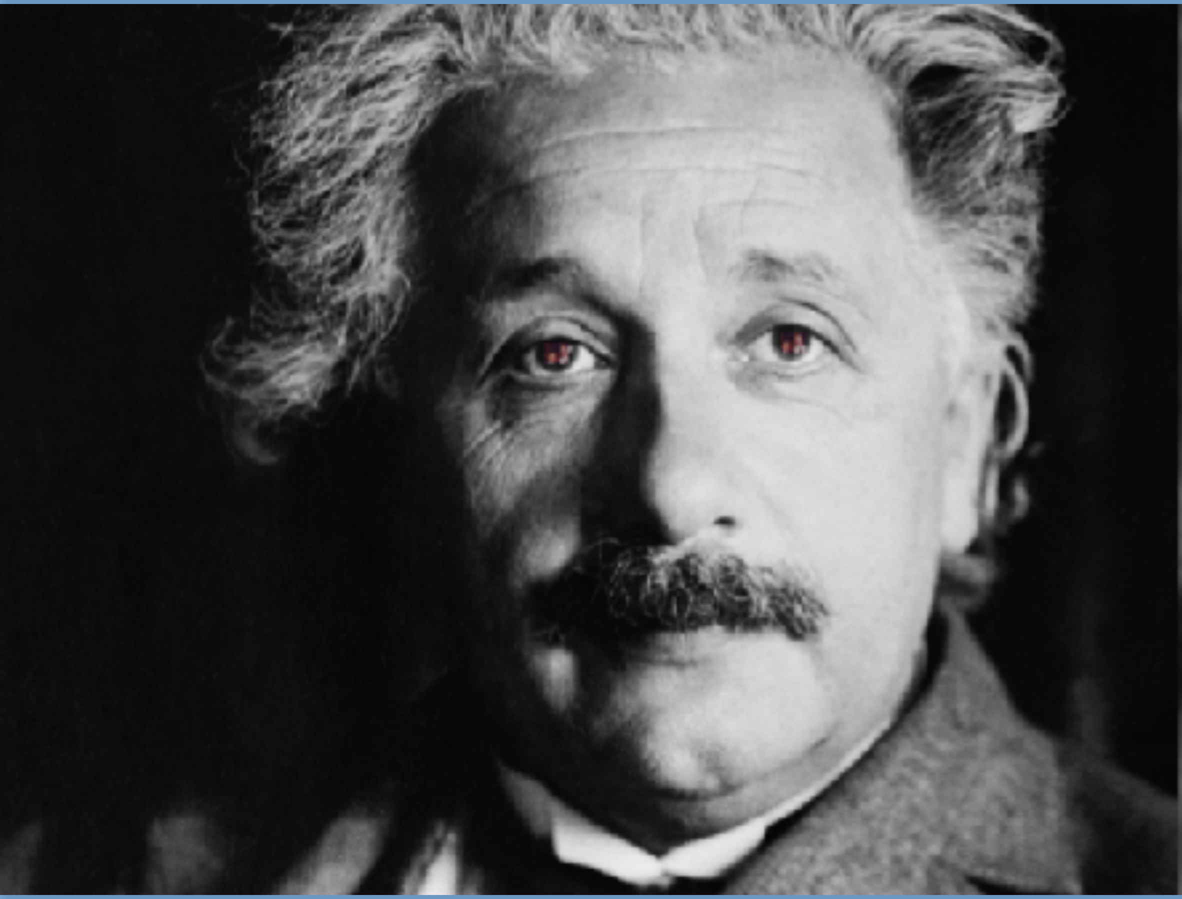
Pietro Ferrero
University of Oxford

On the Lagrangian formulation of the double copy to cubic order

Based on work with **Dario Francia**
arXiv: **2012.00713** [hep-th]

