What can we learn from genuine S/P-wave interference observables?

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Theory

Structure of $B \to K\pi\ell^+\ell^-$ decay

Differential decay rate of the full decay

$q^2 \equiv$ dilepton invariant mass
$\theta_K, \theta_\ell \equiv$ angles final particles
$\phi \equiv$ angle dilepton-plane

$$\frac{d^5\Gamma}{dq^2\, dm^2_{K\pi}\, d\cos\theta_K\, d\cos\theta_\ell\, d\phi} = \boxed{W_P} + \boxed{W_S}$$

Contributions from $B \to K^*(\to K\pi)\ell^+\ell^-$ and $B \to K_0^*(\to K\pi)\ell^+\ell^-$

We parametrise the decay as:

$$\begin{aligned}
W_P = \frac{9}{32\pi} & \left[J_{1s}\sin^2\theta_K + J_{1c}\cos^2\theta_K + (J_{2s}\sin^2\theta_K + J_{2c}\cos^2\theta_K)\cos 2\theta_l \right. \\
& + J_3 \sin^2\theta_K \sin^2\theta_l \cos 2\phi + J_4 \sin 2\theta_K \sin 2\theta_l \cos\phi + J_5 \sin 2\theta_K \sin\theta_l \cos\phi \\
& + (J_{6s}\sin^2\theta_K + J_{6c}\cos^2\theta_K)\cos\theta_l + J_7 \sin 2\theta_K \sin\theta_l \sin\phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin\phi \\
& \left. + J_9 \sin^2\theta_K \sin^2\theta_l \sin 2\phi \right]
\end{aligned}$$

$J_i, \tilde{J}_i \equiv$ functions of $q^2, m^2_{K\pi}$

$$\begin{aligned}
W_S = \frac{1}{4\pi} & \left[\tilde{J}^c_{1a} + \tilde{J}^c_{1b}\cos\theta_K + (\tilde{J}^c_{2a} + \tilde{J}^c_{2b}\cos\theta_K)\cos 2\theta_\ell + \tilde{J}_4 \sin\theta_K \sin 2\theta_\ell \cos\phi \right. \\
& \left. + \tilde{J}_5 \sin\theta_K \sin\theta_\ell \cos\phi + \tilde{J}_7 \sin\theta_K \sin\theta_\ell \sin\phi + \tilde{J}_8 \sin\theta_K \sin 2\theta_\ell \sin\phi \right]
\end{aligned}$$

Structure of $B \to K\pi\ell^+\ell^-$ decay

P-wave described by 6 complex $A^{L,R}_{\|,\perp,0}$

S-wave described by 2 complex $A'^{L,R}_0$

+

2 additional A_t, A_S if $m_\ell \neq 0$ and scalar op.

$L, R \equiv$ chirality of outgoing lepton current
$\|$, $\perp$,0 $\equiv$ helicity of K^* meson (P-wave)

→ Amplitudes multiplied by lineshape $BW_{P,S}(m^2_{K\pi})$

Observables $J_i, \tilde{J}_i$ described as spin-summed squared amplitudes
→ structure $A_i^{L*}A_j^L \pm A_i^{R*}A_j^R$

Useful to define complex 2-component vectors combining amplitudes

$$n_\| = \begin{pmatrix} A_\|^L \\ A_\|^{R*} \end{pmatrix}, \quad n_\perp = \begin{pmatrix} A_\perp^L \\ -A_\perp^{R*} \end{pmatrix}, \quad n_0 = \begin{pmatrix} A_0^L \\ A_0^{R*} \end{pmatrix}, \quad n_S = \begin{pmatrix} A_0'^L \\ A_0'^{R*} \end{pmatrix}$$

$$n'_S = \begin{pmatrix} A_0'^L \\ -A_0'^{R*} \end{pmatrix}$$

n_i vectors used to obtain symmetries among $J_i, \tilde{J}_i$ coefficients

Structure of $B \to K\pi\ell^+\ell^-$ decay

Definitions of $J_i, \tilde{J}_i$ (examples):

