Multi-Winding Magnetic Flux Tubes in Colour-Superconducting Quark Matter

(Based on this work[1])

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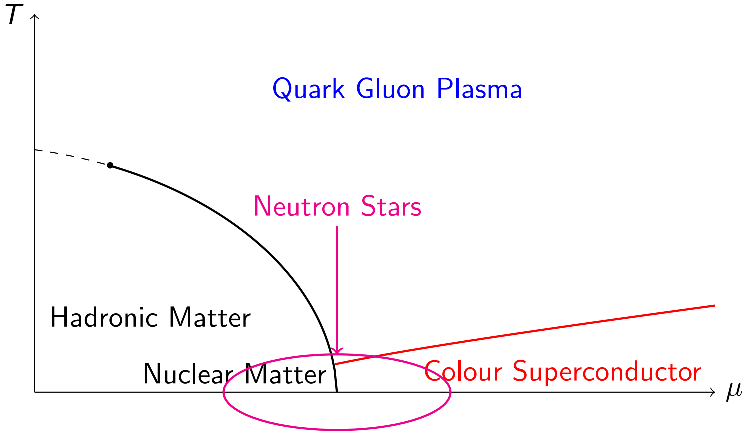
[1]G. W. Evans and Andreas Schmitt arXiv:2009.01141

Schematic QCD Phase Diagram

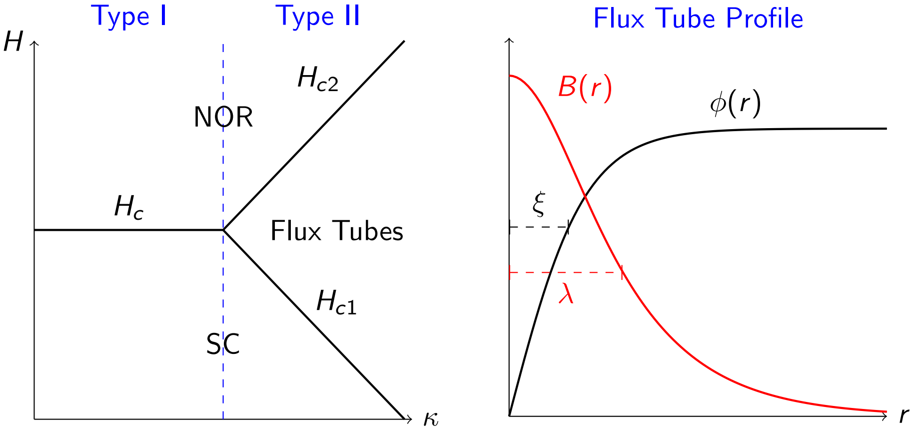


Project Motivation

- Focus on the high μ, low T regime of the diagram
- Study dense, colour superconducting quark matter in the presence of an external magnetic field
- Previous project analysed colour-magnetic defects and constructed phase diagrams using a Ginzburg-Landau approach with massless quarks[2]
- Introduce a correction term that accounts for the strange quark mass for a more realistic approximation

[2] A. Haber and A. Schmitt, JoP G - N&PP 45, 065001, 2018

Superconductivity: Refresher



Type-II when $\kappa = \lambda/\xi > 1/\sqrt{2}$ (blue dashed line) and superconductor admits magnetic flux through defects

Colour Superconductivity

- Meissner Effect: Expulsion of colour-magnetic fields
- More than one way of forming Cooper Pairs e.g. Colour-Flavour Locking (CFL) and Two-Flavour Pairing (2SC)
- CFL: Cooper pairs are $rd - gu$, $bu - rs$, $bd - gs$ and same pairing with the colours/flavours switched ($ru - gd$ etc.)
- 2SC: Only two flavours and two colours participate in pairing for example: u,d with r,g charge pair (all b and s quarks unpaired)
- Can be both electromagnetic and colour superconductors simultaneously

Ginzburg-Landau Potential

Three condensates and three gauge fields (pre-rotation) with additional correction terms introduced,

$$U = \left|\left(\nabla + i\tilde{q}_3\tilde{A}_3 + i\tilde{q}_{81}\tilde{A}_8\right)\phi_1\right|^2 + \left|\left(\nabla - i\tilde{q}_3\tilde{A}_3 + i\tilde{q}_{82}\tilde{A}_8\right)\phi_2\right|^2$$
$$+ \left|\left(\nabla - i\tilde{q}_{83}\tilde{A}_8\right)\phi_3\right|^2 - (\mu^2 - m_1^2)|\phi_1|^2 - (\mu^2 - m_2^2)|\phi_2|^2$$
$$- (\mu^2 - m_3^2)|\phi_3|^2 + \lambda(|\phi_1|^4 + |\phi_2|^4 + |\phi_3|^4)$$
$$- 2h(|\phi_1|^2|\phi_2|^2 + |\phi_1|^2|\phi_3|^2 + |\phi_2|^2|\phi_3|^2) + \frac{\tilde{B}_3^2}{2} + \frac{\tilde{B}_8^2}{2} + \frac{\tilde{B}^2}{2},$$

$$m_1^2 \propto \frac{m_s^2}{2\mu_q} - \frac{2\mu_e}{3}, \quad m_2^2 \propto \frac{m_s^2}{2\mu_q} + \frac{\mu_e}{3}, \quad m_3^2 \propto \frac{\mu_e}{3}.$$

Mass correction[3], coupled rotated field(s) and uncoupled rotated field contributions.

