

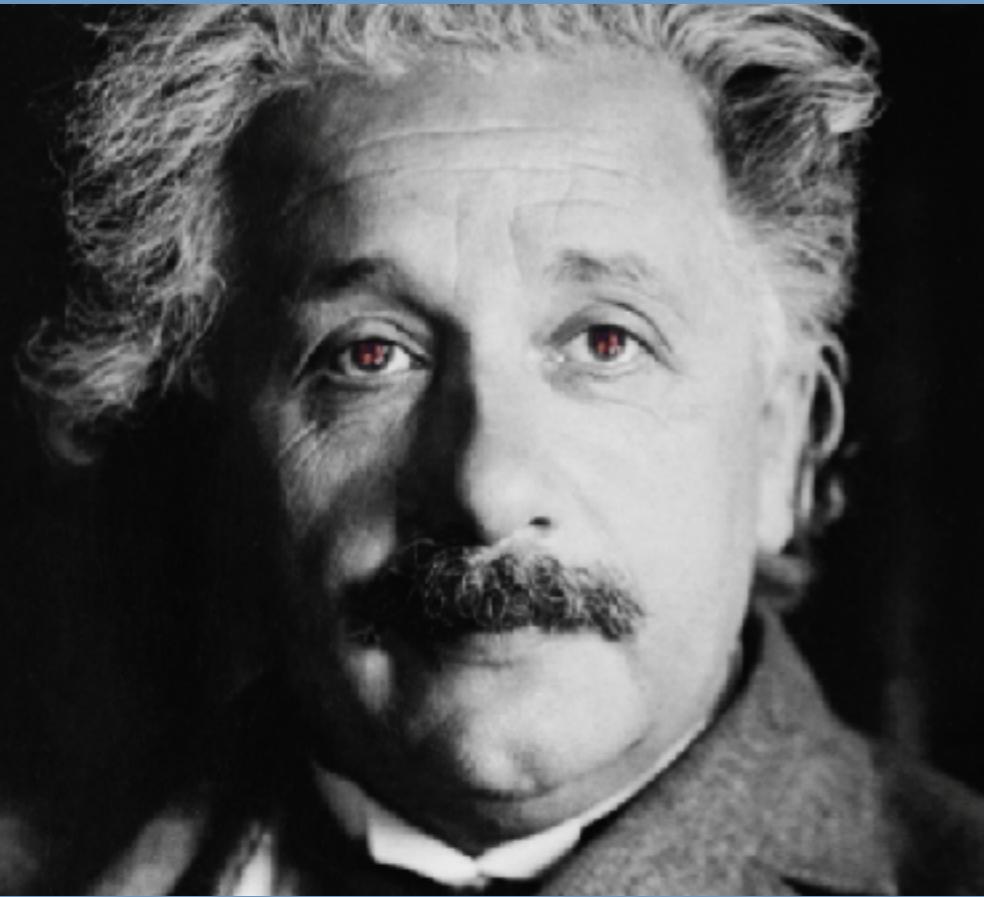


Pietro Ferrero
University of Oxford



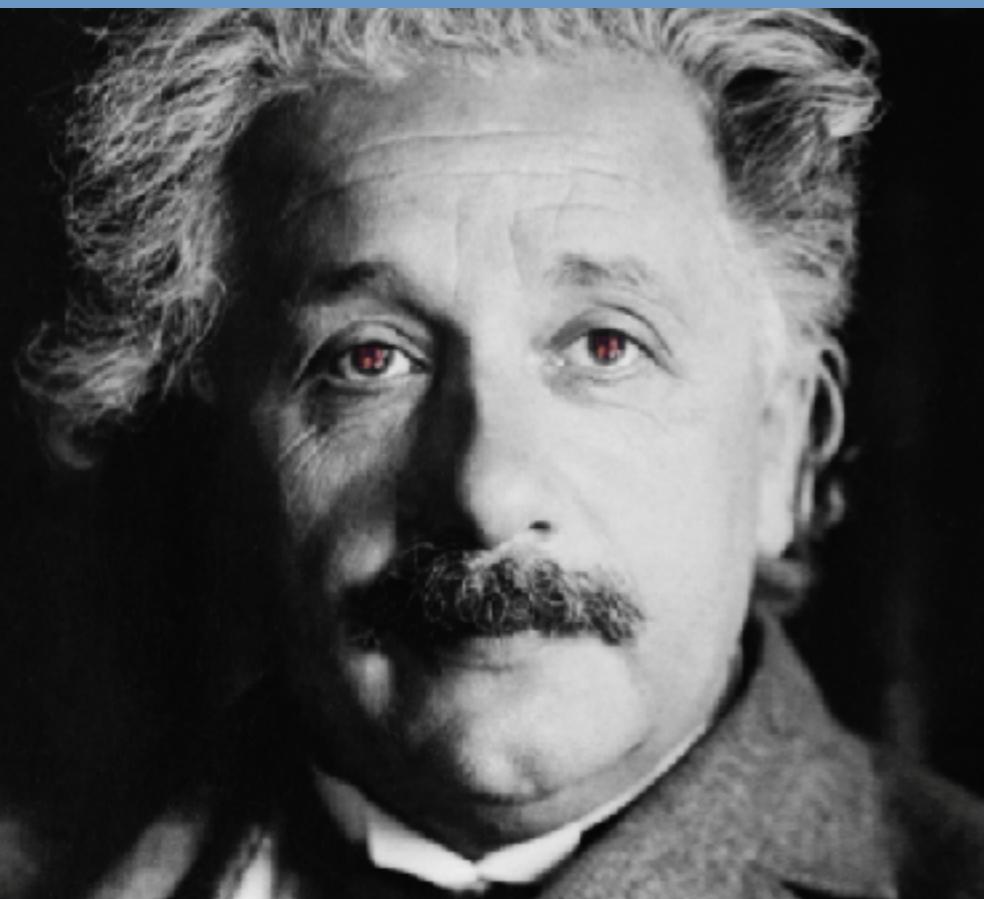
On the Lagrangian formulation of the double copy to cubic order

