$B \to K\pi\ell^+\ell^-$ S-wave and all that

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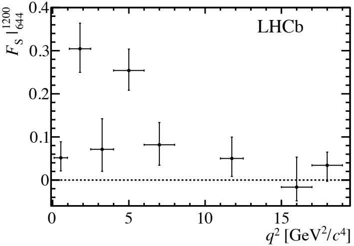
Setting the scene

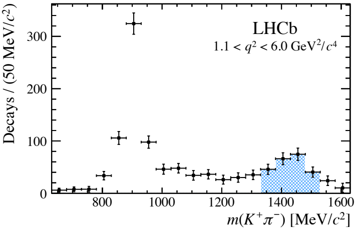
$$
\begin{aligned}
\frac{d\Gamma}{dm_{K\pi}\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi}\Big[& J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_{ell} + J_{2s}\sin^2\theta_K\cos 2\theta_\ell + J_{2c}\cos^2\theta_K\cos\theta_\ell \\
& + J_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + J_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi \\
& + J_5\sin 2\theta_K\sin\theta_\ell\cos\phi + J_{6s}\sin^2\theta_K\cos\theta_\ell \\
& + J_{6c}\cos^2\theta_K\cos\theta_\ell + J_7\sin 2\theta_K\sin\theta_\ell\sin\phi + J_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi \\
& + J_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big] \times |BW_P(m_{K\pi})|^2 \qquad \text{P-wave}
\end{aligned}
$$

$$
\begin{aligned}
+\frac{1}{4\pi}\Big[& (\tilde{J}^c_{1a} + \tilde{J}^c_{2a}\cos 2\theta_\ell)|BW_S|^2 \qquad \text{S-wave} \\
& + [\tilde{J}^{c,r}_{1b}\mathrm{Re}(BW_S BW_P^*) - \tilde{J}^{c,i}_{1b}\mathrm{Im}(BW_S BW_P^*)]\cos\theta_K \\
& + [\tilde{J}^{c,r}_{2b}\mathrm{Re}(BW_S BW_P^*) - \tilde{J}^{c,i}_{2b}\mathrm{Im}(BW_S BW_P^*)]\sin 2\theta_\ell\cos\theta_K \\
& + [\tilde{J}^{r}_{4}\mathrm{Re}(BW_S BW_P^*) - \tilde{J}^{i}_{4}\mathrm{Im}(BW_S BW_P^*)]\sin 2\theta_l\sin\theta_K\cos\phi \\
& + [\tilde{J}^{r}_{5}\mathrm{Re}(BW_S BW_P^*) - \tilde{J}^{i}_{5}\mathrm{Im}(BW_S BW_P^*)]\sin\theta_l\sin\theta_K\cos\phi \\
& + [\tilde{J}^{r}_{7}\mathrm{Im}(BW_S BW_P^*) + \tilde{J}^{i}_{7}\mathrm{Re}(BW_S BW_P^*)]\sin\theta_l\sin\theta_K\sin\phi \\
& + [\tilde{J}^{r}_{8}\mathrm{Im}(BW_S BW_P^*) + \tilde{J}^{i}_{8}\mathrm{Re}(BW_S BW_P^*)]\sin 2\theta_l\sin\theta_K\sin\phi\Big] \qquad \text{S-P-interference}
\end{aligned}
$$

- $B^0 \to K^{*0}(892)\mu^+\mu^-$ measurements performed in particular $m(K\pi)$ range
- S-wave component accounted for in LHCb measurements

Measurements of S-wave so far





- LHCb has also performed dedicated measurements of the S- and D-wave components
  - Measurement of S-wave fraction F_s in $m_{K\pi} \in [6.4, 1.2]$GeV and $m_{K\pi} \in [0.796, 0.996]$ GeV using using model for $m_{K\pi}$ lineshapes [JHEP11(2016)047]
  - S-P-D moment analysis around $m_{K\pi} \in [1.3, 1.5]$GeV [JHEP12(2016)065]

$\rightarrow$ S-wave mostly treated as nuisance parameter but given its large contribution, it could play an important role!
How S-wave observables could be useful? See talk by Mark and Marcel!

