

GETTING A NUMBER OUT OF A QCD AMPLITUDE

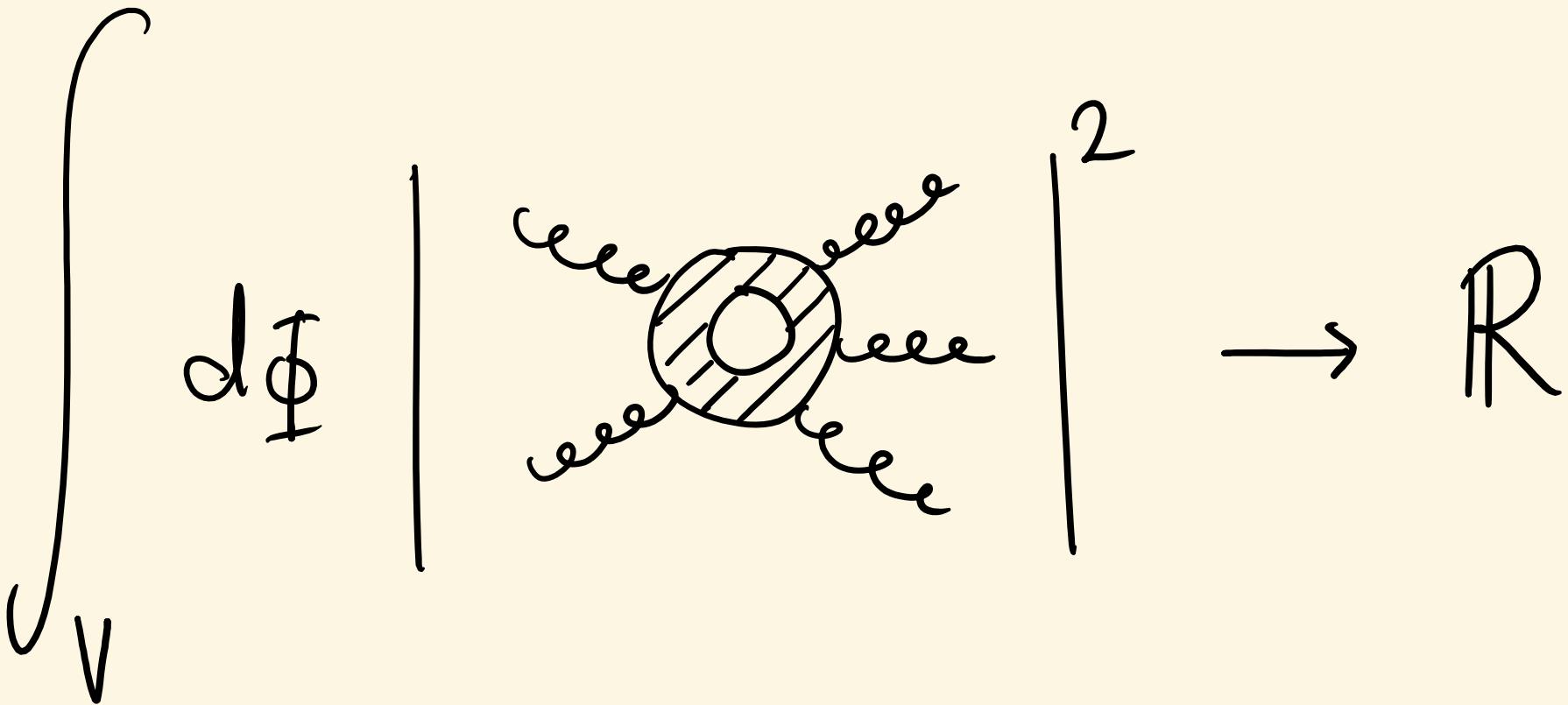
Ryan Moodie

OUTLINE

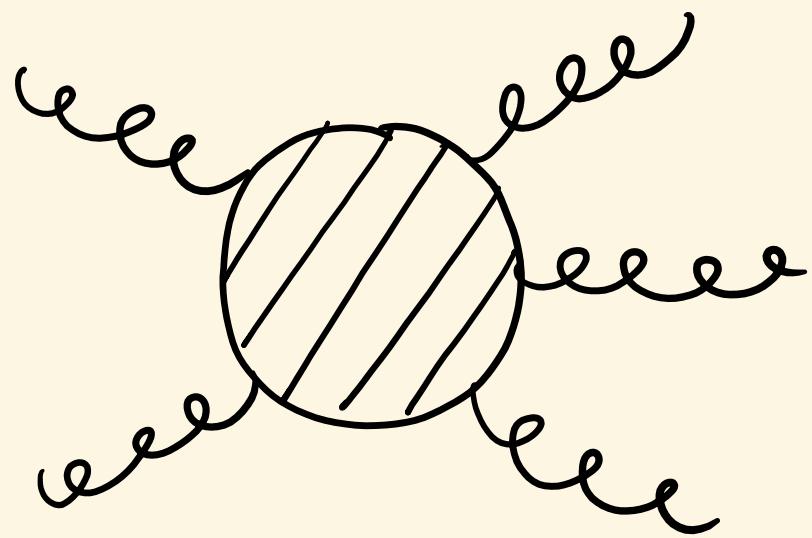
- Tree level
- Higher order corrections
- Infrared divergences
- Loops
- Computation

