Cosmological implications of EW vacuum instability: constraints on the Higgs curvature coupling from inflation

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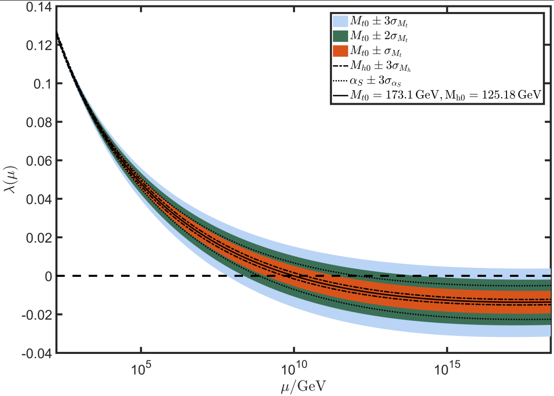
YTF 20: Young Theorists' Forum 2020, Durham

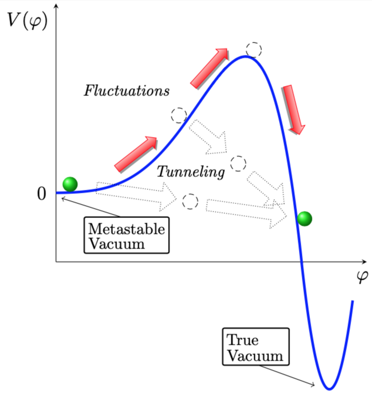
16 December 2020

Introduction

Experimental values of SM particle masses m_h, m_t indicate that:

- SM may be valid up to $\mu_{\rm QG}$; early Universe consistent minimal model.
- currently in metastable EW vacuum $\rightarrow$ constrain fundamental physics.





Markkanen *et al*, "Cosmological Aspects of Higgs Vacuum Metastability", 2018.

Introduction

- Decay expands at c with singularity within $\rightarrow$ true vacuum bubbles:

$$d\langle\mathcal{N}\rangle = \Gamma d\mathcal{V} \;\Rightarrow\; \langle\mathcal{N}\rangle = \int_{\text{past}} d^4x\sqrt{-g}\,\Gamma(x)\,.$$

- Low decay rate Γ today, but higher rates in the early Universe.
- Universe still in metastable vacuum $\rightarrow$ no bubbles in past light-cone:

$$P(\mathcal{N}=0) \propto e^{-\langle\mathcal{N}\rangle} \sim \mathcal{O}(1) \;\Rightarrow\; \langle\mathcal{N}\rangle \lesssim 1\,.$$

Vacuum bubbles expectation value (during inflation)

$$\langle\mathcal{N}\rangle = \frac{4\pi}{3}\int_0^{N_{\text{start}}} dN \left(\frac{a_{\text{inf}}\left(\eta_0 - \eta\left(N\right)\right)}{e^N}\right)^3 \frac{\Gamma(N)}{H(N)} \leq 1$$

Introduction

Aims

- study EW vacuum decay during inflation.
- provide a (stronger) constraint on the Higgs-curvature coupling ξ.

Improvements/differences to previous approaches

- RGI Higgs V_{eff}, 3-loop running, pole-matching, 1-loop curvature.
- Latest SM data (PARTICLE DATA GROUP 2020).
- Inflationary models with time-dependent H, instead of dS.

Fumagalli *et al* '19, Markkanen *et al* '18, Markkanen - Rajantie - Stopyra '18, Espinosa '18, Rajantie - Stopyra '17, East *et al* '17, Czerwińska *et al* '16, Espinosa *et al* '15, Hook *et al* '15, Kamada '15, Kearney *et al* '15, Czerwińska *et al* '15 , Herranen *et al* '14, Fairbairn - Hogan '14, Enqvist *et al* '14, Bhattacharya *et al* '14, Lebedev - Westphal '13, Kobakhidze - Spencer-Smith '13, ...

Tree-level curvature corrections

- Classical solutions to the tunneling process from false to true vacuum.
- High H's during inflation, CdL$\to$HM instanton with action difference

$$B_{\rm HM}(R) \approx \frac{384\pi^2 \Delta V_{\rm H}}{R^2}\,,$$

where $\Delta V_{\rm H} = V_{\rm H}(h_{\rm bar}) - V_{\rm H}(h_{\rm fv})$: barrier height $\to$ decay rate

$$\Gamma_{\rm HM}(R) \approx \left(\frac{R}{12}\right)^2 e^{-B_{\rm HM}(R)}\,.$$

- Curvature effects enter at tree level via non-minimal coupling ξ:

$$V_{\rm H}(h,\mu,R) = \frac{\xi}{2} R h^2 + \frac{\lambda(\mu)}{4} h^4\,.$$

Coleman (1977), Coleman and De Lucia (1980), Hawking and Moss (1987).

