Analytic understanding of boosted top tagging with n-subjettiness

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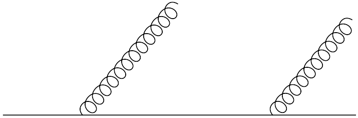
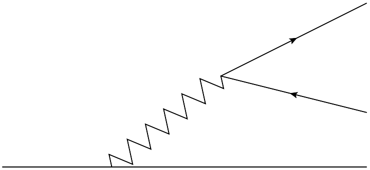
- Tag hadronically decaying top quarks, which are reconstructed as a single jet.
- Machine learning, jet shapes, "prong finders"...
- Focus on n-subjettiness (τ_{32}) (J. Thaler, K, Tilburg 2011), Y_m-Splitter(M Dasgupta, M Guzzi, J Rawling, G Soyez, 2018) and mMDT (M Dasgupta, A Fregoso, S Marzani, G Salam, 2013) .





$\mathrm{Y_m}$-Splitter

1. Cluster jet using Gen-k$_t(p = 1/2)$ $d_{ij} = \min(z_i, z_j)\theta_{ij}^2$
2. Undo last two clusterings
3. Check that $\min(m_{12}, m_{13}, m_{23}) > m_{min}$
4. Check that $\min(z_1, z_2, z_3) > \zeta$

$$\tau_{32} = \frac{\tau_3}{\tau_2}$$

$$\tau_N^{(\beta)} = \frac{1}{p_t R_{jet}^{\beta}} \sum_i p_t^i \min\left((\Delta R_{1,i})^{\beta}, (\Delta R_{2,i})^{\beta}, ..., (\Delta R_{N,i})^{\beta}\right)$$

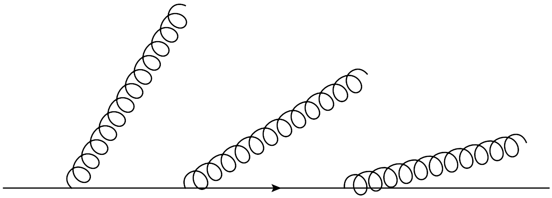
Use $\beta = 2$

$$\tau_N^{(2)} \simeq \sum_i z_i \min\left((\theta_{1,i})^2, (\theta_{2,i})^2, ..., (\theta_{N,i})^2\right)$$

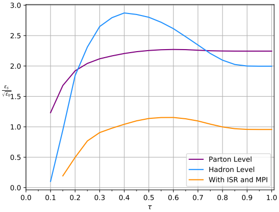
Use Gen-k$_t(p = 1/2)$ axes.

mMDT

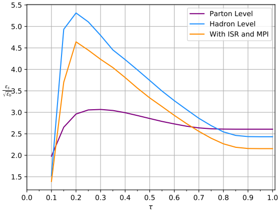
1. Cluster jet using Cambridge Aachen algorithm
2. Undo last clustering
3. Check if lower p_t sub-jet has $z > z_{cut}$. If not, discard this sub jet and go back to 2.
4. If it does, this is the jet.



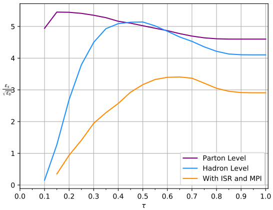
Signal significance



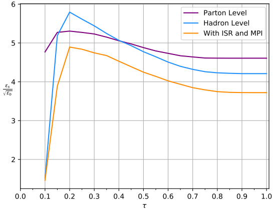
Cut on τ_{32}



Cut on τ_{32} after grooming with mMDT.

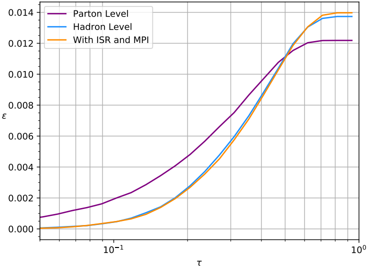


Cut on τ_{32} after application of $\mathrm{Y_m}$-Splitter.

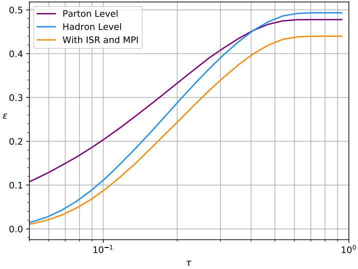


Cut on τ_{32} after application of $\mathrm{Y_m}$-Splitter and grooming.