AIM

AIM



TREE LEVEL



HELICITY SUM

$$\frac{1}{4} \sum_{h_i \in \{+, -\}}$$

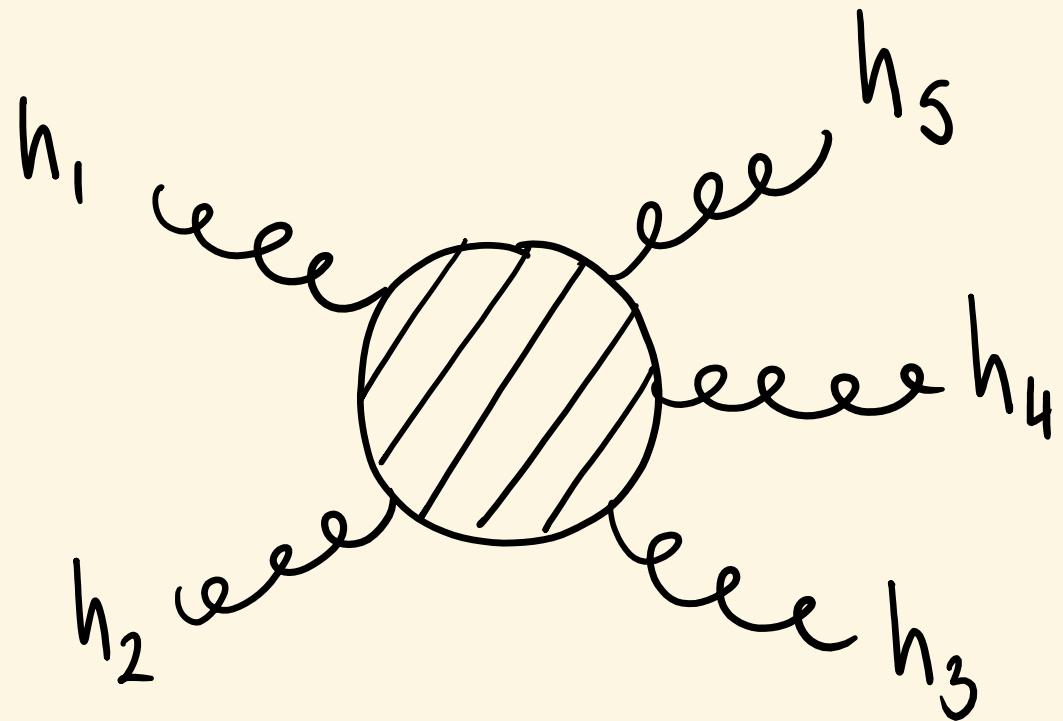
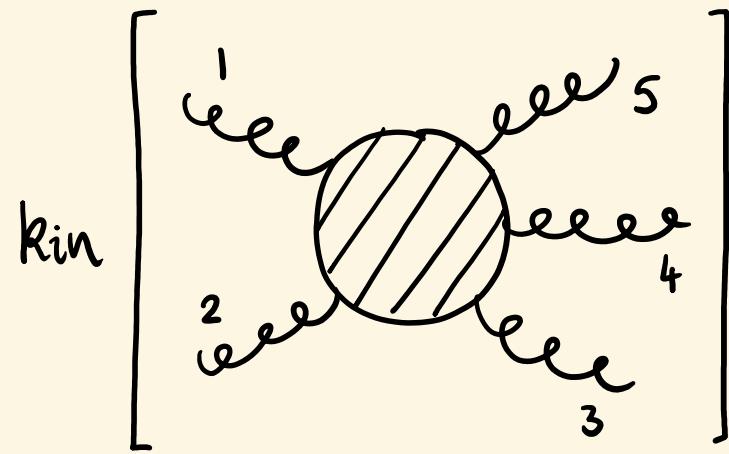
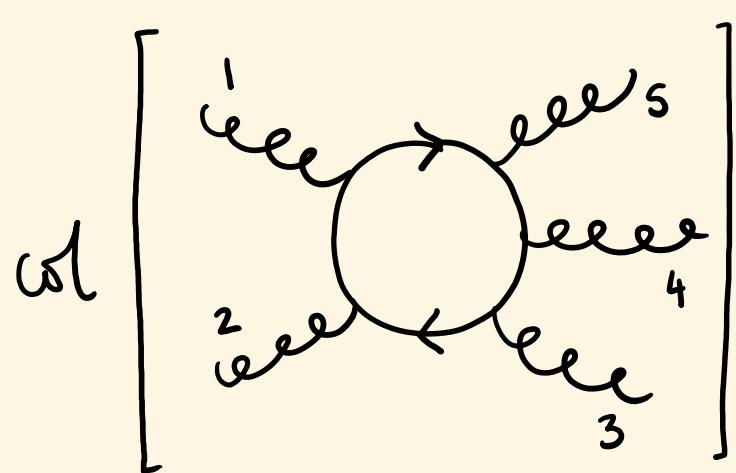


DIAGRAM SUM

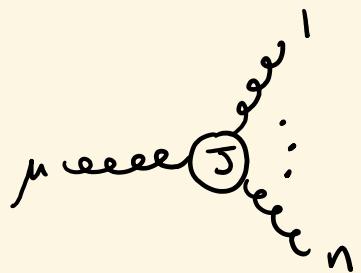
$$\text{Diagram} + \text{Diagram} + \dots \quad (25)$$
The image shows a mathematical expression for a diagram sum. It consists of two hand-drawn Feynman-like diagrams separated by a plus sign, followed by another plus sign and three dots, also preceded by a plus sign. The diagrams are composed of wavy lines and curly braces, representing different contributions to a sum. To the right of the entire expression is the label '(25)'.

