Colour/ Kinematics Duality in AdS_4

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based on [2012.02059]

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Outline

- Motivation
- Colour/ Kinematics Recap
- AdS Setup & Spinor Helicity
- AdS colour/ kinematics & amplitudes
- Links to conformal field theory

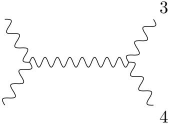
Motivation

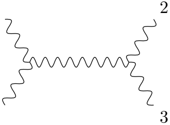
- Aim is to extend flat space relations to curved spacetime, linking gauge and gravity amplitudes
- How much of usual amplitudes 'machinery' works in more general spacetimes?
- Construction of CFT correlation functions
- Links to inflationary cosmology

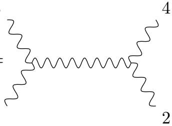
Colour/ Kinematics Duality

- 4pt amplitude, three numerator structures

  Review in 1909.01358

$n_s =$ 

$n_t =$ 

$n_u =$ 

- 4pt colour-dressed YM amplitude:

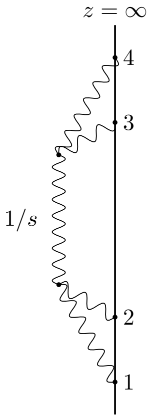
$$\mathcal{A}_4 = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

- If $n_s + n_t + n_u = 0$, then $\mathcal{M}_4 = \frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$
- Can also find relations between colour-ordered expressions

$$uA_{1324} = sA_{1234}, \text{ where } A_{1234} = \frac{n_s}{s} - \frac{n_t}{t}$$

AdS Amplitudes

- We work on the timelike boundary of AdS_4
- Using Witten diagrams, we construct "AdS amplitudes" $\langle jjjj \rangle$
  eg 1011.0780, 1810.12459
- These encode correlators of the boundary CFT



AdS Spinor Helicity

- Construct 4d null momentum
  (analagous to dS case in 1104.2846, 1812.11129, 2005.04234)

$$k^{\mu} = (k^0, k^1, k^2, ik)$$

- Momentum in the radial direction is not conserved,

$$E = \sum_{i=1}^{4} k_i$$

- Flat space limit recovers usual 4d amplitudes

$$\lim_{E \to 0} \langle jjjj \rangle = \frac{A_4}{E}$$

AdS Spinor Helicity

- From null momenta, can make usual spinors

$$k^{\alpha\dot{\alpha}} = k^{\mu}\left(\sigma_{\mu}\right)^{\alpha\dot{\alpha}} = \lambda^{\alpha}\tilde{\lambda}^{\dot{\alpha}}$$

- No longer have full Lorentz symmetry: we can convert spinor indices and extract radial components

$$\bar{\lambda}_i^{\alpha} = -\epsilon^{\alpha\beta}\left(\sigma^3\right)_{\beta\dot{\beta}}\tilde{\lambda}_i^{\dot{\beta}}$$

$$\epsilon_{\alpha\beta}\lambda^{\alpha}\bar{\lambda}^{\beta} = \langle m\bar{m}\rangle = -2ik_m$$

- Large variety of spinor identies but momentum conservation is weaker than in flat space.

Extension to AdS Space

- From Witten diagrams, we find $n_s + n_t + n_u = Q$

  Similar to massive case in 2004.12948
- We are free to shift numerators with generalised gauge transformation

$$\tilde{n}_s = n_s + s\Delta$$

$$A_{1234} = \frac{n_s}{s} - \frac{n_t}{t} = \frac{\tilde{n}_s}{s} - \frac{\tilde{n}_t}{t}$$

- Then BCJ becomes

$$u\langle j_1 j_3 j_2 j_4\rangle = s\langle j_1 j_2 j_3 j_4\rangle + \xi\frac{\tilde{n}_t}{t}$$

Explicit Numerators

- Unlike flat space, all-plus gluon amplitude is non-vanishing

$$n_s^{++++} = \frac{1}{8k_1k_2k_3k_4}\langle\bar{1}\bar{2}\rangle\langle\bar{3}\bar{4}\rangle\Big[i\big(\langle 1\bar{2}\rangle\langle\bar{4}\bar{1}\rangle\langle\bar{1}\bar{3}\rangle + \langle 2\bar{1}\rangle\langle\bar{3}\bar{2}\rangle\langle\bar{2}\bar{4}\rangle\big)$$
$$- k_{\underline{12}}\left(\langle\bar{2}\bar{3}\rangle\langle\bar{4}\bar{1}\rangle - \langle\bar{1}\bar{3}\rangle\langle\bar{2}\bar{4}\rangle\right)$$
$$- \frac{1}{k_{\underline{12}}}\langle\bar{1}\bar{2}\rangle\langle\bar{3}\bar{4}\rangle(k_1 - k_2)(k_3 - k_4)\Big].$$

- From here, can write down full N^{-2}MHV amplitude and find new numerators $\tilde{n}_i$ using permutations

Relation to CFT Correlators

- Witten diagrams calculate the transverse part of the CFT spin-1 correlator
$$\langle JJJJ\rangle = \langle jjjj\rangle + f_1(k_i)\langle jjj\rangle + f_2(k_i)\langle jj\rangle$$
- Longitudinal pieces do not have $1/E$ pole and so vanish in flat space, but are still important here
- We can reconstruct the full correlator using Ward identities by adding lower order contact terms
As in 1304.7760

Next Steps?

- We can construct
$$\langle t_1 t_2 t_3 t_4 \rangle = \frac{k_1 k_2 k_3 k_4}{E} \left(\frac{\tilde{n}_s^2}{s} + \frac{\tilde{n}_t^2}{t} + \frac{\tilde{n}_u^2}{u} \right)$$
- Need to calculate gravitational versions to compare to those obtained from double copy (We might get subleading correction terms)
- Transform to dS space and link to cosmological correlators
- Scattering equation/ worldsheet links