YTF2020

# PRECISE DETERMINATION OF CKM MATRIX ELEMENTS WITH LATTICE QCD+QED

(or: how I learned to stop worrying about non-perturbative QCD)

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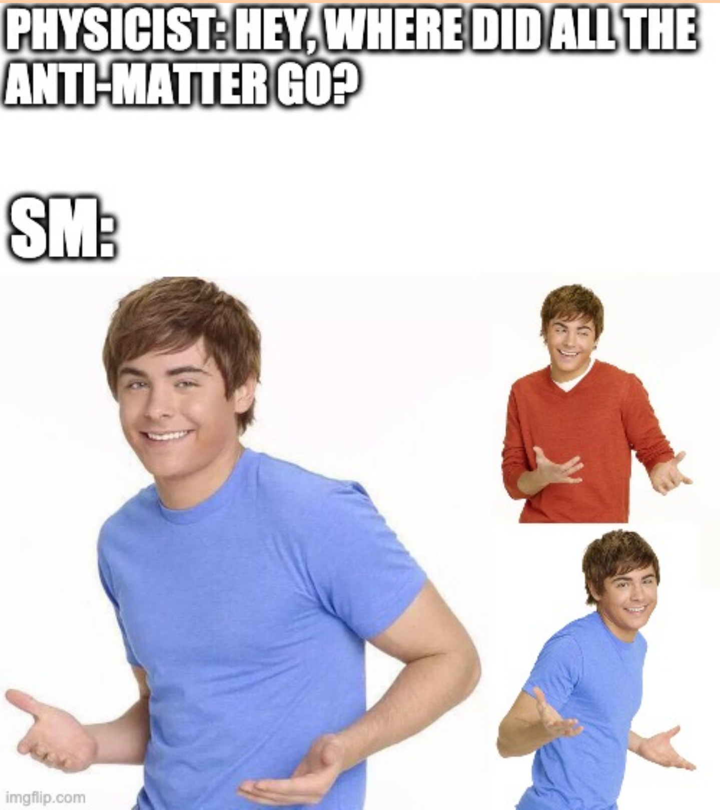
# OVERVIEW

1. Introduction
2. Extracting CKM matrix elements
3. Lattice Formulation of QCD (feat. QED)
4. Strategy
5. Conclusion

# INTRODUCTION

The Standard Model of Particle Physics has its successes (and problems)…

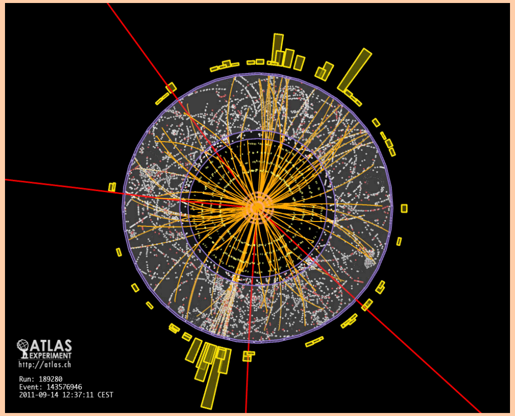



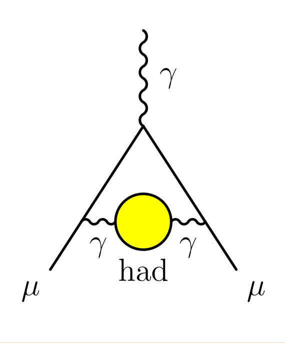

Where do we go from here?

Two ways:

1) High energy/luminosity

2) Quantum corrections in low-energy phenomena



# INTRODUCTION – CKM MATRIX

Nicola Cabibbo

The interaction strength of flavor-changing weak decays in a 3×3 matrix

Makoto Kobayashi

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

SM predicts the CKM matrix to be unitary…

$$\sum_i V_{ij} V_{ik}^* = \delta_{jk} \qquad \text{eg} \quad |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Toshihide Maskawa


…but is it?

