

# QED-effects in $B \rightarrow K\ell\ell$ moments

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## OVERVIEW

- |   |   |     |                   |
|---|---|-----|-------------------|
| I. Brief overview QED frontier<br>& summary of $B \rightarrow K\ell\ell$ results so far                         | 2 | [2] | <b>Theory</b>     |
| II. QED moments in $B \rightarrow K\ell\ell$<br>- 2015 perspective<br>- 2020/21 perspective & new opportunities | 2 | [5] |                   |
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| IV. Conclusions   | 2 | [1] |                   |

**Backup including other work QED-corrections**

# I) QED-corrections in Flavour Physics

“..... becoming an active field and a precision frontier”

- Overview of Methods in increasing structure dependence (resolving the Mesons)

Meson a point particle

Meson with partons!

## “Scalar QED”

PHOTOS MC

Bordone, Pattori, Isidori '16 B->Kll

## Meson EFT B->Kll

Isidori Nabeebaccus, RZ '20

## Chiral Perturbation Theory

K->πlv '00 Cirigliano, Knecht, Neufeld,...

K-> πll, '10 Kubis et al

## Inclusive b-> ell

Huber, Lunghi, Misiak, Wyler '05

Huber, Hurth, Lunghi '15

## Lattice -

proposed '15 - Sachrajda and Rome group

K->l v fist results

more to come other groups Portelli, Guelpers

## SCET-

$B_s \rightarrow \mu\mu$  Beneke, Bobeth, Szafron '17'19

$B \rightarrow K\pi$  Beneke, de Boer, Teolstede, Vos '20

*Analytic approaches (SCET & more to come):*

- Good  $\alpha_{QED}$ -convergence (attention to large logs) ✓
- (Soft)-collinear **Factorisation**, absorbing unphysical IR-divergences) more **complicated** than in **QCD!** ⚡ ⚡ ⚡

# Overview of rare mode B-> Kll

- **scalar QED:** B,K= point particle - PHOTOS & **Bordone, Patteri, Isidori'16**
  - compute real and infer virtual logs from cancellation in rate (good variables)
  - check  $m_{B_{rec}}$ -histogram against PHOTOS and it looks good
- **meson EFT:** B,K partially resolved form factor expansion **Isidori Nabeebaccus, RZ '20**
  - compute real,virtual (“partial finite terms”),  $\ln m_K/m_b$ -logs O(2%) in rate & establish (non)-cancellations of logs
  - fully double differential (no need to resort to kinematic approximations for migration)  
(cf. app A.2.or backups for BIP'16 -comparison)  $R_K$

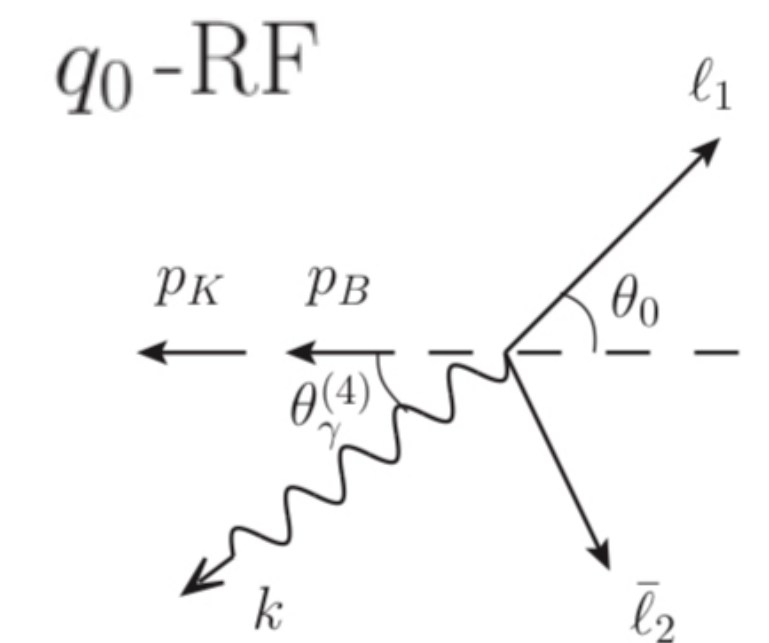
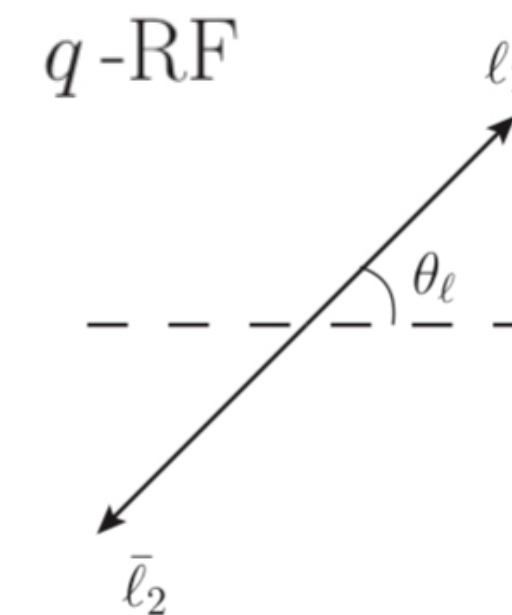
- 1) **Proof** (by gauge invariance) **no** further dangerous **logs** by **structure** dependance !
- 2) Cancellation depends collinear logs depends on kinematic variables

Results

Do collinear logs cancel at differential level (photon inclusive case)?

$q_\ell^2 = (\ell_1 + \ell_2)^2$ ,    [“Hadron **collider**” variables] ,    no

$q_0^2 = (p_B - p_K)^2$     [“B-factory” variables] ,    yes



➔  $R_K$  **theoretically** clean observable, (more) solid ground

- **structure depend approach:** in progress in collinear factorisation  
get finite terms & improved  $\ln m_K/m_b$  ....

hopefully next time .....

