Protected states from AdS_3 integrability

Suvajit Majumder

City, University of London

based on work in collaboration with:
Bogdan Stefanski, Alessandro Torrielli, Olof Ohlsson Sax
arXiv: 2012.xxxxx

Dec 15, 2020

Overview

1. Protected states: why study them?
2. $\mathrm{AdS_{d+1}/CFT_d}$ and integrability
3. Algebraic Bethe Ansatz: XXX spin chain
4. ABA in AdS_3
5. Protected states from ABA
6. Conclusion

Protected states: why study them?

- Protected states are short multiplets of the SUSY algebra.
- Generic n-point CFT correlators are functions of coupling.
- Anomalous dimension for protected operators is zero-overall dimension matches classical dimension.
- Useful tool to test for AdS/CFT dualities.
- Why use integrability to study these?
  - non-renormalisation theorems are weaker/do not exist at low SUSY
  - integrability lets you show exactly (i.e. non-perturbatively in the planar coupling constant) and explicitly that states are protected

$\mathrm{AdS_{d+1}/CFT_d}$: a brief introduction

Conjecture

Closed string theory in AdS_{d+1} *is dual to* a conformal field theory in $\mathbb{R}^d$

- Strong-weak duality
- Most famous example: AdS_5/CFT_4

$$\begin{array}{lccc} & \text{type IIB strings in } AdS_5 \times S_5 & \leftrightarrow & \mathcal{N}=4 \quad \text{SYM} \\ \text{Parameters:} & (g_s, \alpha') & \leftrightarrow & (g_{YM}, N) \end{array}$$

- Large N planar regime:

$$N \to \infty\,, \quad g_{YM} \to 0\,, \quad \lambda \equiv g_{YM}^2 N = \text{finite}$$

AdS_3/CFT_2 setup

- Type IIB superstrings in $AdS_3 \times S_3 \times M_4$ dual to CFT_2 with $\mathcal{N} = (4, 4)$ SUSY.
- Two geometries (preserving max SUSY): $M_4 = T^4$, and $M_4 = S^3 \times S^1$.
- Background supported by non-trivial R-R, and NS-NS 3-form fluxes.
- Integrable spin chain descriptions include massive and massless modes
- Integrable S matrix is exact in α'

Intro to ABA: XXX spin chain

- Heisenberg spin chain: 1+1D, sites in SU(2) fundamental rep., nearest neighbour Hamiltonian

$$H = 4\sum_{i=1}^{L}\left(\frac{1}{4} - \vec{S}_i.\vec{S}_{i+1}\right) = 2\sum_n(\mathbb{I}_{n,n+1} - P_{n,n+1}) = 2L - 2\sum_n P_{n,n+1}$$

- R-matrix: intertwines two arbitrary spaces, labelled a_1, a_2

$$R_{a_1,a_2}(u) = (u + \frac{i}{2})\mathbb{I}_{a_1} \otimes \mathbb{I}_{a_2} + \frac{i}{2}\sum_\alpha \sigma^\alpha_{a_1} \otimes \sigma^\alpha_{a_2} = u\mathbb{I}_{a_1,a_2} + iP_{a_1,a_2}$$

where u is related to momentum p via

$$\frac{u + \frac{i}{2}}{u - \frac{i}{2}} = e^{ip}$$

XXX spin chain contd.

- Monodromy matrix: acts on all spin chain sites, plus an aux. site a

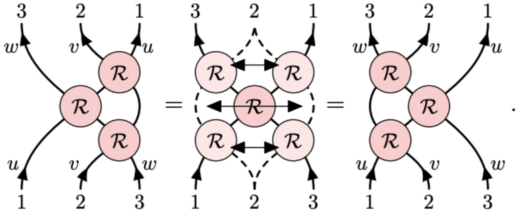
$$T(u) = R_{L,a}(u)R_{L-1,a}(u)...R_{1,a}(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}_a$$