P-wave

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R*} + A_\parallel^L A_\parallel^{R*}\right)$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R*})\right] + \beta_\ell^2\,|A_S|^2$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*}) - \frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^* + A_\parallel^{R*} A_S)\right]$$

Structure of $B \to K\pi\ell^+\ell^-$ decay

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$C_{P,P'}$ contribution inside A_t

$m_\ell \neq 0$ introduces extra amplitudes $A_t^{(\prime)}$ + breaks symmetries

Structure of $B \to K\pi\ell^+\ell^-$ decay

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$m_\ell \neq 0$ introduces extra amplitudes $A_t^{(\prime)}$ + breaks symmetries

S-wave

$$\tilde{J}_{1a}^c = \frac{3}{8}\left[|A_0'^L|^2 + |A_0'^R|^2 + (1-\beta^2)\left(|A_t'|^2 + 2\mathrm{Re}\left[A_0'^L A_0'^{R*}\right]\right)\right]|BW_S|^2$$

$$\tilde{J}_{2a}^c = -\frac{3}{8}\beta^2\left(|A_0'^L|^2 + |A_0'^R|^2\right)|BW_S|^2 = -\frac{3}{8}\beta^2|n_S|^2$$

$$\tilde{J}_5 = \frac{3}{2}\sqrt{\frac{3}{2}}\beta\mathrm{Re}\left[(A_0'^L A_\perp^{L*} - A_0'^R A_\perp^{R*})BW_S BW_P^*\right] = \frac{3}{2}\sqrt{\frac{3}{2}}\beta[\mathrm{Re}(n_\perp^\dagger n_S)\mathrm{Re}(BW_S BW_P^*) - \mathrm{Im}(n_\perp^\dagger n_S')\mathrm{Im}(BW_S BW_P^*)]$$

Structure of $B \to K\pi\ell^+\ell^-$ decay

Definitions of $J_i, \tilde{J}_i$ (examples):

P-wave

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R*} + A_\parallel^L A_\parallel^{R*}\right)$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R*})\right] + \beta_\ell^2\,|A_S|^2$$

$C_{P,P'}$ contribution inside A_t

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*}) - \frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^* + A_\parallel^{R*} A_S)\right]$$

$m_\ell \neq 0$ introduces extra amplitudes $A_t^{(')}$ + breaks symmetries

S-wave

$$\tilde{J}_{1a}^c = \frac{3}{8}\left[|A_0'^L|^2 + |A_0'^R|^2 + (1-\beta^2)\left(|A_t'|^2 + 2\mathrm{Re}\left[A_0'^L A_0'^{R*}\right]\right)\right]|BW_S|^2$$

$$\tilde{J}_{2a}^c = -\frac{3}{8}\beta^2\left(|A_0'^L|^2 + |A_0'^R|^2\right)|BW_S|^2 = -\frac{3}{8}\beta^2|n_S|^2$$

$BW_S BW_P^*$ introduces mixing S-P wave

$$\tilde{J}_5 = \frac{3}{2}\sqrt{\frac{3}{2}}\beta\mathrm{Re}\left[(A_0'^L A_\perp^{L*} - A_0'^R A_\perp^{R*})BW_S BW_P^*\right] = \frac{3}{2}\sqrt{\frac{3}{2}}\beta[\mathrm{Re}(n_\perp^\dagger n_S)\mathrm{Re}(BW_S BW_P^*) - \mathrm{Im}(n_\perp^\dagger n_S')\mathrm{Im}(BW_S BW_P^*)$$

$F_S \propto |n_S|^2$ needs to be extracted from $\tilde{J}_{2a}^c$ if $m_\ell \neq 0$!!

