

Data-driven Approaches to Exclusive $b \rightarrow sll$

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Beyond the Flavor Anomalies II – April 21st, 2021

Anomalies in $b \rightarrow s\ell\ell$ occur across the spectrum of decay observables:

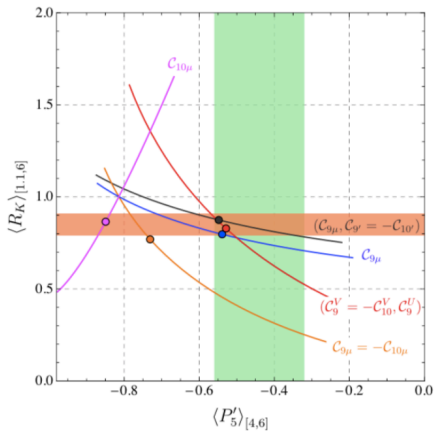
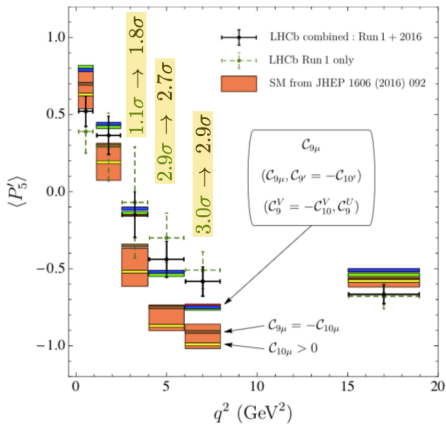
NP “clean” (e.g. R_K) \longleftrightarrow QCD-sensitive observables (e.g. \mathcal{B})

Opportunity to study **BOTH** *New Physics* and *QCD* in “rare” decays.

A point that can be hopefully made is that:

The intensive experimental flavor physics program in the next decade is such that **experimental data** will be used to improve *SM (QCD) predictions*, allowing for finer and very reliable *New Physics analyses*.

Fits to $b \rightarrow s\ell\ell$ (R_K and P_5' measurements from LHCb 2019, 2020)



Algeró et al. Addendum to Eur.Phys.J.C 79 (2019)

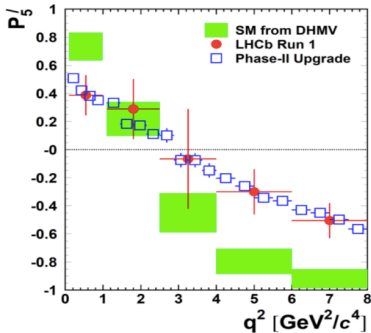
Future expectations

□ By 2030, the (LFUV) anomalies will be either confirmed or ruled-out independently by LHCb Phase-1 and Belle II.

Albrecht et al, 1709.10308

□ Phase-2 will put some experimental errors to negligible levels

CERN-LHCC-2017-003, LHCb EoI



P'_5 defined in

Descotes-Genon, Matias, Ramon, Virto 1207.2753

□ Bottleneck is SM uncertainties:
Assuming vanishing exp uncertainties

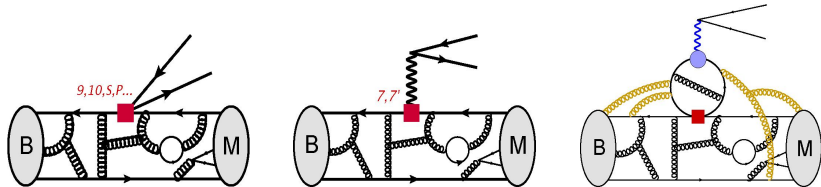
$$\text{Pull}(P'_5[2.5,4.0]) = 3.5\sigma \xrightarrow{2020} 2.8\sigma$$

$$\text{Pull}(P'_5[4.0,6.0]) = 6.5\sigma \xrightarrow{2020} 4.8\sigma$$

$$\text{Pull}(P'_5[6.0,8.0]) = 5.4\sigma \xrightarrow{2020} 4.5\sigma$$

⇒ Need to improve theory uncertainties (“Local” and “Non-local” Form Factors)

