Moduli space of five-dimensional black holes

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YTF 2020

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Motivation

- AdS/CFT correspondence
- Black hole microstate counting and string theory
- Better understanding of gravity by introducing a tuneable parameter (dimension)
- Richer structure

Basic Features of 5d Vacuum GR

- 4d Kerr black hole (S^2 horizon) $\rightarrow$ 5d (doubly rotating) Myers-Perry black hole (S^3 horizon)
- In 5d there is the black ring solution with horizon topology $S^2 \times S^1$ i.e. a non-spherical horizon
- There are also multi-black hole solutions: the black Saturn and di-ring solutions.

4d Vacuum Uniqueness

4d Vacuum Uniqueness [Carter '71, Robinson '75]

All stationary, axisymmetric, asymptotically flat solutions of the vacuum Einstein equations in 4d (subject to technical requirements) are uniquely determined by their mass and angular momentum.

- For a single horizon the unique solution is given by the Kerr solution ("no-hair")
- Can one extend this naively to 5d?

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- Can one extend this naively to 5d?
- No! The black ring is a counterexample (in fact displays 2-fold non-uniqueness)
- What extra information do we need for a uniqueness theorem? Rod structure

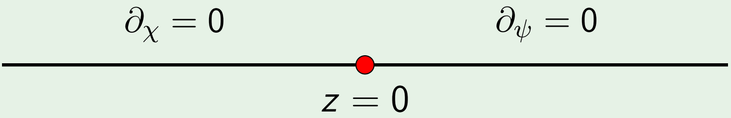
Rod Structure

- By analogy with Kerr solution which is axisymmetric, we now consider "biaxisymmetric" solutions in 5d
- The orbit space (i.e. spacetime quotiented by the Killing vectors) can be identified with a half plane $\{(\rho, z)|\rho > 0\}$ - Weyl-Papapetrou coordinates
- The axis $\rho = 0$ divides into z-intervals (rods) which are either horizons or regions where integer linear combinations of the axial Killing vectors (rod vectors) vanish. This encodes topology
- The lengths of these rods together with these rod vectors gives the rod structure

Rod Structure Examples 1

Example (5d flat space)

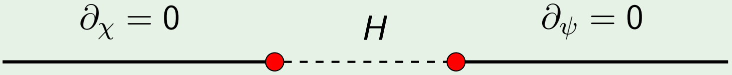
- $ds^2 = -dt^2 + \mu^2(\rho^2 + \mu^2)^{-1}(d\rho^2 + dz^2) + \mu d\psi^2 + \rho^2\mu^{-1}\, d\chi^2$
- ∂_t is the generator of time translations, ∂_ψ and ∂_χ are the generators of rotation and $\mu := \sqrt{\rho^2 + z^2} - z$
- $\rho = 0$ splits up into two (semi-infinite) rods: $z \leq 0$ along which ∂_χ vanishes and $z \geq 0$ along which ∂_ψ vanishes
- Can be represented diagrammatically as:



Rod Structure Examples 2

Example (Myers-Perry)

- $\partial_t, \partial_\psi$ and ∂_χ generators of symmetry as before but now the metric is much more complicated
- Now there is a black hole horizon (the orbit space of which is written as H):

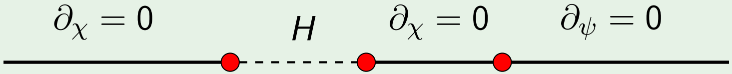


- The horizon rod length here is given by $\frac{4}{3\pi}M$

Rod Structure Examples 3

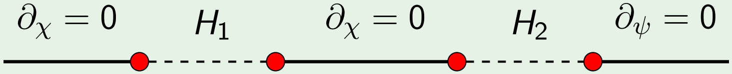
Example (black ring)

- This adds in an extra finite rod:



Example (black Saturn)

- In this case there are 2 horizon components, H_1 and H_2:



Uniqueness Theorem

5d Vacuum Uniqueness [Hollands & Yazadjiev '11]

All stationary, biaxisymmetric, asymptotically flat solutions of the vacuum Einstein equations in 5d are uniquely determined by their rod structure and angular momentum (subject to technical requirements).

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- Note that this only gives uniqueness and not existence. In 4d we have Kerr (at least for single horizons), in 5d it's an open problem

Einstein Equations As A Linear Pair

- Einstein equations:

$$\partial_\rho U + \partial_z V = 0, \tag{1}$$

$$U = \rho \partial_\rho g g^{-1}, \quad V = \rho \partial_z g g^{-1} \tag{2}$$

where g is the matrix of inner product of Killing vectors

- The equations are integrable, which means that they can be written as the integrability condition of a pair of linear equations:

$$\partial_\rho \Psi = A\Psi, \quad \partial_z \Psi = B\Psi \tag{3}$$

i.e. $[\partial_\rho, \partial_z]\Psi = 0$ implies the Einstein equations

- A, B, Ψ are matrix functions of ρ, z and λ, a new complex parameter. A, B are built out of the matrix g

How Does This Help?

- Inverse scattering method:
  - Seed solution + transformation parameters $\rightarrow$ more complicated solution
  - Powerful - 5d solutions given above all generated using this method
- Integrating the LP along $\rho = 0$ and at infinity:
  - In principle, gives solution for metric on $\rho = 0$ for any rod structure subject to constraints on various moduli
  - Gives results for Kerr, Myers-Perry and black ring spacetimes
  - Can be used to rule out double Kerr in 4d and certain black lens solutions in 5d

The End

Thanks for listening!

Questions?

References:

- Review for higher-dimensional gravity: Emparan, Reall (2008), arXiv:0801.3471
- Reviews for uniqueness theorems: Hollands, Ishibashi (2012), arXiv:1206.1164; Chruściel, Costa, Heusler (2012) arxiv:1205.6112
- New 5d classification results: Lucietti, FT (2020), arXiv:2008.12761 and arXiv:2012.00381