[3] K.Iida et al., PRL 93, 132002, 2004

Equations of Motion

Using an over-relaxation method we solve the equations;

$$\tilde{a}_3'' - \frac{\tilde{a}_3'}{R} = \frac{\tilde{q}_3}{\lambda}\left[(\mathcal{N}_1(\tilde{a}_3, \tilde{a}_8) + \Xi R^2) f_1^2 - (\mathcal{N}_2(\tilde{a}_3, \tilde{a}_8) - \Xi R^2) f_2^2\right],$$

$$\tilde{a}_8'' - \frac{\tilde{a}_8'}{R} = \frac{1}{\lambda}\Big[\tilde{q}_{81}(\mathcal{N}_1(\tilde{a}_3, \tilde{a}_8) + \Xi R^2) f_1^2 + \tilde{q}_{82}(\mathcal{N}_2(\tilde{a}_3, \tilde{a}_8) - \Xi R^2) f_2^2 + \tilde{q}_{83}\mathcal{N}_3(\tilde{a}_8) f_3^2\Big],$$

$$f_1'' + \frac{f_1'}{R} + f_1\left[1 - \frac{\alpha}{4} - f_1^2 + \eta(f_2^2 + f_3^2) - \left(\frac{\mathcal{N}_1(\tilde{a}_3, \tilde{a}_8) + \Xi R^2}{R}\right)^2\right] \simeq 0,$$

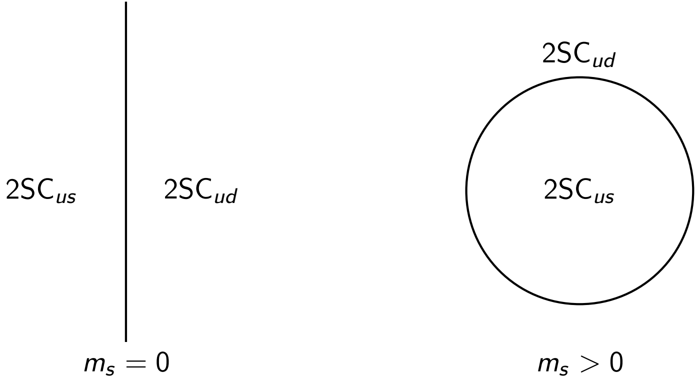
$$f_2'' + \frac{f_2'}{R} + f_2\left[1 - \frac{\alpha}{2} - f_2^2 + \eta(f_3^2 + f_1^2) - \left(\frac{\mathcal{N}_2(\tilde{a}_3, \tilde{a}_8) - \Xi R^2}{R}\right)^2\right] \simeq 0,$$

$$f_3'' + \frac{f_3'}{R} + f_3\left[1 - f_3^2 + \eta(f_1^2 + f_2^2) - \left(\frac{\mathcal{N}_3(\tilde{a}_8)}{R}\right)^2\right] = 0,$$

for re-scaled, dimensionless condensates $f_i(R)$ and gauge fields, $\tilde{a}_j(R)$ where R is re-scaled, dimensionless radial coordinate .

Massless vs Massive Case

- Preferred configuration has winding number $n \to \infty$ and $R \to \infty$ such that flux tubes become domain walls in massless case
- Within $2SC_{ud}(f_3)$ flux tube cores $2SC_{us}(f_2)$ condensate takes non-zero value



Flux Tube Profiles

- Combination of two condensates (red line) has two minima at high winding (low mass)

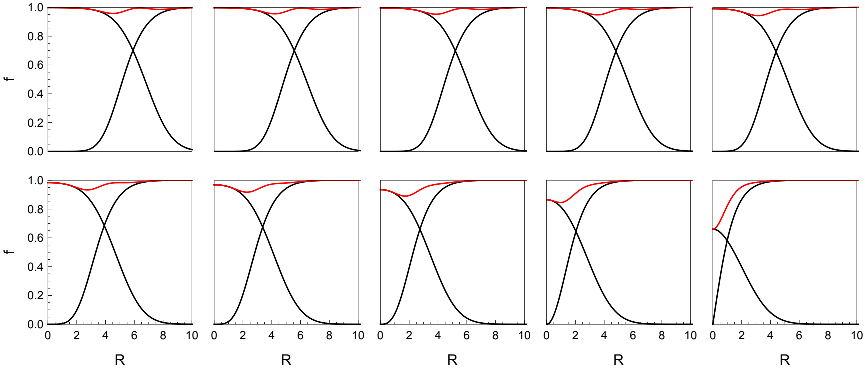
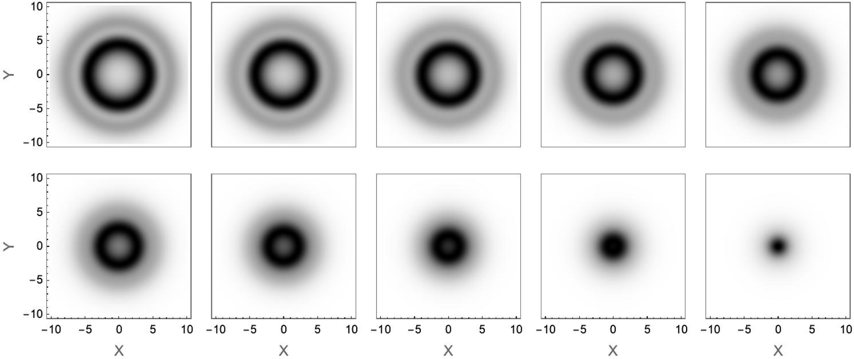