Anatomy of $B \rightarrow M_\lambda \ell^+ \ell^-$ EFT Amplitudes



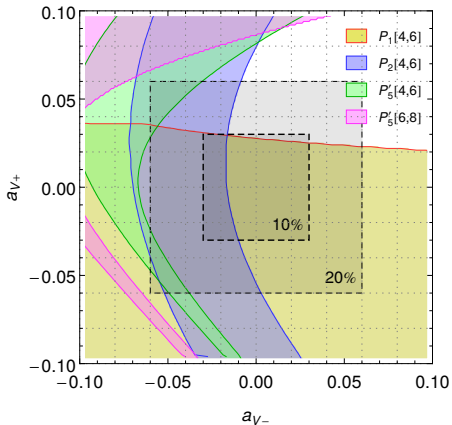
$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\} + \mathcal{O}(\alpha_{\text{em}}^2)$$

► Local (Form Factors): $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \bar{3} \Gamma_\lambda^{(T)} b | \bar{B}(k+q) \rangle$

► Non-Local: $\mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x e^{iq \cdot x} \langle \bar{M}_\lambda(k) | T \{ \mathcal{J}_{\text{em}}^\mu(x), C_i \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$

Experimental Tests: Redundancy provides form factor info

Descotes-Genon, Hofer, Matias, Virto 2016

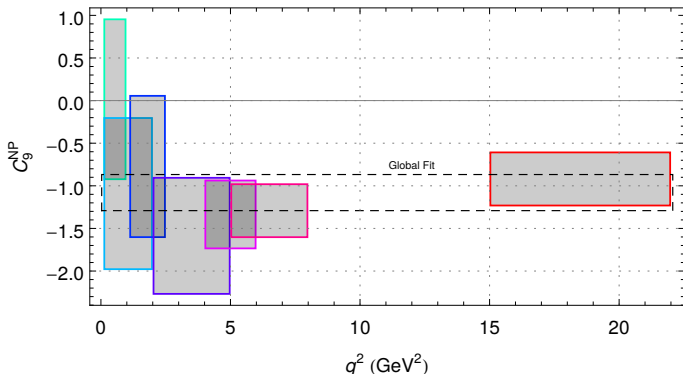


Data will only be consistent with certain form factor values...

.... and may need NP for that!

Experimental Tests: NP contributions to WCs are q^2 -independent

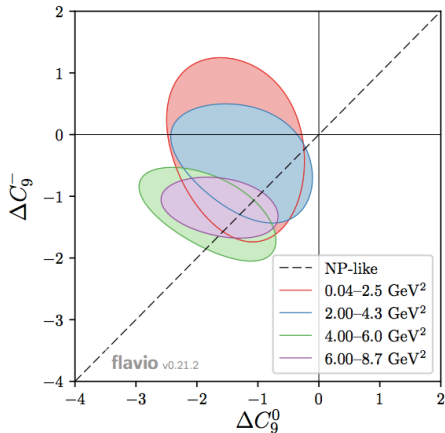
Descotes-Genon, Hofer, Matias, Virto 2016



Fits provide a-posteriori tests that q^2 -dependence of non-local effects is correct
.... and passing this test (with finer binning) will be increasingly harder!

Experimental Tests: NP contributions to WCs are helicity-independent

Altmannshofer, Niehoff, Stangl, Straub 1703.09189

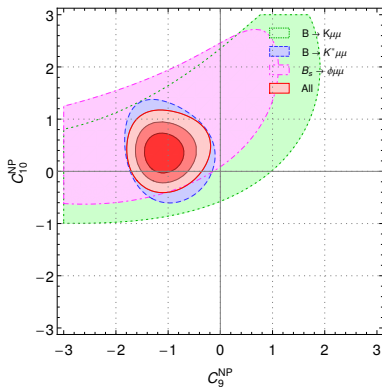


Fits provide a-posteriori tests that λ -dependence of non-local effects is correct

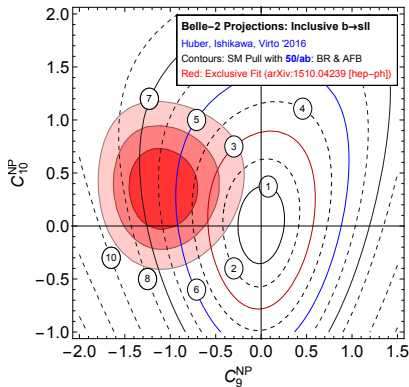
.... and passing this test will be increasingly harder!