One-loop curvature corrections

- Minkowski terms to 3-loops, curvature corrections in dS at 1-loop:

$$V_{\rm H}(h,\mu,R)=\frac{\xi(\mu)}{2}Rh^2+\frac{\lambda(\mu)}{4}h^4+\frac{\alpha(\mu)}{144}R^2+\Delta V_{\rm loops}(h,\mu,R)\,,$$

where the loop contribution can be parametrized as

$$\Delta V_{\rm loops}=\frac{1}{64\pi^2}\sum_{i=1}^{31}\left\{n_i\mathcal{M}_i^4\left[\log\left(\frac{|\mathcal{M}_i^2|}{\mu^2}\right)-d_i\right]+\frac{n_i'R^2}{144}\log\left(\frac{|\mathcal{M}_i^2|}{\mu^2}\right)\right\}$$

- Eliminate μ-dependence by RGI such that $\mu=\mu_*(h,R)$ and

$$\Delta V_{\rm loops}(h,\mu_*,R)=0\,.$$

Markkanen *et al*, "The 1-loop effective potential for the Standard Model in curved spacetime", 2018.

One-loop curvature corrections

RGI effective Higgs potential

$$V_{\mathrm{H}}^{\mathrm{RGI}}(h,R) = \frac{\xi(\mu_*(h,R))}{2}Rh^2 + \frac{\lambda(\mu_*(h,R))}{4}h^4 + \frac{\alpha(\mu_*(h,R))}{144}R^2$$

Calculate barrier height for Γ: entire SM particle spectrum, running of couplings $\lambda, y_t, g', g, \xi, \alpha$ (β-functions, pole-matching)*.

* via numerical code by Fedor Bezrukov (http://www.inr.ac.ru/$\sim$fedor/SM/).

Inflationary Models

- *Quadratic inflation*, where $m = 1.4 \times 10^{13}$ GeV, with

$$V(\phi) = \frac{1}{2} m^2 \phi^2 \, .$$

- *Quartic inflation*, where $\lambda = 1.4 \times 10^{-13}$, with

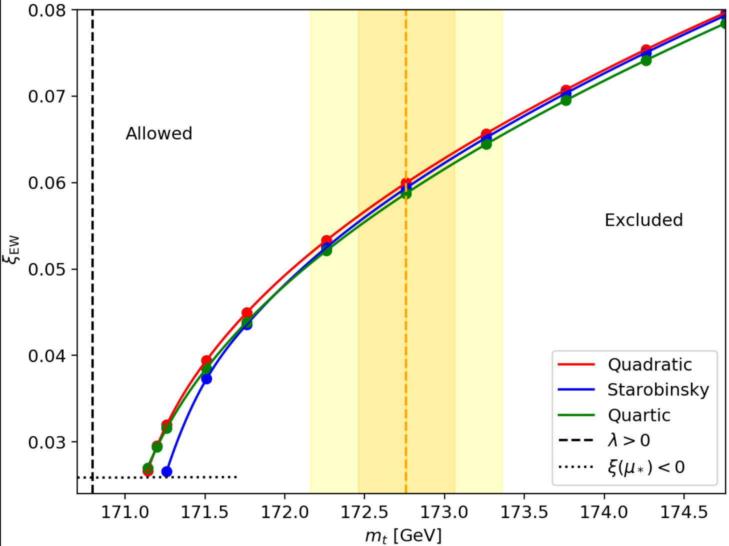
$$V(\phi) = \frac{1}{4} \lambda \phi^4 \, .$$

- *Starobinsky inflation* (Starobinsky-like power-law model), where $\alpha = 1.1 \times 10^{-5}$, with

$$V(\phi) = \frac{3}{4} \alpha^2 M_P^4 \left(1 - e^{-\sqrt{\frac{2}{3}} \frac{\phi}{M_P}} \right)^2 \, .$$

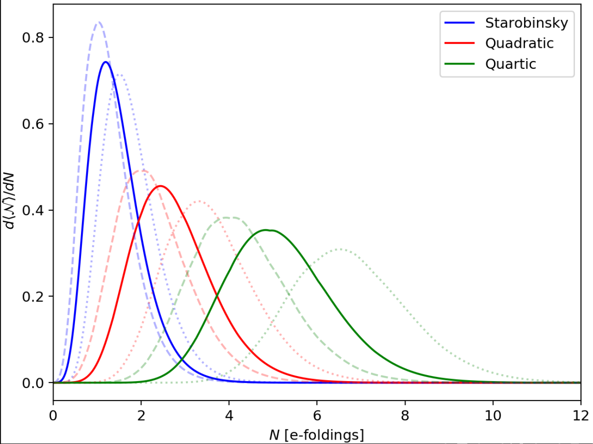
Quadratic and quartic models are simple but not realistic; Starobinsky inflation complies with data and can link different inflationary models.