Testing the unitarity of the CKM matrix is a probe for BSM physics in the flavor sector

# EXTRACTING CKM MATRIX ELEMENTS

For a pseudoscalar, $P^-$, decaying into $l^-\bar{\nu}_l$ pair, the tree-level width is

$$\Gamma(P^- \to l^-\bar{\nu}_l) \equiv \Gamma_0^{\text{tree}} = \frac{G_F^2}{8\pi}\left|V_{q_1 q_2}\right|^2 f_P^2 m_{l^-}^2 \left(1 - \frac{m_{l^-}^2}{M_{P^-}^2}\right)^2 .$$

$$\mathcal{A} \equiv \langle 0|\bar{q}_2\gamma^0\gamma^5 q_1|P^-\rangle = M_P f_P$$

In practice, use inclusive rates from experiments

$\mathcal{O}(\alpha)$ corrections

$$\Gamma(P^- \to l^-\bar{\nu}_l[\gamma]) = \Gamma_0^{\text{tree}}(1 + \boxed{\delta R_P}).$$

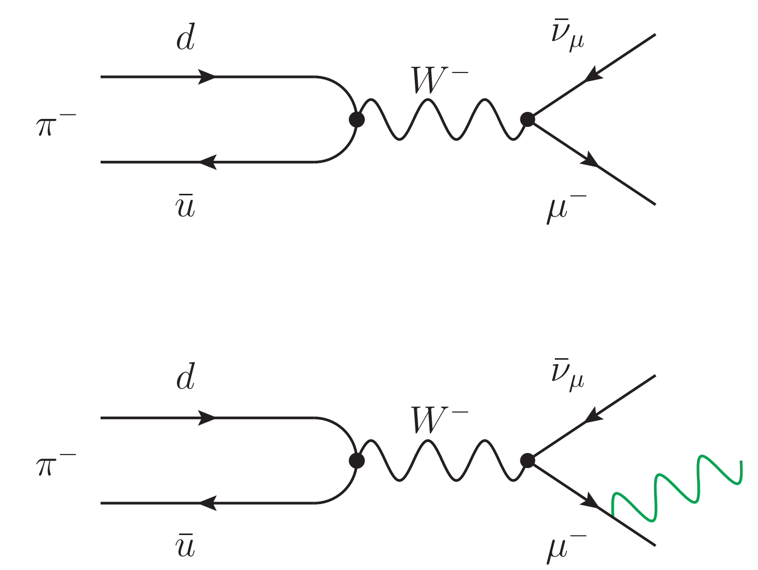



Figure 1: A $\pi^-$ decaying into a $\mu^-\bar{\nu}_\mu$ pair, possibly with a soft photon (in green) in the final state.

For example, the following ratio of CKM matrix elements can be obtained via:

$$\frac{|V_{us}|^2}{|V_{ud}|^2} = \boxed{\frac{\Gamma(K^- \to \mu^-\bar{\nu}_\mu[\gamma])}{\Gamma(\pi^- \to \mu^-\bar{\nu}_\mu[\gamma])}\frac{M_{K^-}^3}{M_{\pi^-}^3}\frac{(M_{\pi^-}^2 - m_{\mu^-}^2)^2}{(M_{K^-}^2 - m_{\mu^-}^2)^2}} \boxed{\frac{(f_\pi/f_K)^2}{1 + \delta R_K - \delta R_\pi}}$$