Experimental Tests: Same with mode-independence

Descotes-Genon, Hofer, Matias, Virto 2016



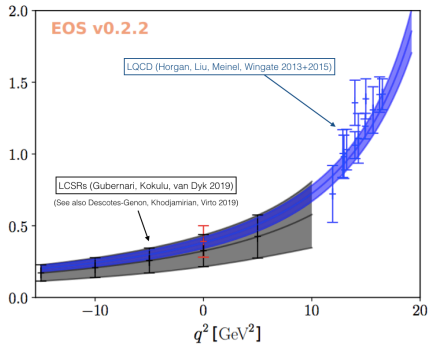
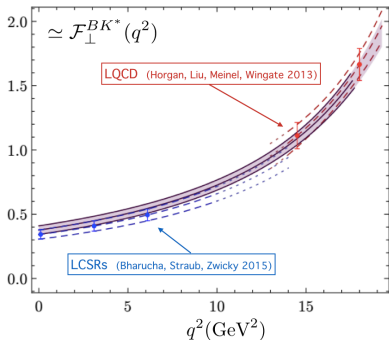
Huber, Ishikawa, Virto, Belle-II Physics Book



Consistency at EFT-level is Model-Indep. and a data-driven test of Theory (QCD).

[Note: Inclusive mode has its own non-local effect]

Local form factors

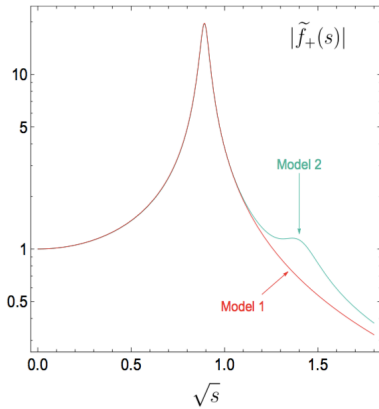
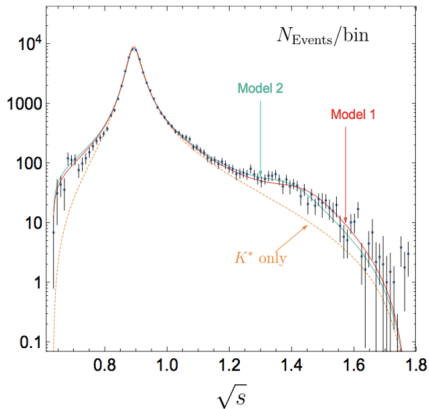


- ▶ Two main approaches: (1) **Lattice QCD** (large q^2) (2) **LCSRs** (low q^2)
- ▶ Two approaches to **LCSRs**, in terms of (1) K^* LCDAs (2) B LCDAs
- ▶ q^2 dependence can be parametrized model-independently

$$\int_{s_{\text{th}}}^{s_0} ds e^{-s/M^2} \omega_i(s, q^2) f_+^*(s) F_i^{(T)(\ell=1)}(s, q^2) = \mathcal{P}_i^{(T), \text{OPE}}(q^2, \sigma_0, M^2)$$

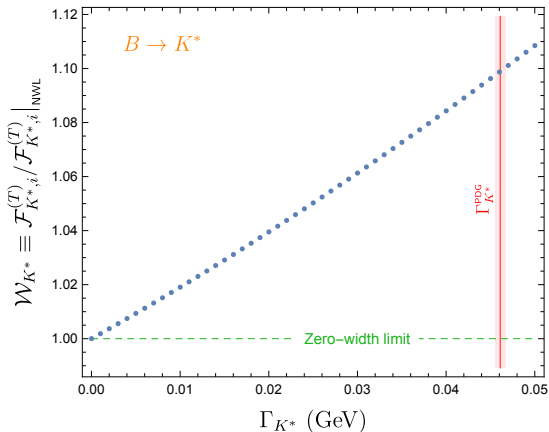
$K\pi$ form factor $f_+(s)$ from $\tau \rightarrow K\pi\nu_\tau$

Data from Belle, arXiv:0706.2231 [hep-ex]



