

# Long distance charm rescattering and connections to $B \rightarrow D\bar{D}h$

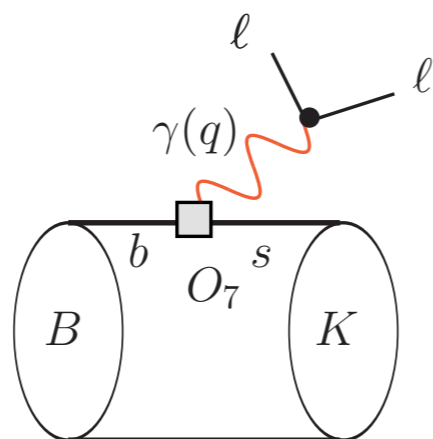
Daniel Johnson (CERN),  
Méril Reboud (TUM),  
Patricia Magalhães (UOB/ITA)

**Beyond Flavour Anomalies II**  
**April 2021**

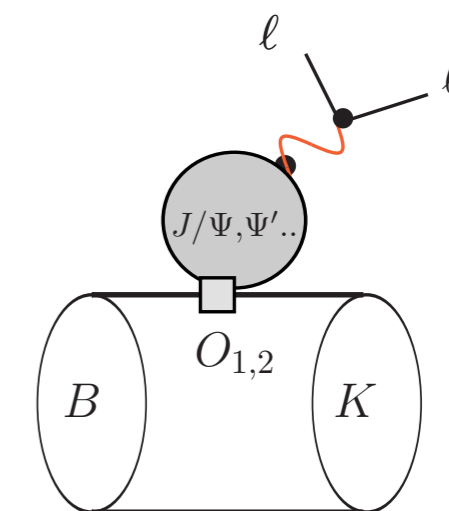
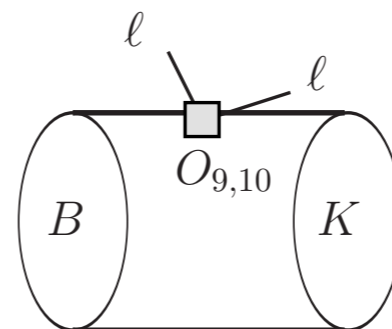
- Anomalies in  $b \rightarrow s \ell \bar{\ell}$ 
  - tensions with SM predictions

↪  $B \rightarrow K^{(*)} \ell^+ \ell^-$

- QCD picture



short



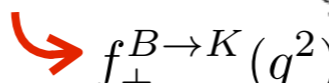
short + long

→ charm loop and the nonperturbative contribution is trick to account  
**theoretical challenge !**

→ understanding the impact of intermediate  $c\bar{c}$  contributions, both resonant and non resonant, is of paramount importance in order to be able to provide reliable and precise SM predictions.

- QCD parametrization

$$A_{\lambda}^{L,R}(B \rightarrow M_{\lambda} \ell \ell) = \mathcal{N}_{\lambda} \left\{ (C_9 \mp C_{10}) \mathcal{F}_{\lambda}(q^2) + \frac{2m_b M_B}{q^2} \left[ C_7 \mathcal{F}_{\lambda}^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_{\lambda}(q^2) \right] \right\}$$


 $f_{+}^{B \rightarrow K}(q^2)$

- non-local form-factors:

$$\mathcal{H}_{\lambda}(q^2) = i\mathcal{P}_{\mu}^{\lambda} \int d^4x e^{iq \cdot x} \langle \bar{M}_{\lambda}(k) | T \{ \mathcal{J}_{\text{em}}^{\mu}(x), C_i \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$$

- size of non-factorisable corrections in  $b \rightarrow s c \bar{c}$  can be significant

 not colour suppressed

- naive factorization

- charm loops are described by  $c\bar{c}$  resonances given by charm vacuum polarization  $h_c(q^2)$

$$\rightarrow \langle K | C_1 \mathcal{O}_1^c + C_2 \mathcal{O}_2^c | B \rangle |_{\text{FA}} \propto (C_1 + C_2/3) f_+^{B \rightarrow K}(q^2) h_c(q^2)$$

- [Lyon- Zwicky]

used BESII data  $\rightarrow R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu\mu)} \propto \text{Im } h_c(q^2)$

*DD̄, D\*D̄, D\*D̄\* ...*

- $R_{\text{fit}}(s) = R_{\text{res}}(s) + R_{\text{con}}(s)$

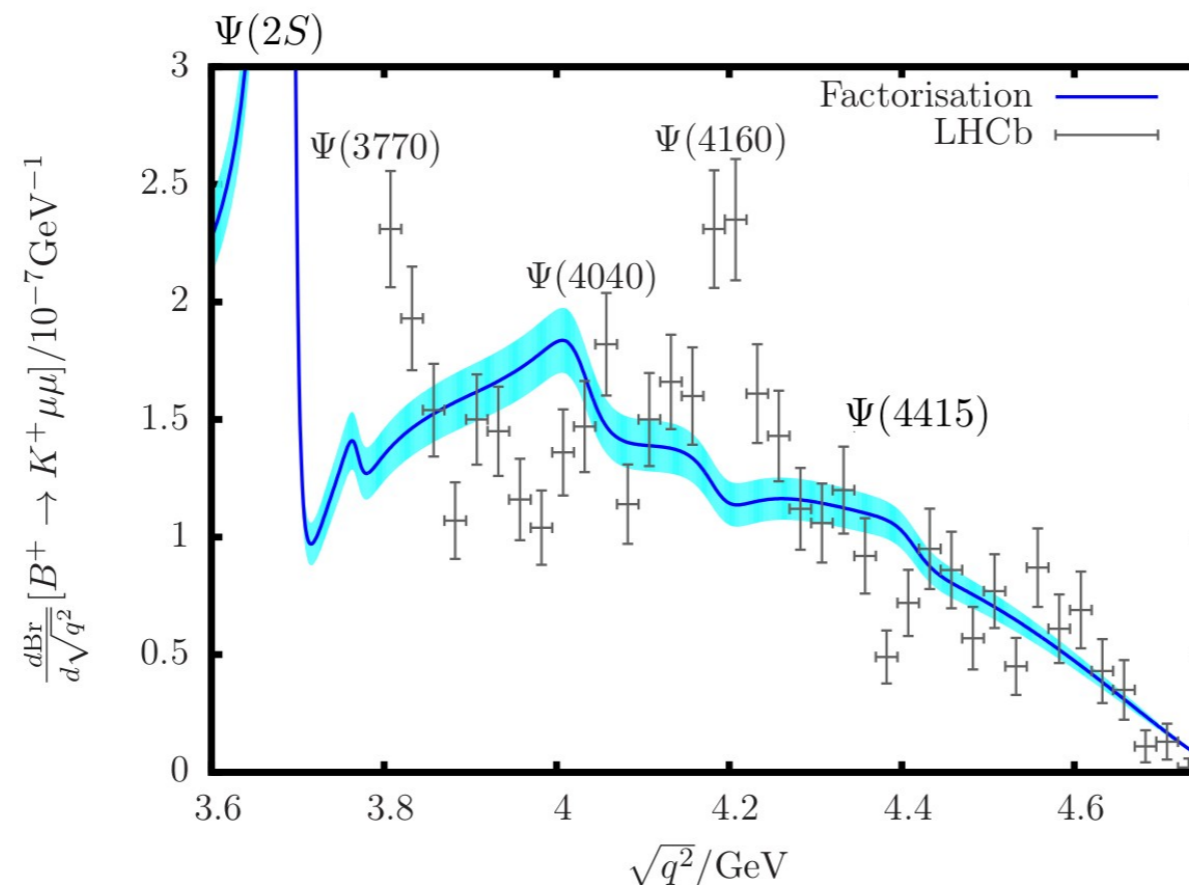
$\hookrightarrow$  as Breit-Wigner

$\rightarrow$  non-factorizable effects are important (high- $q^2$  OPE)!