Structure of $B \to K\pi\ell^+\ell^-$ decay

Writing $\tilde{J}_i$ in terms of n_i + making mass dependence in BW explicit

split $\tilde{J}_i$ in $\mathrm{Re}[BW_S BW_P^*]$ / $\mathrm{Im}[BW_S BW_P^*]$

Take BW as external input to disentangle $\mathrm{Re}[], \mathrm{Im}[]$

extra d.o.f. from the splitting of $\tilde{J}_i$

$$S^r_{S1} = 2\sqrt{3}\frac{1}{\Gamma'_{full}}\beta^2\mathrm{Re}(n_0^\dagger n_S), \qquad S^i_{S1} = 2\sqrt{3}\frac{1}{\Gamma'_{full}}\beta^2\mathrm{Im}(n_0^\dagger n'_S),$$

$$S^r_{S2} = \sqrt{\frac{3}{2}}\frac{1}{\Gamma'_{full}}\beta^2\mathrm{Re}(n_\parallel^\dagger n_S), \qquad S^i_{S2} = \sqrt{\frac{3}{2}}\frac{1}{\Gamma'_{full}}\beta^2\mathrm{Im}(n_\parallel^\dagger n'_S),$$

$$S^r_{S3} = 2\sqrt{\frac{3}{2}}\frac{1}{\Gamma'_{full}}\beta\mathrm{Re}(n_\perp^\dagger n_S), \qquad S^i_{S3} = 2\sqrt{\frac{3}{2}}\frac{1}{\Gamma'_{full}}\beta\mathrm{Im}(n_\perp^\dagger n'_S),$$

$$S^r_{S4} = 2\sqrt{\frac{3}{2}}\frac{1}{\Gamma'_{full}}\beta\mathrm{Re}(n_\parallel^\dagger n'_S), \qquad S^i_{S4} = 2\sqrt{\frac{3}{2}}\frac{1}{\Gamma'_{full}}\beta\mathrm{Im}(n_\parallel^\dagger n_S),$$

$$S^r_{S5} = \sqrt{\frac{3}{2}}\frac{1}{\Gamma'_{full}}\beta^2\mathrm{Re}(n_\perp^\dagger n'_S), \qquad S^i_{S5} = \sqrt{\frac{3}{2}}\frac{1}{\Gamma'_{full}}\beta^2\mathrm{Im}(n_\perp^\dagger n_S),$$

In massive case $S^{r,i}_{S1}$ defined through $\tilde{J}^c_{2b}$ because $\tilde{J}^c_{1b} \neq \tilde{J}^c_{2b}$

Notice different products $n_0^\dagger n_S$ VS $n_0^\dagger n'_S$

Take Home 1 🏠

Complex interference terms $BW_S BW_P^*$ give rise to **new S-wave** observables for Real and Imaginary parts of the $\tilde{J}_i$ using the full $m_{K\pi}$ lineshape

⇩

Neglecting Imaginary interference observables S_{Si} could hide missing information so they **must be considered**

Structure of $B \to K\pi\ell^+\ell^-$ decay

Angular distribution (P-wave) in terms of optimised observables:

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^4(\Gamma+\bar{\Gamma})}{\mathrm{d}q^2\, d\vec{\Omega}}\Bigg|_{\mathrm{P}} = \frac{9}{32\pi}\Big[\hat{F}_T M_1 \sin^2\theta_K + \hat{F}_L M_2 \cos^2\theta_K + (\frac{1}{4}\hat{F}_T \sin^2\theta_K - \hat{F}_L \cos^2\theta_K)\cos 2\theta_l$$

$$+\frac{1}{2}P_1\hat{F}_T \sin^2\theta_K \sin^2\theta_l \cos 2\phi + \sqrt{\hat{F}_T\hat{F}_L}\left(\frac{1}{2}P_4' \sin 2\theta_K \sin 2\theta_l \cos\phi + P_5' \sin 2\theta_K \sin\theta_l \cos\phi\right)$$

$$+2P_2\hat{F}_T \sin^2\theta_K \cos\theta_l - \sqrt{\hat{F}_T\hat{F}_L}\left(P_6' \sin 2\theta_K \sin\theta_l \sin\phi + \frac{1}{2}P_8' \sin 2\theta_K \sin 2\theta_l \sin\phi\right)$$

$$-P_3\hat{F}_T \sin^2\theta_K \sin^2\theta_l \sin 2\phi\Big]$$

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$$-P_3\hat{F}_T \sin^2\theta_K \sin^2\theta_l \sin 2\phi\Big]$$