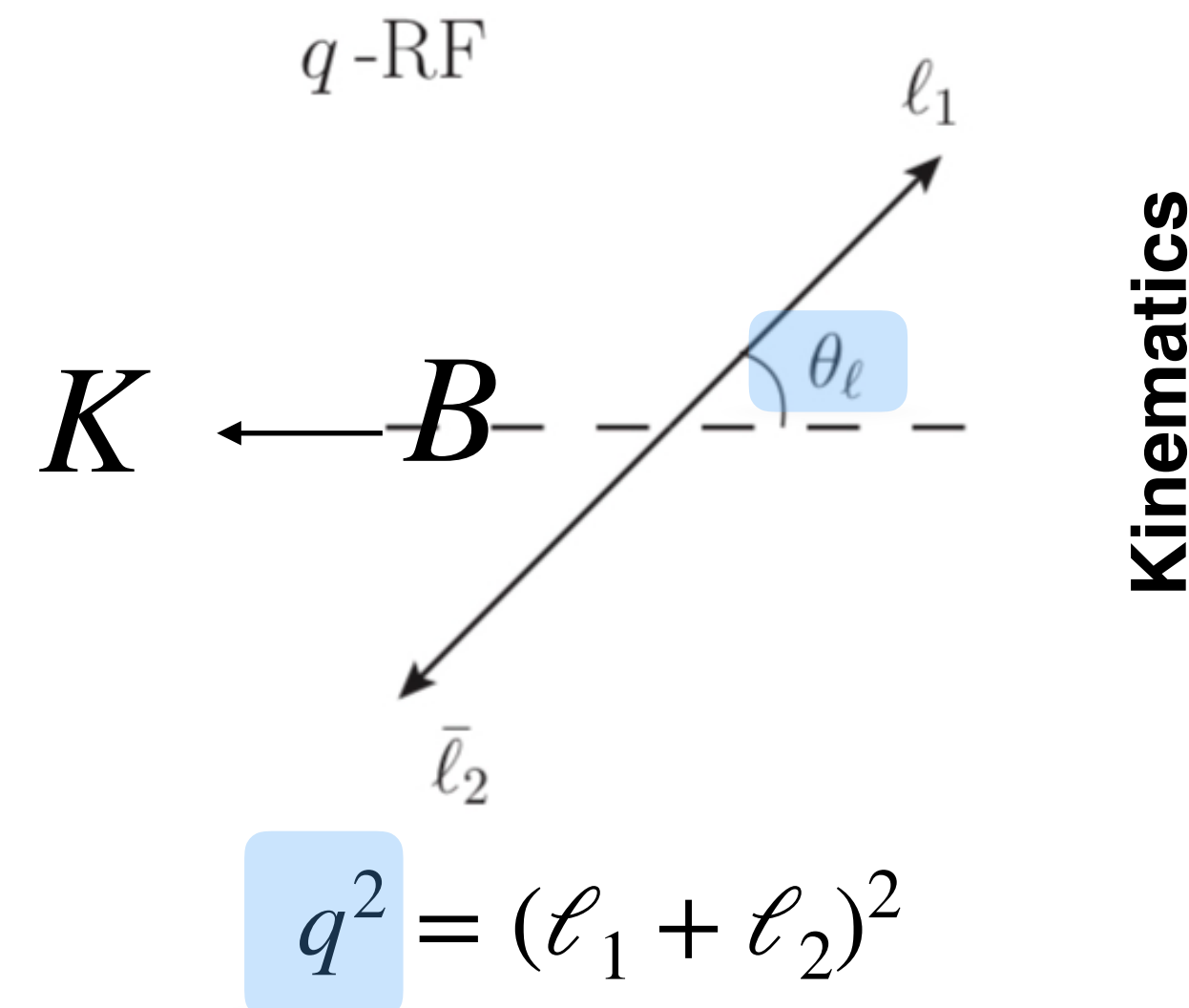
**After summary - main talk starts**

## Moments in B → KII

$$\frac{d^2}{dq^2 dc_{\theta_\ell}} \Gamma = \sum_{l=0}^{l_{\max}} \sum_{m=-l}^l M_{lm}(q^2, c_{\theta_\ell}) P_\ell^m(c_{\theta_\ell})$$

$$M_{lm} \propto \int_{-1}^1 dc_{\theta_\ell} P_l^m(c_{\theta_\ell}) \frac{d^2}{dq^2 dc_{\theta_\ell}} \Gamma$$

- Hypothesis: (a) no QED-corrections  
(b) dim=6 Heff (standard one)



Then it's well understood that  $l_{\max} = 2$  as only S- & P-wave in amplitude

## QED-corrections in $B \rightarrow K\ell\ell$

$$\Gamma(B \rightarrow K\ell\ell) + \Gamma(B \rightarrow K\ell\ell\gamma) = \Gamma^{(0)} \left( 1 + \frac{\alpha}{\pi} O(1) \right)$$

- However, sizeable (logs)
  - 1) differential in  $(q^2, \theta_\ell)$
  - 2) differential in photon e.g. photon energy cut-off  $\Delta E$

small in photon inclusive rate  
(no logs by unitarity e.g. Bloch-Nordsieck, KLN)

$$\frac{\alpha}{\pi} O(1) \rightarrow \frac{\alpha}{\pi} O(1) \ln \frac{m_\ell}{m_b} \ln \frac{\Delta E}{m_b}$$