Based on work with Dario Francia
arXiv: 2012.00713 [hep-th]



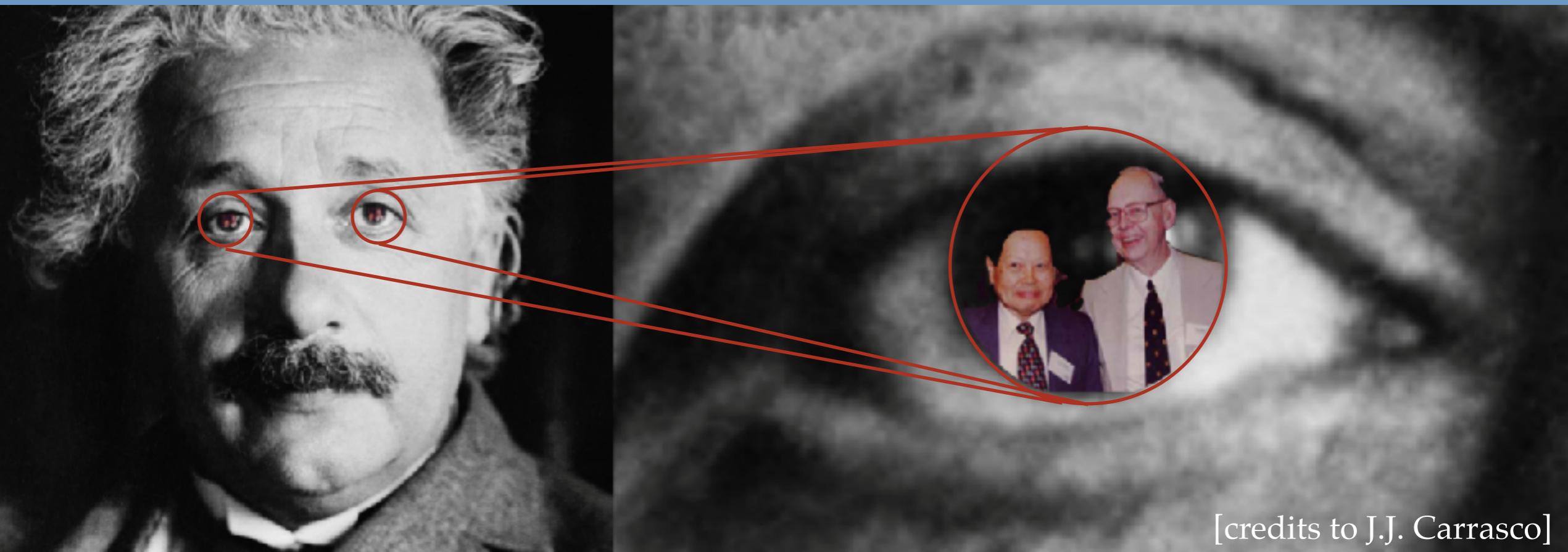
[credits to J.J. Carrasco]

Gravity



[credits to J.J. Carrasco]