$\rightarrow$  not enough to cover the anomaly!

[Lyon-Zwicky] (see also [Brass, Hiller & Nisandzic]): *a posteriori* check of the factorization approach

- Requires additional factors to match measurements



- R ratio **cannot** correctly reproduce the resonances:  
e.g.  $\Psi(3770)$  is a D-wave resonance so its decay-constant vanishes in the non-relativistic limit!

- More general approach (in collaboration with D. van Dyk and S. Kürten):

$$\mathcal{H}_\lambda^{\text{res}}(q^2) = \sum_\psi \frac{\mathcal{A}(\psi \rightarrow \ell\ell) \mathcal{A}(BK^{(*)} \rightarrow \psi)}{m_\psi^2 - q^2 + i\Gamma_\psi m_\psi} \quad (\text{for illustration only!})$$

- Fix parameters using:

$$\mathcal{A}(e^+e^- \rightarrow D\bar{D}) \propto \sum_\psi \frac{\mathcal{A}(\psi \rightarrow D\bar{D}) \mathcal{A}(\psi \rightarrow e^+e^-)}{m_\psi^2 - q^2 + i\Gamma_\psi m_\psi} \quad (\text{BES, BaBar, Belle})$$

$$\mathcal{A}(B \rightarrow K^{(*)} D\bar{D}) \propto \sum_\psi \frac{\mathcal{A}(\psi \rightarrow D\bar{D}) \mathcal{A}(BK^{(*)} \rightarrow \psi)}{m_\psi^2 - q^2 + i\Gamma_\psi m_\psi} \quad (\text{LHCb, BaBar, Belle})$$

- Number of parameters:
  - *Naive approach* (Breit-Wigners): widths and masses are channel dependent → **hundreds** of parameters
  - *K matrix approach*: impose unitarity of the  $S$  matrix

$$S = 1 + 2i T = 1 + 2i \rho^{1/2} \hat{T} \rho^{1/2}$$

$$\hat{T} = \hat{K} (1 - i\rho \hat{K})^{-1}$$

[Chung, Brose, et al. 1995]

[Uglov, Kalashnikova, et al. 2019]

$$\hat{K}_{ij} = \sum_{\psi_r} \frac{g_{ri}^0 g_{rj}^0}{m_{\psi_r}^2 - q^2} + \hat{c}_{ij} \quad i, j \in \{ee, \text{eff}, D^{(*)} \bar{D}^{(*)}, BK^{(*)}\}$$

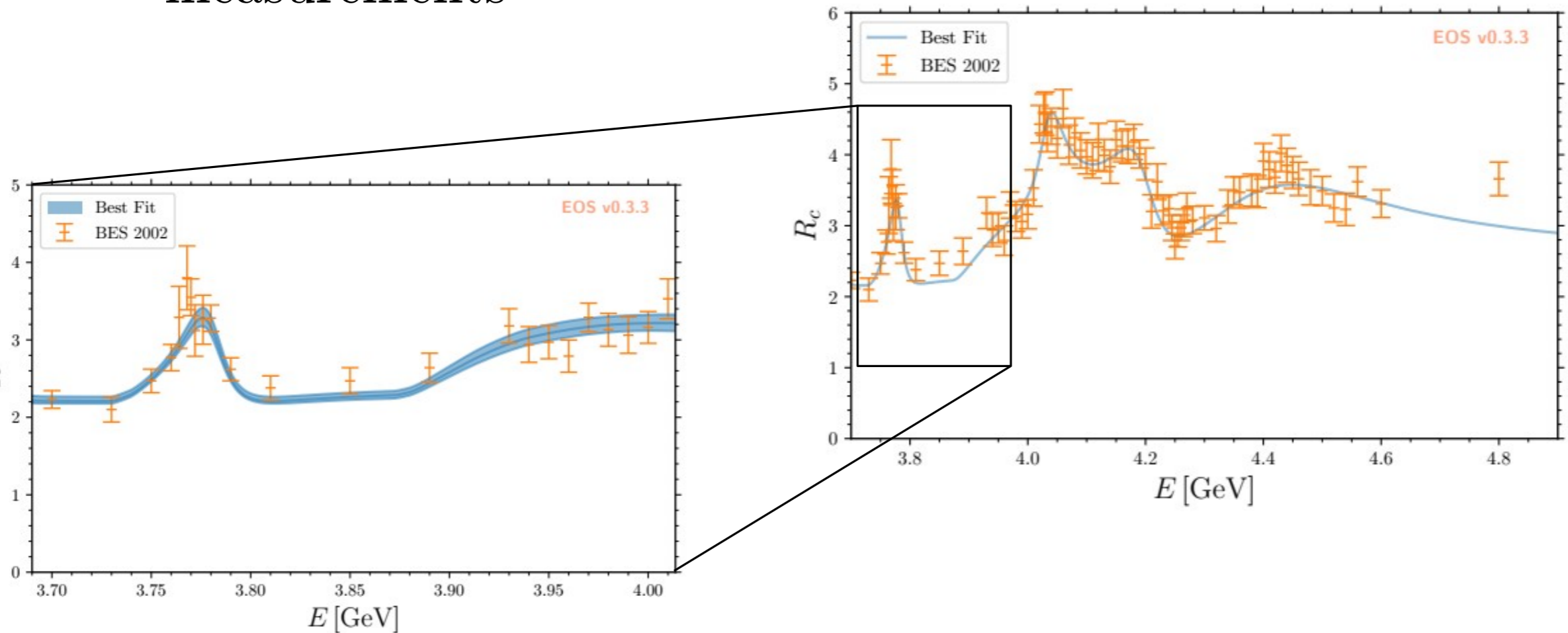
$g_{ri}^0$  real-valued couplings

$\hat{c}_{ij}$  non-resonant contributions

eff effective channel to light hadrons (needed for the  $\psi(2S)$ )

- Strategy:

1) Fit the  $R$  ratio and  $ee \rightarrow D^{(*)} \bar{D}^{(*)}$  cross-section measurements



2) Use it with  $B \rightarrow K^{(*)} D \bar{D}$  Dalitz plot analysis to predict  $B \rightarrow K^{(*)} \ell \ell$  spectrum



- First  $B \rightarrow D\bar{D}h$  results from LHCb in 2020:

JHEP12 139

Measurement of branching fraction ratios for  $B^+ \rightarrow D^{*+}D^-K^+$ ,  $B^+ \rightarrow D^{*-}D^+K^+$ , and  $B^0 \rightarrow D^{*-}D^0K^+$  decays