Results: Bounds on ξ



Results: Bubble nucleation time

- If bubble production at the end $N < 1$, our bounds may not be reliable because $B_{\rm HM}$ is calculated in dS.
- If bubble formation at $N \gg 60$, our bounds would depend significantly on the the early stages of inflation.



Results: Significance of the total duration of inflation

- We study early time behavior by splitting the $\langle\mathcal{N}\rangle$-integral

$$\langle\mathcal{N}\rangle(N_{\text{start}}) = \langle\mathcal{N}\rangle(60) + \int_{60}^{N_{\text{start}}} \frac{d\mathcal{V}}{dN}\Gamma(N)\, dN \ ,$$

  where we set $\langle\mathcal{N}\rangle(60) = 1$ and slow roll applies to the 2nd term.
- $B_{\text{HM}} \approx$ constant at early times, so that

$$\langle\mathcal{N}\rangle(N_{\text{start}}) \approx 1 + \frac{4\pi e^{-B_{\text{HM}}}}{3} N_{\text{start}} \, .$$

- Contributing if $N_{\text{start}} \gtrsim e^{B_{\text{HM}}} \sim 10^{60} \gg 60\, e$-folds but not infinite.

Conclusions

Consistent inclusion of 1-loop curvature corrections beyond dS $\rightarrow$ most accurate constraints to date:

ξ-bounds for $m_t \pm 2\sigma$ in each model (numerical errors$< 1\%$)

$$\begin{aligned} \text{Quadratic}: \ \xi_{\text{EW}} &\geq 0.060^{+0.007}_{-0.008}, \\ \text{Quartic}: \ \xi_{\text{EW}} &\geq 0.059^{+0.007}_{-0.008}, \\ \text{Starobinsky}: \ \xi_{\text{EW}} &\geq 0.059^{+0.007}_{-0.009}, \end{aligned}$$

with the minimal assumption that inflation lasts $N = 60$ e-foldings.

that are $V(\phi)$-independent, N_{start}-independent and m_t-dependent.

Additional slides

Additional slides - Overview of computation

1. Calculate ΔV_{H} and plug it in Γ.
2. Choose inflationary model by specifying $V(\phi)$ for the inflaton.
3. Complete calculation of $\langle \mathcal{N} \rangle$ imposing the condition $\langle \mathcal{N} \rangle \leq 1$.
4. Result: constraint on $\xi \geq \xi_{\langle \mathcal{N} \rangle = 1}$.

Additional slides - Large Higgs field approximation

- At high field values $h \gg 10^{10}$ GeV, a reasonable approximation for the Higgs potential is having constant ξ and constant $\lambda < 0$:

$$V_{\rm H}(h,R) = \frac{1}{2}\xi R h^2 - \frac{1}{4}|\lambda| h^4 \,.$$

- At the top of the barrier, $\left.\frac{dV_{\rm H}}{dH}\right|_{h=h_{\rm bar}} = 0$, we have

$$h_{\rm bar}^2 = \frac{12\xi H^2}{|\lambda|}\,; \qquad V_{\rm H}(h_{\rm bar},R) = \frac{\xi^2 R^2}{4|\lambda|}\,.$$

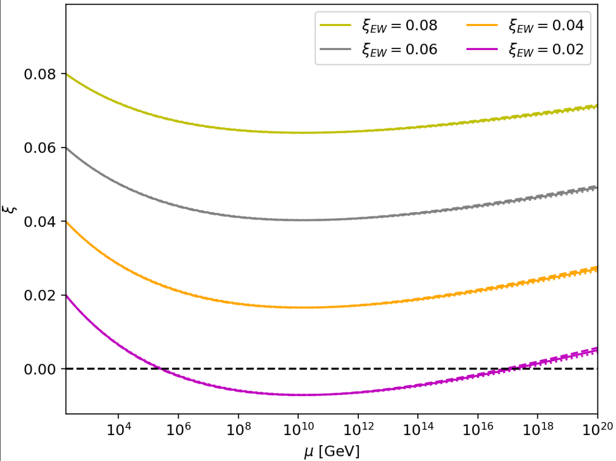
- HM action approximately constant and independent of background:

$$B_{\rm HM} = \frac{96\pi^2\xi^2}{|\lambda|}\,.$$

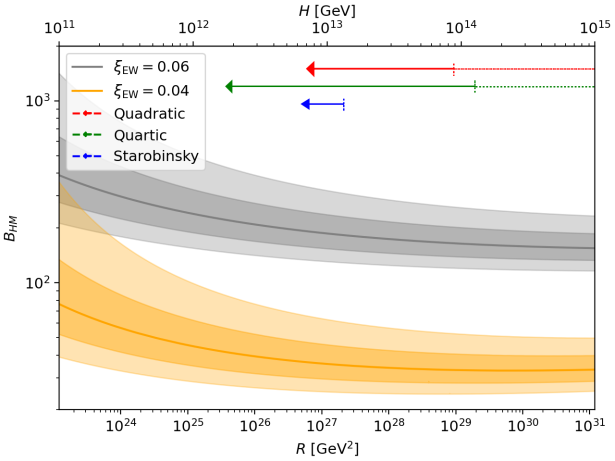
Herranen *et al*, "Spacetime curvature and the Higgs stability during inflation", 2014.