Experiment

Theory

# LATTICE QCD+QED

Your quintessential intro-to-lattice slide

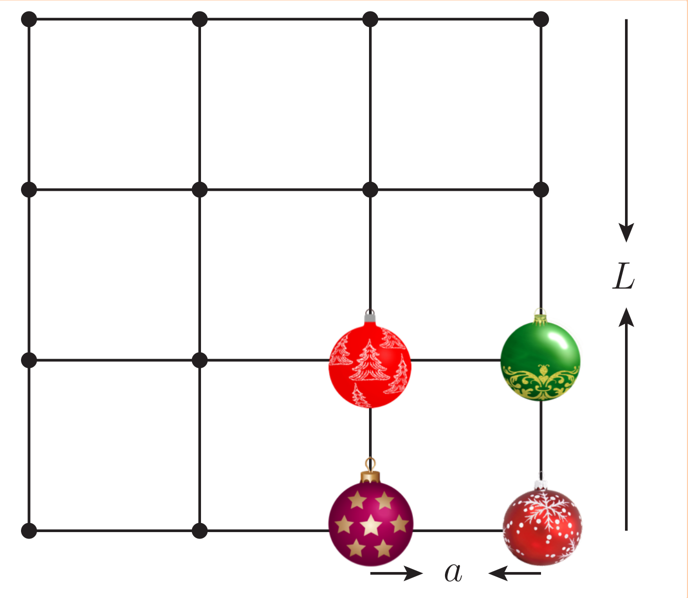



Figure 2: A pictorial representation of a lattice with spacing, *a*, and spatial extent, *L*.
The quarks live on the sites (baubles) and the gauge fields are the links connecting the sites.

An elegant idea first proposed by Kenneth Wilson in 1974 [1].
The central idea is two-fold:

1) to **discretise 4D spacetime** into a hypercube with finite lattice spacing and
2) to make the transition from `Minkowskian' to Euclidean field theory via **Wick rotation,** $t \rightarrow it$ .

Then, the VEV of operators can be expressed with the path integral formalism:

$$\boxed{\langle 0|O_1 O_2 \ldots O_n|0\rangle} = \frac{1}{Z}\int D\psi D\overline{\psi} DGDA \quad O_1 O_2 \ldots O_n \boxed{e^{-S_f - S_G - S_A},}$$

where $S_{f,G,A}$ are the fermion, gluon and photon action, respectively.


[1] Kenneth G. Wilson. Confinement of quarks. Phys. Rev. D, 10:2445–2459,Oct 1974

# LATTICE QCD+QED

In practice…

Lattice simulations performed in isospin-symmetric limit, $\delta m \equiv m_u - m_d = 0$

$\implies$ compute isospin-breaking (IB) corrections

Since $\alpha \sim \frac{m_u - m_d}{\Lambda_{QCD}} \sim 1\%$, treat IB effects perturbatively in path integral expansion [5] :

$$\langle O \rangle = \langle O \rangle_0 + \frac{1}{2!} e^2 \frac{\partial}{\partial e^2} \langle O \rangle \Big|_{e=0} + \sum_{f \in \{u,d,s\}} \Delta m_f \frac{\partial}{\partial m_f} \langle O \rangle \Big|_{e=0} + \mathcal{O}\left(\alpha^2, \left(m_f - m_f^{sim}\right)^2\right)$$

Extra step in QCD+QED simulations: remove spatial zero modes of the photon - $\mathrm{QED_L}$ [6]

Electro-quenched approximation: sea quarks are electrically-neutral

[5] G.M. de Divitiis, *et al.* Leading isospin breaking effects on the lattice. Phys. Rev. D, 87(11):114505, 2013

[6] Masashi Hayakawa and Shunpei Uno. QED in finite volume and finite size scaling effect on electromagnetic properties of hadrons. Prog. Theor. Phys.,120:413–441, 2008

# LATTICE QCD+QED

In *practice* practice…

Ideally…

$$L^{-1} \ll E \ll a^{-1}$$

⇒ Small lattice spacing + large box volume = Big computers

The simulation utilises a C++ mathematical object library known as **Grid** [2,3] and a Grid-based workflow management library called **Hadrons** [4].

It is performed on a 1468-node HPC system called **Tesseract**, provided by the DiRAC Extreme Scaling services.


Figure 3: The Tesseract at the DiRAC facility.
Image from https://www.epcc.ed.ac.uk/facilities/dirac

[2] Peter Boyle, Azusa Yamaguchi, Guido Cossu, and Antonin Portelli. Grid: A next generation data parallel C++ QCD library. 12 2015.
[3] https://github.com/paboyle/Grid
[4] https://github.com/aportelli/Hadrons

GRID

Hadrons

# STRATEGY

New challenge: Compute $\delta R$ at (near) physical point simulation!