PRD102 051102

First observation of the decay  $B^0 \rightarrow D^0\bar{D}^0K^+\pi^-$

R. Aaij *et al.*<sup>\*</sup>  
(LHCb Collaboration)

(Received 9 July 2020; accepted 27 August 2020; published 22 September 2020)



The LHCb collaboration

4 world-best BFs

1 first observation

- First study of amplitude structure, also in 2020, using  $B^+ \rightarrow D^+D^-K^+$

Model-Independent Study of Structure in  $B^+ \rightarrow D^+D^-K^+$  Decays

PRL125 242001

R. Aaij *et al.*<sup>\*</sup>  
(LHCb Collaboration)

(Received 2 September 2020; accepted 7 October 2020; published 7 December 2020)

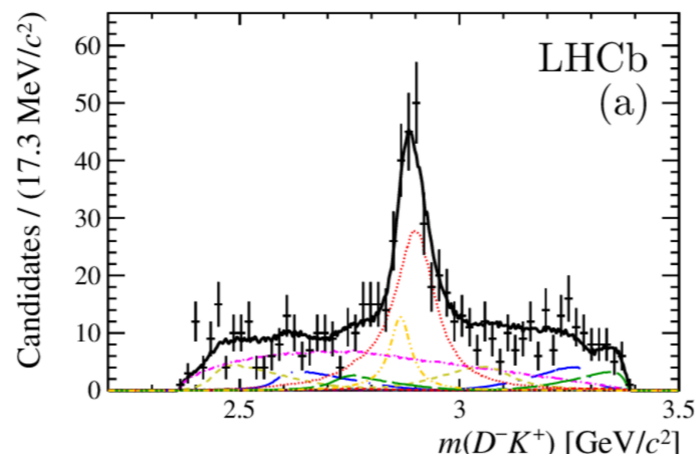
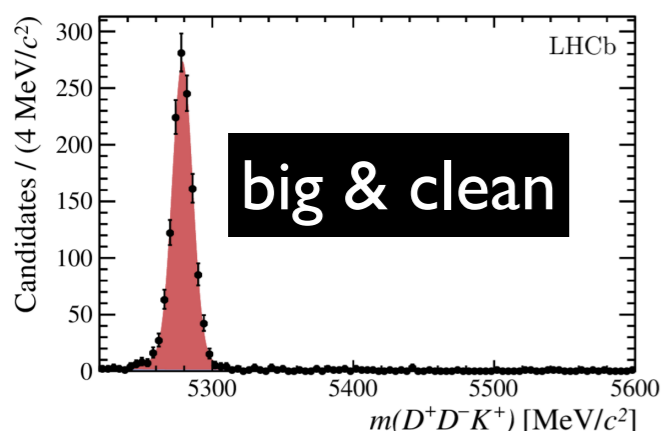
Amplitude analysis of the  $B^+ \rightarrow D^+D^-K^+$  decay

PRD102 112003

R. Aaij *et al.*<sup>\*</sup>  
(LHCb Collaboration)

(Received 2 September 2020; accepted 8 October 2020; published 7 December 2020)

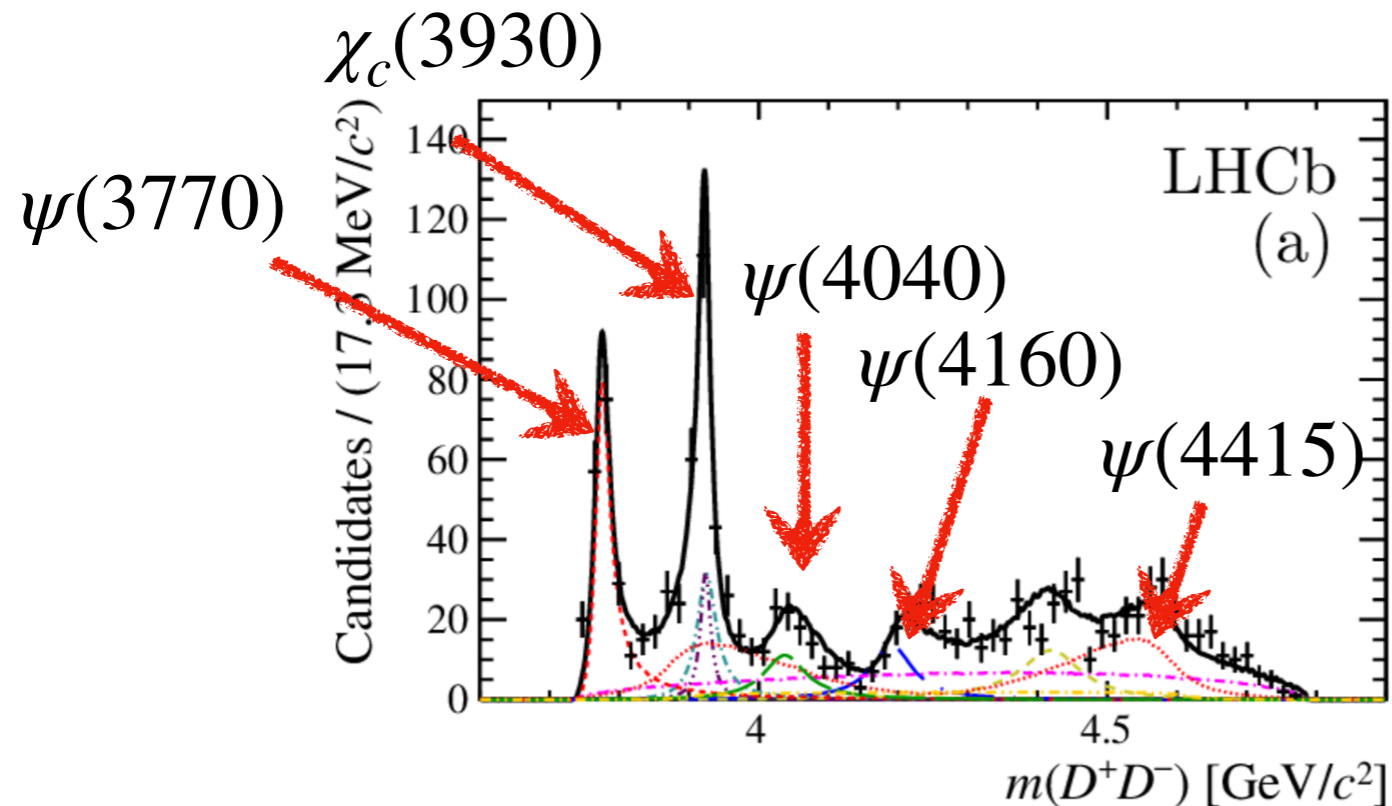
Conceived as a clean insight into charmonium spectrum