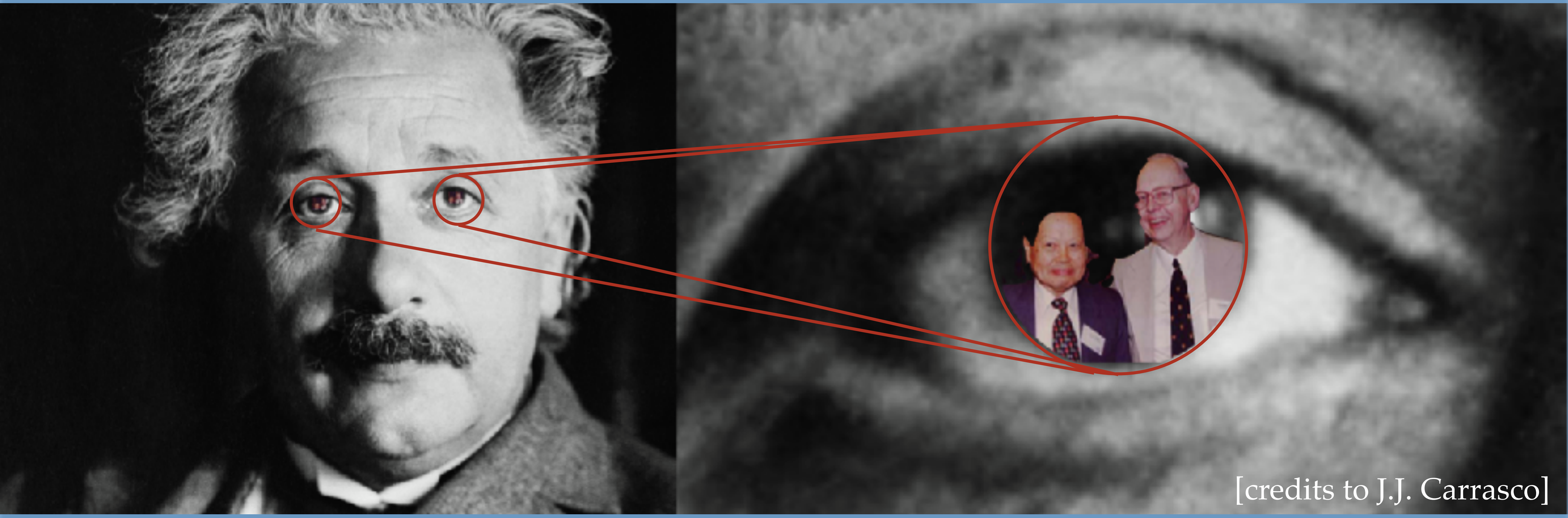
[credits to J.J. Carrasco]

Gravity



[credits to J.J. Carrasco]

Gravity

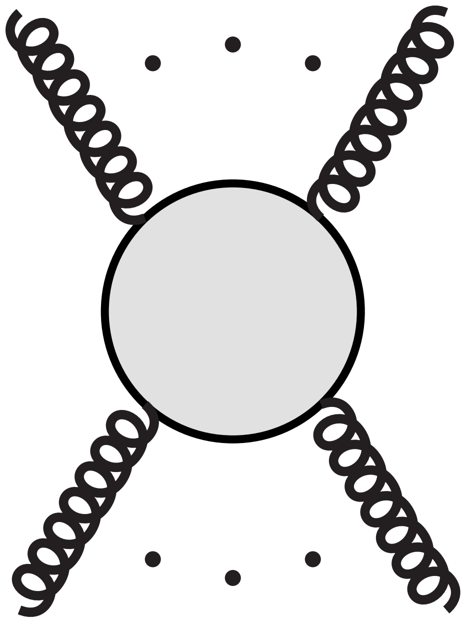


[credits to J.J. Carrasco]

=(Yang-Mills)^2

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

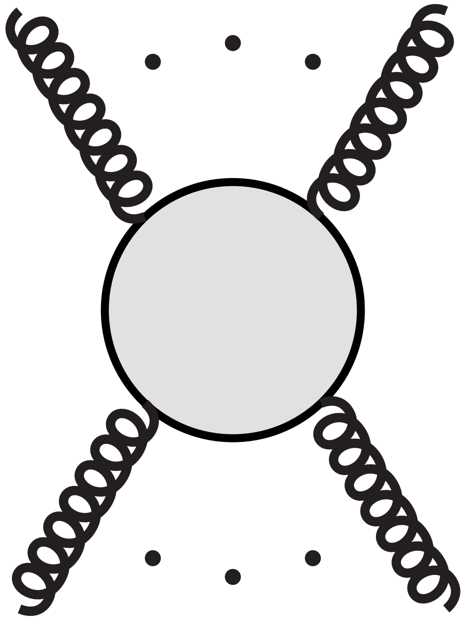


$$= g^{n-2} \sum_{i \in \Gamma_3} \frac{c_i\, n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

Yang-Mills Amplitude

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]



$$= g^{n-2} \sum_{\boxed{i \in \Gamma_3}} \frac{c_i \, n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