Additional slides - Beta function of non-minimal coupling

$$16\pi^2\beta_\xi = 16\pi^2\frac{d\xi}{d\ln\mu} = \left(\xi - \frac{1}{6}\right)\left(12\lambda + 6y_t^2 - \frac{3}{2}g'^2 - \frac{9}{2}g^2\right)$$



Additional slides - Curvature effects on the bounce action



Shaded areas: 1σ, 2σ deviation from the central m_t; a heavier top quark decreases the value of B_{HM} and vice versa.

Solid red, blue and green arrows: last 60 e-foldings in quadratic, Starobinsky and quartic inflation.

Additional slides - Numerical solution

- General single-field (ϕ) inflationary model: space-time geometry determined by the Friedmann eq. and the field's EoM in FRW:

$$\begin{aligned} H^2 &= \frac{1}{3M_P^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \\ \ddot{\phi} &= -3H\dot{\phi} - V'(\phi)\,. \end{aligned}$$

- In terms of e-foldings N and the inflaton field ϕ:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{V(\phi)}{3M_P^2}\left[1 - \frac{1}{6M_P^2}\left(\frac{d\phi}{dN}\right)^2\right]^{-1},$$

$$R = 6\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right) = 12H^2\left[1 - \frac{1}{4M_P^2}\left(\frac{d\phi}{dN}\right)^2\right].$$

Additional slides - Numerical solution

Solve the system of coupled differential equations

$$\frac{d^2\phi}{dN^2} = \frac{V(\phi)^2}{M_P^2 H^2}\left(\frac{d\phi}{dN} - M_P^2\frac{V'(\phi)}{V(\phi)}\right),$$

$$\frac{d\tilde{\eta}}{dN} = -\tilde{\eta}(N) - \frac{1}{a_{\rm inf} H(N)},$$

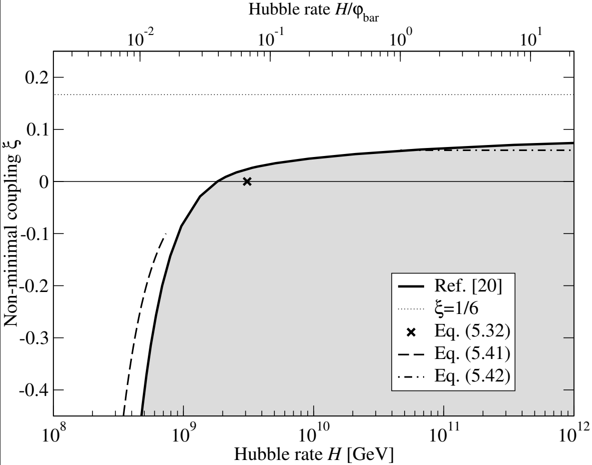
$$\frac{d\langle\mathcal{N}\rangle}{dN} = \gamma(N) = \frac{4\pi}{3}\left[a_{\rm inf}\left(\frac{3.21e^{-N}}{a_0 H_0} - \tilde{\eta}(N)\right)\right]^3 \frac{\Gamma(N)}{H(N)},$$

where $\tilde{\eta} = e^{-N}\eta$, η: conformal time and we set the end of inflation at

$$\left.\frac{\ddot{a}}{a}\right|_{\phi=\phi_{\rm inf}} = \left. H^2\left[1 - \frac{1}{2M_P^2}\left(\frac{d\phi}{dN}\right)^2\right]\right|_{\phi=\phi_{\rm inf}} = 0\,.$$

Additional slides - Results: Comparison with past bounds

- Constraints in general agreement with previous studies (Herranen *et al*, 2014) and (Markkanen *et al*, 2018).



Markkanen *et al*, "Cosmological Aspects of Higgs Vacuum Metastability", 2018.

Additional slides - Results: Bubble nucleation time

- Clear localised peak in all cases $\rightarrow$ definite, fairly well-defined time during inflation, when vacuum decay is most likely.
- Geometric factor decreases exponentially, dynamical factor $\Gamma(N)$ increases not exponentially $\rightarrow$ their product $\gamma(N)$ has a maximum.