Gravity

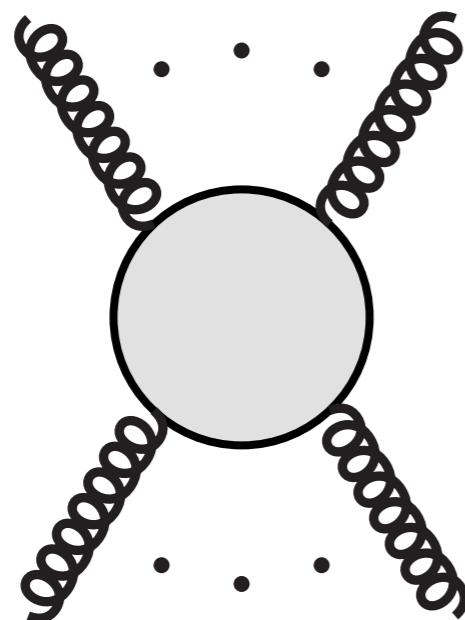


[credits to J.J. Carrasco]

$$= (\text{Yang-Mills})^2$$

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

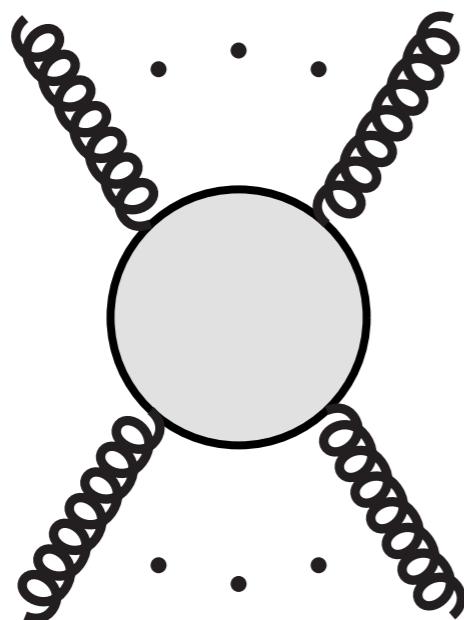


$$= g^{n-2} \sum_{i \in \Gamma_3} \frac{c_i n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

Yang-Mills
Amplitude

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]



Yang-Mills
Amplitude

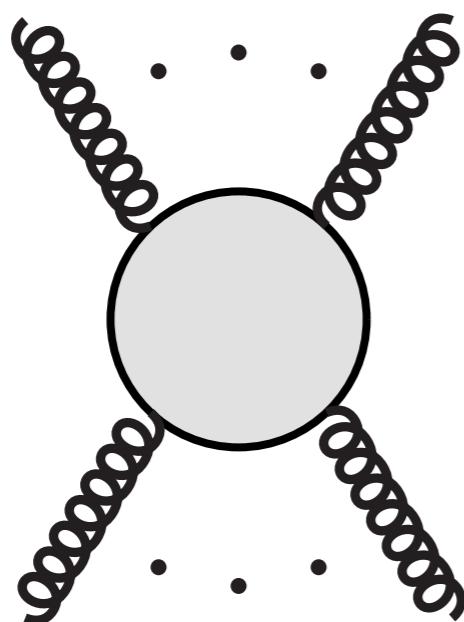
$$= g^{n-2} \sum_{i \in \Gamma_3} \frac{c_i n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

$i \in \Gamma_3$

Diagrams with
cubic vertices

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]



Yang-Mills
Amplitude

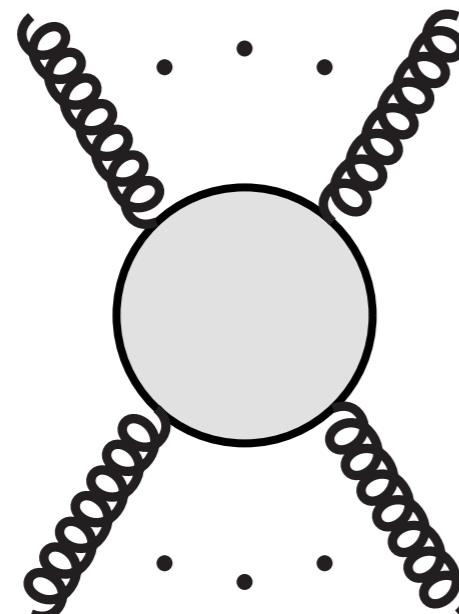
$$= g^{n-2} \sum_{i \in \Gamma_3} \frac{c_i n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

Colour factor:
Structure constants

Diagrams with
cubic vertices

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]



Yang-Mills Amplitude

$$= g^{n-2} \sum_{i \in \Gamma_3} \frac{c_i[n_i]}{\prod_{\alpha_i} s_{\alpha_i}}$$