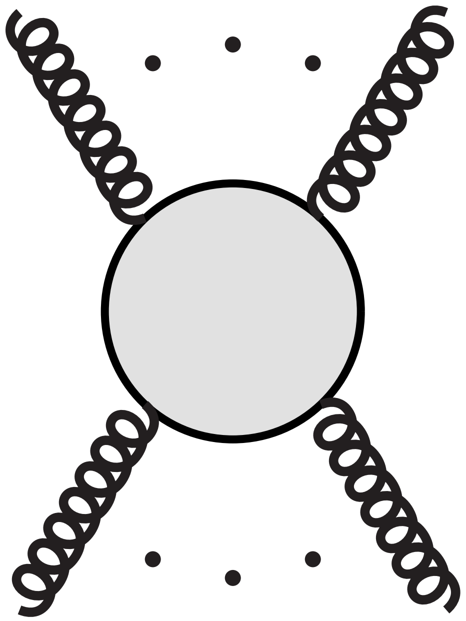
Yang-Mills Amplitude

Diagrams with cubic vertices

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

Colour factor:
Structure constants



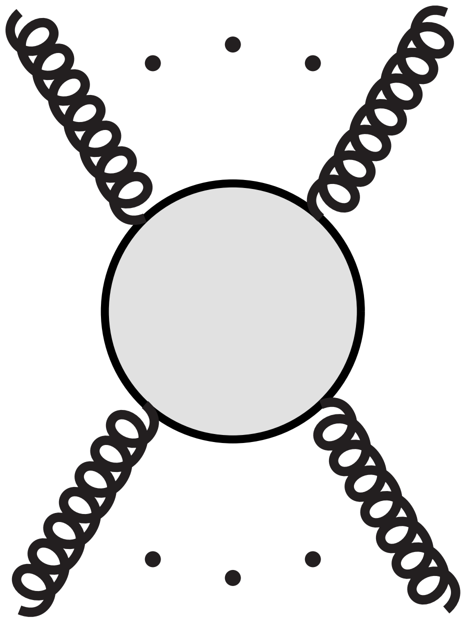
$$= g^{n-2} \sum_{\boxed{i\in\Gamma_3}} \frac{\boxed{c_i}\, n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

Yang-Mills Amplitude

Diagrams with
cubic vertices

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]



Yang-Mills Amplitude

$$= g^{n-2}\sum_{\boxed{i\in\Gamma_3}} \frac{\boxed{c_i}\,\boxed{n_i}}{\prod_{\alpha_i} s_{\alpha_i}}$$

Colour factor:
Structure constants

Kinematic factor:
Polarisations
& momenta

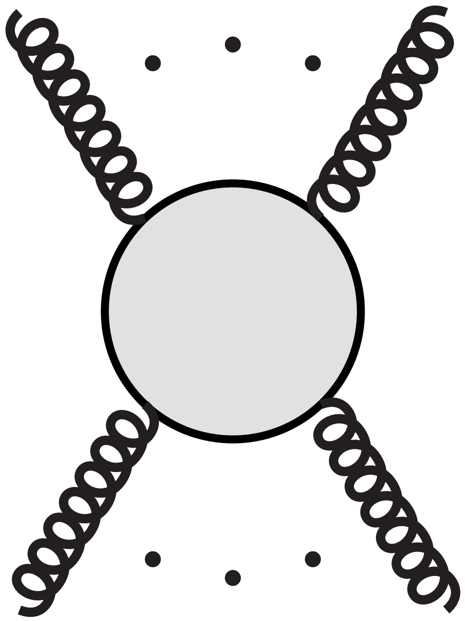
Diagrams with
cubic vertices

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

Kinematic factor:
Polarisations
& momenta

Colour factor:
Structure constants



$$= g^{n-2}\sum_{\boxed{i\in\Gamma_3}}\frac{\boxed{c_i}\,\boxed{n_i}}{\boxed{\prod_{\alpha_i} s_{\alpha_i}}}$$

Yang-Mills
Amplitude

Diagrams with
cubic vertices

Scalar propagators

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

Kinematic factor:
Polarisations
& momenta

Colour factor:
Structure constants

$$g^{n-2}\sum_{\boxed{i\in\Gamma_3}}\frac{\boxed{c_i}\,\boxed{n_i}}{\boxed{\prod_{\alpha_i} s_{\alpha_i}}}$$

Diagrams with
cubic vertices

Scalar propagators

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

Colour factor:
Structure constants

Kinematic factor:
Polarisations
& momenta

$$g^{n-2}\sum_{\boxed{i\in\Gamma_3}}\frac{\boxed{c_i}\,\boxed{n_i}}{\boxed{\prod_{\alpha_i} s_{\alpha_i}}}$$

Diagrams with
cubic vertices

Scalar propagators

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

$$i\left(\frac{\kappa}{2}\right)^{n-2}\sum_{\underbrace{i\in\Gamma_3}_{\substack{\text{Diagrams with}\\\text{cubic vertices}}}}\frac{\overbrace{c_i}^{\substack{\text{Colour factor:}\\\text{Structure constants}}}\overbrace{n_i}^{\substack{\text{Kinematic factor:}\\\text{Polarisations}\\\text{\& momenta}}}}{\underbrace{\prod_{\alpha_i}s_{\alpha_i}}_{\text{Scalar propagators}}}$$

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

Kinematic factor:
Polarisations
& momenta

$$i\left(\frac{\kappa}{2}\right)^{n-2}\sum_{\boxed{i\in\Gamma_3}}\frac{c_i\,\boxed{n_i}}{\boxed{\prod_{\alpha_i} s_{\alpha_i}}}$$

