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Based on work with **JP Gauntlett, JM Perez Ipina, D Martelli, JF Sparks** arXiv: **2011.10579** [hep-th] arXiv: **2012.xxxx** [hep-th] (soon!)

A brief overview of the M2-branes picture

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See e.g. Benini, Hristov, Zaffaroni 1511.04085 [hep-th] Zaffaroni 1902.07176 [hep-th] for a review

...



Plebanski-Demianski accelerating, rotating, dyonically charged black holes

Asymptotically AdS₄ in the UV

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Extremal

Near the horizon: $AdS_2 \times WCP^1_{[n_-, n_+]}$

The WCP¹_[n_{-}, n_{+}] horizon has conical deficits:











They can be uplifted to **completely smooth** solutions of **11d supergravity!**

UV



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$$\mathbf{UV}$$
 $AdS_4 \times SE_7$



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Gauntlett, Kim, Waldram 0612253 [hep-th]







3d SCFT compactified on the spindle: 1d SCQM





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- **O Entropy** of the extremal and supersymmetric black hole:

$$S_{BH} = \frac{\pi}{G_{(4)}} \frac{J}{Q_e} = \frac{\pi}{4 G_{(4)}} \left(\sqrt{\chi^2 + 16 \left(Q_e^2 + Q_m^2 \right)} - \chi \right)$$

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 - In the 1d SQM, the **R-symmetry** of the 3d SCFT mixes with the U(1) isometry of the spindle (even without rotation!):

$$R_{1d} = R_{3d} + 2\sqrt{2} \frac{n_+ n_- \sqrt{8 n_-^2 n_+^2 Q_e^2 + n_-^2 + n_+^2}}{\sqrt{16 n_-^2 n_+^2 Q_e^2 + n_-^2 + n_+^2}} \partial_{\varphi}$$

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For **D3-branes** on a spindle: we reproduce entropy and mixing with a **Field Theory computation** (anomaly polynomial)

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Near-horizon limit: entropy, spinors and R-symmetry

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D3-branes on spindles: $AdS_3 \times WCP^1_{[n_-, n_+]}$ in 5d

Plebanski-Demianski black holes and 11d uplift

PD black holes in 4d

Most general solution to 4d Einstein-Maxwell* theory (no NUT charge)
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$$ds_4^2 = \frac{1}{\Omega^2} \left\{ -\frac{Q}{\Sigma} \left[dt - a\sin^2\theta d\phi \right]^2 + \frac{\Sigma}{Q} dr^2 + \frac{\Sigma}{P} d\theta^2 + \frac{P}{\Sigma} \sin^2\theta \left[adt - \left(r^2 + a^2\right) d\phi \right]^2 \right\}$$
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$$\begin{aligned} \Omega &= 1 - \alpha r \cos \theta \\ \Sigma &= r^2 + a^2 \cos^2 \theta \\ P &= 1 - 2 \alpha m \cos \theta + \left(\alpha^2 \left(a^2 + e^2 + g^2 \right) - a^2 \right) \cos^2 \theta \\ Q &= \left(r^2 - 2 m r + a^2 + e^2 + g^2 \right) \left(1 - \alpha^2 r^2 \right) + \left(a^2 + r^2 \right) r^2 \end{aligned}$$

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 $\alpha \leftrightarrow$ acceleration

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Here $P_{\pm} = P(\theta_{\pm})$. Locally \mathbb{R}^2 , but globally?

Due to the acceleration, $P_{-} \neq P_{+}$ we cannot make it regular! We have conical deficits at the two poles (with different deficit at each pole).



The 4d solutions are singular! Shall we throw them away?