- Transfer matrix: monodromy matrix traced over aux. site

$$F(u) = \mathrm{Tr}_a T(u) = A(u) + D(u)$$

- Yang-Baxter equation

$$R_{a_1,a_2}(\lambda-\mu)T_{a_1}(\lambda)T_{a_2}(\mu) = T_{a_2}(\mu)T_{a_1}(\lambda)R_{a_1,a_2}(\lambda-\mu)$$



XXX spin chain: ABA

- The algebraic Bethe ansatz for the XXX spin chain gives the eigenvectors of the Hamiltonian

$$|u_1, \ldots, u_M\rangle = B(u_1)B(u_2)...B(u_M)\,|\uparrow\uparrow .. \uparrow\rangle$$

where $0 < M \leq L$.

- Energy of solution

$$E(u_1, \ldots, u_M) = \sum_{k=1}^{M} \frac{2}{(u_k)^2 + \frac{1}{4}}$$

- Eigenstate constraint: Bethe equation

$$\left(\frac{u_j + \frac{i}{2}}{u_j - \frac{i}{2}}\right)^L = \prod_{k \neq j} S(u_j, u_k)$$

for j=1,. . . ,M, where $S(u_j, u_k) = \frac{u_j - u_k + i}{u_j - u_k - i}$

AdS_3 massless ABA: $\mathrm{psu}(1|1)^4_{\mathrm{c.e.}}$ spin chain

- $\mathrm{psu}(1|1)^4_{c.e.}$: tensor product of two commuting copies of $\mathrm{psu}(1|1)^2_{c.e.}$ and has 8 supercharges S^i_I, Q^i_I (with $I = L, R$ and $i = 1, 2$), which satisfy

$$\{\mathrm{Q}^i_L, \mathrm{S}^i_L\} = \mathrm{H}^i_L \quad \{\mathrm{Q}^i_R, \mathrm{S}^i_R\} = \mathrm{H}^i_R \quad ,$$
$$\{\mathrm{Q}^i_L, \mathrm{Q}^i_R\} = \mathrm{P}^i \quad \{\mathrm{S}^i_L, \mathrm{S}^i_R\} = \mathrm{K}^i$$

  where $\mathrm{H}^i_L, \mathrm{H}^i_R, \mathrm{P}^i, \mathrm{K}^i$ are central elements.

- The massless modes are in a tensor product representation

$$\rho_{\mathrm{psu}(1|1)^4} = \rho_L \otimes \tilde{\rho}_L$$

- Graded 2D vector spaces, short reps. of two $\mathrm{psu}(1|1)^2_{c.e.}$

$$\mathcal{V}_{\rho_L} = \{|\phi\rangle, |\psi\rangle\}, \qquad \mathcal{V}_{\tilde{\rho}_L} = \left\{\left|\tilde{\psi}\right\rangle, \left|\tilde{\phi}\right\rangle\right\}$$

AdS_3 massless ABA: $\mathrm{psu}(1|1)^4_{\mathrm{c.e.}}$ spin chain

- $\mathrm{psu}(1|1)^4$ R-matrix

$$R_{\mathrm{psu}(1|1)^4} \approx R^{LL}_{\mathrm{psu}(1|1)^2} \hat{\otimes} R^{\tilde{L}\tilde{L}}_{\mathrm{psu}(1|1)^2}$$

- Constituent $\mathrm{psu}(1|1)^2$ R-matrices

$$R^{LL}_{\mathrm{psu}(1|1)^2}(\gamma_1, \gamma_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -b & a & 0 \\ 0 & a & b & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$R^{\tilde{L}\tilde{L}}_{\mathrm{psu}(1|1)^2}(\gamma_1, \gamma_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -b & -a & 0 \\ 0 & -a & b & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

where $a(\gamma_1, \gamma_2) = \mathrm{sech}\, \frac{\gamma_1 - \gamma_2}{2}$, $b(\gamma_1, \gamma_2) = \tanh \frac{\gamma_1 - \gamma_2}{2}$, $\gamma = \log \tan \frac{p}{4}$

$\mathrm{psu}(1|1)^4_{\mathrm{c.e.}}$ ABA contd.