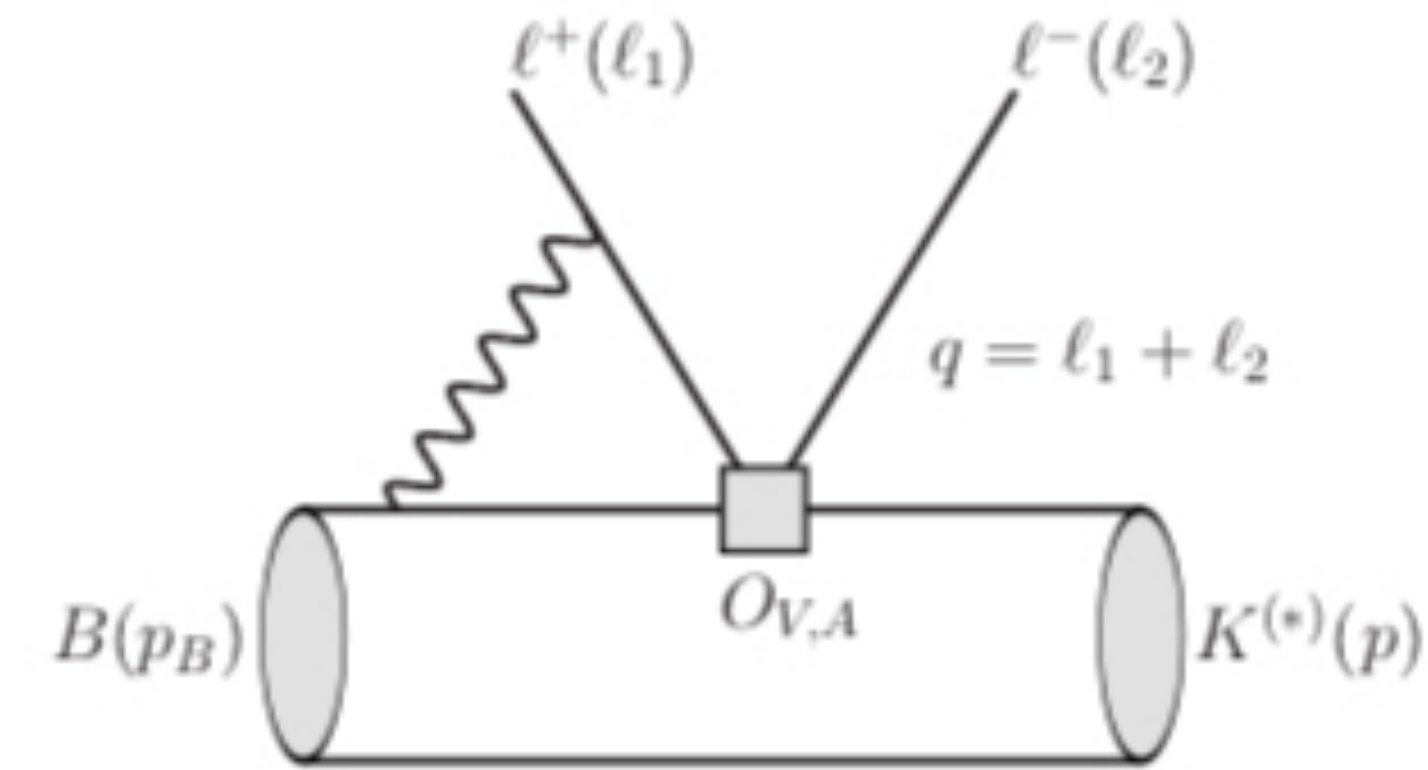
$O(1\%) \qquad O(10 + \%)$

# QED-Moments in B->Kll the 2015 perspective

Hopfer, Gratex, RZ'15

Generalised helicity formalism for EFT

- **with** (no) QED B->Kll 1-> 3(2) process}
  - no:** (ll)-pair = "1-particle" :  $l_{max} = 2$
  - with:** richer:  $l_{max} = \infty$



⇒ Proposed to assess QED in higher moments (i.e.  $l > 2$ ) (soft)-collinear logs would lead to sizeable moments

$$M_{l>2}^{ee} \gg M_{l>2}^{\mu\mu}$$

relax condition a)



## QED-Moments in B → Kll the 2020 perspective

- A splendid **formula (hard-collinear logs)** from LO-differential rate

$$\frac{d\Gamma}{dq^2} \Big|_{hard-col.} \propto Q_{\ell_1}^2 \frac{\alpha}{\pi} \frac{1}{\Gamma^{LO}} \left( \int_{\hat{q}^2}^1 \frac{dz}{z} P_{f \rightarrow f\gamma}(z) \frac{d\Gamma^{LO}(\hat{q}^2/z)}{d\hat{q}^2/z} \right) \ln \frac{\Lambda_b}{m_\ell}$$

$$P_{f \rightarrow f\gamma}(z) = \frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z)$$

universal splitting  
function for fermion to  
photons

$$\int_0^1 dz P_{f \rightarrow f\gamma}(z) = 0.$$

cancellation of logs in  
photon inclusive rate

also holds when double differential

⇒ No sizeable QED in higher moments (i.e.  $l > 2$ )  
as no higher moments @LO !!

$$M_{l>2}^{ee} \approx M_{l>2}^{\mu\mu}$$



## QED-Moments in B-> Kll the 2021 chase of perspective

*What is the new bottom line?*

1. Test experimentally  $M_{l>2}^{ee} \approx M_{l>2}^{\mu\mu}$  (hard for electrons)

2. Predict  $M_{l>2}^{\ell\ell}$  from theory  $\Rightarrow$  **need structure dependent approach**

*Meson EFT QED-moment  
paper makes limited sense*

(A) Compare  $M_{l>2}^{\mu\mu}$  to experiment.

(B) Test for New Physics (NP) relax assumptions b) i then **dim= 8,10 Heff**

$$H_{eff} \supset \frac{C_8}{\Lambda_{NP}^2} \bar{b} \gamma_{\{\mu} \overleftrightarrow{D}_{\nu\}} s \bar{\ell} \gamma_{\{\mu} \overleftrightarrow{D}_{\nu\}} \ell \quad \Delta_{C_8} = \begin{cases} \frac{m_b^2}{\Lambda_{NP}^2} \approx O(10^{-5})? & \text{UV-NP} \\ O(\text{not-so-small}) & \text{light-NP} \end{cases}$$

• Can we test for light NP using higher muon moments?

If new physics really light then what  $\Delta_{C_{10,12}}$ ? (which we can also predict)

- Theorists wish:  $M_{l>2}^{\ell\ell}(q^2)|_{\text{photon-cuts}}$  &  $M_{l>2}^{\ell\ell}(q_0^2)|_{\text{photon-cuts}}$  from experiment (Bresmstrahlung removed)

⇒ A lot of useful information can be extracted

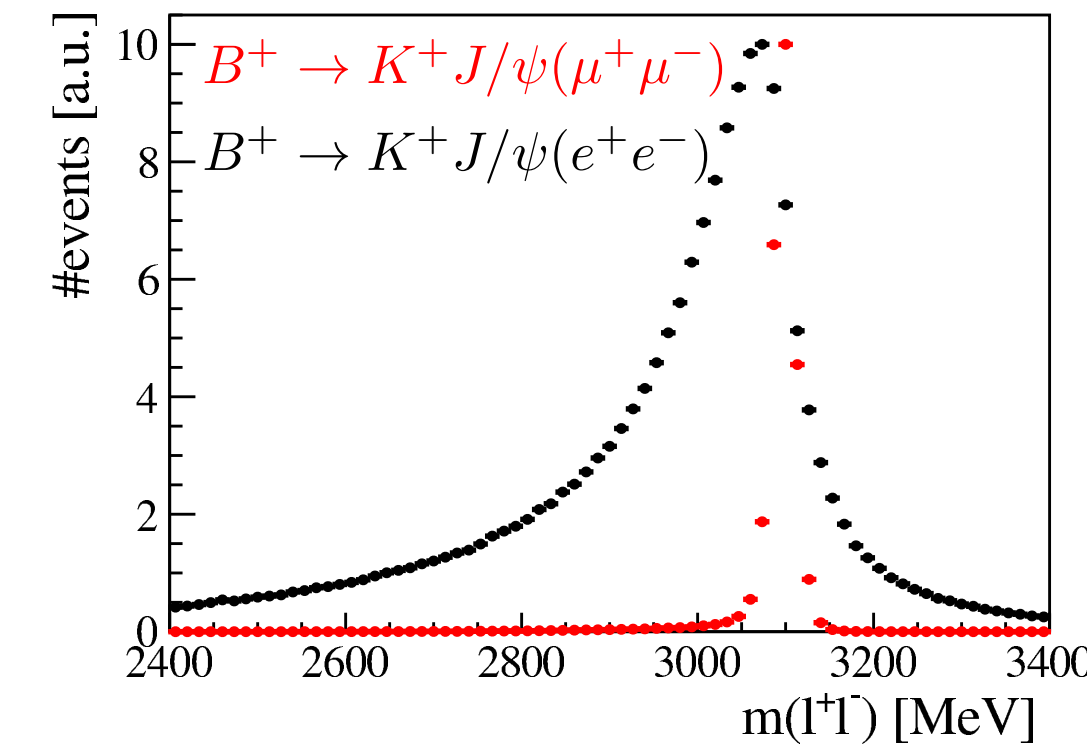
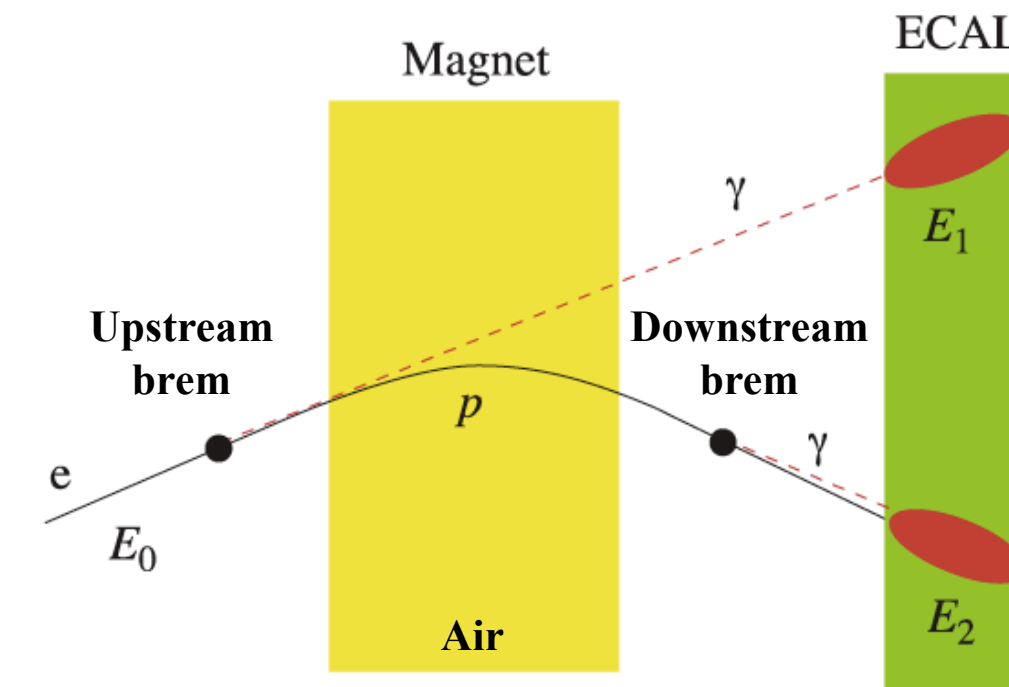
Now, Patrick will give you an idea why this is not straightforward....