COLOUR SUM

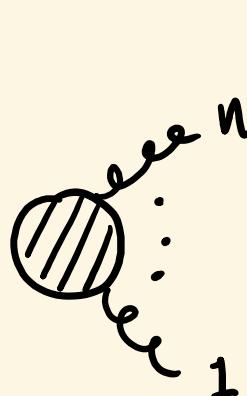
$$\sum_{\sigma}$$



RECURSION



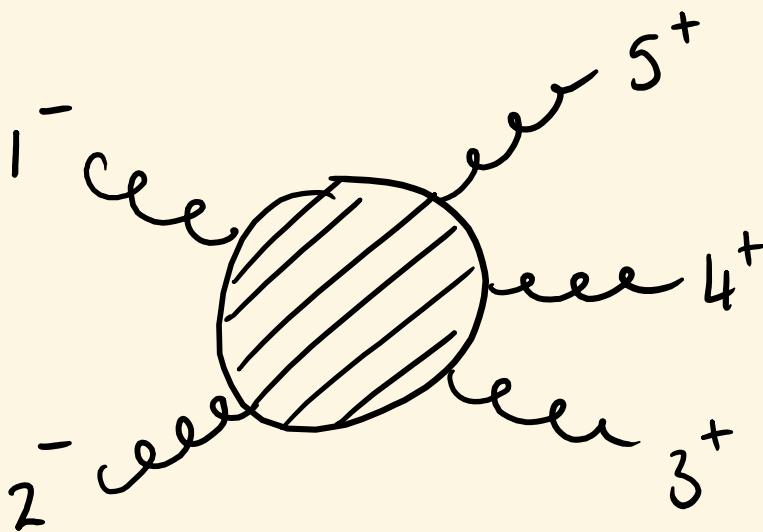
$$\mu \text{eeee} \circlearrowleft \text{eee}^n = \frac{-i}{S_{1\dots n}} \left[\sum_{i=1}^{n-1} \mu \text{eeee} \text{ee}^i \circlearrowleft \text{eee}^{i+1} + \sum_{i=1}^{n-2} \sum_{j=i+1}^{n-1} \mu \text{eeee} \text{ee}^i \circlearrowleft \text{eee}^{j+1} \right]$$



$$= \sum_{i=3}^{n-1} \sum_h \frac{1}{\left(\sum_{j=1}^{i-1} p_j \right)^2} \begin{array}{c} \text{eeee}^{i-1} \\ \vdots \\ \text{eeee}^1 \end{array} \circlearrowleft \hat{A}_L \begin{array}{c} \text{eeee}^{-h} \\ \vdots \\ \text{eeee}^1 \end{array} \begin{array}{c} \text{eeee}^h \\ \vdots \\ \text{eeee}^i \end{array} \circlearrowleft \hat{A}_R \begin{array}{c} \text{eeee}^n \\ \vdots \\ \text{eeee}^1 \end{array}$$

SPINOR HELICITY

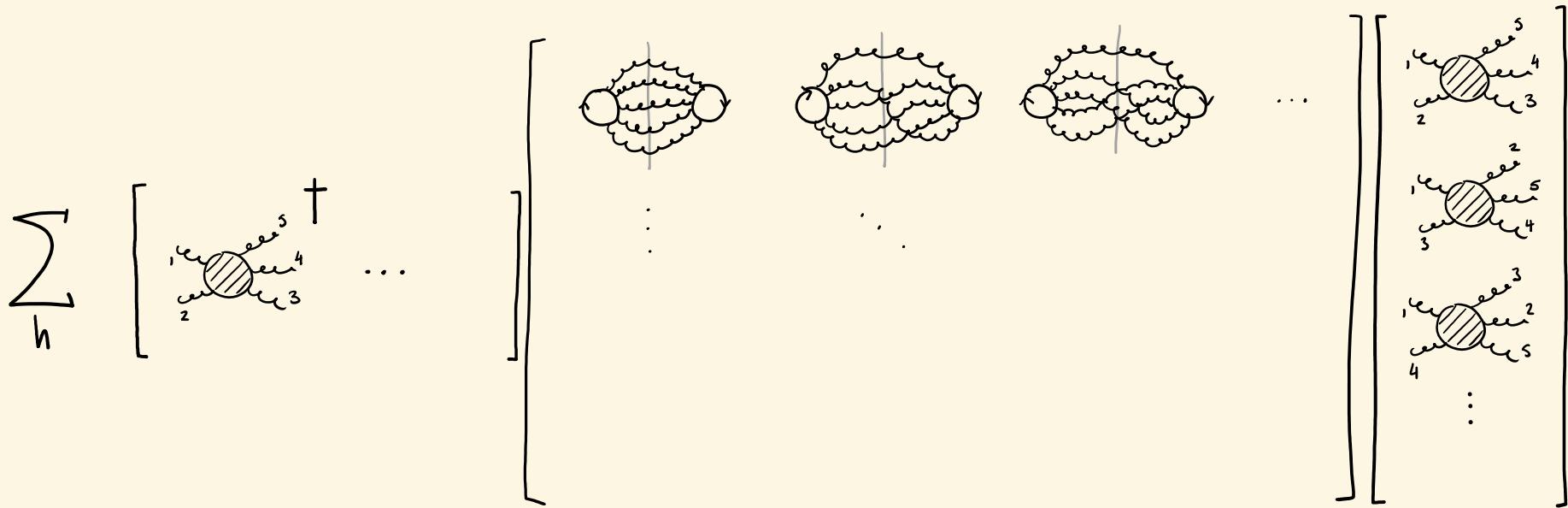
$$\psi = \begin{pmatrix} |1\rangle \\ |1\rangle \end{pmatrix}$$