Diagrams with
cubic vertices

Scalar propagators

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

Kinematic factor:
Polarisations
& momenta

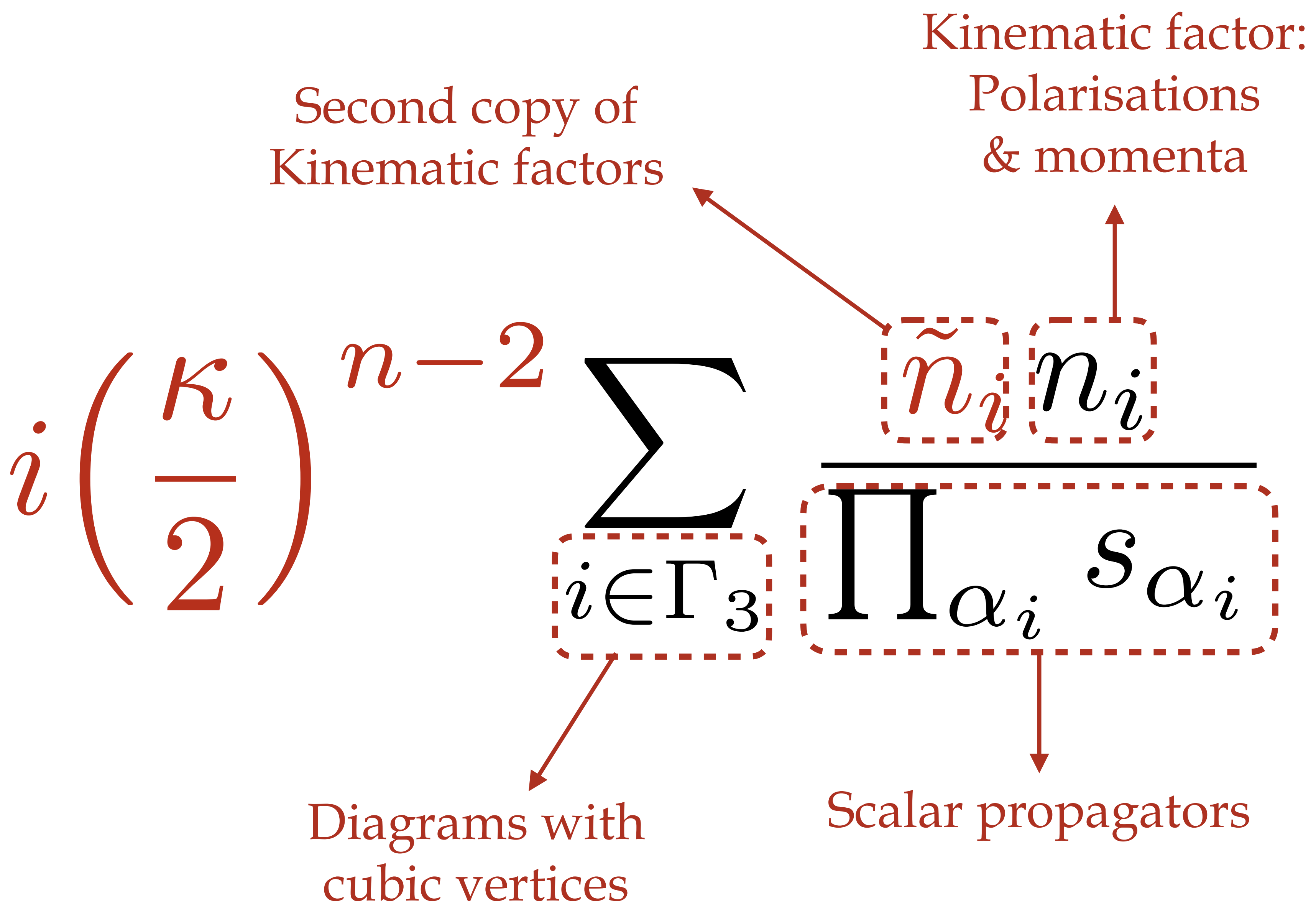
$$i\left(\frac{\kappa}{2}\right)^{n-2}\sum_{\boxed{i\in\Gamma_3}}\frac{\tilde{n}_i\,\boxed{n_i}}{\boxed{\prod_{\alpha_i} s_{\alpha_i}}}$$

Diagrams with
cubic vertices

Scalar propagators

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]



Second copy of
Kinematic factors

Kinematic factor:
Polarisations
& momenta

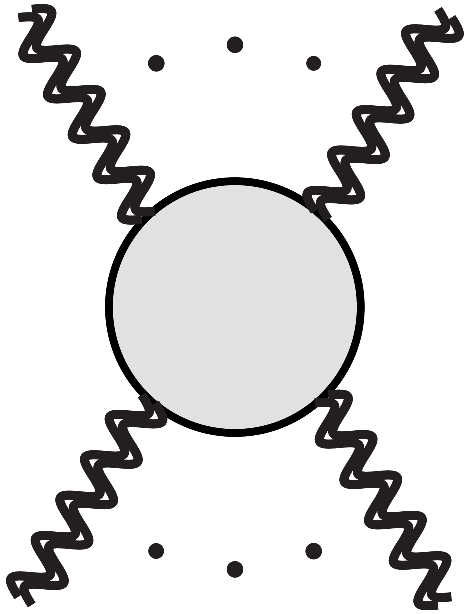
$$i\left(\frac{\kappa}{2}\right)^{n-2}\sum_{\boxed{i\in\Gamma_3}}\frac{\boxed{\tilde{n}_i}\,\boxed{n_i}}{\boxed{\prod_{\alpha_i} s_{\alpha_i}}}$$