Treatment of lineshape

- 5D differential decay rate of S-wave related observables
  - Typical choice of LASS and relativistic Breit–Wigner parametrisations for S- and P-wave lineshapes
  - Variations of S-wave lineshape as systematic
  - Impacts S-P interference observables $\tilde{J}_i$

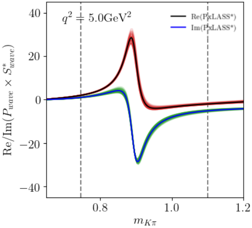
$$\frac{d\Gamma_S}{d\cos\theta_\ell \cos\theta_K \phi m_{K\pi}} = \frac{1}{4\pi}\Bigg(\Big[\tilde{J}^c_{1a} + \tilde{J}^c_{2a}\cos 2\theta_\ell\Big]|LASS(m_{K\pi})|^2 + \Big[\tilde{J}^c_{1b}\cos\theta_K + \tilde{J}^c_{2b}\cos\theta_K\cos 2\theta_\ell + \tilde{J}_4\sin\theta_K\sin 2\theta_\ell\cos\phi + \tilde{J}_5\sin\theta_K\sin\theta_\ell\cos\phi + \tilde{J}_7\sin\theta_K\sin\theta_\ell\sin\phi + \tilde{J}_8\sin\theta_K\sin 2\theta_\ell\sin\phi\Big] LASS(m_{K\pi})^* BW(m_{K\pi})\Bigg)$$

- Too few candidates to obtain LASS parameters from rare mode (though in wide q^2 bin we probably have some constraining power)
- Instead take from $B^0 \to J/\psi(\psi')K^+\pi^-$ amplitude analyses $\to$ depends on treatment of exotic states...

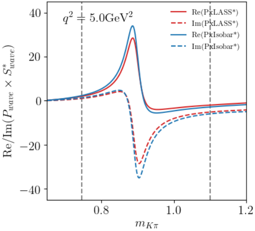
Importance of lineshape

Measurements of P-wave observables are independent of the lineshape.
But S-wave observables suffer from large systematic uncertainty.

- Statistical uncertainty of LASS parameters taken from $B \to \psi K^+\pi^-$



- Uncertainty of choice of LASS vs Isobar with $K^{*0}(1430)$, $K^{*0}(800)$ and N.R



Thanks to Alex Marshall and Mark Smith

$\to$ Induce systematic uncertainty on $\tilde{J}_i$ commensurate to statistical precision of Run1+2 [Preliminary toy studies]

Direct Fits to Wilson Coefficients]

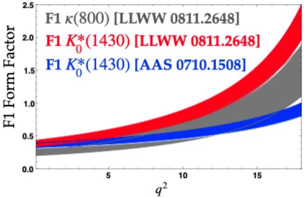
- Model a broad resonant component at the amplitude level:

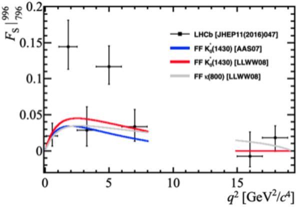
$$A_{00}^{L,R}(q^2) \propto \sqrt{\beta_\ell}\lambda_{K_0^*}\left[(C_9^{eff} \mp C_{10})f_+(q^2) + C_7^{eff} 2m_b \frac{f_T(q^2)}{(m_B + m_{K_0^*})}\right]$$

- The $K\pi$ dependence included as factor into the amplitude

$$\mathcal{A}_{00}(q^2, m_{K\pi}^2) = A_{00}^{L,R}(q^2)G(m_{K\pi}^2)$$

- In absence of consensus on $B \to K\pi$ FFs can
  - Take variations of existing FFs as systematic?
  - Try to float FF parametrisation?
  - Decouple S-wave from P-wave amplitudes?





Light-Cone Sum Rules for S-wave $B \to K\pi$ Form Factors

Sébastien Descotes-Genon, Alexander Khodjamirian, Javier Virto and K. Keri Vos

in progress...

The plan...

- Constrain $B \to K\pi$ S-wave form factor by imposing what we know of QCD
- Light-cone sum rule analysis (as done for P-wave) See also talk Javier 2020

  [J. Virto, A. Khodjamirian, S. Descotes-Genon JHEP 1912, 083 (2019)] [arXiv:1908.02267]

  - Improvement over assuming K^* is a stable state
  - Finite width effects in P wave at 20% level for BR
  - Higher resonances large impact $\to$ can be constrained by moment analysis

    See also talk Javier 2020/21
- S wave even more challenging; generally broad resonances
- Relevant for $B \to K^*\ell\ell$, but also $B \to K\pi\pi$!