Pioneering work done by RM123 collaboration.

PoS, CD15:023, June-July 2016

Phys. Rev. D, 95:034504, Feb 2017

Phys. Rev. D, 100:034514, Aug 2019

$$\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu[\gamma]) \equiv \Gamma = \Gamma_0 + \Gamma_1$$
$$= \lim_{L\to\infty} \left(\underbrace{\boxed{\Gamma_0(L)}}_{\text{Evaluated on the lattice}} - \underbrace{\boxed{\Gamma_0^{\mathrm{pt}}(L)}}_{\text{FV correction}}\right) + \underbrace{\boxed{\lim_{m_\gamma\to 0} \Gamma_1^{\mathrm{pt}}(m_\gamma, \Delta E_\gamma)}}_{\text{Final state photon}}$$

$$\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu[\gamma]) = \Gamma_0^{\mathrm{tree}} \left(1 + \underbrace{\frac{\delta\Gamma_0}{\Gamma_0^{\mathrm{tree}}} - \frac{\delta\Gamma_0^{\mathrm{pt}}}{\Gamma_0^{\mathrm{tree}}} + \frac{\delta\Gamma_1^{\mathrm{pt}}}{\Gamma_0^{\mathrm{tree}}}}_{\delta R_\pi}\right)$$

# STRATEGY

- Four-fermion operator with neutrino leg amputated.
  *Eg* tree-level weak Hamiltonian

$$H_W = (\bar{\mu}\gamma_l \nu_\mu)(\overline{q_1}\gamma_L q_2) \rightarrow \widetilde{H}_{W,\alpha} = (\bar{\mu}\gamma_l)_\alpha(\overline{q_1}\gamma_L q_2)$$

$\Rightarrow$ match back to SM using W-regularisation [7].

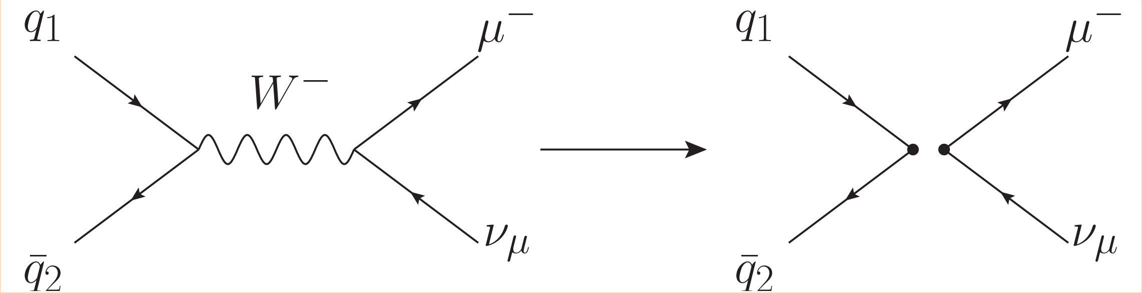



Figure 4: At $E \ll M_{W^\pm}$, the effective weak Hamiltonian is a four-fermion operator with pinched interaction vertices.