Form factors for unstable mesons (e.g., K^*): width effects

Descotes-Genon, Khodjamirian, Virto 2019



Crucial input: $\tau \rightarrow K\pi\nu$

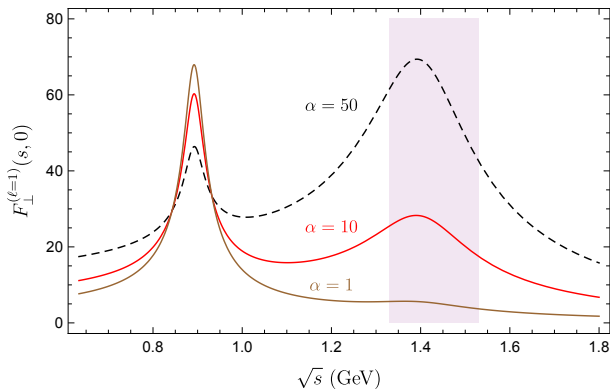
$$\mathcal{W}_{K^*} \simeq 1 + 1.9 \frac{\Gamma_{K^*}}{m_{K^*}}$$

$$\mathcal{W}_{K^*} = 1.09 \pm 0.01$$

\Rightarrow BRs are corrected by a factor $|\mathcal{W}_{K^*}|^2 \simeq 1.2$ (increasing anomalies)

Form factors for unstable mesons (e.g., K^*)

Descotes-Genon, Khodjamirian, Virto 2019



$\alpha = 1$: $\mathcal{F}_{K^*,\perp}(0) = 0.28$; $\alpha = 10$: $\mathcal{F}_{K^*,\perp}(0) = 0.22$; $\alpha = 50$: $\mathcal{F}_{K^*,\perp}(0) = 0.11$.

High $K\pi$ -Mass Moments in $B \rightarrow K\pi\ell\ell$

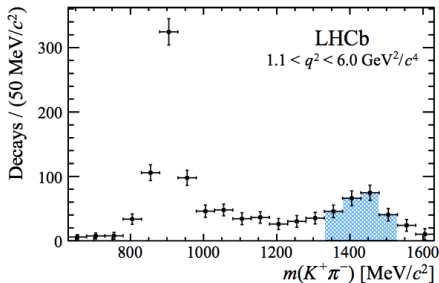
Descotes-Genon, Khodjamirian, Virto 2019

Differential decay rate including S, P, D waves -- [$d\Omega = d\cos\theta_\ell d\cos\theta_K d\phi$]

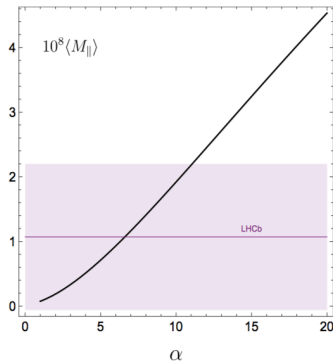
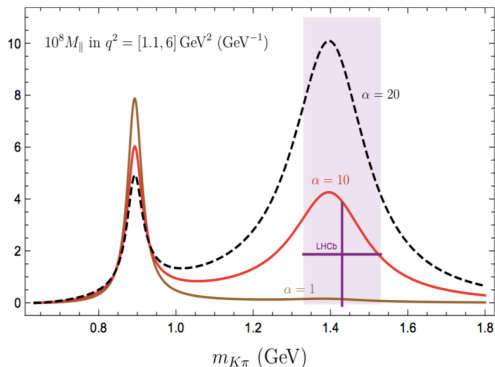
$$\frac{d\Gamma}{dq^2 dk^2 d\Omega} = \frac{1}{4\pi} \sum_{i=1}^{41} f_i(\Omega) \tilde{\Gamma}_i(q^2, k^2)$$

The 41 moments $\tilde{\Gamma}_i(q^2, k^2)$ have been measured by LHCb ([arXiv:1609.04736](https://arxiv.org/abs/1609.04736)) in the bins

$$\sqrt{k^2} \in [1.33, 1.53] \text{ GeV}, \quad q^2 \in [1.1, 6] \text{ GeV}^2$$



Example: $\langle M_{\parallel} \rangle$ (= some smart combination of moments):



Bounds: From $\langle M_{\parallel} \rangle$: $\alpha \lesssim 11$; From $\langle M_{\perp} \rangle$: $\alpha \lesssim 17$; From $\langle M_{\text{re}} \rangle$: $\alpha \lesssim 18$.

So far bounds from dBR are stronger (something like $\alpha \lesssim 3$).