Tagging Efficiency for jets groomed with mMDT and tagged with Y_m-Splitter and τ_{32}



Light quark initiated jets



Top initiated jets

$$160\text{GeV} < m_j < 225\text{GeV}$$

Large Logs

$$\rho = \frac{m^2}{R^2 p_t^2}$$

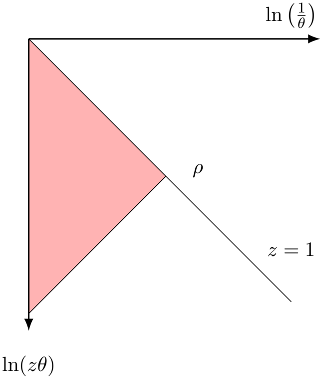
$$\Sigma^1_{\mathsf{Real}}(\rho) \simeq \frac{C_F \alpha_s}{\pi} \int_0^1 \frac{\mathrm{d}z}{z} \int_0^1 \frac{\mathrm{d}\theta^2}{\theta^2} \Theta(z\theta^2 < \rho)$$

$$\Sigma^1_{\mathsf{Virt.}}(\rho) \simeq -\frac{C_F \alpha_s}{\pi} \int_0^1 \frac{\mathrm{d}z}{z} \int_0^1 \frac{\mathrm{d}\theta^2}{\theta^2}$$

$$\Sigma^1(\rho) \simeq \frac{-C_F \alpha_s}{2\pi} \ln^2(\rho)$$

Resummation

- Large logs occur at all orders, need to re-sum.
- Logs come from region of phase space vetoed for real emissions
- Dominant contribution from emissions strongly ordered in angle.



$$\Sigma(\rho) \simeq \exp[-\frac{C_F \alpha_s}{2\pi} \ln^2(\rho)]$$

- $\mathrm{Y_m}$-Splitter requires 3 particles (2 emissions)
- $\tau_2 = \min(z_1 \min(\theta_1^2, \theta_{12}^2), z_2\theta_2^2) \simeq \min(z_1\theta_1^2, z_2\theta_2^2)$
- $\tau_3 = 0$ at $\mathcal{O}(\alpha_s^2)$.

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\rho} = \left(\frac{\alpha_s C_F}{\pi}\right)^2 \times$$

$$\int_\zeta^1 \frac{\mathrm{d}z_1}{z_1} \int_\zeta^1 \frac{\mathrm{d}z_2}{z_2} \int_0^1 \frac{\mathrm{d}\theta_1^2}{\theta_1^2} \int_0^{\theta_1^2} \frac{\mathrm{d}\theta_2^2}{\theta_2^2} \delta(\rho - \max(z_1\theta_1^2, z_2\theta_2^2)\Theta(\min(z_2\theta_2^2, z_1 z_2\theta_1^2) > \rho_{min})$$

$$\rho\frac{\mathrm{d}\Sigma}{\mathrm{d}\rho} \overset{\zeta > \frac{\rho_{min}}{\rho}}{=} \left(\frac{\alpha_s C_F}{\pi}\right)^2 \ln\left(\frac{\rho}{\rho_{min}}\right)\ln^2(\zeta)\,.$$

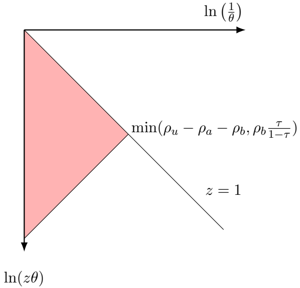
$$\overset{\zeta < \frac{\rho_{min}}{\rho}}{=} \left(\frac{\alpha_s C_F}{\pi}\right)^2 \ln^2\left(\frac{\rho_{min}}{\rho}\right)\left(\frac{3}{2}\ln(\zeta) + \ln\left(\frac{\rho_{min}}{\rho}\right)\right)$$

$$\rho_i = z_i \theta_i^2$$

$$\tau_{32}(\{p_i\}) = \frac{\sum_{i=1} \rho_i - \rho_a - \rho_b}{\sum_{i=1} \rho_i - \rho_a} \rightarrow \sum_{i \neq a,b} \rho_i < \rho_b \frac{\tau}{1-\tau}$$

a and b are the emissions which set the largest and second largest ρ_i.
(Valid for $\tau < \frac{1}{2}$)