A Feynman diagram showing a five-particle vertex. Five external lines, each labeled with a helicity value, meet at a central circular vertex. The top-right line is labeled 5^+ , the top-left 1^- , the bottom-left 2^- , the bottom-right 3^+ , and the middle-right 4^+ . The central vertex is shaded with diagonal lines.

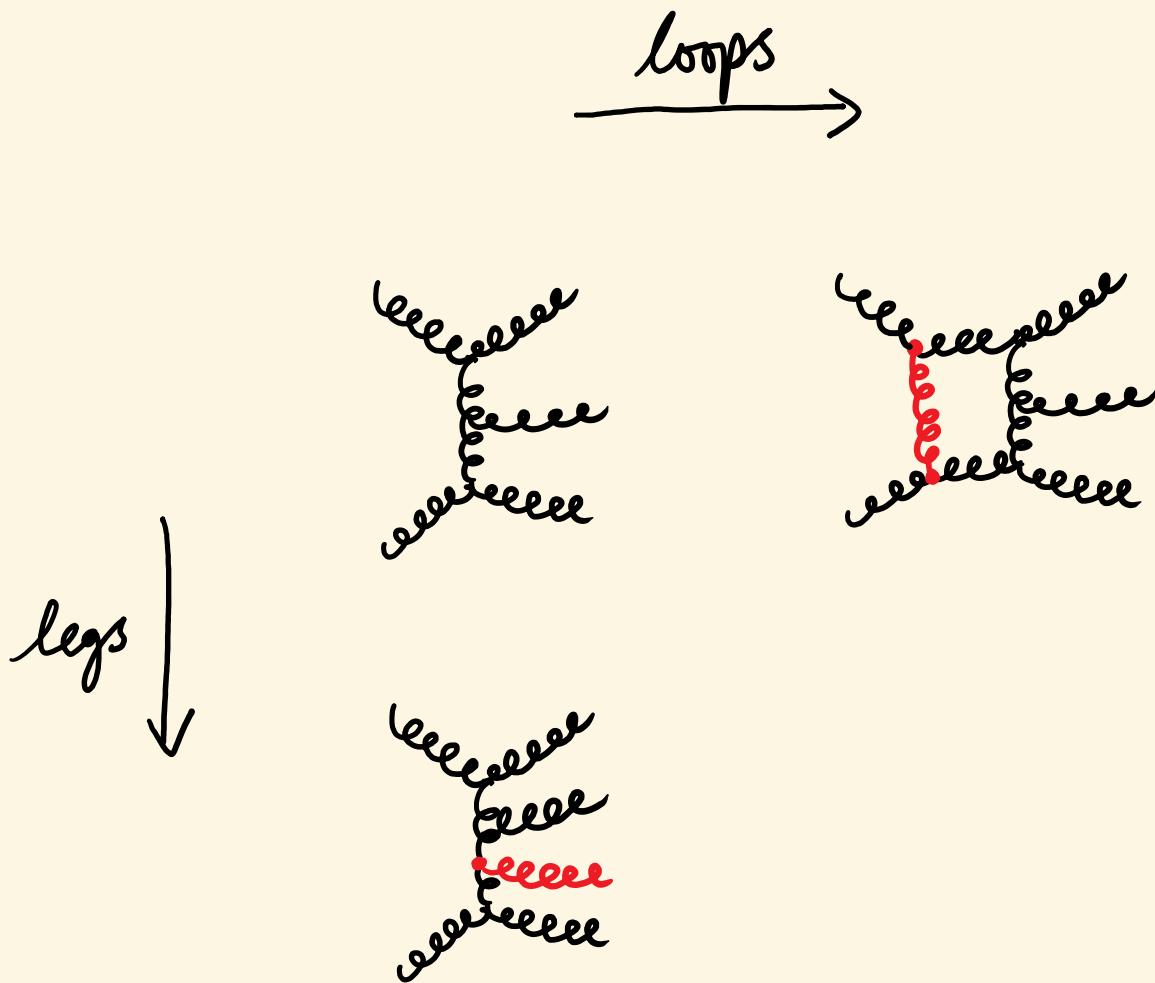
$$= \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