Big surprise in the  $D^-K^+$  spectrum

Model independent and dependent discovery of exotic  $cs\bar{u}d$  structure

- Despite the exotic reflection, clean access to the charmonium spectrum



## Discovery of scalar state

Needed both the expected  $\chi_{c2}(3930)$  \*and\* a new  $\chi_{c0}(3930)$

- Exploration of the new  $D^-K^+$  structures is a high priority: incorporating a wide array of  $B \rightarrow D^{(*)}\bar{D}^{(*)}h^{(*)}$  decays to confirm and probe it
- Will gain further handles on the charmonium states, beyond  $D^0\bar{D}^0$  and  $D^+D^-$ , through tricky  $>3$ -body analyses to access  $D^{*+}D^-$  and  $D^{*+}D^{*-}$

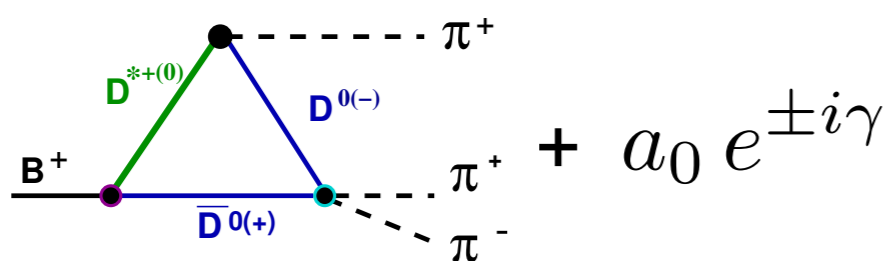
## Key questions:

- Theoretical models for  $D^-K^+$  states: tetra quark? molecule? threshold enhancement?
- Modelling of  $D^0K^+$ : several broad, overlapping states

work in progress  
Kostas and Roman

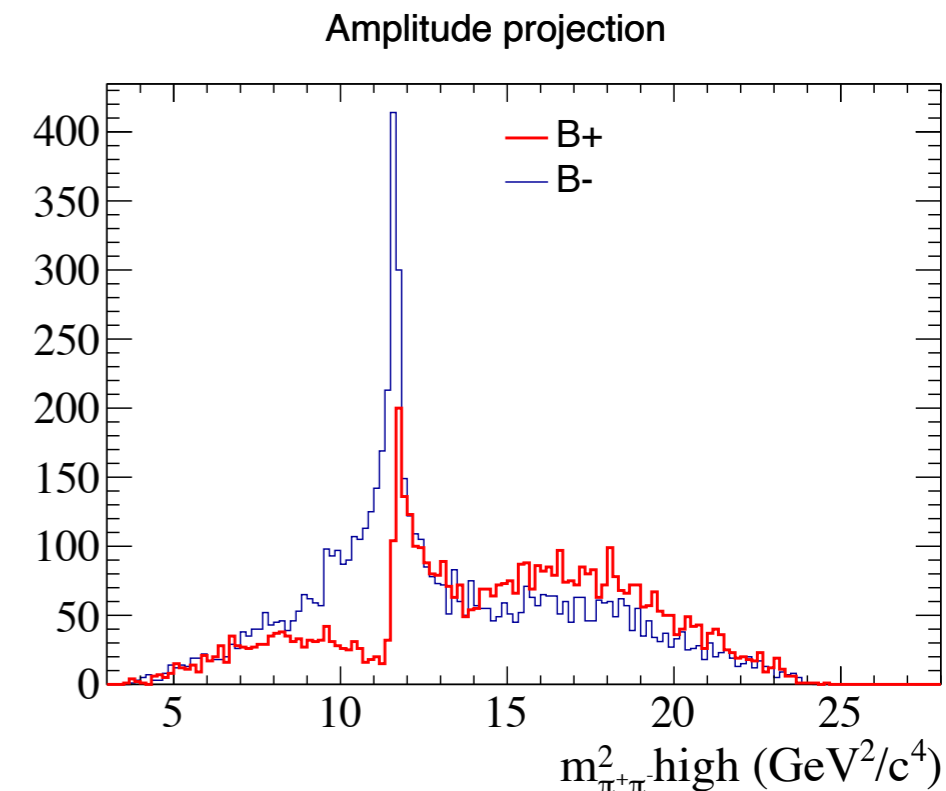
- Why we think charm rescattering can play a role?
  - FSI are sources of strong phase variation that can interfere with other sources

- observed this effect in  $B \rightarrow \pi\pi\pi$



Bediaga, Frederico, Magalhaes - PLB 806 (2020) 135490

→ explain the observed CP violation (LHCb data) in the middle of the phase space!

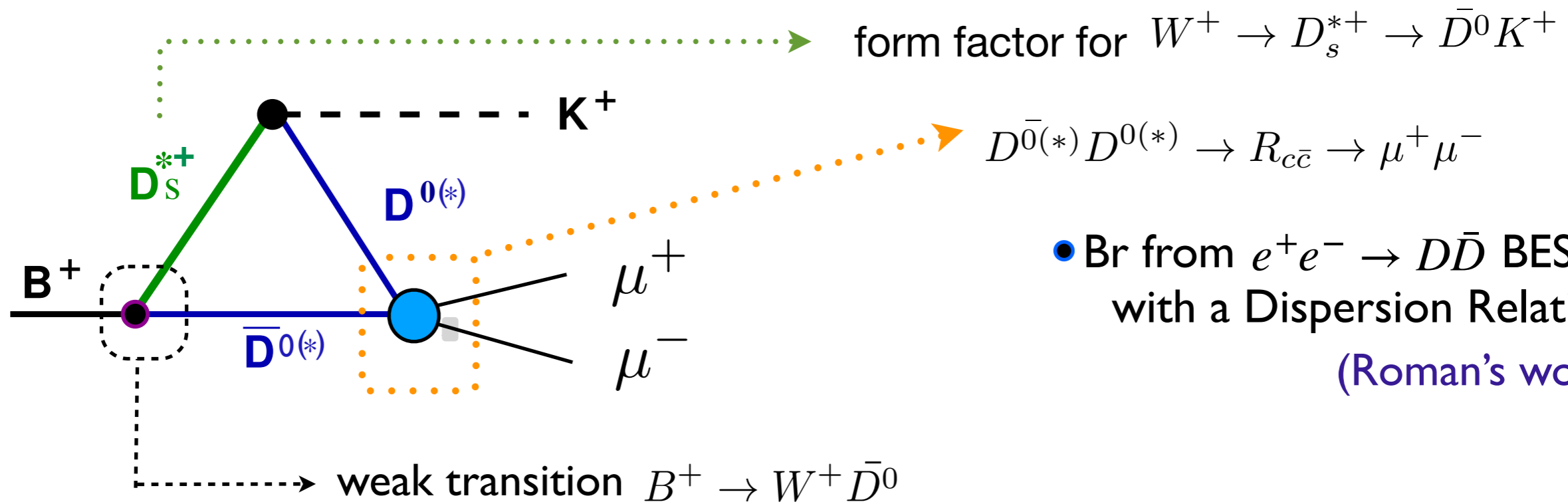


- Similar interaction can happen in  $B \rightarrow K\mu\mu$

- amplitude interferes with short-distance component →  $C_9^{\text{eff}}(q^2) = C_9 + Y(q^2)$

→ Propose an alternative data-driven approach to calculate this exclude charm-rescattering contribution!