Colour factor:
Structure constants

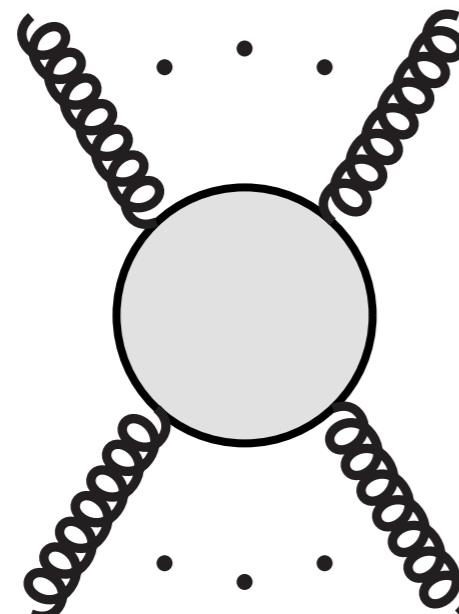
Kinematic factor:
Polarisations
& momenta

Diagrams with
cubic vertices

Detailed description: The equation shows the decomposition of a Yang-Mills amplitude. On the left is a Feynman diagram with a central shaded circle and four external gluon lines (represented by wavy lines) meeting at a vertex. An equals sign follows. To the right is a sum over cubic vertices ($i \in \Gamma_3$) of a ratio. The numerator contains a colour factor c_i and a kinematic factor $[n_i]$, both enclosed in red dashed boxes. The denominator is a product of kinematic factors s_{α_i} for each vertex i . Red arrows point from the text labels to their corresponding parts in the equation.

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]



A Feynman diagram representing a Yang-Mills amplitude. It consists of a central circular vertex connected to four external lines, each represented by a wavy line. Ellipses indicate additional external lines.

Yang-Mills Amplitude

$$= g^{n-2} \sum_{i \in \Gamma_3} \frac{c_i n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

Colour factor:
Structure constants

Kinematic factor:
Polarisations
& momenta

Diagrams with
cubic vertices

Scalar propagators

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

$$g^{n-2} \sum_{i \in \Gamma_3} \frac{c_i n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

Colour factor:
Structure constants

Kinematic factor:
Polarisations
& momenta

Diagrams with
cubic vertices

Scalar propagators

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

$$g^{n-2} \sum_{i \in \Gamma_3} \frac{c_i n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

Colour factor:
Structure constants

Kinematic factor:
Polarisations
& momenta

Diagrams with
cubic vertices

Scalar propagators

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

$$i \left(\frac{\kappa}{2} \right)^{n-2} \sum_{i \in \Gamma_3} \frac{c_i n_i}{\prod_{\alpha_i} s_{\alpha_i}}$$

Colour factor:
Structure constants

Kinematic factor:
Polarisations
& momenta

Diagrams with
cubic vertices

Scalar propagators

The diagram illustrates the double copy relation. The left side shows a kinematic factor involving a sum over cubic vertices and scalar propagators. The right side shows a colour factor involving structure constants and diagrams with cubic vertices. Arrows point from the terms in the kinematic factor to their corresponding components in the colour factor.

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

$$i \left(\frac{\kappa}{2} \right)^{n-2} \sum_{i \in \Gamma_3} \frac{c_i [n_i]}{\prod_{\alpha_i} s_{\alpha_i}}$$

Kinematic factor:
Polarisations
& momenta

Diagrams with
cubic vertices

Scalar propagators

The diagram shows a mathematical expression involving a sum over diagrams in Γ_3 . The expression is multiplied by a kinematic factor $i \left(\frac{\kappa}{2} \right)^{n-2}$. The term $c_i [n_i]$ is enclosed in a dashed red box and has a red arrow pointing to the text 'Diagrams with cubic vertices'. The denominator $\prod_{\alpha_i} s_{\alpha_i}$ is also enclosed in a dashed red box and has a red arrow pointing to the text 'Scalar propagators'. The text 'Kinematic factor: Polarisations & momenta' is positioned above the expression.

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

$$i \left(\frac{\kappa}{2} \right)^{n-2} \sum_{i \in \Gamma_3} \frac{\tilde{n}_i[n_i]}{\prod_{\alpha_i} s_{\alpha_i}}$$

Kinematic factor:
Polarisations
& momenta

Diagrams with
cubic vertices

Scalar propagators

The diagram shows the mathematical expression for the double copy relation. On the left, the term $i \left(\frac{\kappa}{2} \right)^{n-2}$ is shown with a red arrow pointing to the left side of the equation, labeled "Diagrams with cubic vertices". On the right, the denominator $\prod_{\alpha_i} s_{\alpha_i}$ is shown with a red arrow pointing to the right side of the equation, labeled "Scalar propagators". The middle part of the equation contains a summation symbol \sum over $i \in \Gamma_3$, with a red bracket enclosing the term $\tilde{n}_i[n_i]$.