- Monodromy matrices

$$T_1 = \prod R^{LL}_{\mathrm{psu}(1|1)^2} = \begin{pmatrix} \mathfrak{a}^1 & \mathfrak{b}^1 \\ \mathfrak{c}^1 & \mathfrak{d}^1 \end{pmatrix} , \quad T_2 = \prod R^{\tilde{L}\tilde{L}}_{\mathrm{psu}(1|1)^2} = \begin{pmatrix} \mathfrak{a}^2 & \mathfrak{b}^2 \\ \mathfrak{c}^2 & \mathfrak{d}^2 \end{pmatrix} ,$$

- B operators: generators of magnon excitations

$$B_1 \equiv \mathfrak{b}^1 \hat{\otimes} 1^2 , \qquad B_3 \equiv 1^1 \hat{\otimes} \mathfrak{b}^2$$

- Algebraic Bethe ansatz

$$|\Psi\rangle = \prod_{i=1}^{N_3} B_3(\beta_{3,i}) \prod_{j=1}^{N_1} B_1(\beta_{1,j}) \, |\chi..\chi\rangle$$

- Bethe equations

$$e^{-iLp_k} = (-1)^{N_0-1} \prod_{i\neq k}^{N_0} S^2(\gamma_{kj}) \prod_{j=1}^{N_1} \coth \frac{\beta_{1,jk}}{2} \prod_{l=1}^{N_3} \coth \frac{\beta_{3,lk}}{2}$$

$$1 = \prod_{i=1}^{N_0} \tanh \frac{\beta_{1,ki}}{2}\,, \qquad k = 1, ..., N_1$$

$$1 = \prod_{i=1}^{N_0} \tanh \frac{\beta_{3,ki}}{2}\,, \qquad k = 1, ..., N_3$$

where

$$\beta_{l,jk} \equiv \beta_{l,j} - \gamma_k\,, \qquad l = 1, 3\,,$$

and $S(\gamma)$ is the famous Zamolodchikov sine-Gordon scalar factor.

T^4 protected states from $\mathrm{psu}(1|1)^4_{\mathrm{c.e.}}$ ABA

Protected states $\Leftrightarrow$ Bethe states with ONLY fermionic zero-momentum massless excitations $\chi, \tilde{\chi}$

- Hodge diamond (for each L):

$$\left|Z^L\right\rangle$$

$$\left|Z^L\chi^\pm\right\rangle \qquad \left|Z^L\tilde{\chi}^\pm\right\rangle$$

$$\left|Z^L\chi^+\chi^-\right\rangle \qquad \left|Z^L\chi^\pm\tilde{\chi}^{\pm'}\right\rangle \qquad \left|Z^L\tilde{\chi}^+\tilde{\chi}^-\right\rangle$$

$$\left|Z^L\chi^+\chi^-\tilde{\chi}^\pm\right\rangle \qquad \left|Z^L\tilde{\chi}^+\tilde{\chi}^-\chi^\pm\right\rangle$$

$$\left|Z^L\chi^+\chi^-\tilde{\chi}^+\tilde{\chi}^-\right\rangle$$

- Hodge numbers of T^4: $h^{0,0} = h^{2,2} = h^{2,0} = h^{0,2} = 1$, $h^{1,1} = 4$, $h^{0,1} = h^{1,0} = h^{2,1} = h^{1,2} = 2$

Conclusion

- Obtained Hodge numbers for T^4 by computing protected spectrum using integrability.
- Integrability based proof of non-renormalizability of the protected operators.
- Each background has a moduli space(20 D) and our results hold across full moduli space
- New results: Protected spectrum for K^3, realised as T^4 orbifolds.
- Generalisation to mixed flux backgrounds.

Thank You!