- Use the same FSI approach as used in  $B \rightarrow \pi\pi\pi$  for  $B \rightarrow K\mu\mu$



- Br from  $e^+e^- \rightarrow D\bar{D}$  BES data with a Dispersion Relation (Roman's work)

- The charm rescattering amplitude helicity average:

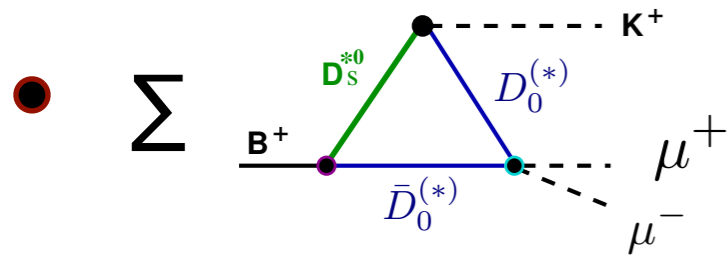
$$A(q^2) = C_W \int \frac{d^4\ell}{(2\pi)^4} \frac{\alpha + \Delta\bar{D}_0 - \Delta_a}{\Delta_{D^0} \Delta_{\bar{D}^0} \Delta_{D^*} \Delta_a} T_{D\bar{D} \rightarrow \mu^+ \mu^-}(q^2, \ell)$$

$$C_W = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* m_a^2 m_{D_s^*}^2 F^{BD}(0) F^{D_s^*}(0) \sim 10^{-5}$$

$$\alpha = M_B^2 + M_K^2 - 2s_{12} + M_{D_0}^2 + M_{\bar{D}_0}^2 - m_a^2;$$

$$\Delta_{D_i} = P_{D_i} - m_{D_i}^2 + i\epsilon$$

tensor integral to deal with angular distribution

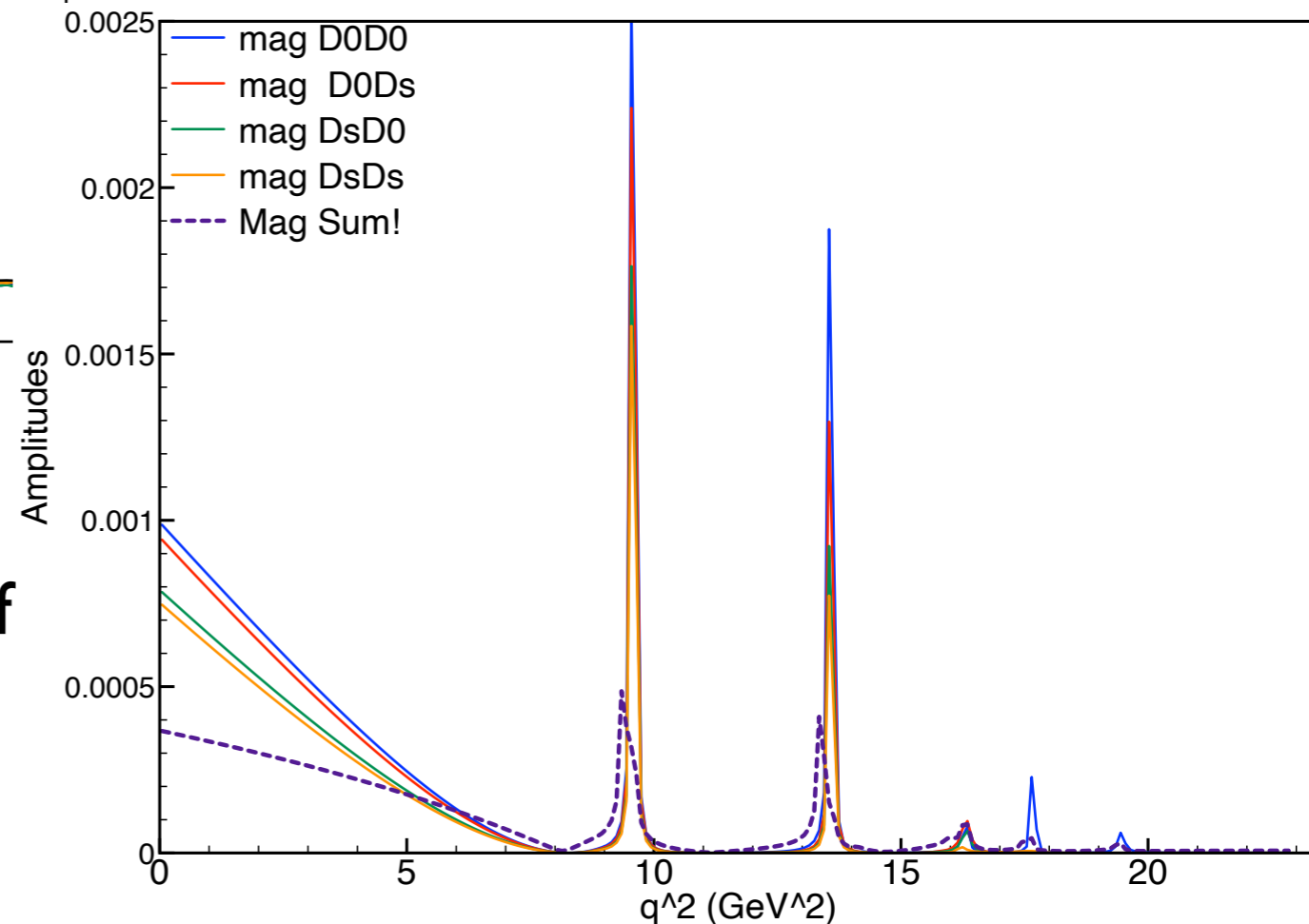
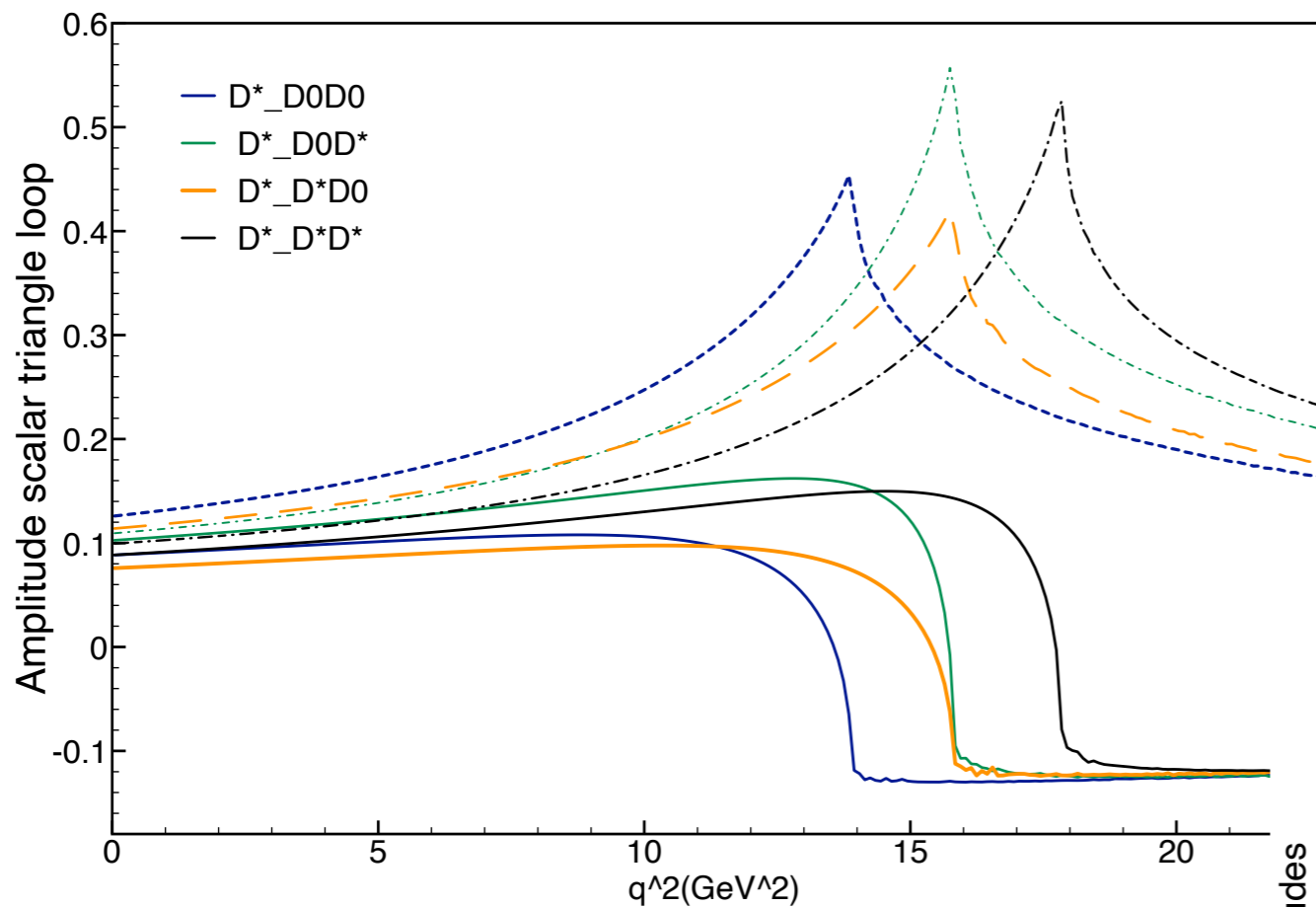