Non-local Form Factors: OPE + dispersion relations

$$\mathcal{H}_{\lambda,x}(q^2) = \mathcal{H}_{\lambda,x}^{\text{OPE}}(q_0^2 < 0) + (q^2 - q_0^2) \int_{s_{\text{th}}}^{\infty} dt \frac{\rho_{\lambda,x}(t)}{(t - q^2 - i\epsilon)(t - q_0^2)}$$

- $\mathcal{H}_{\lambda,x}^{\text{OPE}}(q_0^2)$: Theory

e.g. Khodjamirian et al 2010, 2012; Asatrian, Greub, Virto 2019; Gubernari, van Dyk, Virto 2020

- $\rho_{\lambda,c}(t)$: $B \rightarrow K^{(*)} J/\psi$, $B \rightarrow K^{(*)} \psi(2S)$, $B \rightarrow K^{(*)} D\bar{D}$, ...
- $\rho_{\lambda, sb}(t)$: $B \rightarrow K^{(*)} \phi$, $B \rightarrow K^{(*)} \bar{K}K$, ...
- $\rho_{\lambda, ud}(t)$: $B \rightarrow K^{(*)} \rho$, $B \rightarrow K^{(*)} \omega$, $B \rightarrow K^{(*)} \pi\pi$, $B \rightarrow K^{(*)} \pi\pi\pi$, ...

Charm contribution \longrightarrow numerically leading

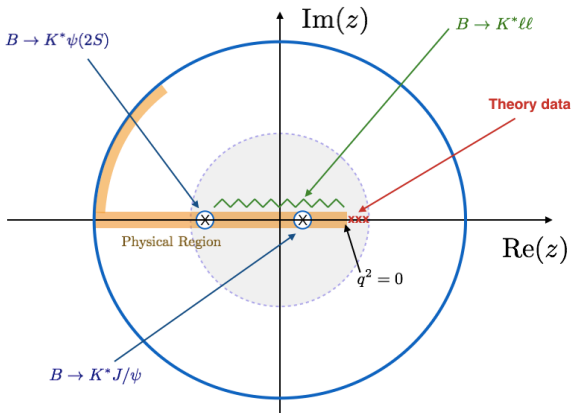
From OPE region to physical region requires DATA ($B \rightarrow K^{(*)} X_{1--}$)

Non-local Form Factors: analyticity

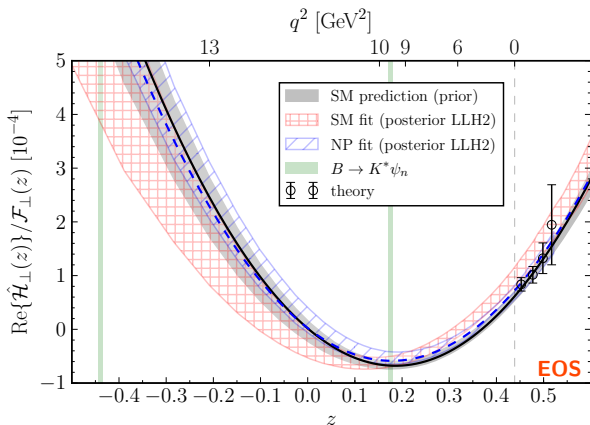
Bobeth, Chrzaszcz, van Dyk, Virto 1707.07305

Gubernari, van Dyk, Virto 2011.09813

$$\hat{\mathcal{H}}_{\lambda}^{B \rightarrow M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B \rightarrow M} p_n^{B \rightarrow M}(z)$$



Constrain non-local effect with $B \rightarrow K^* \psi_n$ | Use interresonance $B \rightarrow K^* \ell \ell$ DATA



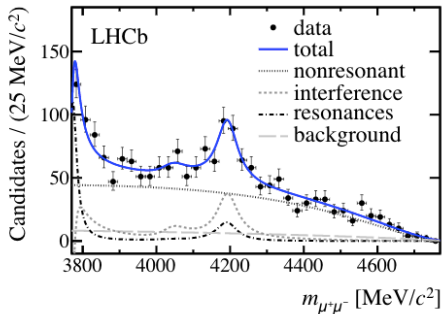
PRIOR \Rightarrow **SM prediction for $B \rightarrow K^* \mu \mu$**

POSTERIOR \Rightarrow **SM prediction for $B \rightarrow K^* e^+ e^-$** \Rightarrow **LFU tests!**

\Rightarrow **$SU(3)$ prediction for $b \rightarrow d \ell \ell$** -- (requires more discussion)

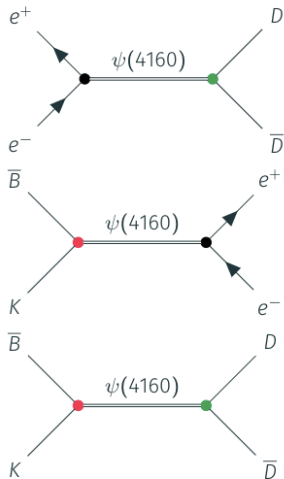
P-vector (K -matrix) approach to $B \rightarrow K^* \ell \ell$ @ high- q^2

Kürten, van Dyk, et al, w.i.p



$\psi(4160)$:

- expectation: ● small!
- empirically: ● > ●
- prediction: $\psi(4160)$ should be more even more visible in $B \rightarrow K D \bar{D}$!