Diagrams with
cubic vertices

Scalar propagators

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]



Gravity Amplitude

Second copy of
Kinematic factors

Kinematic factor:
Polarisations
& momenta

$$= i\left(\frac{\kappa}{2}\right)^{n-2}\sum_{i\in\Gamma_3}\frac{\tilde{n}_i\, n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

Diagrams with
cubic vertices

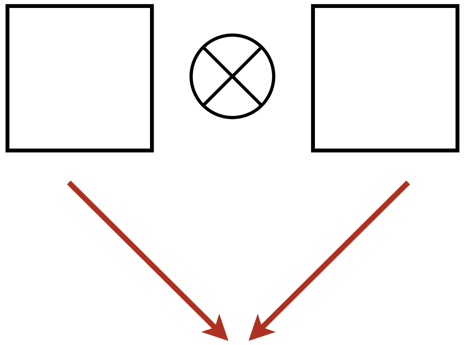
Scalar propagators

Not only gravity

In the **non supersymmetric** case:

Not only gravity

In the **non supersymmetric** case:

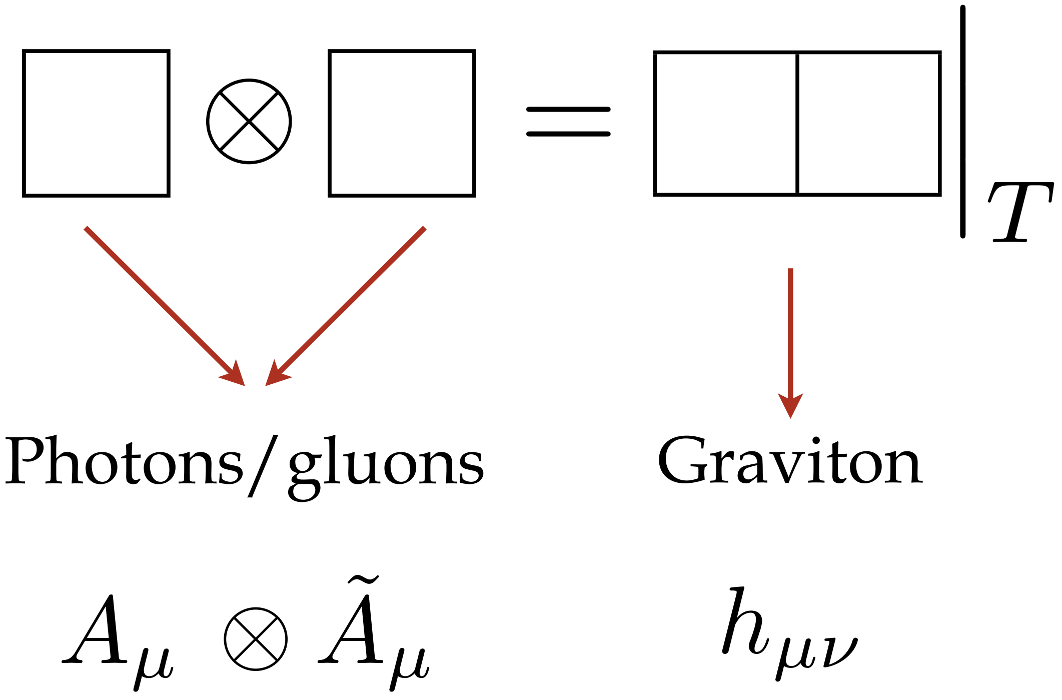


Photons/gluons

$$A_\mu \otimes \tilde{A}_\mu$$

Not only gravity

In the **non supersymmetric** case:



$$\Box \otimes \Box = \Box\!\Box \Big|_T$$

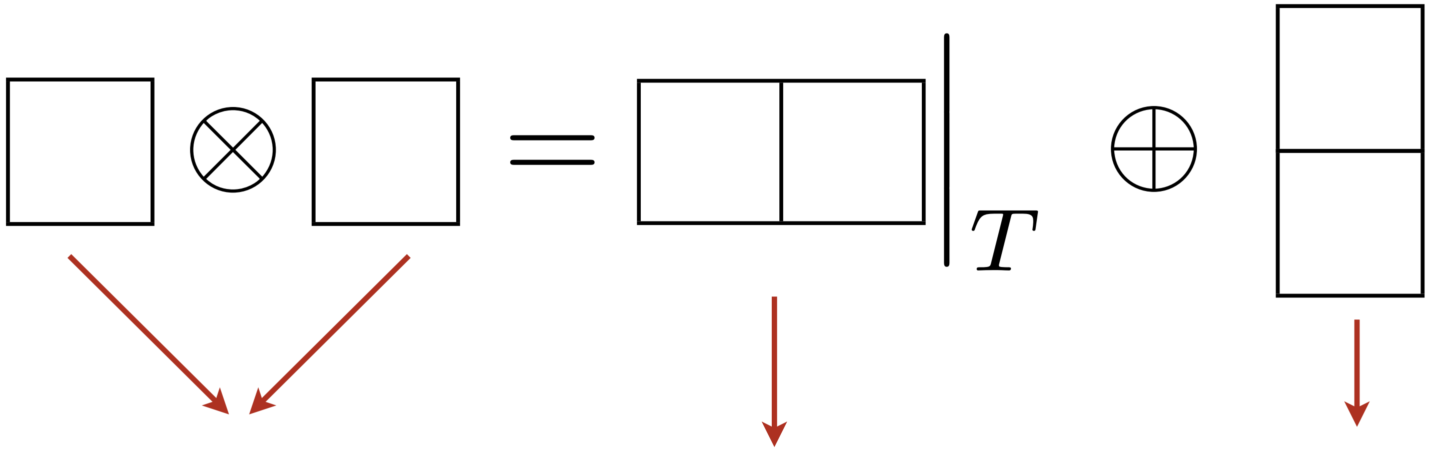
Photons/gluons

Graviton

$$A_\mu \otimes \tilde{A}_\mu \qquad h_{\mu\nu}$$

Not only gravity

In the **non supersymmetric** case:



Photons/gluons

Graviton

Two-form

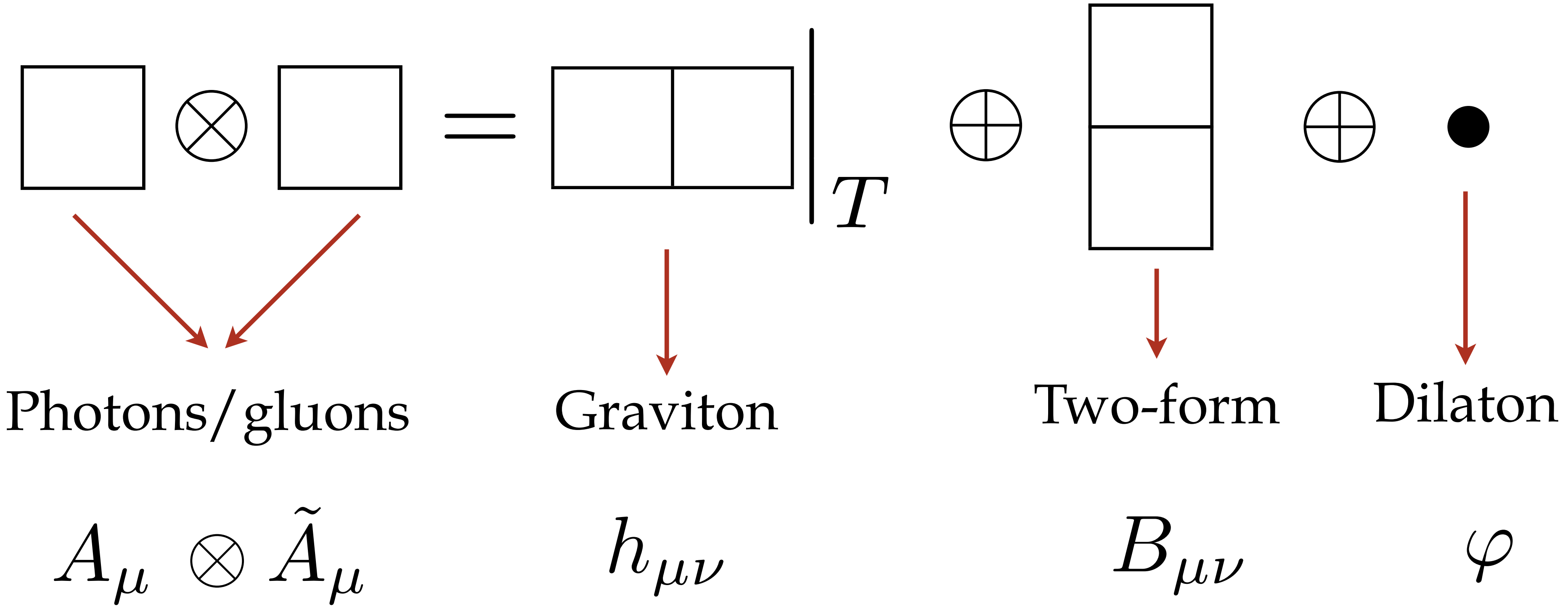
$A_\mu \otimes \tilde{A}_\mu$

$h_{\mu\nu}$

$B_{\mu\nu}$

Not only gravity

In the **non supersymmetric** case:



$$A_\mu \otimes \tilde{A}_\mu \qquad h_{\mu\nu} \qquad B_{\mu\nu} \qquad \varphi$$