Basis of observables

$n_c - n_{rel} = 2n_A - n_{sym}$

Massless case $(m_\ell = 0)$

P-wave: $\frac{d\Gamma}{dq^2} + 7\,P_i + F_L = 9$ observables

Experimentally $\rightarrow \frac{d\Gamma}{dq^2} = 1$

S-wave: $5\,S_i^{\mathrm{Re}} + 5\,S_i^{\mathrm{Im}} + F_S = 11$ observables

Total: $19 + \left(\frac{d\Gamma}{dq^2} = 1\right)$ observables

14 d.o.f. ≡ 6 non-trivial relations ✅

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$$\mathcal{O}_{m_\ell=0} = \left\{d\Gamma/dq^2, F_L, P_1, P_2, P_3, P_4', P_5', P_6', P_8', F_S, S^r_{S1}, S^r_{S2}, S^r_{S3}, S^r_{S4}, S^r_{S5}, S^i_{S1}, S^i_{S2}, S^i_{S3}, S^i_{S4}, S^i_{S5}\right\}$$

"Basis" of 20 obs is redundant → 6 non-trivial relations → 14 indep. obs

Basis of observables

$$n_c - n_{rel} = 2n_A - n_{sym}$$

Massive case $(m_\ell \neq 0)$

P-wave: $\frac{d\Gamma}{dq^2} + 7\,P_i + F_L + M_{1,2} \;=\; 11$ observables

Experimentally $\rightarrow \frac{d\Gamma}{dq^2} = 1$

S-wave: $5\,S_i^{\rm Re} + 5\,S_i^{\rm Im} + F_S + M'_{3,4,5} \;=\; 14$ observables

Total: $24 + \left(\frac{d\Gamma}{dq^2} = 1\right)$ observables

18 d.o.f. $\equiv$ 7 non-trivial relations

✅

Basis of observables

$n_c - n_{rel} = 2n_A - n_{sym}$

Massive case $(m_\ell \neq 0)$

P-wave: $\dfrac{d\Gamma}{dq^2} + 7\,P_i + F_L + M_{1,2} \;=\; 11$ observables

Experimentally $\rightarrow \dfrac{d\Gamma}{dq^2} = 1$

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Total: $24 + \left(\dfrac{d\Gamma}{dq^2} = 1\right)$ observables