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No: uplift to 11d and regularize!



$$ds_{11}^2 = \frac{1}{4} ds_4^2 + ds^2 (SE_7) = \frac{1}{4} ds_4^2 + \left(\eta + \frac{1}{2}A\right)^2 + ds^2 (KE_6)$$
$$G = \frac{3}{8} \operatorname{vol}_4 - \frac{1}{2} *_4 F \wedge J$$

Gauntlett, Varela 0707.2315 [hep-th]

4d metric













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This solves the eom of 11d supergravity and preserves SUSY

1. "Quantize" the conical deficits, making the space in the θ , ϕ directions into a **spindle** $WCP^{1}_{[n_{-}, n_{+}]}$

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2. Make the space in the θ , ϕ , ψ directions into a **Lens space** S^3/\mathbb{Z}_q , seen as a Hopf-like fibration over $\mathbb{WCP}^1_{[n_-, n_+]}$. This requires

$$m = \frac{g}{\alpha}$$

Compatible with susy

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3. At fixed *t*, *r* the space (*Y*₉) now looks like a **completely smooth** Lens space **fibration** over the *KE*₆ !

$$S^3/\mathbb{Z}_q \hookrightarrow Y_9 \to KE_6$$

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Make the space in the θ, ϕ, ψ directions into a **Lens space** S^3/\mathbb{Z}_a , seen 2. as a Hopf-like fibration over $WCP^1_{[n_-, n_+]}$. This requires

$m = \frac{g}{\alpha}$ Compatible with susy

At fixed *t*, *r* the space (Y_9) now looks like a **completely smooth** Lens space 3. fibration over the *KE*₆ !

$$S^3 / \mathbb{Z}_q \hookrightarrow Y_9 \to KE_6$$

The **flux** *G* and its dual $*_{11}$ *G* are properly **quantized** 4.

Near-horizon limit

The BPS & extremal horizon

$$ds^{2} = \frac{1}{4} \left(y^{2} + j^{2} \right) \left(-\rho^{2} d\tau^{2} + \frac{d\rho^{2}}{\rho^{2}} \right) + \frac{y^{2} + j^{2}}{q(y)} dy^{2} + \frac{q(y)}{4 \left(y^{2} + j^{2} \right)} (dz + j \rho \, d\tau)^{2}$$
$$A = h(y)(dz + j \rho \, d\tau)$$

The BPS & extremal horizon



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Two parameters:

- a is fixed by n_{\pm} and $j \rightarrow$ **acceleration**
- $j \in (0, 1/\sqrt{2})$ is continuous \rightarrow **rotation**

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$$\Delta z = \frac{\sqrt{2}\sqrt{n_{+}^{2} + n_{-}^{2}}}{n_{+}n_{-}\sqrt{1 - 2j^{2}}}\pi$$

For a generic PD black hole we find entropy

$$S_{BH} = \frac{1}{4G_{(4)}}A = \frac{\left(r_{+}^{2} + a^{2}\right)\Delta\phi}{2G_{(4)}\left(1 - \alpha^{2}r_{+}^{2}\right)}$$

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$$J = \text{angular momentum}$$

$$Q_e = \text{electric charge}$$

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How is supersymmetry realized in the dual Field Theory?

The UV geometry is $AdS_4 \times SE_7$, with metric

$$ds_{11}^2 = \frac{1}{4} ds^2 (AdS_4) + ds^2 (SE_7) = \frac{1}{4} ds^2 (AdS_4) + \left(\frac{1}{4} d\psi + \sigma + \frac{1}{2}A\right)^2 + ds^2 (KE_6)$$

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First example of mixing between higher and lower dimensional R-symmetry (with no rotation)!

D3-branes on spindles

 $AdS_3 \times \mathbb{WCP}^1_{[n_-, n_+]}$ solutions of 5d minimal gauged supergravity

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Uplift to 10d Regularization of conical deficits

Completely regular $AdS_3 \times Y_7$ solutions of type IIB supergravity

 $AdS_3 \times \mathbb{WCP}^1_{[n_-, n_+]}$ solutions of 5d minimal gauged supergravity



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Gauntlett, Kim, Waldram 0612253 [hep-th]



Interpretation as (D3-)branes wrapped on spindles



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Lack the full geometry: we conjecture the AdS_3 solutions to arise as near-horizon geometry of a 5d black string



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Entropy/central charge and R-symmetry mixing reproduced with a **Field Theory computation**



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Thank you for the attention!