A field theory realization

Anastasiou, Borsten, Duff, Hughes, Nagy 1408.4434 [hep-th]

$$H_{\mu\nu} = A_\mu \circ \tilde{A}_\nu$$

A field theory realization

Anastasiou, Borsten, Duff, Hughes, Nagy 1408.4434 [hep-th]

$$\boxed{H_{\mu\nu}} = A_\mu \circ \tilde{A}_\nu$$

Double copy
Field

A field theory realization

Anastasiou, Borsten, Duff, Hughes, Nagy 1408.4434 [hep-th]

$$\underbrace{\boxed{H_{\mu\nu}}}_{\text{Double copy Field}} = A_\mu \underbrace{\boxed{\circ}}_{\text{Convolution Product}} \tilde{A}_\nu$$

A field theory realization

Anastasiou, Borsten, Duff, Hughes, Nagy 1408.4434 [hep-th]

$$H_{\mu\nu} = A_\mu \circ \tilde{A}_\nu$$

Double copy
Field

Convolution
Product

$$h_{\mu\nu} = H_{(\mu\nu)} - \frac{1}{D-2}\,\eta_{\mu\nu}\,\varphi$$

$$B_{\mu\nu} = H_{[\mu\nu]}$$

$$\varphi = H^{\mu}{}_{\mu} - \frac{1}{\Box}\,\partial^\mu \partial^\nu H_{\mu\nu}$$

A field theory realization

Anastasiou, Borsten, Duff, Hughes, Nagy 1408.4434 [hep-th]

$$\boxed{H_{\mu\nu}} = A_\mu \boxed{\circ} \tilde{A}_\nu$$

Double copy
Field

Convolution
Product

$$h_{\mu\nu} = H_{(\mu\nu)} - \frac{1}{D-2}\eta_{\mu\nu}\,\varphi$$

$$B_{\mu\nu} = H_{[\mu\nu]}$$

$$\varphi = H^{\mu}_{\ \mu} - \frac{1}{\Box}\partial^\mu\partial^\nu H_{\mu\nu}$$

Gravitational local symmetries are correctly reproduced from spin one gauge transformations at the **linearized level**

A field strength for the DC

$$\mathcal{R}_{\mu\nu\rho\sigma} = -\frac{1}{2}\,F_{\mu\nu} \circ \tilde{F}_{\rho\,\sigma}$$

A field strength for the DC

$$\mathcal{R}_{\mu\nu\rho\sigma} = -\frac{1}{2}\boxed{F_{\mu\nu}} \circ \boxed{\tilde{F}_{\rho\,\sigma}}$$

Linearized spin-one
field strengths

A field strength for the DC

$$\mathcal{R}_{\mu\nu\rho\sigma} = -\frac{1}{2}\boxed{F_{\mu\nu}}\circ\boxed{\tilde{F}_{\rho\,\sigma}}$$

Linearized spin-one
field strengths

Invariant under:

A field strength for the DC

$$\mathcal{R}_{\mu\nu\rho\sigma} = -\frac{1}{2}\,\boxed{F_{\mu\nu}} \circ \boxed{\tilde{F}_{\rho\,\sigma}}$$

Linearized spin-one
field strengths

Invariant under:

$$\delta\, H_{\mu\nu} = \partial_\mu\, \alpha_\nu + \partial_\nu\, \tilde{\alpha}_\mu = \delta h_{\mu\nu} + \delta B_{\mu\nu}$$

Is there a way to write

$$\mathcal{L}_{\text{Gravity}} = \mathcal{L}_{\text{YM}} \otimes \widetilde{\mathcal{L}}_{\text{YM}}$$

?

Is there a way to write

$$\mathcal{L}_{\text{Gravity}} = \mathcal{L}_{\text{YM}} \otimes \widetilde{\mathcal{L}}_{\text{YM}}$$

?

For independent attempts, see
Bern, Dennen, Huang, Kiermaier 1004.0694 [hep-th]
Borsten, Nagy 2004.14945 [hep-th]
Borsten, Jurco, Kim, Macrelli, Saemann, Wolf 2007.13803 [hep-th]

Free Lagrangians for the DC

Free Lagrangians for the DC

1) **Maxwell-like** Lagrangian **Campoleoni, Francia 1206.5877 [hep-th]**

Free Lagrangians for the DC

1) **Maxwell-like** Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} \,=\, \frac{1}{2}\, H^{\mu\nu} \, \left(\eta_{\rho\mu}\, \eta_{\nu\sigma}\, \Box \,-\, \eta_{\rho\mu}\, \partial_\sigma\, \partial_\nu \,-\, \eta_{\sigma\nu}\, \partial_\rho\, \partial_\mu \, \right) \, H^{\rho\sigma}$$

Free Lagrangians for the DC

1) **Maxwell-like** Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} = \frac{1}{2} H^{\mu\nu} \left(\eta_{\rho\mu} \eta_{\nu\sigma} \Box - \eta_{\rho\mu} \partial_\sigma \partial_\nu - \eta_{\sigma\nu} \partial_\rho \partial_\mu \right) H^{\rho\sigma}$$

Local

Relation to tensionless strings ✓

Free Lagrangians for the DC

1) **Maxwell-like** Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} \;=\; \frac{1}{2}\, H^{\mu\nu}\,\left(\,\eta_{\rho\mu}\,\eta_{\nu\sigma}\,\Box \;-\; \eta_{\rho\mu}\,\partial_\sigma\,\partial_\nu \;-\; \eta_{\sigma\nu}\,\partial_\rho\,\partial_\mu\,\right)\, H^{\rho\sigma}$$