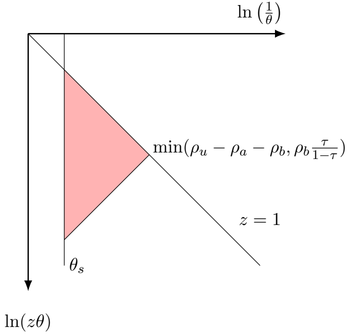
$$\sum_i \rho_i < \rho_u \rightarrow \sum_{i \neq a,b} \rho_i < \rho_u - \rho_a - \rho_b$$

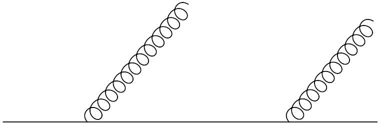


$$\text{LO} \times \exp[-\ln^2(\min(\rho_b \frac{\tau}{1-\tau}, \rho_u - \rho_a - \rho_b))]$$

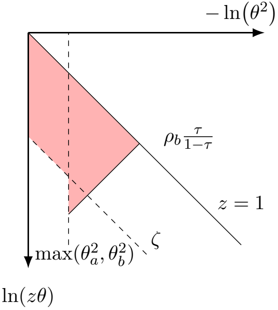
Secondary emissions

- There is also a veto on secondary emissions
- Coherence: emissions from $a(b)$ only up to $\theta_a(\theta_b)$

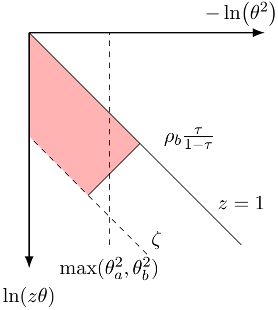




- mMDT and Y_m-Splitter use different clustering sequences
- Grooming only affects primary emissions
- Removes sensitivity to most non perturbative region of phase space



(a) $\max(\theta_a^2, \theta_b^2)\zeta > \rho_b \frac{\tau}{1-\tau}$



(b) $\max(\theta_a^2, \theta_b^2)\zeta < \rho_b \frac{\tau}{1-\tau}$

Improve accuracy of LO pre-factor:

- Correct phase space and kinematics
- Restore full splitting functions

$$\Sigma^{\mathsf{LO}}(\tau) = \left(\frac{C_F\alpha_s}{2\pi}\right)^2 \times$$

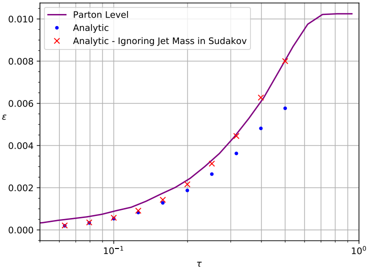
$$\int P(z)P(z_p)\mathrm{d}z\mathrm{d}z_p\Delta^{-1/2}\frac{\mathrm{d}\theta_{12}^2}{2\pi}\frac{\mathrm{d}\theta_{13}^2}{\theta_{13}^2}\frac{\mathrm{d}\theta_{23}^2}{\theta_{23}^2}\Theta(\rho_l < \frac{s_{123}}{p_t^2 R^2} < \rho_u)\Theta_{\mathsf{Y_m\text{-}Splitter}}\Theta(\theta_{13} > \theta_{23})$$

Results for background distribution

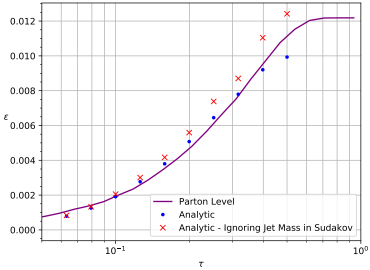
$$\Sigma(\rho_l, \rho_u, \rho_{min}, \zeta, \tau) = \left(\frac{C_F}{2\pi}\right)^2$$