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

$$i \left(\frac{\kappa}{2} \right)^{n-2} \sum_{i \in \Gamma_3} \frac{[\tilde{n}_i | n_i]}{\prod_{\alpha_i} s_{\alpha_i}}$$

Second copy of Kinematic factors

Kinematic factor:
Polarisations & momenta

Diagrams with cubic vertices

Scalar propagators

The double copy relations

Bern, Carrasco, Johansson 0805.399, 1004.0576 [hep-th]

The diagram illustrates the double copy relations. On the left, a grey circle with four wavy lines (representing a gravity amplitude) is equated to a sum of diagrams on the right. The right side consists of a red factor $i \left(\frac{\kappa}{2}\right)^{n-2}$ followed by a summation over $i \in \Gamma_3$. The term inside the summation is a ratio of two products: the numerator is $\tilde{n}_i |n_i|$ and the denominator is $\prod_{\alpha_i} s_{\alpha_i}$. Arrows point from various parts of the equation to their descriptions:

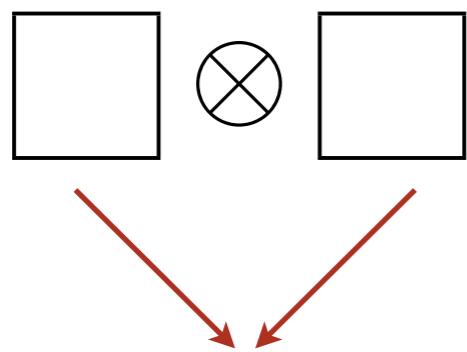
- A red arrow points from the term $\tilde{n}_i |n_i|$ to the text "Kinematic factor: Polarisations & momenta".
- A red arrow points from the term $\prod_{\alpha_i} s_{\alpha_i}$ to the text "Scalar propagators".
- A red arrow points from the summation index $i \in \Gamma_3$ to the text "Diagrams with cubic vertices".
- A red arrow points from the text "Second copy of Kinematic factors" to the term $i \left(\frac{\kappa}{2}\right)^{n-2}$.
- A red arrow points from the text "Gravity Amplitude" to the grey circle on the left.

Not only gravity

In the **non supersymmetric** case:

Not only gravity

In the **non supersymmetric** case:



$$A_\mu \otimes \tilde{A}_\mu$$

Not only gravity

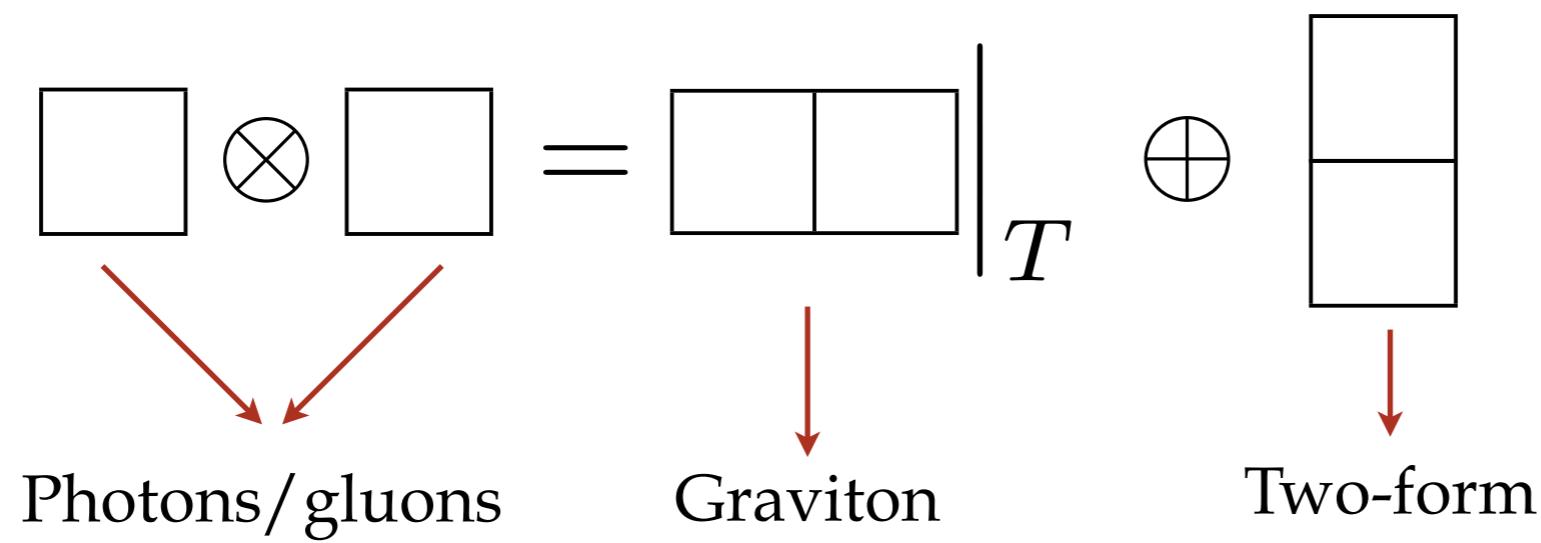
In the **non supersymmetric** case:

$$\begin{array}{c} \square \otimes \square = \square | T \\ \searrow \quad \swarrow \\ \text{Photons/gluons} \qquad \qquad \qquad \text{Graviton} \end{array}$$

$$A_\mu \otimes \tilde{A}_\mu \qquad h_{\mu\nu}$$

Not only gravity

In the **non supersymmetric** case:



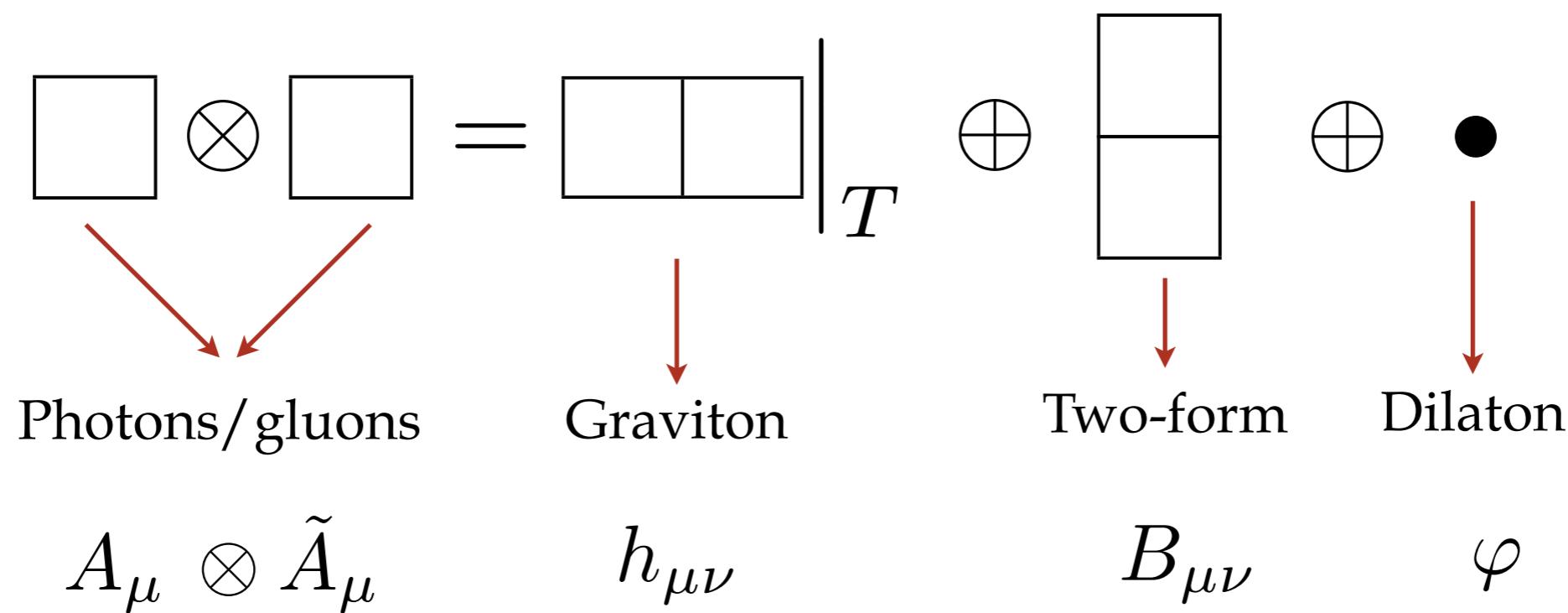
$$A_\mu \otimes \tilde{A}_\mu$$

$$h_{\mu\nu}$$

$$B_{\mu\nu}$$

Not only gravity

In the **non supersymmetric** case:



A field theory realization

Anastasiou, Borsten, Duff, Hughes, Nagy 1408.4434 [hep-th]

$$H_{\mu\nu} = A_\mu \circ \tilde{A}_\nu$$

A field theory realization

Anastasiou, Borsten, Duff, Hughes, Nagy 1408.4434 [hep-th]

$$\boxed{H_{\mu\nu}} = A_\mu \circ \tilde{A}_\nu$$

↓
Double copy
Field

A field theory realization

Anastasiou, Borsten, Duff, Hughes, Nagy 1408.4434 [hep-th]

$$\boxed{H_{\mu\nu}} = A_\mu \circledast \tilde{A}_\nu$$

↓
Double copy
Field

↓
Convolution
Product

The diagram illustrates the double copy relation. It features three main components: a dashed red box labeled $H_{\mu\nu}$, a solid black A_μ , and a dashed red circle labeled \tilde{A}_ν . An arrow points from the box to the text "Double copy Field". Another arrow points from the circle to the text "Convolution Product". Between the A_μ and the circle is a symbol resembling a convolution product (\circledast).