Figure: $R = 10$ corresponds to $r \simeq 7.7$fm

Domain Wall Remnants

- At high winding (low mass) magnetic field becomes ring-like in X-Y plane



$H - T_c/\mu_q$ Phase Diagram

- Preferred n depends on T_c/μ_q for given m_s

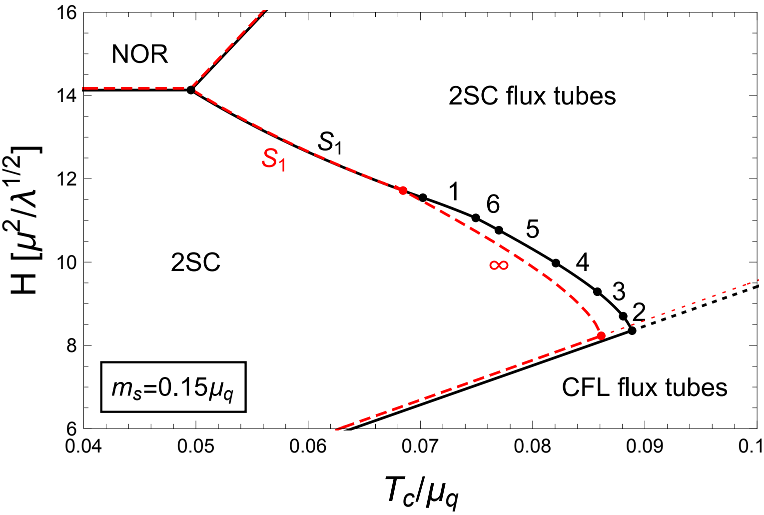
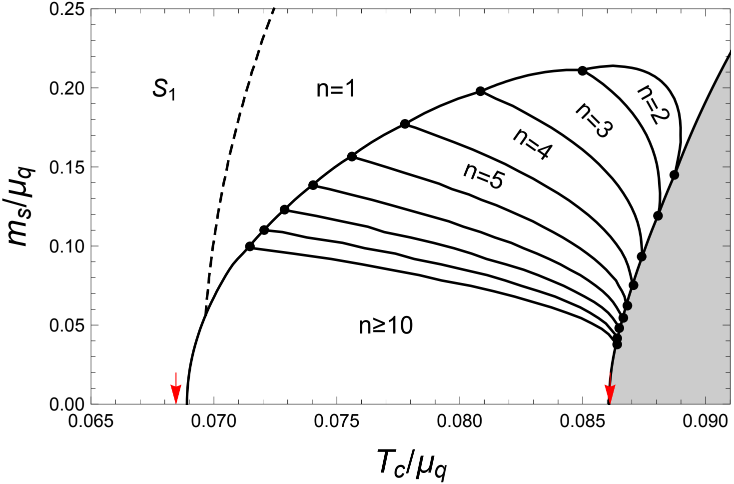


Figure: $H/(\mu^2/\lambda^{1/2}) \simeq 14$ corresponds to $H \sim 10^{19}$G at $T_c/\mu_q = 0.05$

$m_s/\mu_q - T_c/\mu_q$ Phase Diagram

- Dense region of high n preferred configurations at very low masses



Summary and Outlook

Summary

- Constructed phase diagrams for colour-superconducting quark matter in an external magnetic field H
- In the presence of a strange quark mass multi-winding flux tubes are preferred over domain walls

Outlook

- Flux tubes effect on Neutron Star ellipticity and amplitude of Gravitational Waves?[4]
- Similarities to magnetic ring-like structures in other models?[5,6]
- Improve temperature treatment
- Explore the mass effect on CFL phase

[4] K. Glampedakis, D.I. Jones, L. Samuelsson, PRL 109, 081103, 2012

[5] M.N. Chernodub and A.S. Nedelin, Phys. Rev. D 81, 125022, 2010

[6] D.Bazeia, M.A. Marques and D. Melnikov, Phys. Lett. B 785, 2018