AMPLITUDE SQUARED



HIGHER ORDER CORRECTIONS

NEXT-TO-LEADING-ORDER



CROSS-SECTION

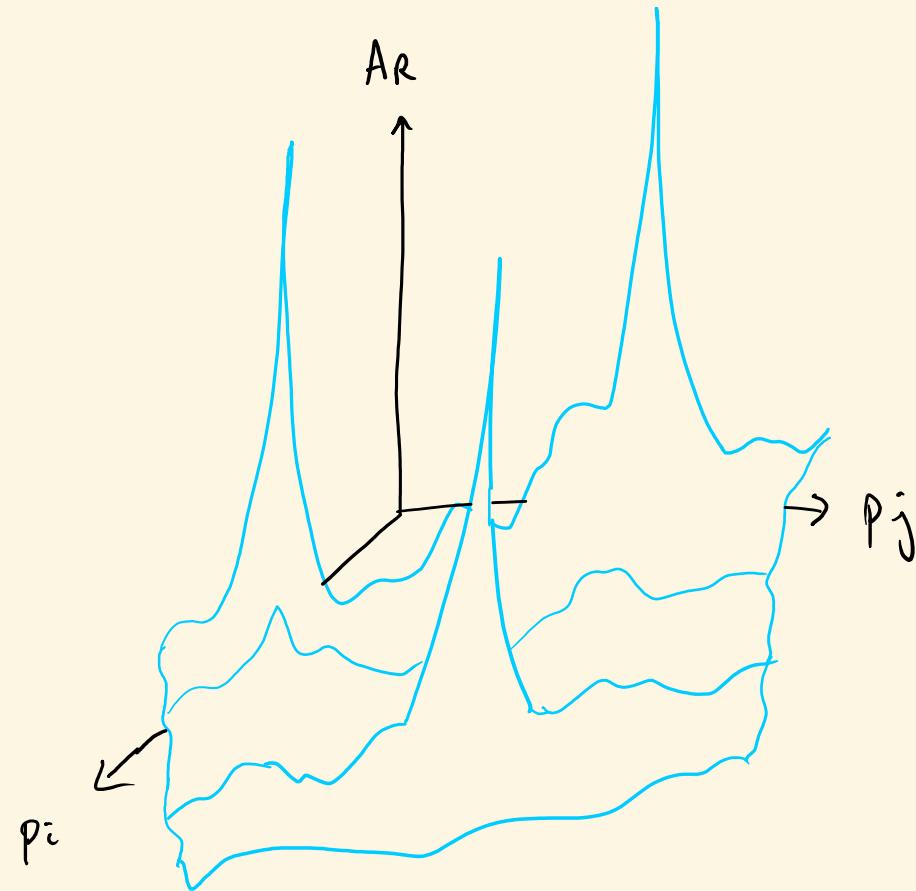
$$\int d\tilde{\Phi}_5 \left[\left| \text{Diagram} \right|^2 + 2 \operatorname{Re} \left(\text{Diagram}^+ \text{Diagram} \right) \right]$$

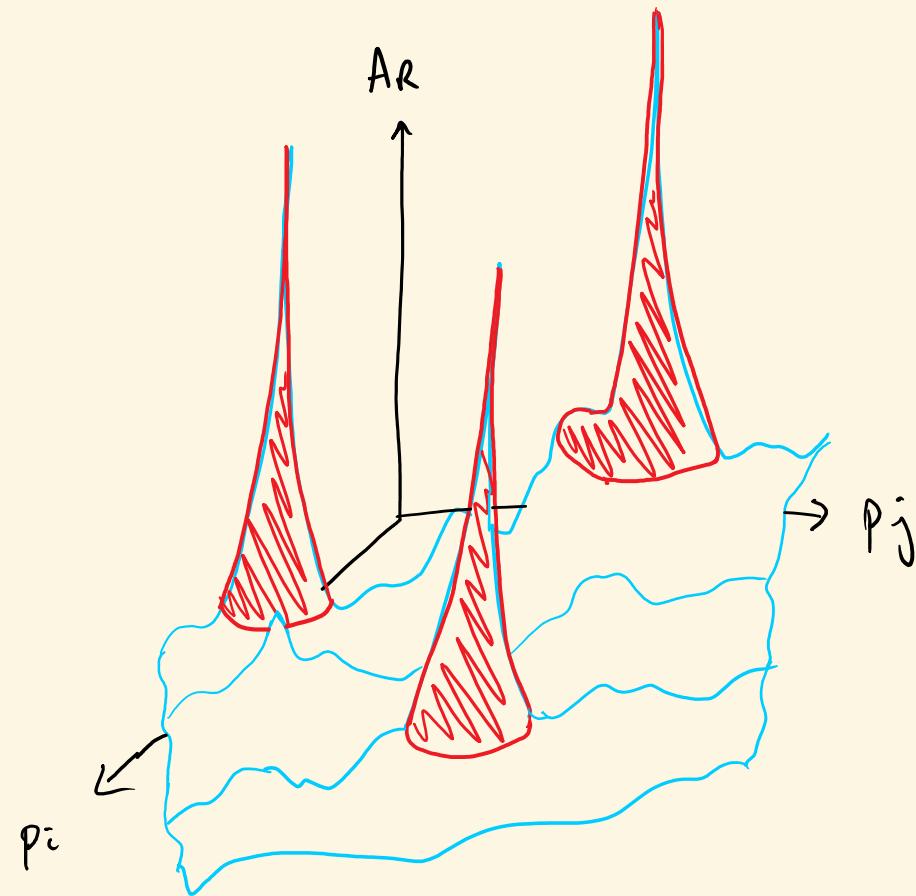
$$+ \int d\tilde{\Phi}_6 \left[\left| \text{Diagram} \right|^2 \right]$$