A field theory realization

Anastasiou, Borsten, Duff, Hughes, Nagy 1408.4434 [hep-th]

$$H_{\mu\nu} = A_\mu \circledast \tilde{A}_\nu$$

↓
Double copy
Field

↓
Convolution
Product

$$h_{\mu\nu} = H_{(\mu\nu)} - \frac{1}{D-2} \eta_{\mu\nu} \varphi$$

$$B_{\mu\nu} = H_{[\mu\nu]}$$

$$\varphi = H^\mu_\mu - \frac{1}{\Box} \partial^\mu \partial^\nu H_{\mu\nu}$$

A field theory realization

Anastasiou, Borsten, Duff, Hughes, Nagy 1408.4434 [hep-th]

$$H_{\mu\nu} = A_\mu \circledast \tilde{A}_\nu$$

Double copy
Field

Convolution
Product

$$h_{\mu\nu} = H_{(\mu\nu)} - \frac{1}{D-2} \eta_{\mu\nu} \varphi$$

$$B_{\mu\nu} = H_{[\mu\nu]}$$

$$\varphi = H^\mu_\mu - \frac{1}{\Box} \partial^\mu \partial^\nu H_{\mu\nu}$$



Gravitational local symmetries are correctly reproduced from spin one gauge transformations at the **linearized level**

A field strength for the DC

$$\mathcal{R}_{\mu\nu\rho\sigma} = -\frac{1}{2} F_{\mu\nu} \circ \tilde{F}_{\rho\sigma}$$

A field strength for the DC

$$\mathcal{R}_{\mu\nu\rho\sigma} = -\frac{1}{2} \begin{matrix} F_{\mu\nu} \\ \circ \\ \tilde{F}_{\rho\sigma} \end{matrix}$$

Linearized spin-one
field strengths

A field strength for the DC

$$\mathcal{R}_{\mu\nu\rho\sigma} = -\frac{1}{2} \begin{matrix} F_{\mu\nu} \\ \circ \\ \tilde{F}_{\rho\sigma} \end{matrix}$$

Linearized spin-one
field strengths

Invariant under:

A field strength for the DC

$$\mathcal{R}_{\mu\nu\rho\sigma} = -\frac{1}{2} \begin{matrix} F_{\mu\nu} \\ \circ \\ \tilde{F}_{\rho\sigma} \end{matrix}$$

Linearized spin-one
field strengths

Invariant under:

$$\delta H_{\mu\nu} = \partial_\mu \alpha_\nu + \partial_\nu \tilde{\alpha}_\mu = \delta h_{\mu\nu} + \delta B_{\mu\nu}$$

Is there a way to write

$$\mathcal{L}_{\text{Gravity}} = \mathcal{L}_{\text{YM}} \otimes \tilde{\mathcal{L}}_{\text{YM}}$$

?

Is there a way to write

$$\mathcal{L}_{\text{Gravity}} = \mathcal{L}_{\text{YM}} \otimes \widetilde{\mathcal{L}}_{\text{YM}}$$

?

For independent attempts, see
Bern, Dennen, Huang, Kiermaier 1004.0694 [hep-th]

Borsten, Nagy 2004.14945 [hep-th]

Borsten, Jurco, Kim, Macrelli, Saemann, Wolf 2007.13803 [hep-th]

Free Lagrangians for the DC

Free Lagrangians for the DC

1) Maxwell-like Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

Free Lagrangians for the DC

1) Maxwell-like Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} = \frac{1}{2} H^{\mu\nu} (\eta_{\rho\mu} \eta_{\nu\sigma} \square - \eta_{\rho\mu} \partial_\sigma \partial_\nu - \eta_{\sigma\nu} \partial_\rho \partial_\mu) H^{\rho\sigma}$$

Free Lagrangians for the DC

1) Maxwell-like Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} = \frac{1}{2} H^{\mu\nu} (\eta_{\rho\mu} \eta_{\nu\sigma} \square - \eta_{\rho\mu} \partial_\sigma \partial_\nu - \eta_{\sigma\nu} \partial_\rho \partial_\mu) H^{\rho\sigma}$$

