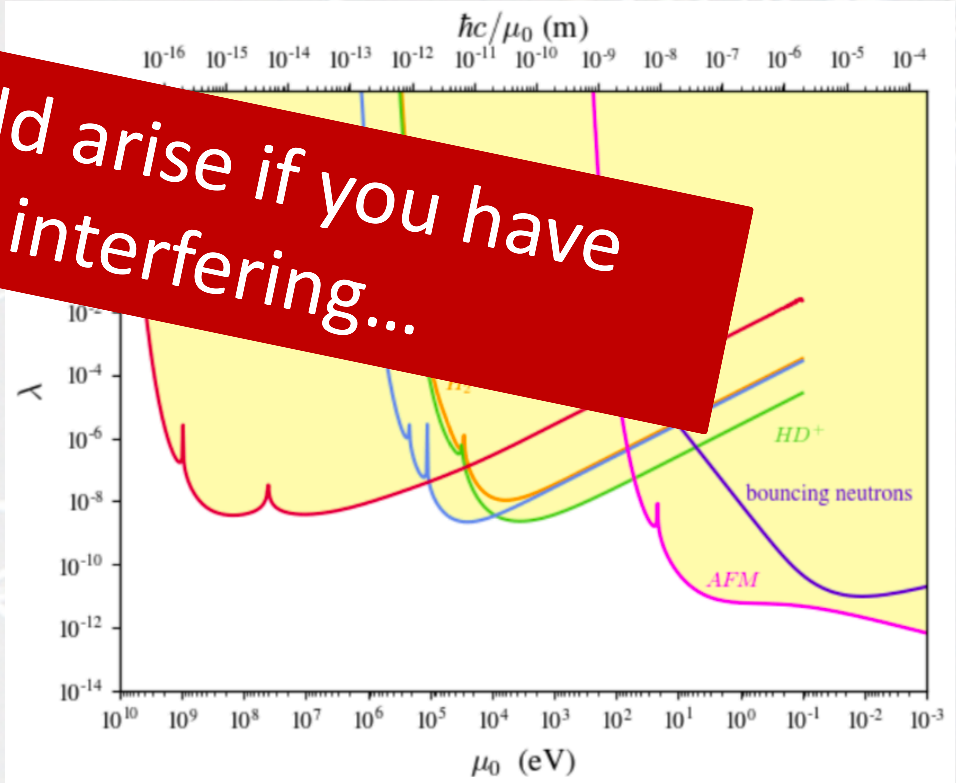
Note that similar effects could arise if you have different spin operators interfering…

Corresponds to violations of causality, unitar

Interesting Cases:

- IFF a = b, potential is “screened” with finite value in short distance limit
- If b > a potential develops a turning point in r





Summary and Conclusions

- **All** possible scalar fifth forces can be encapsulated by a single, real, positive definite spectral function
- Combined with the optical theorem, this framework facilitates a straightforward route to obtaining the potentials from loop exchange
- Experimental observables can be expressed in completely general terms, allowing straightforward extraction of limits to any model
- This description provides a unique framework to consider more speculative scenarios such as violation of QFT fundamentals
- The landscape of possible scalar fifth forces is much richer than the simple Yukawa scenario – and definitely worth pursuing!