# Electrons at LHCb

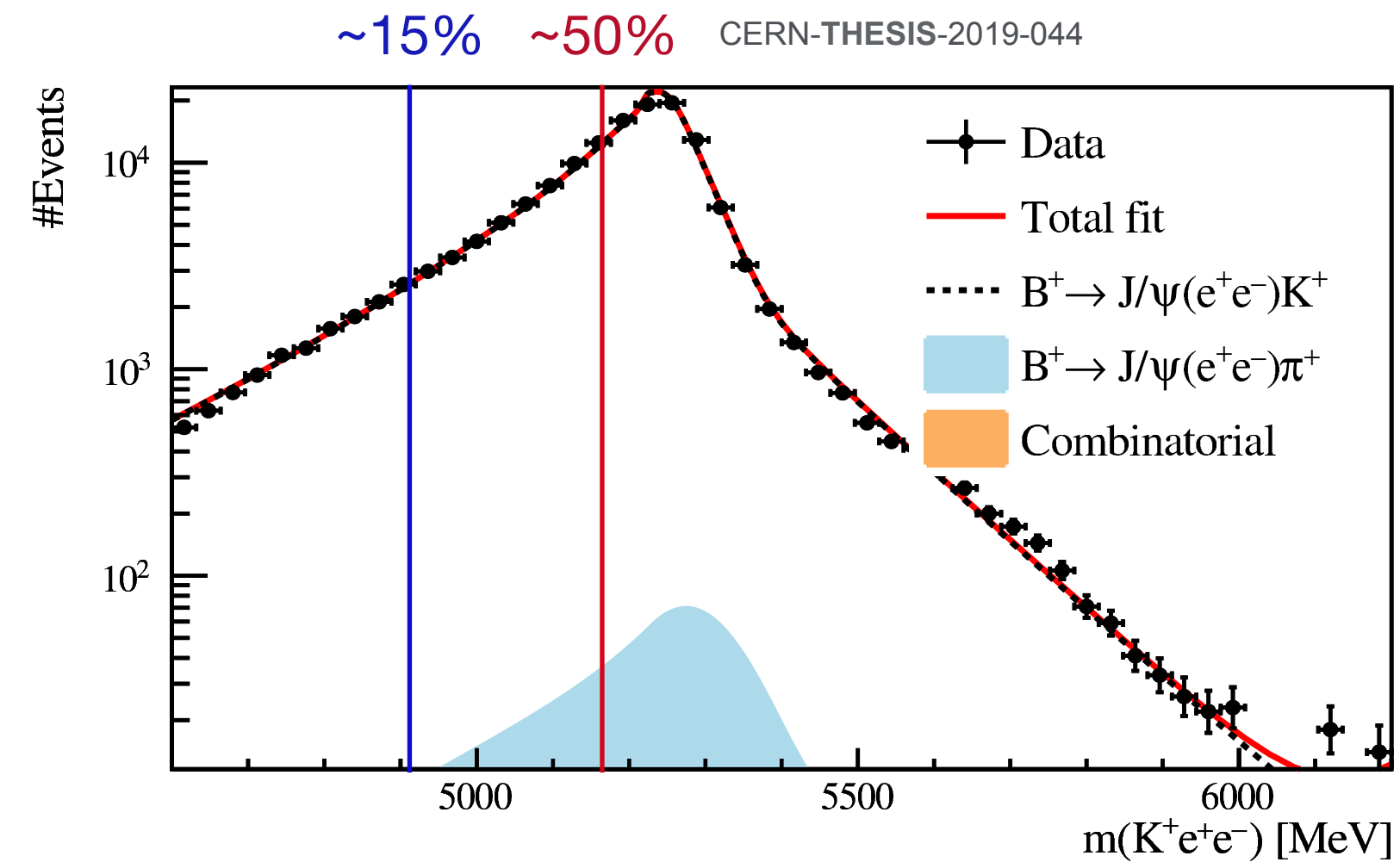
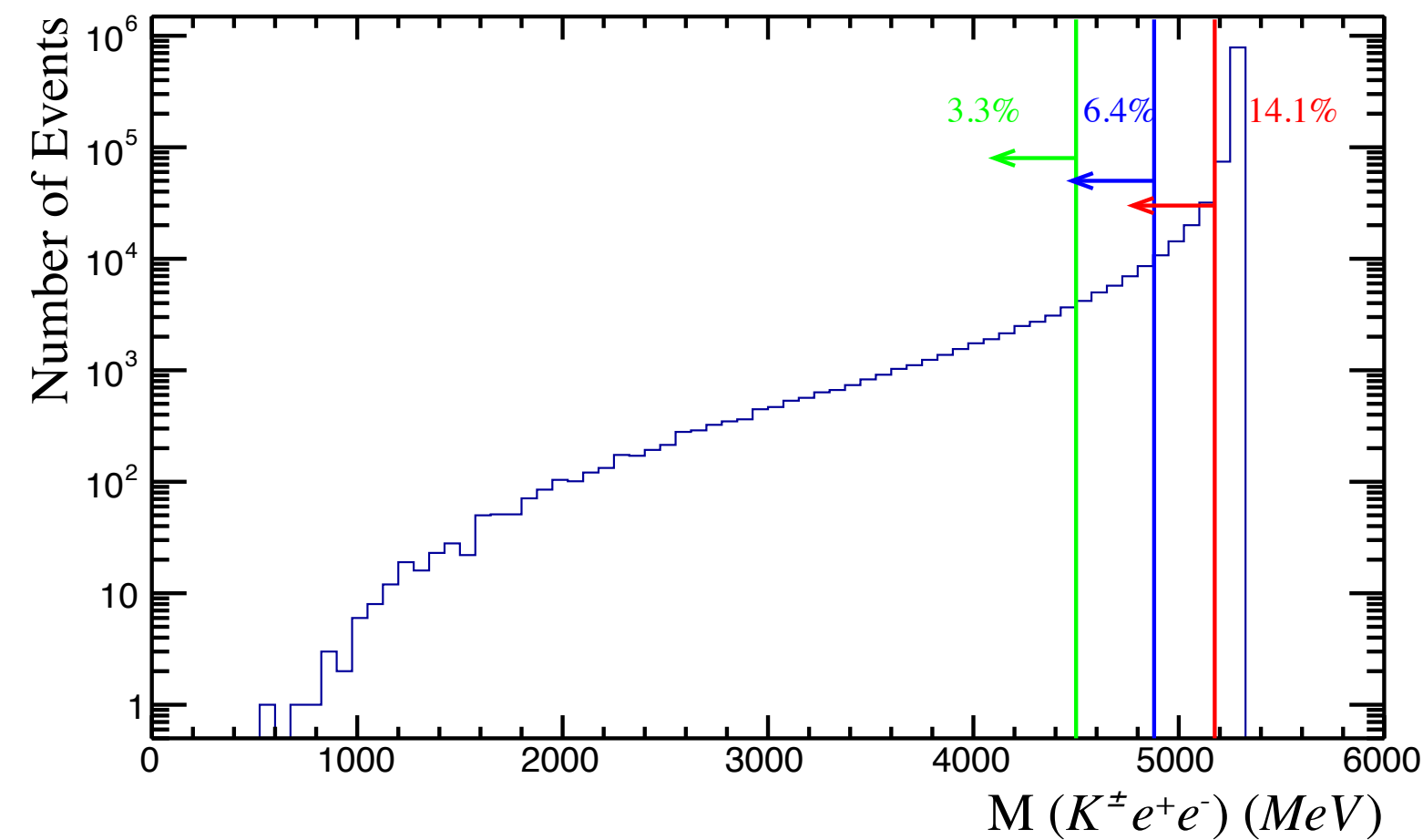
Many plots have been taken from Thibaud Humair's thesis: [CERN-THESIS-2019-044](#)