Local

Relation to tensionless strings



Free Lagrangians for the DC

1) Maxwell-like Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} = \frac{1}{2} H^{\mu\nu} (\eta_{\rho\mu} \eta_{\nu\sigma} \square - \eta_{\rho\mu} \partial_\sigma \partial_\nu - \eta_{\sigma\nu} \partial_\rho \partial_\mu) H^{\rho\sigma}$$

Local

Relation to tensionless strings



Requires constraint

$$\partial^\mu (\alpha_\mu + \tilde{\alpha}_\mu) = 0$$

Free Lagrangians for the DC

1) Maxwell-like Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} = \frac{1}{2} H^{\mu\nu} (\eta_{\rho\mu} \eta_{\nu\sigma} \square - \eta_{\rho\mu} \partial_\sigma \partial_\nu - \eta_{\sigma\nu} \partial_\rho \partial_\mu) H^{\rho\sigma}$$

Local

Relation to tensionless strings



Requires constraint

$$\partial^\mu (\alpha_\mu + \tilde{\alpha}_\mu) = 0$$

2) Non-local Lagrangian

Francia 1001.5003 [hep-th]

Free Lagrangians for the DC

1) Maxwell-like Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} = \frac{1}{2} H^{\mu\nu} (\eta_{\rho\mu} \eta_{\nu\sigma} \square - \eta_{\rho\mu} \partial_\sigma \partial_\nu - \eta_{\sigma\nu} \partial_\rho \partial_\mu) H^{\rho\sigma}$$

Local

Relation to tensionless strings



Requires constraint

$$\partial^\mu (\alpha_\mu + \tilde{\alpha}_\mu) = 0$$

2) Non-local Lagrangian

Francia 1001.5003 [hep-th]

$$\mathcal{L}_{NL} = \frac{1}{2} \mathcal{R}_{\mu\nu\rho\sigma} \frac{1}{\square} \mathcal{R}^{\mu\nu\rho\sigma} = \frac{1}{8} \left(F_{\mu\nu} \circ \tilde{F}_{\rho\sigma} \right) \frac{1}{\square} \left(F^{\mu\nu} \circ \tilde{F}^{\rho\sigma} \right)$$

Free Lagrangians for the DC

1) Maxwell-like Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} = \frac{1}{2} H^{\mu\nu} (\eta_{\rho\mu} \eta_{\nu\sigma} \square - \eta_{\rho\mu} \partial_\sigma \partial_\nu - \eta_{\sigma\nu} \partial_\rho \partial_\mu) H^{\rho\sigma}$$

Local



Relation to tensionless strings

Requires constraint

$$\partial^\mu (\alpha_\mu + \tilde{\alpha}_\mu) = 0$$

2) Non-local Lagrangian

Francia 1001.5003 [hep-th]

$$\mathcal{L}_{NL} = \frac{1}{2} \mathcal{R}_{\mu\nu\rho\sigma} \frac{1}{\square} \mathcal{R}^{\mu\nu\rho\sigma} = \frac{1}{8} \left(F_{\mu\nu} \circ \tilde{F}_{\rho\sigma} \right) \frac{1}{\square} \left(F^{\mu\nu} \circ \tilde{F}^{\rho\sigma} \right)$$

Manifest DC structure



Extends to massive theory

Relation to tensionless strings

Free Lagrangians for the DC

1) Maxwell-like Lagrangian

Campoleoni, Francia 1206.5877 [hep-th]

$$\mathcal{L}_{ML} = \frac{1}{2} H^{\mu\nu} (\eta_{\rho\mu} \eta_{\nu\sigma} \square - \eta_{\rho\mu} \partial_\sigma \partial_\nu - \eta_{\sigma\nu} \partial_\rho \partial_\mu) H^{\rho\sigma}$$

Local



Relation to tensionless strings

Requires constraint

$$\partial^\mu (\alpha_\mu + \tilde{\alpha}_\mu) = 0$$

2) Non-local Lagrangian

Francia 1001.5003 [hep-th]

$$\mathcal{L}_{NL} = \frac{1}{2} \mathcal{R}_{\mu\nu\rho\sigma} \frac{1}{\square} \mathcal{R}^{\mu\nu\rho\sigma} = \frac{1}{8} \left(F_{\mu\nu} \circ \tilde{F}_{\rho\sigma} \right) \frac{1}{\square} \left(F^{\mu\nu} \circ \tilde{F}^{\rho\sigma} \right)$$

Manifest DC structure



Extends to massive theory

Non-local

Relation to tensionless strings

Interactions?

The two Lagrangians are equivalent,
and both can be extended to include cubic
interactions by means of the Noether procedure

Thank you
for the attention!