Local

Relation to tensionless strings

✓

Requires constraint

$$\partial^\mu\,\left(\alpha_\mu \,+\, \widetilde{\alpha}_\mu\right) \,=\, 0$$

Free Lagrangians for the DC

1) **Maxwell-like** Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} = \frac{1}{2} H^{\mu\nu} \left(\eta_{\rho\mu} \eta_{\nu\sigma} \Box - \eta_{\rho\mu} \partial_\sigma \partial_\nu - \eta_{\sigma\nu} \partial_\rho \partial_\mu \right) H^{\rho\sigma}$$

Local

Relation to tensionless strings

✓

Requires constraint

$$\partial^\mu \left(\alpha_\mu + \widetilde{\alpha}_\mu \right) = 0$$

2) **Non-local** Lagrangian

Francia 1001.5003 [hep-th]

Free Lagrangians for the DC

1) **Maxwell-like** Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} = \frac{1}{2} H^{\mu\nu} \left(\eta_{\rho\mu}\,\eta_{\nu\sigma}\,\Box - \eta_{\rho\mu}\,\partial_\sigma\,\partial_\nu - \eta_{\sigma\nu}\,\partial_\rho\,\partial_\mu \right) H^{\rho\sigma}$$

Local

Relation to tensionless strings

✓

Requires constraint

$$\partial^\mu \left(\alpha_\mu + \widetilde{\alpha}_\mu\right) = 0$$

2) **Non-local** Lagrangian

Francia 1001.5003 [hep-th]

$$\mathcal{L}_{NL} = \frac{1}{2}\,\mathcal{R}_{\mu\nu\rho\sigma}\,\frac{1}{\Box}\,\mathcal{R}^{\mu\nu\rho\sigma} = \frac{1}{8}\left(F_{\mu\nu}\circ\widetilde{F}_{\rho\sigma}\right)\frac{1}{\Box}\left(F^{\mu\nu}\circ\widetilde{F}^{\rho\sigma}\right)$$

Free Lagrangians for the DC

1) **Maxwell-like** Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} \,=\, \frac{1}{2}\, H^{\mu\nu} \left(\eta_{\rho\mu}\, \eta_{\nu\sigma}\, \Box \,-\, \eta_{\rho\mu}\, \partial_\sigma\, \partial_\nu \,-\, \eta_{\sigma\nu}\, \partial_\rho\, \partial_\mu \right) H^{\rho\sigma}$$

Local

Relation to tensionless strings ✓

Requires constraint

$$\partial^\mu \left(\alpha_\mu \,+\, \widetilde{\alpha}_\mu\right) \,=\, 0$$

2) **Non-local** Lagrangian

Francia 1001.5003 [hep-th]

$$\mathcal{L}_{NL} \,=\, \frac{1}{2}\, \mathcal{R}_{\mu\nu\rho\sigma}\, \frac{1}{\Box}\, \mathcal{R}^{\mu\nu\rho\sigma} \,=\, \frac{1}{8}\, \Big(F_{\mu\nu} \circ \widetilde{F}_{\rho\sigma} \Big)\, \frac{1}{\Box}\, \Big(F^{\mu\nu} \circ \widetilde{F}^{\rho\sigma} \Big)$$

Manifest DC structure

Extends to massive theory ✓

Relation to tensionless strings

Free Lagrangians for the DC

1) **Maxwell-like** Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} \;=\; \tfrac{1}{2}\, H^{\mu\nu} \,\left(\eta_{\rho\mu}\,\eta_{\nu\sigma}\,\Box \;-\; \eta_{\rho\mu}\,\partial_\sigma\,\partial_\nu \;-\; \eta_{\sigma\nu}\,\partial_\rho\,\partial_\mu\,\right)\, H^{\rho\sigma}$$

Local

Relation to tensionless strings

✓

Requires constraint

$$\partial^\mu\,(\alpha_\mu + \widetilde{\alpha}_\mu) \,=\, 0$$

2) **Non-local** Lagrangian

Francia 1001.5003 [hep-th]

$$\mathcal{L}_{NL} \;=\; \tfrac{1}{2}\,\mathcal{R}_{\mu\nu\rho\sigma}\,\frac{1}{\Box}\,\mathcal{R}^{\mu\nu\rho\sigma} \;=\; \tfrac{1}{8}\,\Big(F_{\mu\nu}\,\circ\,\widetilde{F}_{\rho\sigma}\Big)\,\frac{1}{\Box}\,\Big(F^{\mu\nu}\,\circ\,\widetilde{F}^{\rho\sigma}\Big)$$

Manifest DC structure

Extends to massive theory

Relation to tensionless strings

✓

Non-local

Interactions?

The two Lagrangians are equivalent,
and both can be extended to include cubic
interactions by means of the Noether procedure

Thank you
for the attention!