S-wave $B \to K\pi$ form factors

- Generated by the axial-vector and pseudotensor $b \to s$ transition currents

$$j_A^\mu = \bar{s}\gamma^\mu\gamma_5 b\,, \quad j_T^\mu = \bar{s}\sigma^{\mu\nu}q_\nu\gamma_5 b\,.$$

- Form factors $F_i(k^2, q^2, q\cdot\bar{k})$ defined as

$$\begin{aligned} -i\langle K^-(k_1)\pi^+(k_2)|\bar{s}\gamma^\mu\gamma_5 b|\bar{B}^0(p)\rangle &= F_t\, k_t^\mu + F_0\, k_0^\mu + \ldots\,, \\ \langle K^-(k_1)\pi^+(k_2)|\bar{s}\sigma^{\mu\nu}q_\nu\gamma_5 b|\bar{B}^0(p)\rangle &= F_0^T\, k_0^\mu + \ldots \end{aligned}$$

- S-wave isolated via partial wave expansion:

$$F_{0,t}(k^2, q^2, q\cdot\bar{k}) = F_{0,t}^{(\ell=0)}(k^2, q^2) + \sum_{\ell=1}^{\infty}\sqrt{2\ell+1}\; F_{0,t}^{(\ell)}(k^2, q^2)\; P_\ell^{(0)}(\cos\theta_K)\,,$$

- In progress: LCSR expressions for $B \to (K\pi)_S$ form factors $F_{0,t}^{(\ell=0)}$ and $F_0^{T(\ell=0)}$

LCSR I [Analyticity + Unitarity + Duality]

- Start with correlation function:

$$\Pi_b(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_S(x), j_b(0)\}|\bar{B}^0(q+k)\rangle\,,$$

- Use dispersion relation in the variable k^2:

$$\Pi^{(\mathrm{OPE})}(k^2,q^2) = \frac{1}{\pi}\int\limits_{(m_K+m_\pi)^2}^{\infty} ds\,\frac{\mathrm{Im}\Pi(s,q^2)}{s-k^2}\,.$$

- Obtain spectral density by inserting a full set of states

$$2\,\mathrm{Im}\Pi_b^{(K\pi)}(k,q) \;=\; \sum_{K\pi}\int d\tau_{K\pi}\langle 0|j_S\,|K(k_1)\pi(k_2)\rangle^*\langle K(k_1)\pi(k_2)|j_b|\bar{B}^0(q+k)\rangle\;,$$

$$\mathrm{Im}\Pi(s,q^2) \;=\; \mathrm{Im}\Pi^{(K\pi)}(s,q^2) + \mathrm{Im}\Pi^{(h)}(s,q^2)\theta(s-s_h)\,.$$

LCSR II [Analyticity + Unitarity + Duality]

- Assume quark-hadron duality

$$\int_{s_h}^{\infty} ds\, \frac{\mathrm{Im}\Pi^{(h)}(s,q^2)}{s-k^2} = \int_{s_0}^{\infty} ds\, \frac{\mathrm{Im}\Pi^{(OPE)}(s,q^2)}{s-k^2}\,,$$

- Perform Borel transformation in the variable k^2

$$\frac{1}{\pi}\int_{(m_K+m_\pi)^2}^{s_0} ds\, e^{-s/M^2}\,\mathrm{Im}\Pi^{(K\pi)}(s,q^2) = \frac{1}{\pi}\int_{m_s^2}^{s_0} ds\, e^{-s/M^2}\,\mathrm{Im}\Pi^{(OPE)}(s,q^2)$$

$$\equiv \Pi^{(\mathrm{OPE})}(q^2,s_0,M^2)$$

- $\Pi^{\mathrm{OPE}}(q^2,s_0,M^2)$ OPE expression after subtracting the above-threshold contribution from the dispersive integral

LCSR for $B \to (K\pi)_S$

$$\int_{(m_K+m_\pi)^2}^{s_0} ds\, e^{-s/M^2} \omega_{0,t}(s,q^2) F_S(s) F_{0,t}^{(\ell=0)}(s,q^2) = i\Pi_{0,t}^{(\mathrm{OPE})}(q^2, s_0, M^2)$$

