Amplitudes and Backgrounds in Split Signature

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Motivation

- Classical gravity from amplitudes $\Delta p^\mu, \; \Delta s^\mu, \; F^{\mu\nu}_{\rm rad}, \; \Psi_{\alpha\beta\gamma\delta}$

KMOC [1811.10950]

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- Classical/quantum double copy

BCJ [0805.3993]

Radiation in EM and gravity

$$F_{\mu\nu} \rightarrow \phi_{\alpha\beta} = F_{\mu\nu}\, \sigma^{\mu\nu}{}_{\alpha\beta} = (\mathbf{E} - i\mathbf{B}) \cdot \boldsymbol{\sigma}$$

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$$F_{\mu\nu} \to \phi_{\alpha\beta} = F_{\mu\nu}\, {\sigma^{\mu\nu}}_{\alpha\beta} = (\mathbf{E} - i\mathbf{B}) \cdot \boldsymbol{\sigma}$$

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$$W_{\mu\nu\rho\sigma} \to \Psi_{\alpha\beta\gamma\delta} = W_{\mu\nu\rho\sigma}\, \sigma^{\mu\nu}{}_{\alpha\beta}\, \sigma^{\rho\sigma}{}_{\gamma\delta}$$

NP basis and amplitudes

$$\Psi = \Psi_0|n\rangle^4 + \Psi_1|k\rangle|n\rangle^3 + \Psi_2|k\rangle^2|n\rangle^2 + \Psi_3|k\rangle^3|n\rangle + \Psi_4|k\rangle^4$$

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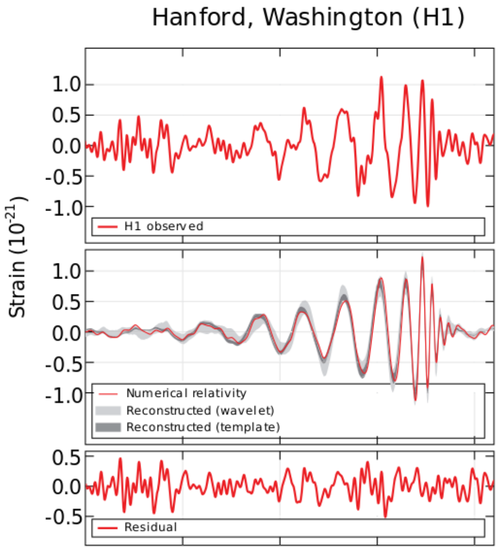
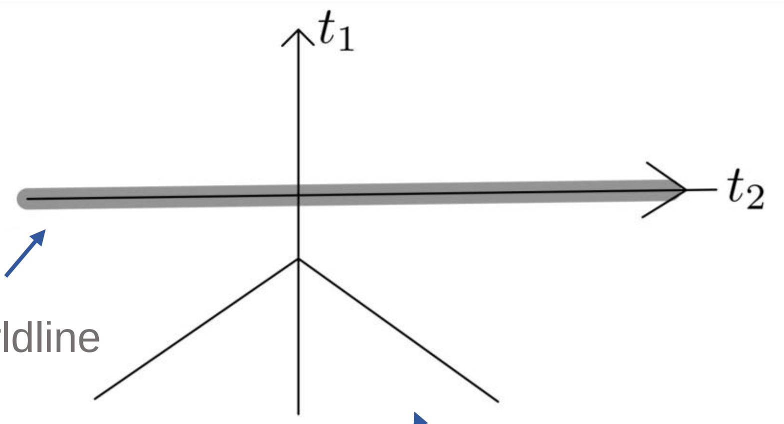


"knows" about asymptotic evolution, can compute it with amplitudes!

LIGO collaboration [1602.03837]

Asymptotic radiation in (2,2)



Massive scalar particle worldline
with $u^\mu = \frac{p^\mu}{m} = (0, 1, 0, 0)$

Vacuum: $\langle \psi | \Psi(x) | \psi \rangle = 0$

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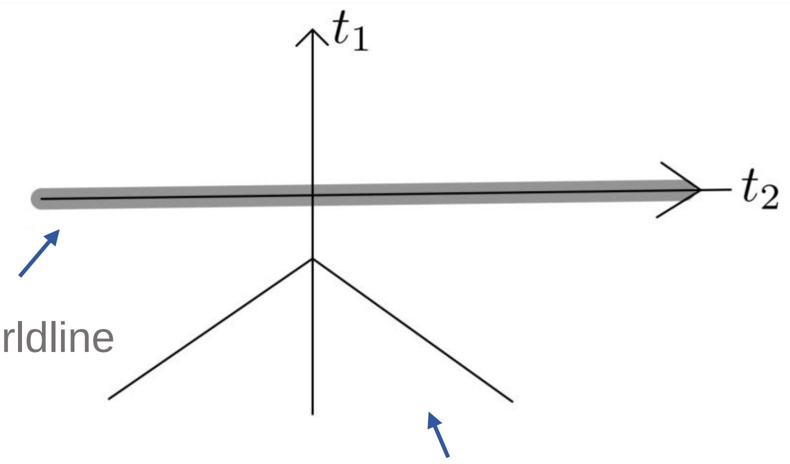