we considered in the HQ spin symmetry 4 different triangle contributions and weight the different contributions of BR

$$\text{BR}[B \rightarrow D_s^* \bar{D}_0] = 7.6 \times 10^{-3}$$

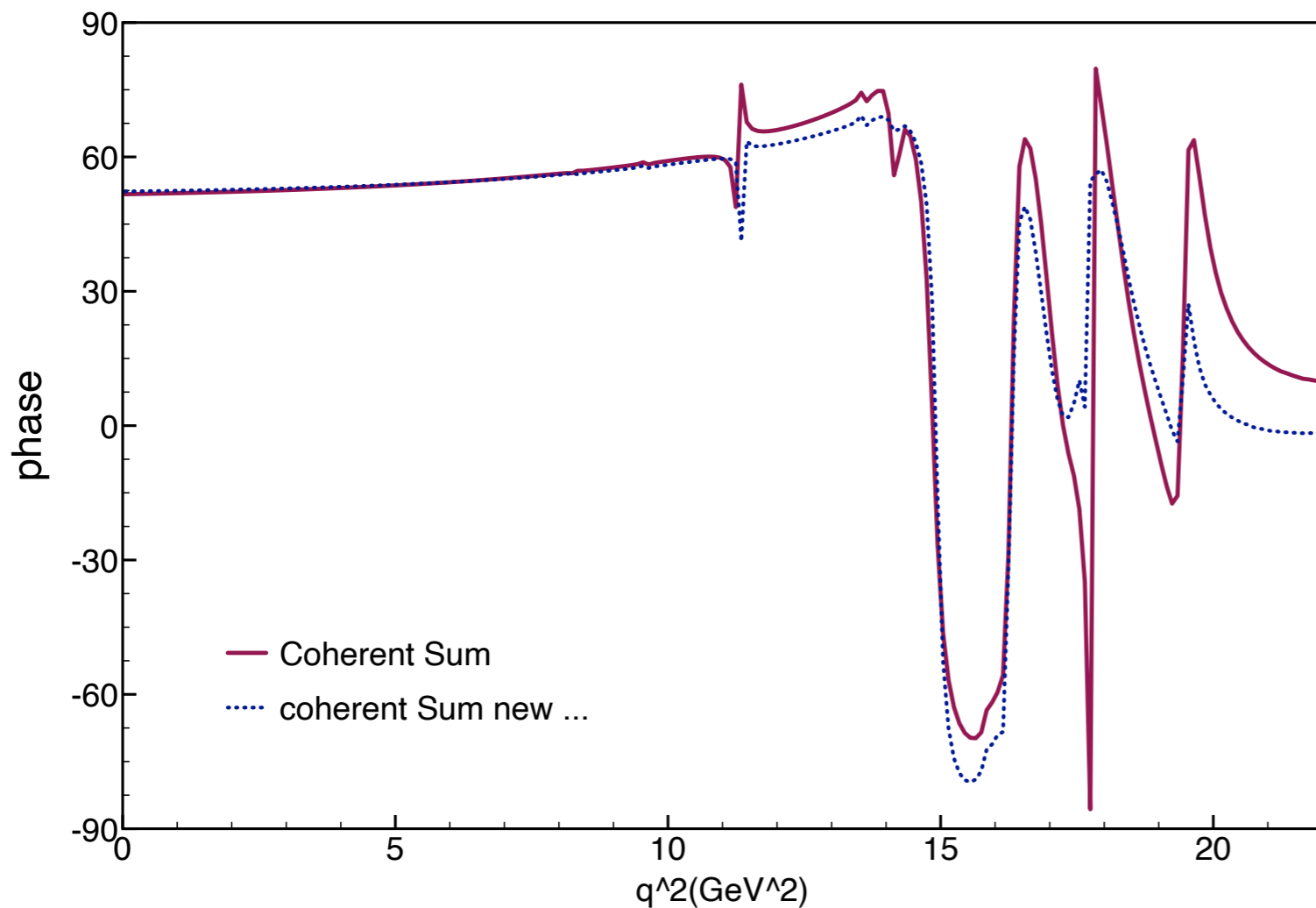
$$\text{BR}[B \rightarrow D_s^* D_0^* (\bar{2007})] = 1.7 \times 10^{-2}$$

(PDG) - The other variations don't make difference - backup slide



we could also float the overall size of the amplitude in a fit to the data

- Phase highlight the  $q^2$  dependence effect



- significant even at low  $q^2$  !