- Sequential insertion of E.M vector and SIB scalar current

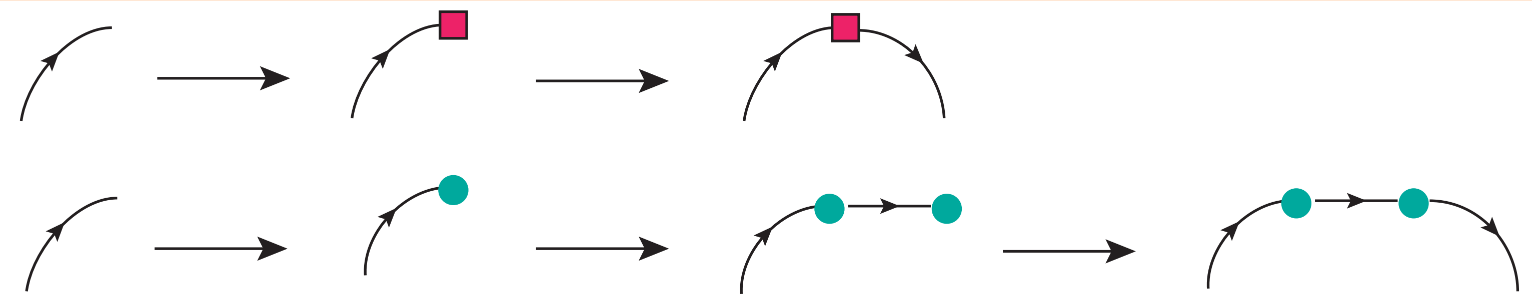


Figure 5: Building quark propagators with scalar (red square) and photon vector (green circle) current insertions sequentially.


[7] A. Sirlin. 'Large Mw, Mz behaviour of the o($\alpha$) corrections to semileptonic processes mediated by W'. Nuclear Physics B, 196(1):83 – 92, 1982

# ISOSPIN-BREAKING DIAGRAMS

Previously…

Kinematic factors from phase space integral

$$\Gamma_0^{\text{tree}} \equiv \Gamma(\pi^- \to \mu^- \bar{\nu}_\mu) = K \sum_{r,s} |\mathcal{M}^{r,s}|^2$$

IB corrections to the amplitudes

0 (if using experimental masses)

$$\delta\Gamma_0 = \left(1 + \cancel{\delta K} \sum_{r,s} |\mathcal{M}^{r,s}|^2 + 2K \sum_{r,s} \mathrm{Re}[\mathcal{M}^{r,s*}\delta\mathcal{M}^{r,s}]\right)$$

At $\mathcal{O}(\alpha, \delta m)$, this gives…

- 3 non-factorisable diagrams
- 5 factorisable diagrams

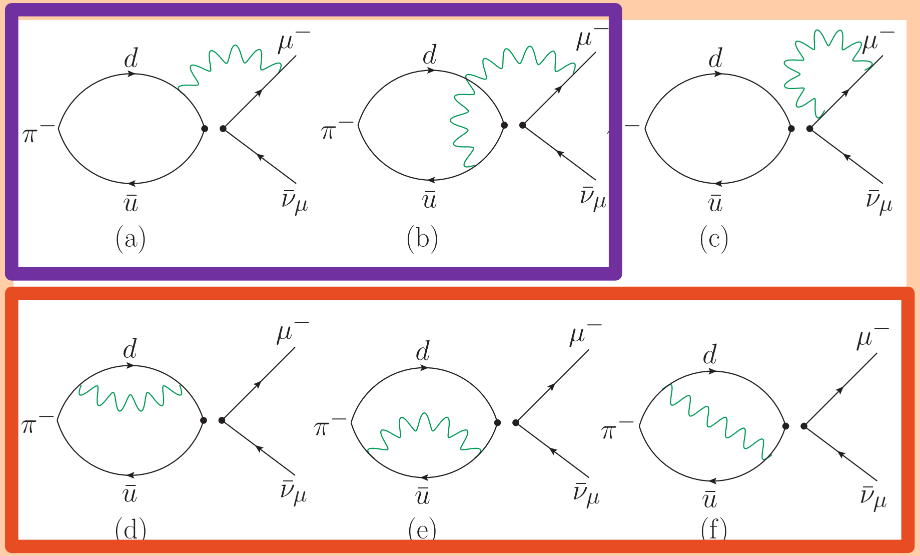


Figure 6: The six possible virtual QED correction at $\mathcal{O}(\alpha)$.