- s_0 effective threshold
- $\omega_{0,t}(s,q^2)$ kinematic factors
- $F_S(s)$ scalar form factor:$(m_s - m_d)\langle K^-(k_1)\pi^+(k_2)|\bar{s}d|0\rangle \equiv F_S((k_1+k_2)^2)$
- $\Pi_{0,t}^{(\mathrm{OPE})}$ pert. calculable in terms of B-LCDA parameters

LCSR for $B \to (K\pi)_S$

$$\int_{(m_K+m_\pi)^2}^{s_0} ds\, e^{-s/M^2} \omega_{0,t}(s,q^2) F_S(s) F_{0,t}^{(\ell=0)}(s,q^2) = i\Pi_{0,t}^{(\text{OPE})}(q^2, s_0, M^2)$$

Key points:

- No closed expression for the $F_{0,t}^{(\ell=0)}(s,q^2)$!
- Only information on a weighted integral over the $K\pi$ invariant mass
- Use sum rule to constrain parameters of your favourite $K\pi$ S-wave model

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Inputs:

- $F_S(s)$ from data
- s_0 from two-point sum rule using scalar $K\pi$ form factor from data

QCD to constrain S wave models

- Use QCD sumrules to constrain $B \to (K\pi)_S$ parametrizations/models
- Simple sum of Breit-wigners (used for P-wave case) does not suffice

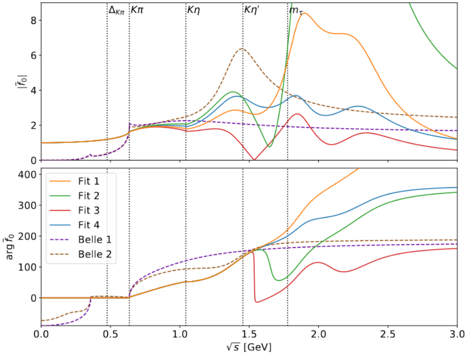
Model requirements:

- appropriate analytical properties
- poles corresponding to known resonances
- cuts for the revevant open channels
- simple (linear) dependence on the parameters to be constrained by the sum rules

S wave form factor [preliminary]

Hanhart, Kubis [ArXiV:2103.01966]

- Based on rescattering πK phase shifts + parametrization for higher resonances
- Applied to $\tau^- \to K_S \pi^- \nu_\tau$ data to fit resonance parameters



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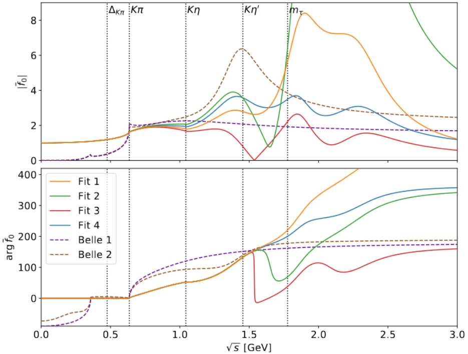


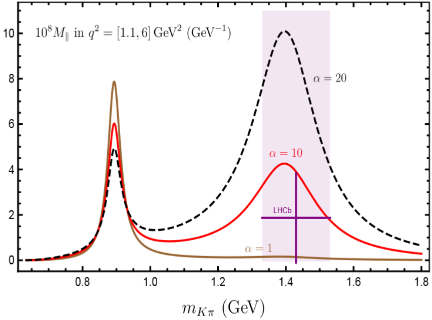
In progress:

- Use parametrization and constrain its parameters using LCSRs
- Less parameters to constrain by the sumrule (as the parameterization is more fixed)

What can data do for us?

LHCb [JHEP12(2016)065] [arXiV:1609.04736]

Example of use of the data to constrain higher-partial waves:



from: [J. Virto, A. Khodjamirian, S. Descotes-Genon JHEP 1912, 083 (2019)] [arXiv:1908.02267]

- 41 angular moments depending on S, P, D waves
- Higher P-wave resonances large impact on $B \to K\pi$
- Can be constrained by moment analysis
- Similar approach for the S wave?

Discussion:

In progress:

- Hybrid theory + data driven approach using LCSR

Questions:

- Moment analyses in q^2 and k^2 bins over the full spectrum possible?
- Optimal bin sizes? (precision versus information)
- Can we minimise dependence on BSM contributions?

Remarks:

- R_{K^*}, $R_{K\pi}$ only clean in SM
  - S-wave form factors could be important to go beyond an SM-null test
- How should we treat the S-wave amplitude in direct fits to Wilson Coefficients ? (see talk A. Mauri)

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Thank you for your attention!