Electrons lose energy from two ways in LHCb:

- Bremsstrahlung
- FSR - modelled with PHOTOS



R. Coutinho's talk at Munich

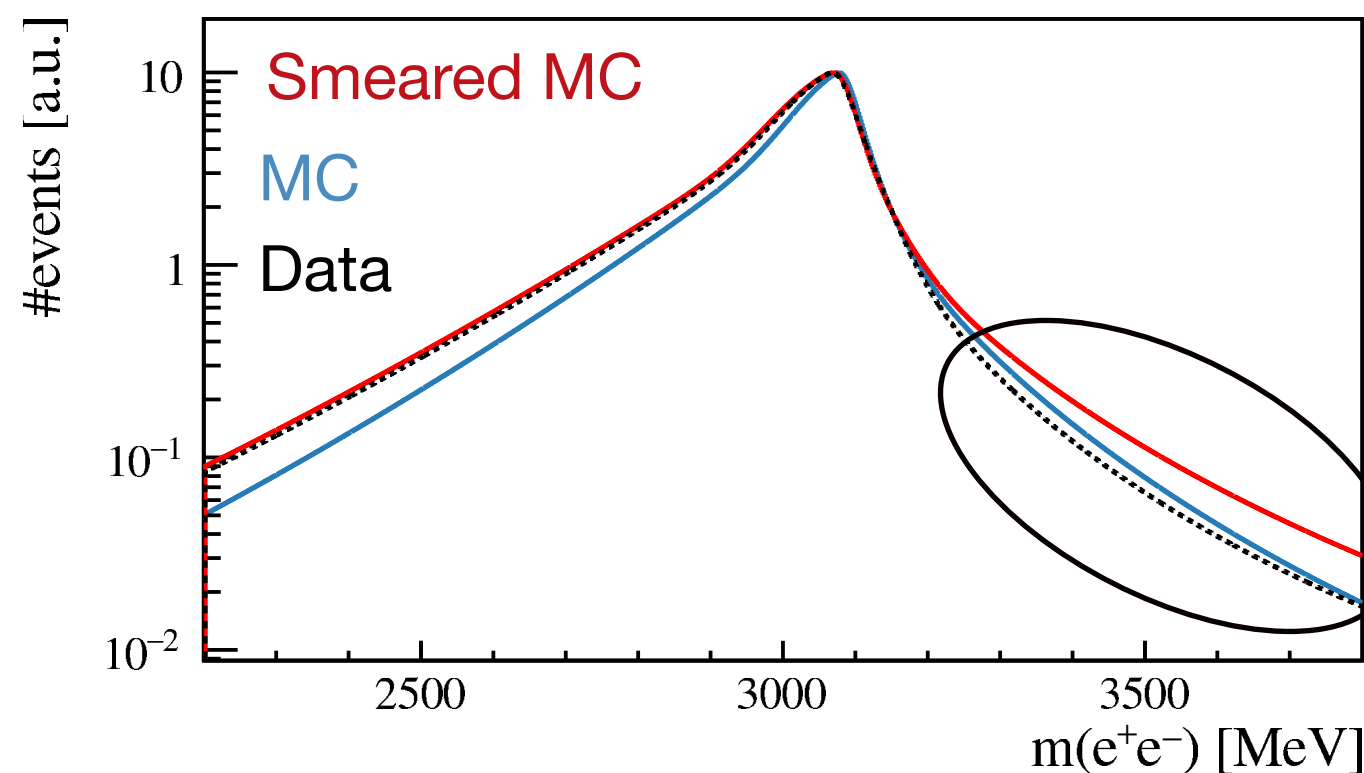


Effect of QED is sub-dominant with respect to bremsstrahlung - but that doesn't mean it doesn't matter!

# How is it controlled

- Shape difference between data/MC obtained from J/psi mode. This difference is assumed to translate to rare mode.
- Simulation shows that the correction is nicely portable.
- What would happen if QED effects are vastly different between the two modes?

CERN-THESIS-2019-044



Imperfection here leads to negligible systematic.

- Migration in and out of  $q^2$  bin very small, unlikely to be an issue in any case.

CERN-THESIS-2019-044

	up→in [%]	down→in [%]	in→in [%]	in→up [%]	in→down [%]
Run 1					
No smearing	$8.00 \pm 0.23$	$0.34 \pm 0.05$	$96.94 \pm 0.14$	$1.31 \pm 0.10$	$1.75 \pm 0.11$
smearing	$7.90 \pm 0.23$	$0.43 \pm 0.06$	$96.83 \pm 0.15$	$1.53 \pm 0.10$	$1.65 \pm 0.11$

- Only possible problem would be in the B mass shape. This was checked for rare mode\*. Should we also check the J/psi?