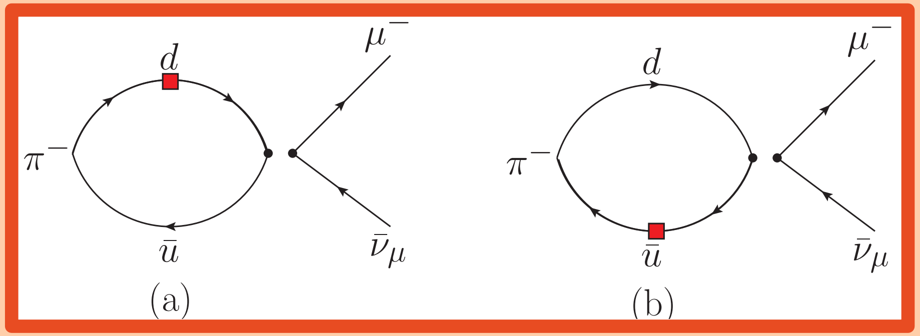


Figure 7: The two possible scalar insertion diagrams at $\mathcal{O}(\delta m)$.

# PRESS ENTER AND DRUM ROLL…

$$\delta R_\pi = \frac{\alpha}{\pi}\log\left(\frac{M_Z^2}{M_W^2}\right) + 2\,\frac{\delta\mathcal{A}_\pi}{\mathcal{A}_\pi} - 2\,\frac{\delta M_\pi}{M_\pi} + \delta\Gamma_\pi^{\mathrm{pt}}$$

FLAG/AY

$$\frac{|V_{us}|^2}{|V_{ud}|^2} = \boxed{\frac{\Gamma(K^- \to \mu^-\,\bar\nu_\mu[\gamma])}{\Gamma(\pi^- \to \mu^-\,\bar\nu_\mu[\gamma])}\,\frac{M_{K^-}^3}{M_{\pi^-}^3}\,\frac{\left(M_{\pi^-}^2 - m_{\mu^-}^2\right)^2}{\left(M_{K^-}^2 - m_{\mu^-}^2\right)^2}}\;\frac{\boxed{(f_\pi/f_K)^2}}{1+\boxed{\delta R_K - \delta R_\pi}}$$