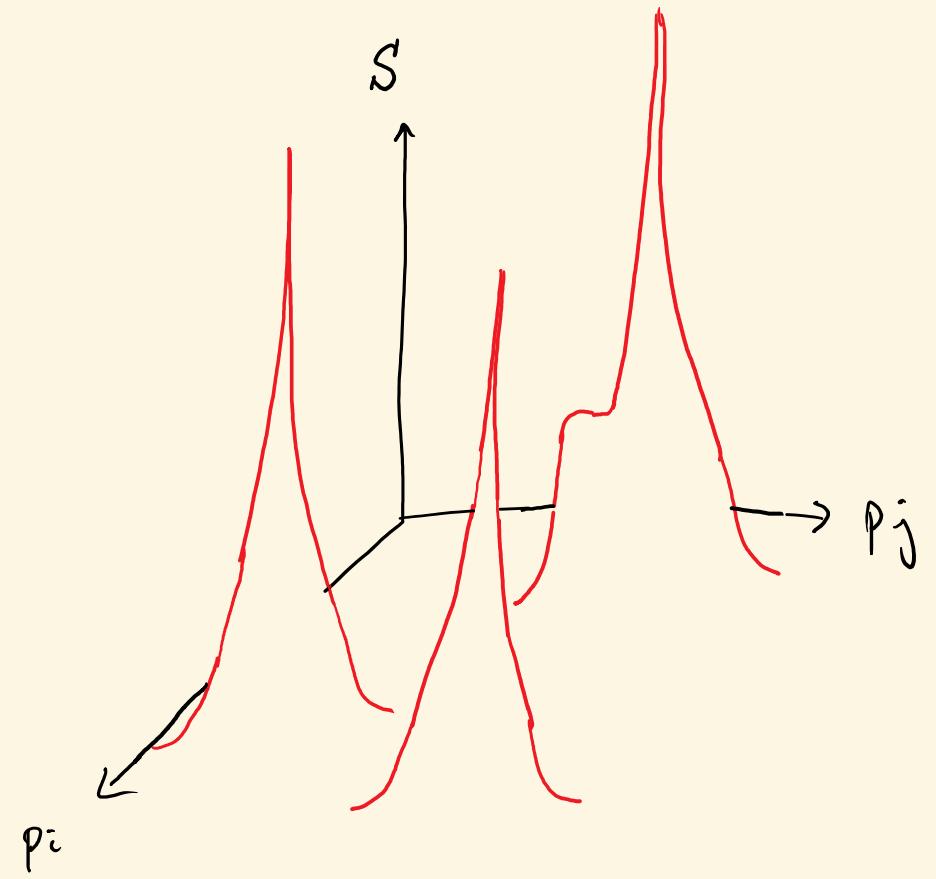
INFRARED DIVERGENCES

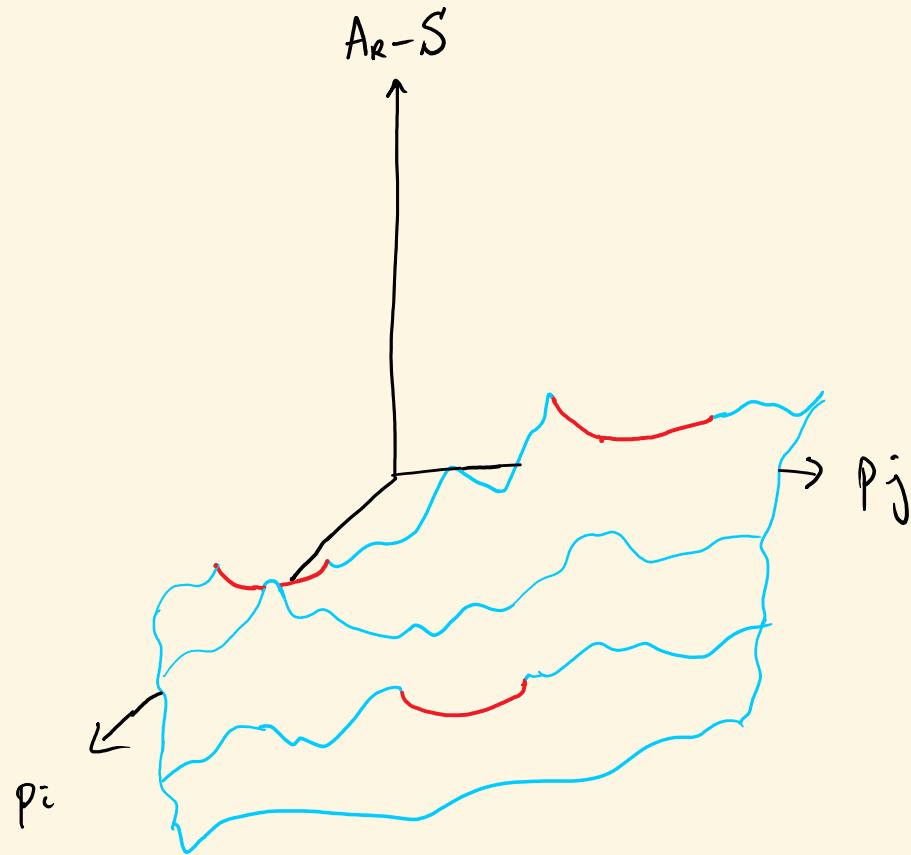
SUBTRACTION SCHEME

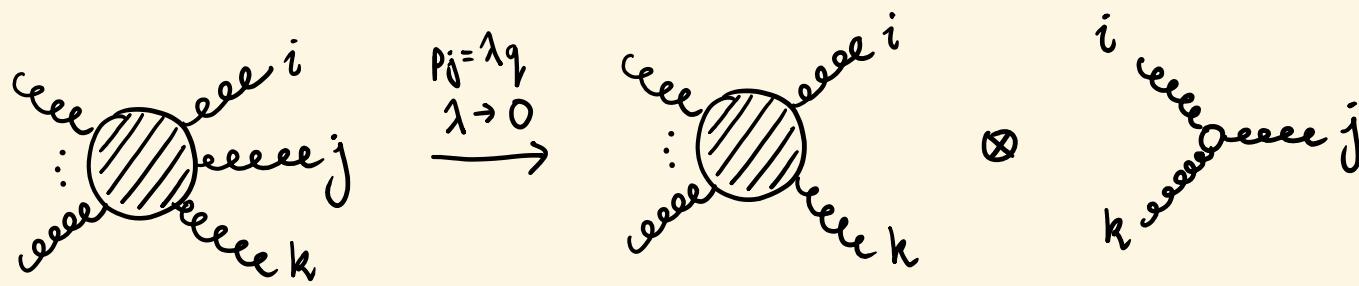
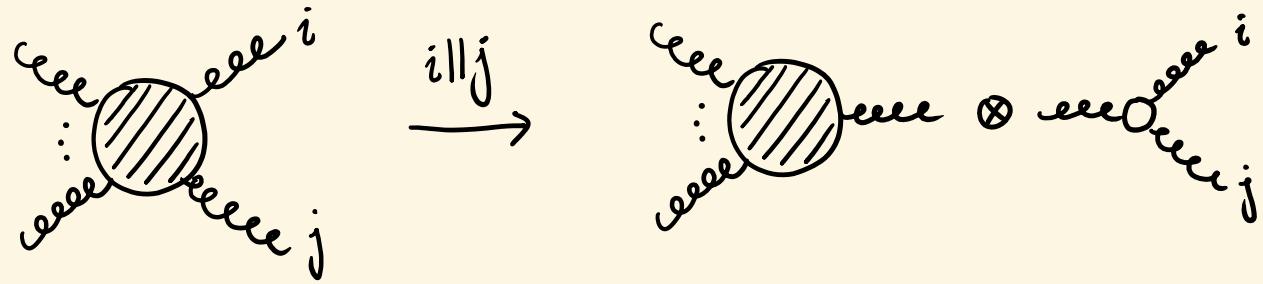
$$\int d\Phi_5 \left[| \text{Diagram} |^2 + 2 \operatorname{Re} \left(\text{Diagram}^+ \right) + I \right]$$
$$+ \int d\Phi_6 \left[| \text{Diagram} |^2 - S \right]$$