# Why it's probably fine

PHOTOS cross-checked with independent calculation, things look under control.

Bordone, Isidori, Pattori  
arXiv:1605.07633

Only impact would be on Mass shape (bin migration barely affected).

Shapes between  $J/\psi$  and rare mode very similar in simulation.

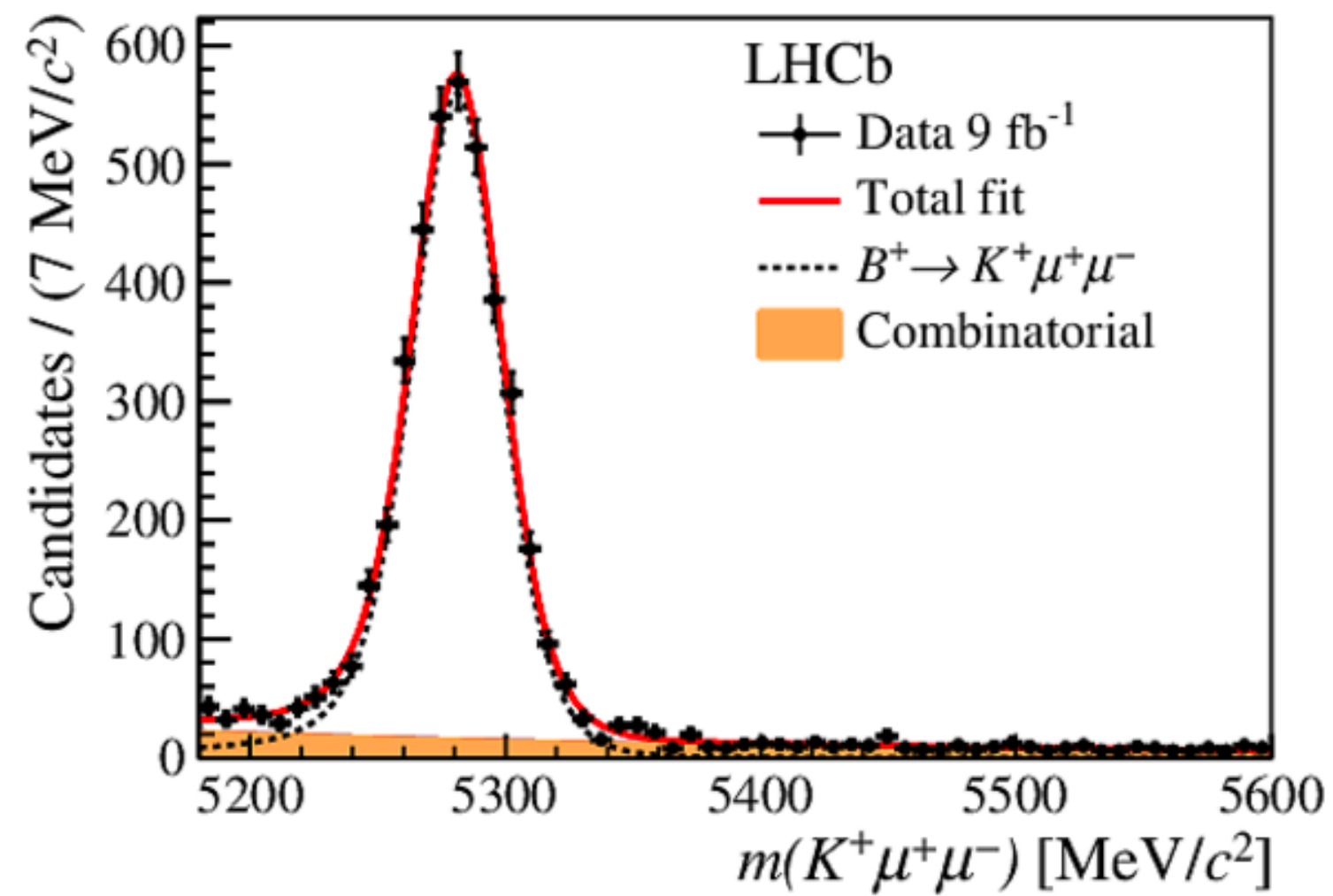
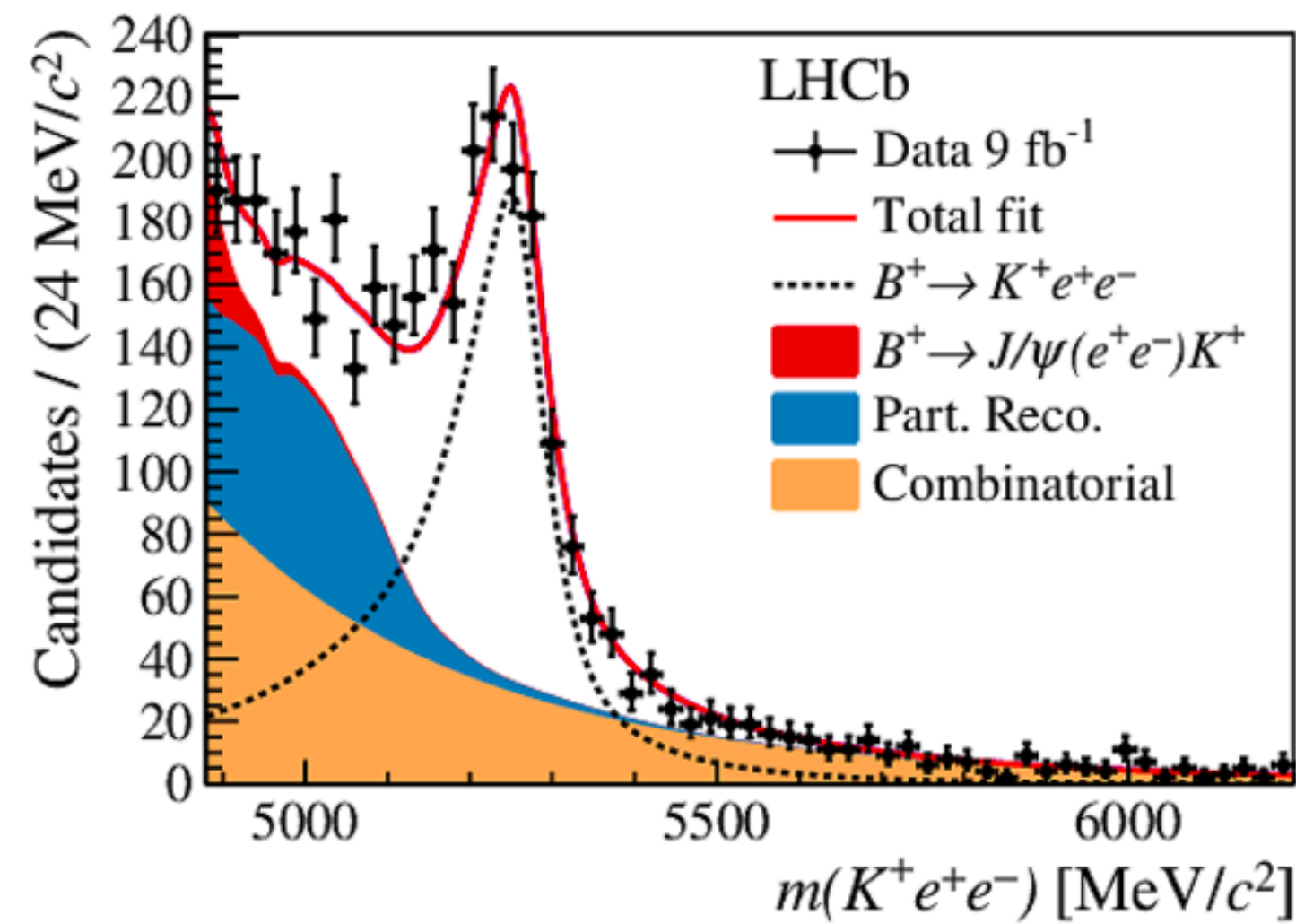
After correction  $J/\psi$  shape looks good.