Latest values from PDG 2020

PRELIMINARY

$$\frac{V_{us}}{V_{ud}} = 0.23176(4)(28)\,(37)$$

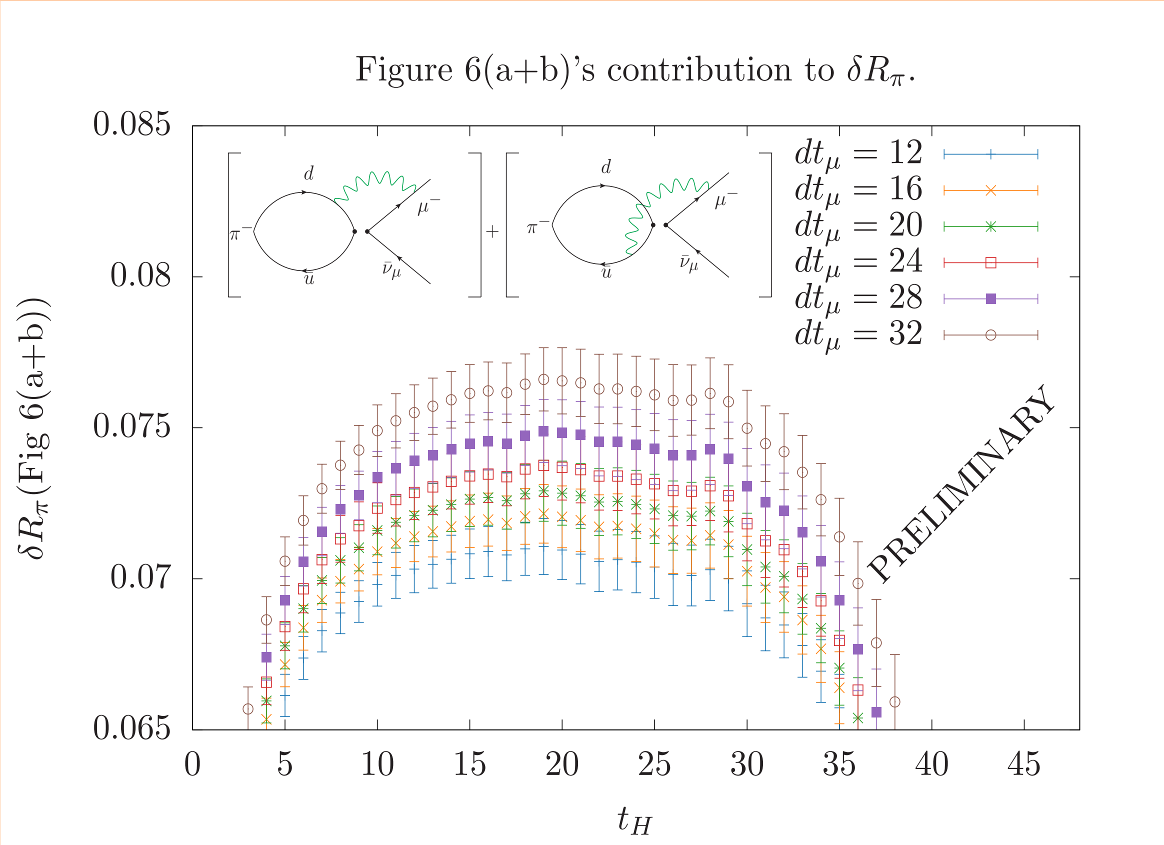



Figure 8: Data corresponding to Figure 6(a+b). A combined fit to the plateaux gives a contribution to $\frac{\delta\mathcal{A}_\pi}{\mathcal{A}_\pi}$.

Inset: The relevant diagrams that contribute to this data.

# CONCLUSION & OUTLOOK

PRELIMINARY

$$\frac{V_{us}}{V_{ud}} = 0.23176(4)(28)\,(37)$$

- Search for hints of BSM physics by testing unitarity of CKM
- First principle calculation of hadronic observables possible with Lattice QCD+QED
- Near-physical point lattice determination of $\frac{V_{us}}{V_{ud}}$ with per-mille precision!

Next….

- [ Sleep for days ]
- Renormalise $H_W$ to obtain $V_{us}$ and $V_{ud}$ individually
- Semi-leptonic decays, $K^{\pm} \to \pi^0 l^{\pm} \nu_l$…

# THANK YOU FOR LISTENING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# BUT WAIT THERE’S MORE?

# SCALE SETTING & TUNING

- Simulation input bare quark mass ≠ physical renormalized quark mass
- Correct for this mistuning by matching hadronic observables to their physical counterparts.

*Eg.*

$$M_{\pi^-}^2 = (aM_\pi^0)^2 + \alpha \frac{\partial (aM_{\pi^-})^2}{\partial \alpha} + \sum_{f \in u,d,s} \Delta m_f \frac{\partial (aM_\pi)^2}{\partial m_f} + \mathcal{O}\left(\alpha^2, \delta m_f^2, \alpha \delta m_f\right)$$

$$M_{K^-}^2 = (aM_K^0)^2 + \alpha \frac{\partial (aM_{K^-})^2}{\partial \alpha} + \sum_{f \in u,d,s} \Delta m_f \frac{\partial (aM_{K^-})^2}{\partial m_f} + \mathcal{O}\left(\alpha^2, \delta m_f^2, \alpha \delta m_f\right)$$

$$M_{K^0}^2 = (aM_K^0)^2 + \alpha \frac{\partial (aM_{K^0})^2}{\partial \alpha} + \sum_{f \in u,d,s} \Delta m_f \frac{\partial (aM_{K^0})^2}{\partial m_f} + \mathcal{O}\left(\alpha^2, \delta m_f^2, \alpha \delta m_f\right)$$

$$a = \frac{(aM_{\Omega^-})^{\text{phys}}}{M_{\Omega^-}}$$

# DEFINING (ISOSYMMETRIC) QCD POINT

- Physically, no purely-QCD processes ⇒ prescription-dependent
- BMW mesonic scheme [8].
- RM123 advocates ‘hadronic scheme’
- Intermediate results may be prescription-dependent, but $f_P\sqrt{1+\delta R_P}$ is <u>prescription-independent</u>.