## Caveats

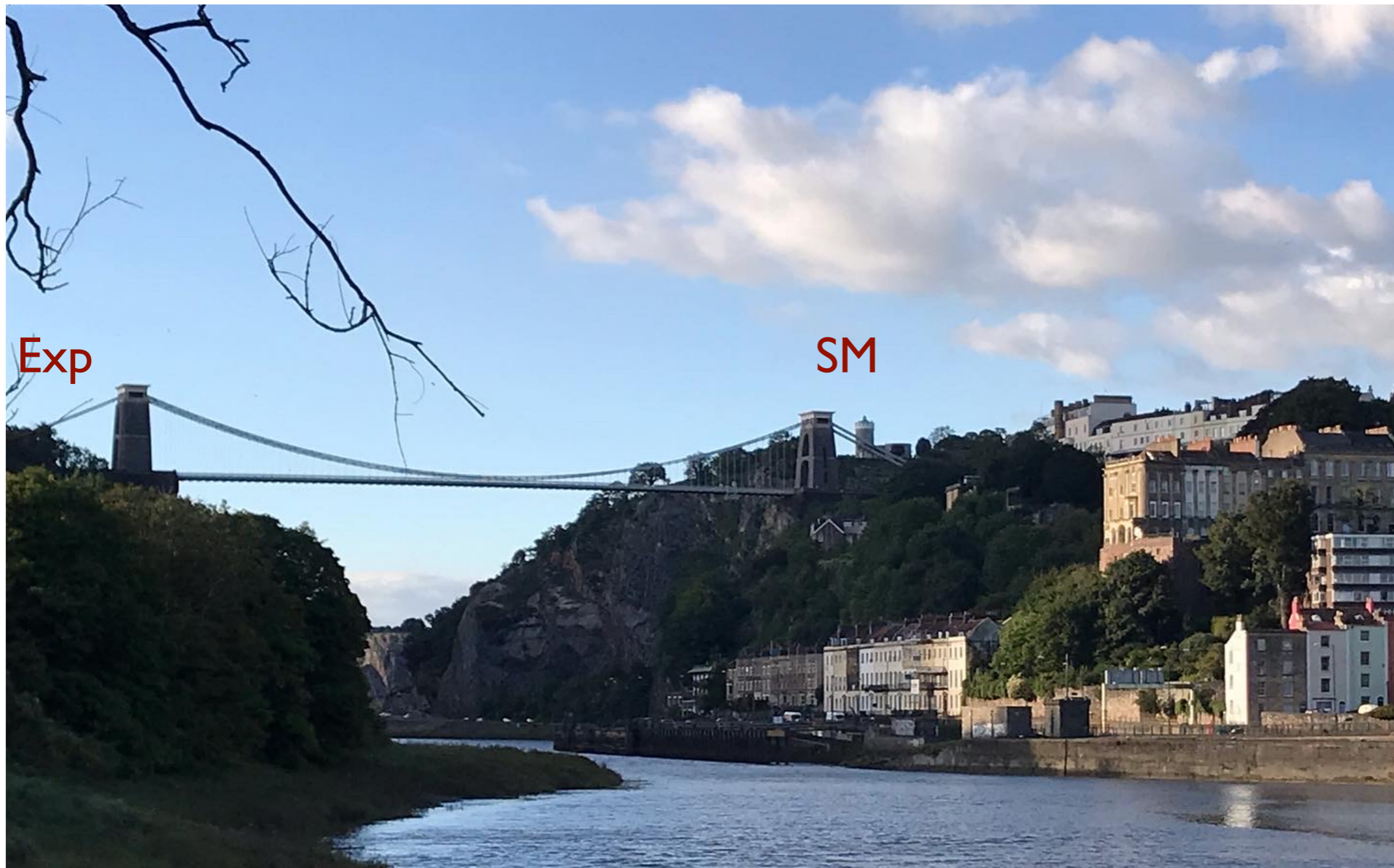
- improve our description of the relative FF and BR (?)
- accuracy on the relative coupling  $D^{\bar{0}(*)} D^{0(*)} \rightarrow R_{c\bar{c}} \rightarrow \mu^+ \mu^-$
- care is needed as BES input already sums over muon helicities

- To include this amplitude in the full description of  $B \rightarrow K\mu\mu$ :

→ add it coherently in a isobar description together with resonant and non resonant  $c\bar{c}$  components to fit data

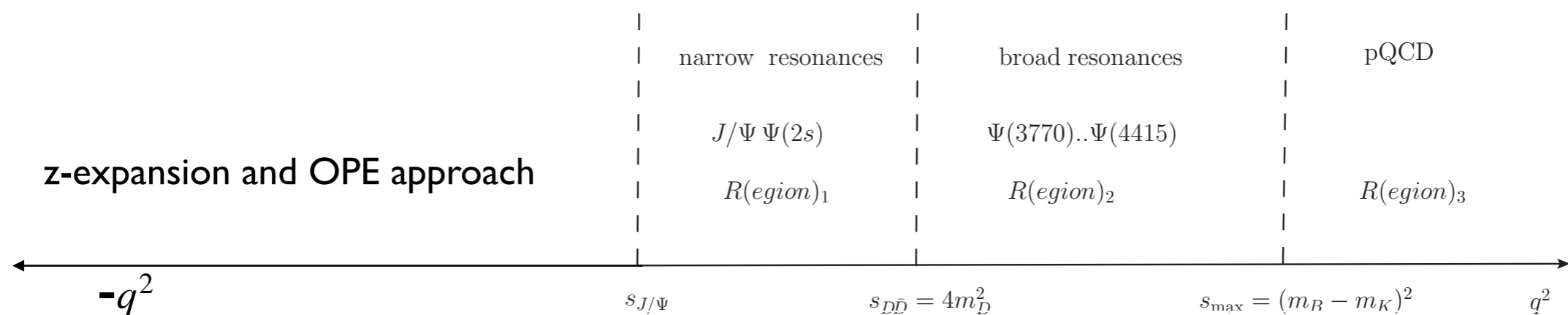
$$\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell} = \sum_{\lambda_1, \lambda_2 = -1/2}^{+1/2} \left| \mathcal{M}_{\lambda_1, \lambda_2}^{B \rightarrow K\mu\mu} + \mathcal{M}_{\lambda_1, \lambda_2}^{B \rightarrow DD^* \rightarrow K\mu\mu} \right|^2$$

how we do the bridge ?



- How to improve the accuracy on the relative couplings
 {

$$\begin{cases}
 D^{\bar{0}(*)} D^{0(*)} \rightarrow R_{c\bar{c}} \\
 R_{c\bar{c}} \rightarrow \mu^+ \mu^-
 \end{cases}$$
  - ↪ LHCb new data
  
- Not possible to avoid the DK resonances in  $B \rightarrow D\bar{D}h$ 
  - ↪ not well defined theoretically
  
- Non-resonant charm contributions:
  - not negligible in hadronic approach!
  - how to model it in QCD approach?
  
- How to connect our knowledge from diff regions?