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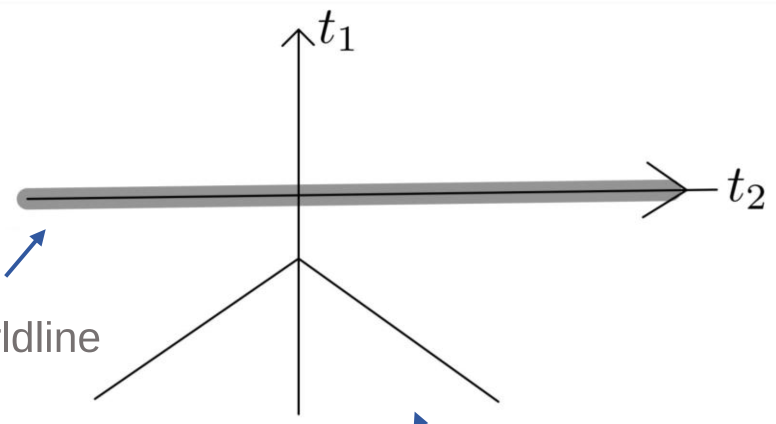

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$$S = \lim_{t_1 \to \infty} U(-t_1, t_1)$$

Measure $\langle\psi|S^\dagger \Psi(x) S|\psi\rangle$ at large t_1 infinity

$$|\psi\rangle \sim \int \varphi(p)|p\rangle$$

$\mathcal{M}^{(3)}$ Is enough classically $(\hbar \to 0)$

- Find the coherent scattered state $\mathcal{M}_h^{(3)} = -\kappa\, m^2 (u \cdot \varepsilon_h(k))^2$

$$S|\psi\rangle = \exp\left(\sum_h \int d\Phi(k)\, \delta(2p\cdot k)\, i\mathcal{M}_{-h}^{(3)}(k)\, a_h^\dagger(k) \right) |0\rangle$$

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- Can also compute asymptotic Weyl spinor

$$\langle\, \Psi(x)\rangle = 2\kappa \mathrm{Re} \int d\Phi(k)\delta(2p\cdot k) i\mathcal{M}_+^{(3)}(k)|k\rangle^4 e^{-ik\cdot x}$$

$$\kappa = \sqrt{32\pi G_N}$$

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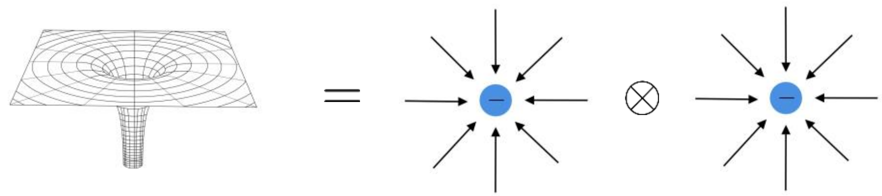
$$\mathcal{M}^{(3)} \leftrightarrow (\mathcal{A}^{(3)})^2 \sim \langle \phi(x)\rangle$$

The Classical double copy



$$A^\mu(x) = \phi(x)\, K^\mu(x), \quad K^2 = 0, \;\; K \cdot \partial\, K^\mu = 0,$$

$$\Rightarrow g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x) = \eta_{\mu\nu} + \phi\, K_\mu K_\nu,$$

MOCW [1410.0239]

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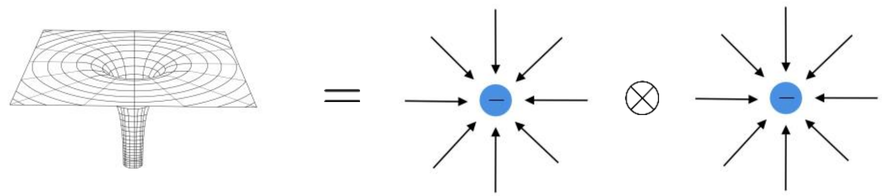

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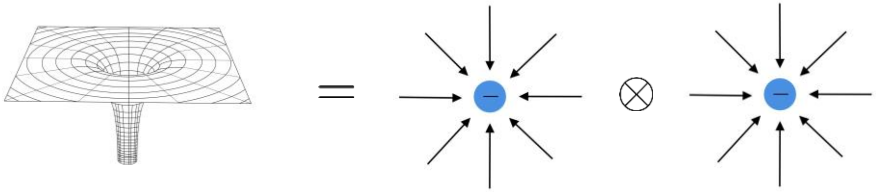

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MOCW [1410.0239]

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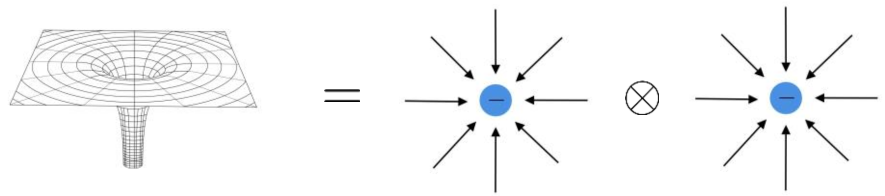

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Fully characterized by three point amplitude!

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Thank you!