[8] Borsanyi, S., et al. (BMW) (2013), Phys. Rev. Lett. 111 (25), 252001, arXiv:1306.2287

# SEQUENTIAL INSERTION

Is old really gold?

Did a cost comparison between:

1. gauge-fixed wall source propagators with sequential insertion
2. $\mathbb{Z}_2$ point source All-to-All propagators

$$\frac{\text{cost}_{\text{A2A}}}{\text{cost}_{\text{seq}}} = \frac{\Delta_{\text{A2A}}}{\Delta_{\text{seq}}} \sqrt{\frac{N_{\text{A2A}}}{N_{\text{seq}}}}$$

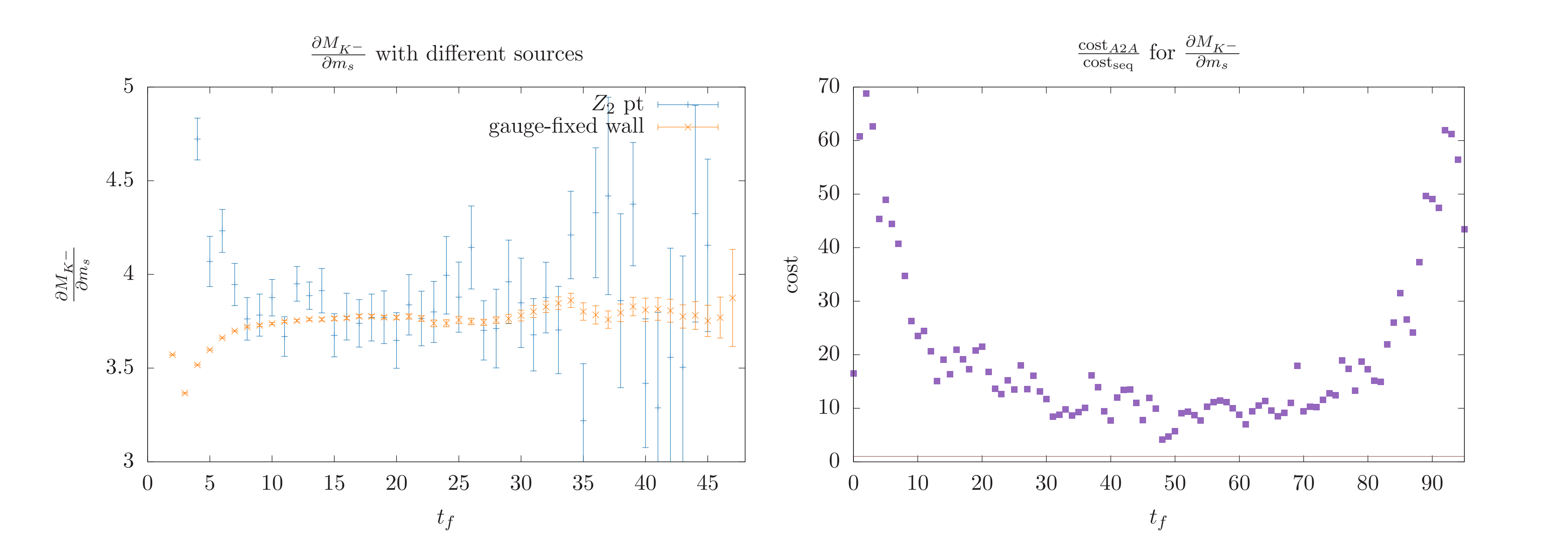


Figure 9: A signal (left) and cost (rght) comparison between two methods of current insertion for $\frac{\partial M_{K^-}}{\partial m_s}$.