—>Everything should be under control.

Still, can we test it to make sure?

# How to test it

- Could we compare  $\cos(\theta_l)$  distributions between  $J/\psi$  and rare mode?
- Problem: QED effect very correlated with B mass.
  - Can we control this enough to make precise enough test?
- Another problem: Veto to reject  $B \rightarrow (X_c \rightarrow K^- \ell^+ \nu_\ell X) \ell^- \nu_\ell X$  cuts out  $\cos(\theta_l)$  region.



LHCb-PAPER-2021-004, arXiv:  
2103.11769

- Would we learn anything from looking at the muon channel?

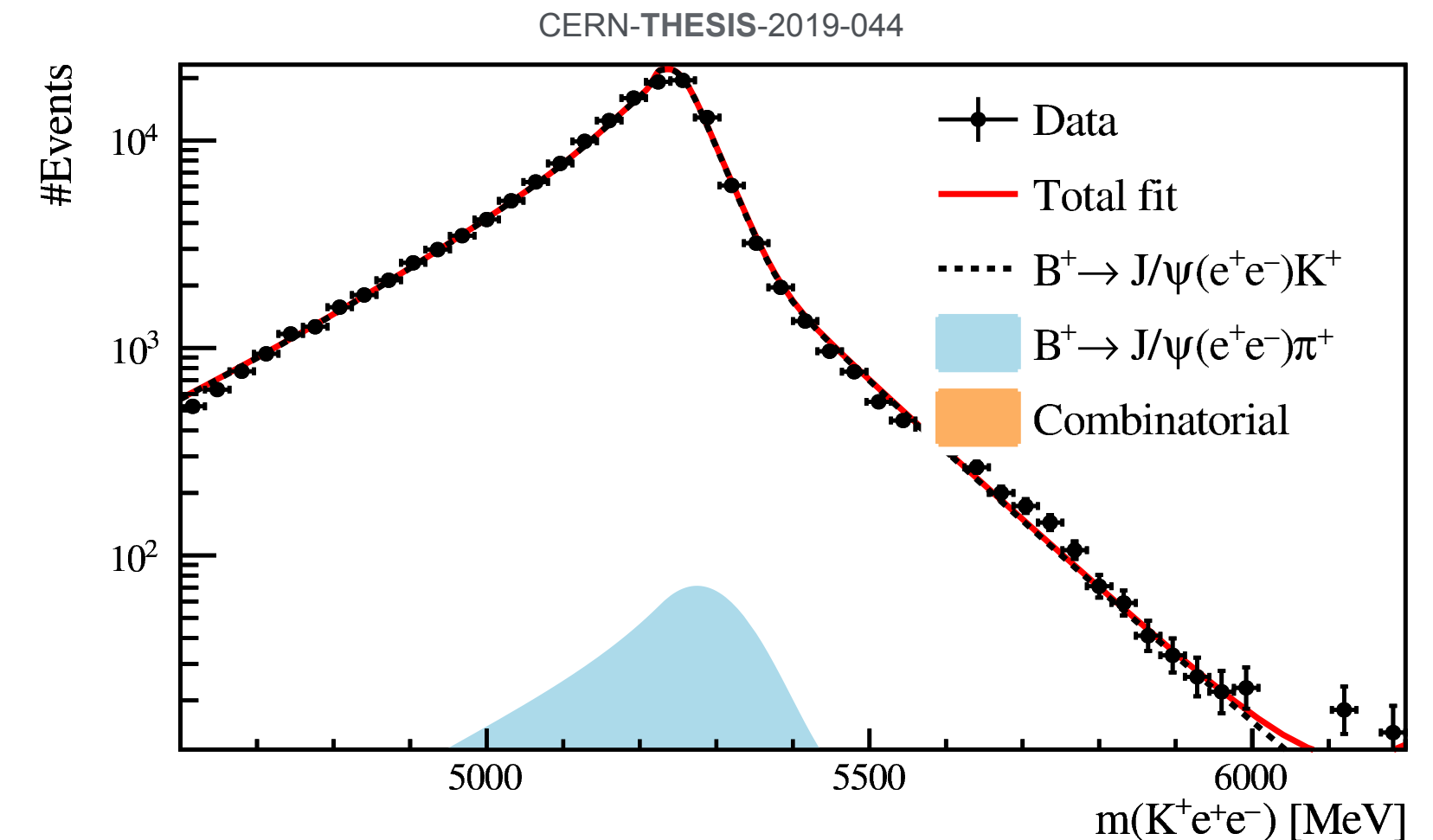


# What about the $J/\psi$ mode?

- $J/\psi$  mode would not suffer from this problem.
- Cannot calculate  $\cos(\theta_l)$  after any mass constraint as that would remove sensitivity to it.

- How about:

- Measure  $\cos(\theta_l)$  distribution with no constraints.
- Turn PHOTOS off and determine migration matrix.
- Publish unfolded spectrum (and efficiency corrected).



- How would one validate the unfolding matrix? Seems difficult without relying on some QED model.
- How do we avoid chasing our tail here? Fold in different QED models?



## Conclusions

- **Cross checking PHOTOS** against dedicated Monte Carlo using INZ'20 at different  $q^2$  for  $R_K$  and  $B \rightarrow K\ell\ell$  differential rate will be a good sanity check

$$\Delta_{\text{QED}} R_K \approx \frac{\Delta\Gamma_{K\mu\mu} \Big|_{m_B^{\text{rec}}=5.175 \text{ GeV}}}{\Gamma_{K\mu\mu} \Big|_{q_0^2 \in [1,6] \text{ GeV}^2}} - \frac{\Delta\Gamma_{Kee} \Big|_{m_B^{\text{rec}}=4.88 \text{ GeV}}}{\Gamma_{Kee} \Big|_{q_0^2 \in [1,6] \text{ GeV}^2}} = 1.7 \% \quad \text{vs} \quad 3.0 \%$$

INZ'20
BIP'16

- 2020-perspective: use **higher moments** to **test light-NP**.  
Theory prediction necessitates structure dependent effects: precision frontier & active field
- The way we correct experimentally means that we implicitly assume no large differences between  $J/\psi$  and rare mode.
- Explicitly checking electron mode in data seems difficult, perhaps intractable.
- Muon or  $J/\psi$  modes more feasible - would those give us the information we want?