$$\int \alpha_s(k_{t1})\alpha_s(k_{t2})P(z)P(z_p)\mathrm{d}z\mathrm{d}z_p\Delta^{-1/2}\frac{\mathrm{d}\theta_{12}^2}{2\pi}\frac{\mathrm{d}\theta_{13}^2}{\theta_{13}^2}\frac{\mathrm{d}\theta_{23}^2}{\theta_{23}^2}\delta(\rho_{123}-\frac{s_{123}}{p_t^2 R^2})\mathrm{d}\rho_{123}\Theta(\rho_u > \rho_{123} > \rho_l)$$

$$\Theta_{\text{Y}_\text{m}\text{-Splitter}}\Theta(\theta_{13} > \theta_{23})\frac{\exp[-R(\min(\rho_b\frac{\tau}{1-\tau}, \rho_u - \rho_{123})) - \gamma_E R'(\min(\rho_b\frac{\tau}{1-\tau}, \rho_u - \rho_{123}))]}{\Gamma[1 + R'(\min(\rho_b\frac{\tau}{1-\tau}, \rho_u - \rho_{123}))]}, \tag{1}$$



Without pre-grooming.



With pre-grooming.

Key difference between signal and background:

- Top has fixed invariant mass

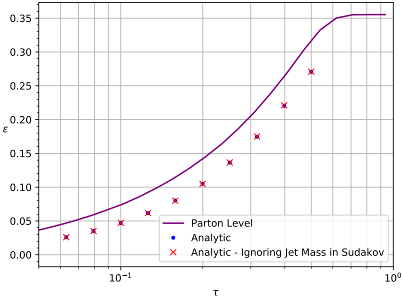
$$\Sigma^{\mathsf{LO}}(\rho, \rho_{min}, \zeta) = \frac{1}{\sigma_0} \int |M_{t\to bq\bar{q}}|^2 \mathrm{d}\Phi_3 \delta\left(\frac{s_{123}}{p_t^2} - \rho_t\right) \Theta_{\mathsf{Clust}} \Theta_{\mathsf{Y_m\text{-}Splitter}},$$

$$R(\min(\rho_b \frac{\tau}{1-\tau}, \rho_u - \rho_{123})) \to R(\min(\rho_b \frac{\tau}{1-\tau}, \rho_u - \rho_t))$$

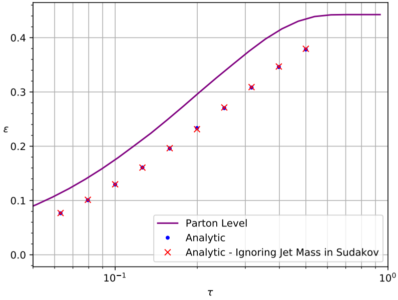
Condition for τ constraint to be stronger than jet mass constraint:

$$\tau_{32} < \frac{M_{max}^2 - M_T^2}{M_{max}^2 - M_T^2 + p_t^2 \text{Min}(d_{12}, d_{13}, d_{23})}$$

Jet mass constraint is irrelevant for $\tau < 0.76$ and $160\text{GeV} < m_{jet} < 225\text{GeV}$.

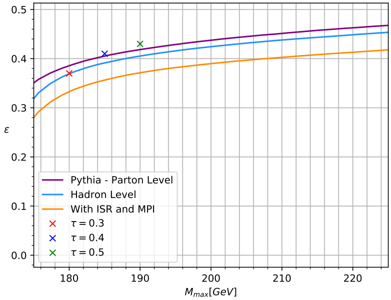


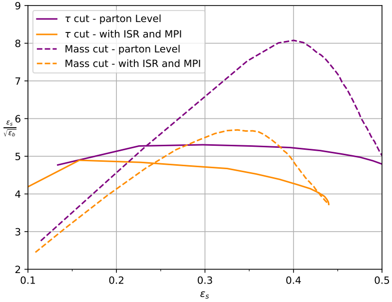
Without pre-grooming.



With pre-grooming.

- We can replace the τ cut with a mass cut which generates the same signal suppression.
- Mass restriction affects the background more than the signal.





Conclusions

- Used resummation to understand the physics driving various tagging procedures.

- mMDT + $\mathrm{Y_m}$-Splitter + τ_{32} is robust and effective.

- Replacing τ_{32} cut with a stringent mass cut is more effective.