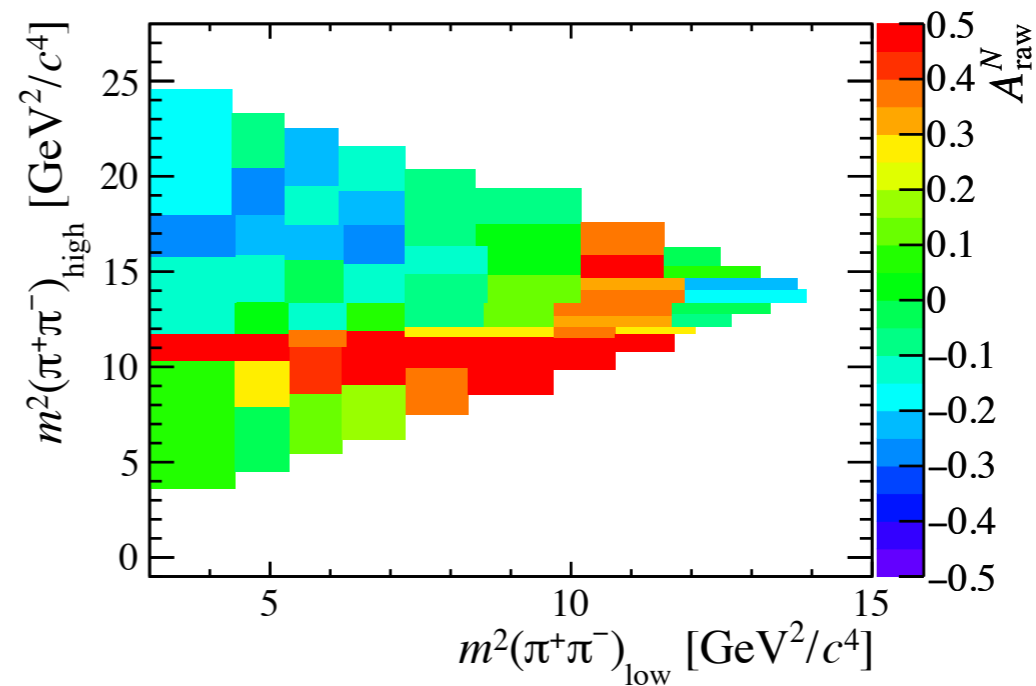
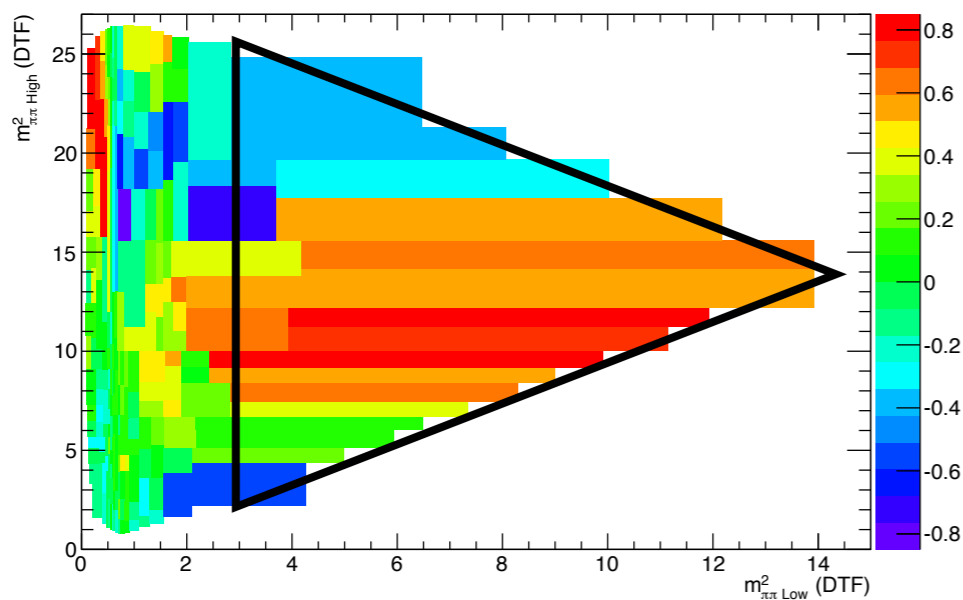


## The charm rescattering as a new mechanism to generate CP violation at high mass region of $B \rightarrow \pi\pi\pi$ phase-space

$\bullet A_{B^\pm \rightarrow \pi^- \pi^+ \pi^\pm}(s_{12}, s_{23}) =$ 
 $+ a_0 e^{\pm i\gamma}$

$\bullet$   Run I

$\bullet$  our model



- Rescattering model

PDG BR

$$B \rightarrow D^{\bar{0}(*)} D_{s1}(2536)^+ \times B(D_{s1}(2536) \rightarrow D^*(2007)^0 K^+) \quad (2.2 \pm 0.7) \times 10^{-4}$$

$$B \rightarrow \bar{D}^*(2007)^0 D_{s1}(2536)^+ \times B(D_{s1}(2536) \rightarrow D^*(2007)^0 K^+) \quad (5.5 \pm 1.6) \times 10^{-4}$$

$$B \rightarrow \bar{D}^0 D_{sJ}(2700)^+ \times B(D_{sJ}(2700)^+ \rightarrow D^0 K^+) \quad (5.6 \pm 1.8) \times 10^{-4}$$

- Open issues (some thought in the following slides)
  - 1) Centrifugal barrier factors (finite size effects)
  - 2) Non-resonant contributions  $\hat{c}_{ij}$
  - 3) Experimental inputs (choice of channels, fit model(s))
  - 4) Relation to the OPE description at small  $q^2$

1) Centrifugal barrier factors (finite size effects) [Blatt, Weisskopf, 1952]

$$\hat{K}_{ij} = \sum_{\psi_r} \frac{g_{ri}^0 g_{rj}^0 B_{ri}^L(q, q_\alpha) B_{rj}^L(q, q_\alpha)}{m_{\psi_r}^2 - q^2} + \hat{C}_{ij}$$

$$F_0(q) = 1$$

$$B_{ai}^l(q, q_\alpha) = \frac{F_l(q)}{F_l(q_\alpha)}$$

$$F_1(q) = \sqrt{\frac{2z}{z+1}}$$

$$F_2(q) = \sqrt{\frac{13z^2}{(z-3)^2 + 9z}}$$

$z = (q/q_R)^2$  and  $q_R$  corresponds to the range of interaction.

Are these factors really needed?

## 2) Non-resonant contributions

$$\hat{K}_{ij} = \sum_{\psi_r} \frac{g_{ri}^0 g_{rj}^0}{m_{\psi_r}^2 - q^2} + \hat{c}_{ij} \quad (\mathbf{N}_{\text{channel}})^2 \text{ parameters}$$

- These constants account for  $\mathbf{R}_{\text{udsc}} - \mathbf{R}_{\text{uds}}$
- $\hat{c}_{ij}$  with  $\mathbf{i} = 0$  are enough to get a good fit of  $ee \rightarrow c\bar{c}$
- Can we safely set the other to zero?

## 3) Experimental inputs (choice of channels, fit model(s))

- The charge assignment in  $B \rightarrow K^{(*)} D \bar{D}$  allows to avoid  $K^{(*)} D$  resonances
- We need access to specific waves contributions:

	DD	DD*	D*D*
S - wave	✗	★	✗
P - wave	★		★
D - wave			✗
F - wave			★

★ Needed contribution

✗ Contribution that need to be under control

4) Relation to the OPE description at small  $q^2$ 

- The  $z$  parametrization describes the spectrum below the  $D\bar{D}$  threshold
- The  $K$  matrix approach describes the spectrum above the  $D\bar{D}$  threshold
- A fit can vary them independently but they are connected to the same OPE!
  - How do we reconcile the two parametrizations?
  - In principle they should be continuous and smooth at the threshold