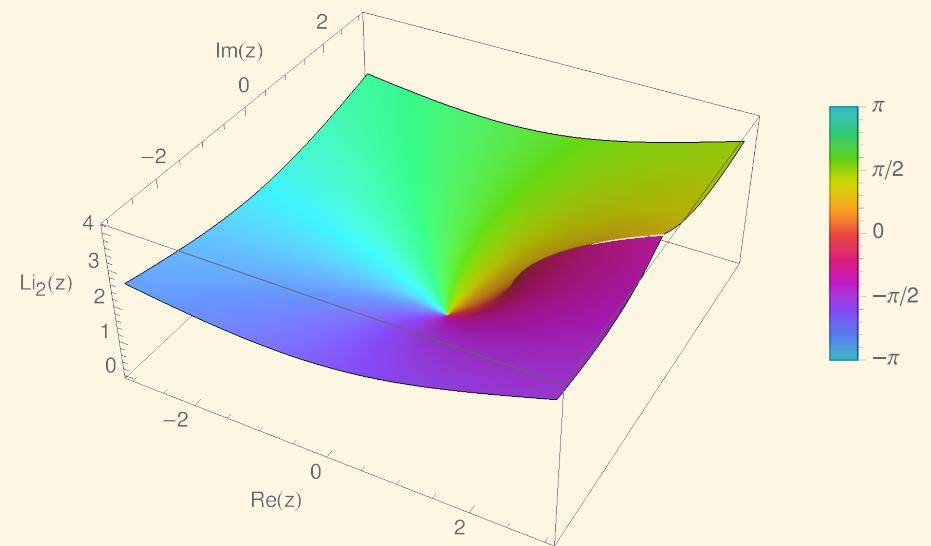
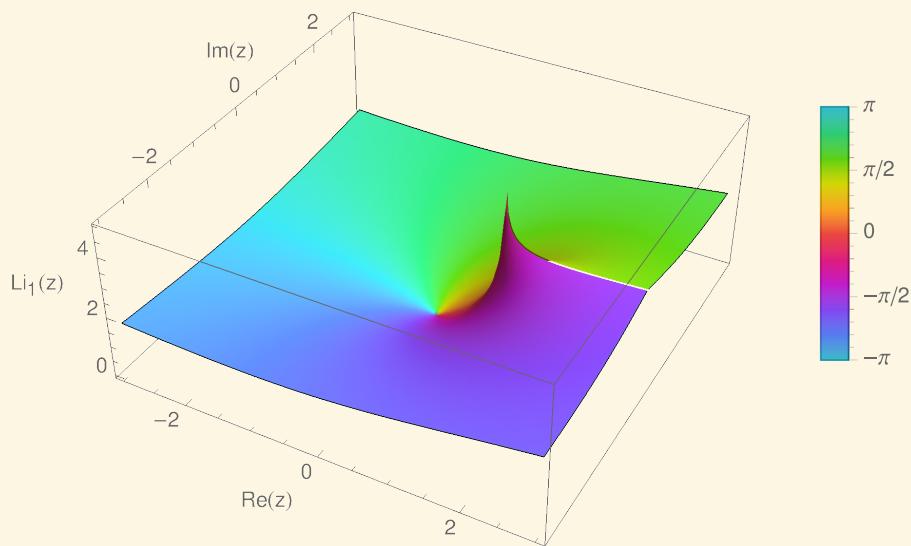
LOOPS

ALGEBRAIC

$$\text{Diagram} = \sum c \text{ Diagram} + \sum b \text{ Diagram} + \sum a \text{ Diagram} + R$$

- Integrand reduction
- Coefficients
 - Generalised unitarity
 - OPP
- Master integrals
 - Integration-by-parts identities
 - Laporta

ANALYTIC



- Polylogarithms
- Laurent expansion

COMPUTATION

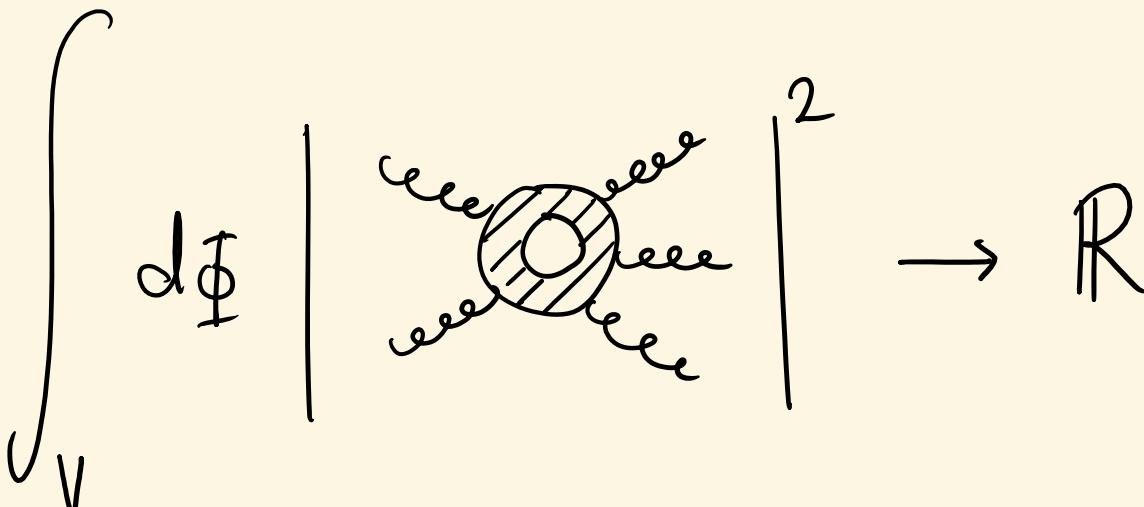
TECHNIQUES

- Large intermediate numbers
 - Finite fields
 - Momentum twistor variables
- High performance
 - FORM
 - C++

OVERVIEW

OVERVIEW

- Calculation organisation
- Infrared regulation
- Loop reduction
- Computation

$$\int_V d\phi \left| \text{Diagram} \right|^2 \rightarrow \mathbb{R}$$


// reveal.js plugins