P-vector (K -matrix) approach to $B \rightarrow K^* \ell \ell$ @ high- q^2

Kürten, van Dyk, et al, w.i.p

Channels: Only $J^{PC} = 1^{--}$ $c\bar{c}$ states, plus an “eff. channel” associated to the ratio R_c .

	e^+e^-	$D^0\bar{D}^0$	$D_{(s)}^+D_{(s)}^-$	$D^0\bar{D}^{*0}$	$D_{(s)}^+D_{(s)}^{*-}$	$D^{*0}\bar{D}^{*0}$	$D_{(s)}^{*+}D_{(s)}^{*-}$	$B\bar{K}^{(*)}$
e^+e^-								♥
$D^0\bar{D}^0$	♣							♣
$D_{(s)}^+D_{(s)}^-$	♣							○
$D^0\bar{D}^{*0}$	♣							○
$D_{(s)}^+D_{(s)}^{*-}$	♣							○
$D^{*0}\bar{D}^{*0}$	♣							○
$D_{(s)}^{*+}D_{(s)}^{*-}$	♣							○

Resonances: $\psi(3770, 4040, 4160, 4415)$, $X(3940, 4230, 4260, 4360, 4660)$

Highly non-trivial fit to all $c\bar{c}$ \Rightarrow LOCAL prediction for $B \rightarrow K^{(*)} \ell \ell$

Amplitude analysis of $B^0 \rightarrow K^* \mu^+ \mu^-$

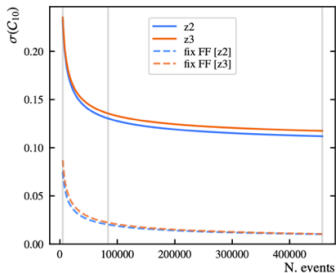
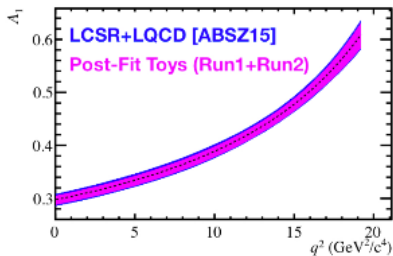
- Nice interplay between theory and experiments
- State-of-the-art inputs from theory to extract best info from data
- Vice-versa: possible determination (validation) of theoretical assumptions from data

Need to parametrize the decay amplitudes:

$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

Form factor treatment in $B^0 \rightarrow K^* \mu^+ \mu^-$ amplitude analysis

- Constrained to **Bharucha, Straub, Zwicky '15**
- Form factor uncertainties will limit the experimental sensitivity on the Wilson coefficients after LHCb Upgrade I [50 fb^{-1}]



Q1: What is the impact of the K^{*0} narrow width approximation in the treatment of the Form Factors in the analysis?

□ Naively parametrized as $FF(q^2) \times BW(m_{K\pi})$

- $BW(m_{K\pi})$ contains phase-space factor $\Rightarrow q^2$ -dependent

\Rightarrow Fit posterior can give good indication about the preferred FF values in data

\Rightarrow Plus, residual (q^2 -dependent) difference can be absorbed by $\mathcal{H}_\lambda(q^2)$

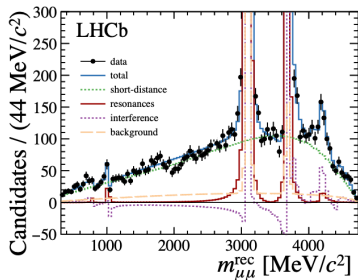
- However, we risk to bias the interpretation of $\mathcal{H}(q^2)$...