Understand what we do,  
 but what exactly does PHOTOS do?  
 experimental treatment  
 slight LFU-violation  
 (needs corrected for)

# **BACKUP THEORY STUFF**

# Cancellation of logs (photon-inclusive)\*

cancel?	$\frac{d^2\Gamma}{dq^2 dc_\ell}$	$\frac{d^2\Gamma}{dq_0^2 dc_0}$	“main result 1”
<b>soft</b>	yes	yes	
<b>soft-collinear</b>	yes	yes	
collinear	no	yes	

• Note: once photon energy cut-off restored (all logs come back)

total rates agree

$$\Gamma(\Delta E) = \int_{\Delta E} \frac{d^2\Gamma}{dq^2 dc_\ell} dq^2 dc_\ell = \int_{\Delta E} \frac{d^2\Gamma}{dq_0^2 dc_0} dq_0^2 dc_0$$

---

\* use photon energy cut-off - all done analytic  
 (technical aspect: soft energy and angular integral shown to be separately Lorentz-invariant!)

Q: Are the collinear logs universal?

$$C(\Delta E)_{method 1} \stackrel{?}{=} C(\Delta E)_{method 2}$$

Or if  $B, K$ -meson resolved (structure dependence), further collinear logs?

A: yes, no new col.-logs  $\ln m_{\ell_1}$  due to gauge invariance

• Write in meson-EFT:  $A^{(1)} = \hat{Q}_{\ell_1} a_{\ell_1} + \delta A^{(1)}$

1)  $\hat{Q}_{\ell_1}^2 \int_{\gamma} |a_{\ell_1}|^2 = O(1) \hat{Q}_{\ell_1}^2 \ln m_{\ell_1} + \dots$  **collinear-log**      whereas  $\int_{\gamma} Rest \xrightarrow{m_{\ell_1} \rightarrow 0} finite^*$ . **IR-safe**

2) Hence  $\delta A \rightarrow \delta A + A_{structure}^{B,K}$ , no new real collinear logs

3) Since real & virtual cancel (in  $q_0^2, c_0$  variables),  
no new virtual collinear logs either

“main result 2”

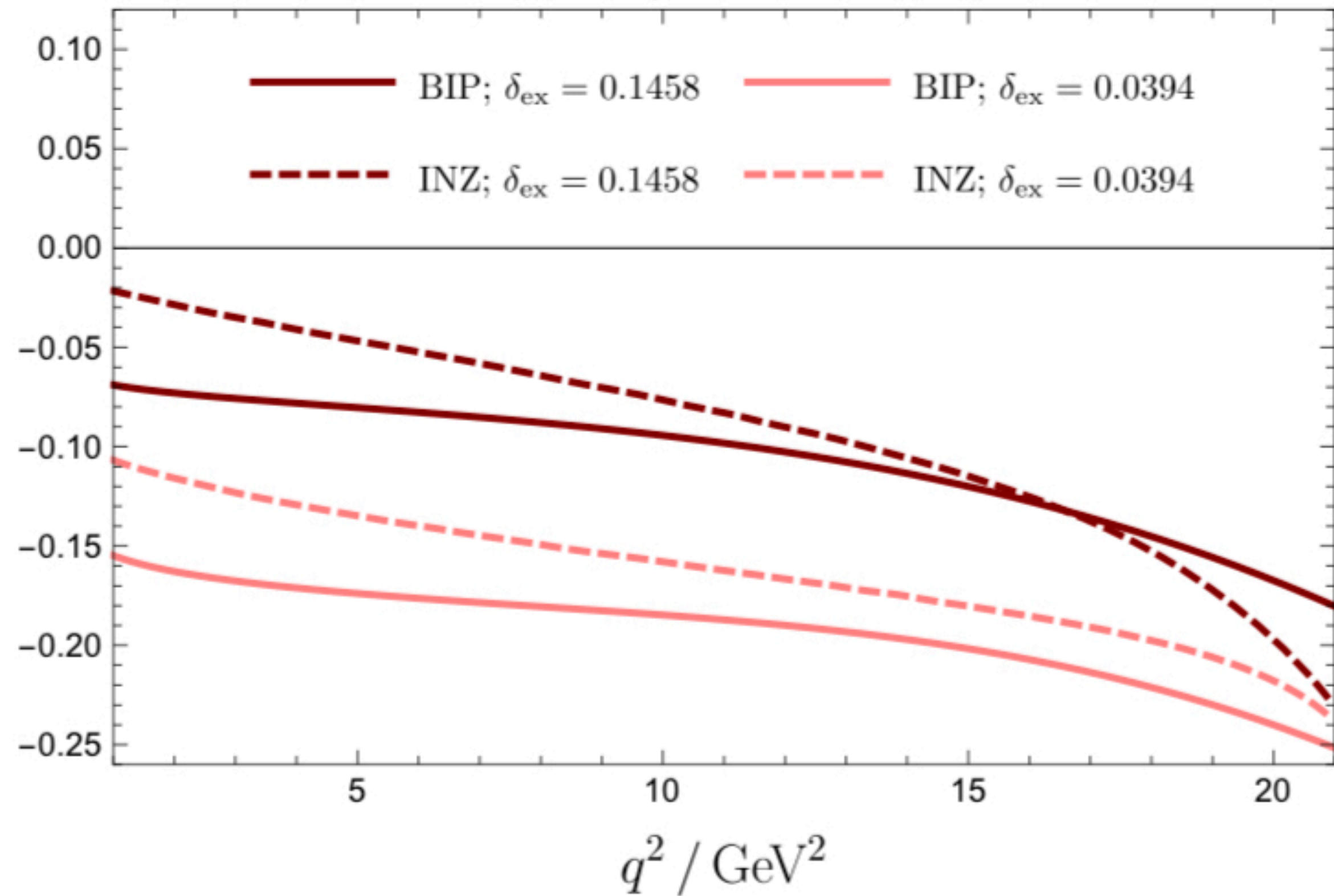
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\* by gauge invariance: collinear region:  $A = \epsilon^\mu A_\mu \Rightarrow \ell_1^\mu A_\mu = \mathcal{O}(m_{\ell_1})$

# Difference between BIP and INZ

sizeable in electron mode at low  $q^2$  ca 8%

$$\Delta^{(\ell)}(q^2; \delta_{\text{ex}}), \bar{B}^0 \rightarrow \bar{K}^0 e^+ e^-$$



$$\Delta^{(0)}(q_0^2; \delta_{\text{ex}}), \bar{B}^0 \rightarrow \bar{K}^0 e^+ e^-$$