Non-local terms $\mathcal{H}(q^2)$ in $B^0 \rightarrow K^* \mu^+ \mu^-$ amplitude analysis

□ Two complementary approaches considered at LHCb: *Isobar-like* [Blake et al, 1709.03921](#); [Cornella et al, 2001.04470](#) and *z-expansion* [Bobeth et al, 1707.07305](#); [Chrzaszcz et al, 1805.06378](#); [Gubernari et al, 2011.09813](#)

□ Level of compatibility between the two can give useful hint on our understanding of non-local hadronic contributions

⇒ Run-I $B^+ \rightarrow K^+ \mu^+ \mu^-$ result suggests non-local terms have little impact outside resonant regions [LHCb, EPJ C \(2017\) 77, 161](#)



Non-local hadronic: z-expansion

- Attempt to be as much model-independent as possible

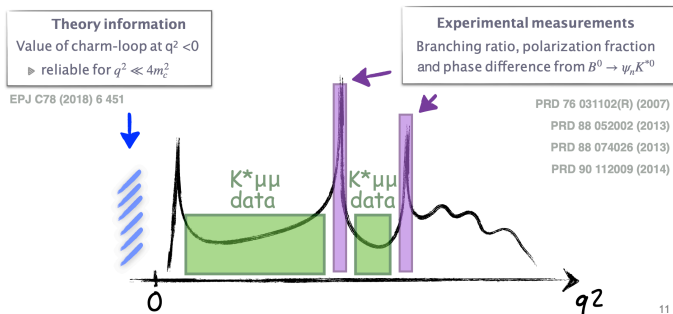
$$\cdot \hat{\mathcal{H}}_\lambda(q^2) \equiv \sum_k \alpha_k^{(\lambda)} z^k$$

- Goal: detect q^2 -dependent non-local contribution

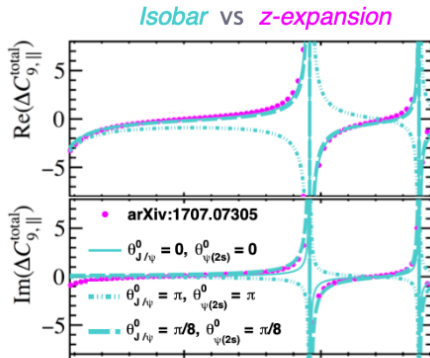
- Constrained from theory ($q^2 < 0$) and $B^0 \rightarrow \psi_n K^*$ measurements

$$\cdot \text{Res}_{q^2 \rightarrow m_\psi} \mathcal{H}_\lambda = \frac{m_\psi f_\psi^* \mathcal{A}_\lambda^\psi}{m_B^2}$$

- Interesting test: remove theory constraints and check compatibility



Q2: What kind of information can we provide to the theory community?



Blake et al, EPJ C78 (2018) 6, 453

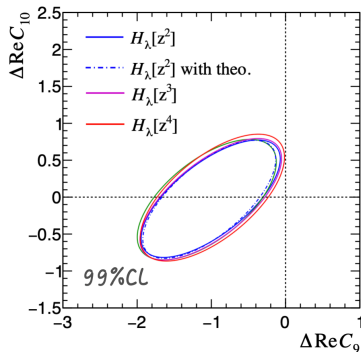
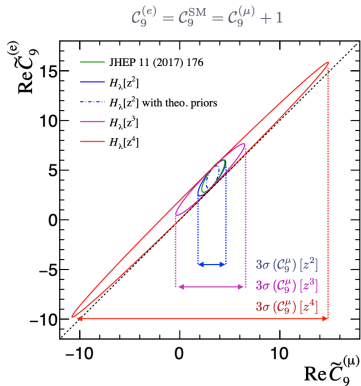
Unbinned LFU amplitude analysis AM, Serra, Coutinho, 1805.06401

□ Simultaneous analysis of $B^0 \rightarrow K^* \mu^+ \mu^-$ and $B^0 \rightarrow K^* e^+ e^-$

⇒ All (nuisance) hadronic parameter are shared

Define $\Delta C_i = C_i^{(\mu)} - C_i^{(e)} \Rightarrow$ model-independent

⇒ Include (unbinned) information from $R_{K^*}, \Delta P'_5, \dots$



Q3: How to preserve the use of available data in the future?

Release nothing and proceed anyway.

No risk of data mis-use.

No extra work.

Cannot update when hadronic information improves

Ultimate sensitivity is lost.

Release background-subtracted data.

Not much work.

Full flexibility given.

Difficult to use, big risk of mis-use.

Will people accept this before data is fully exhausted?

Allow for reinterpretation behind an API.

Hide experimental details.

Flexibility can be defined.

A lot of work.

Relies on some level of consistency between analyses.

By. P. Owen