18 d.o.f. ≡ 7 non-trivial relations ✅

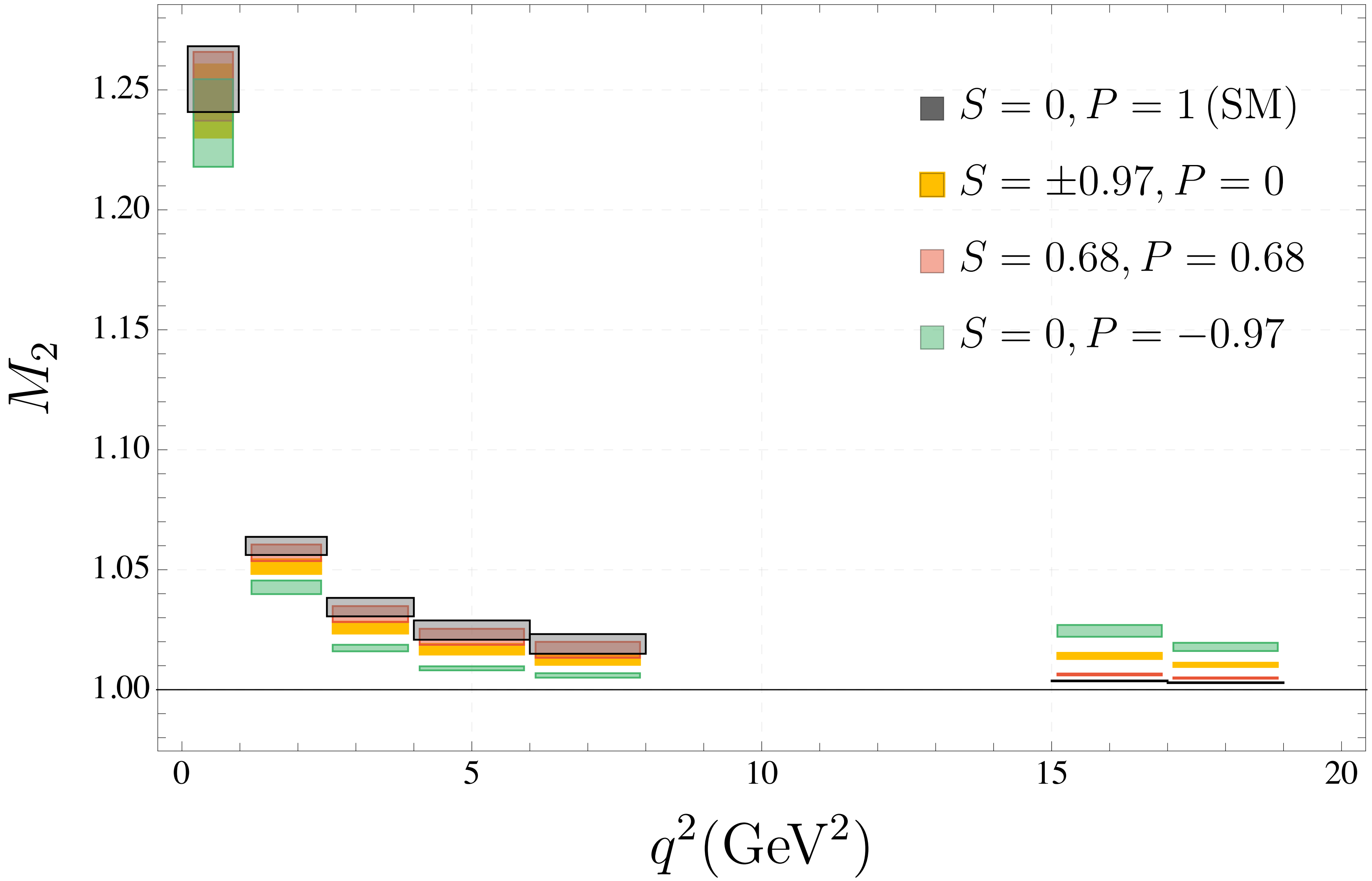
$$\mathcal{O}_{m_{\neq}=0} = \left\{d\Gamma/dq^2, M_1, M_2, F_L, P_1, P_2, P_3, P'_4, P'_5, P'_6, P'_8, M'_3, M'_4, M'_5, F_S, S^r_{S1}, S^r_{S2}, S^r_{S3}, S^r_{S4}, S^r_{S5}, S^i_{S1}, S^i_{S2}, S^i_{S3}, S^i_{S4}, S^i_{S5}\right\}$$

"Basis" of 25 obs is redundant → 7 non-trivial relations → 18 indep. obs

Predictions of M_2

$$M_2 = -\frac{J_{1c}}{J_{2c}}$$

M_2 potentially sensitive to pseudoscalar (C_P) & scalar (C_S) NP contributions but very insensitive to $C_7^{\rm NP}$, $C_{9\mu}^{\rm NP}$



[Fleischer et al. arXiv: 1703.10160]

$$S = \sqrt{1 - 4\frac{m_\mu^2}{m_{B_s}^2}}\;\frac{m_{B_s}^2}{2m_b m_\mu}\left(\frac{C_S - C_{S'}}{C_{10}^{\rm SM}}\right)$$

$$P = \frac{C_{10}^{\rm SM} + C_{10}^{\rm NP} - C_{10'}}{C_{10}^{\rm SM}} + \frac{m_{B_s}^2}{2m_b m_\mu}\left(\frac{C_P - C_{P'}}{C_{10}^{\rm SM}}\right)$$

Reduced sensitivity to C_S, only for extreme values of $C_P = 0.28\,(P = -0.97)$ there is some disentangling between NP scenarios albeit high precision required

Example of new relations

Since $n_{\|}, n_{\perp}$ span space for complex 2D vectors, we can contract other n_i with them

E.g.

$$n_{\perp}^{\dagger} n_S' = a_S'(n_{\perp}^{\dagger} n_{\|}) + b_S' |n_{\perp}|^2$$
$$n_{\|}^{\dagger} n_S' = a_S' |n_{\perp}|^2 + b_S'(n_{\|}^{\dagger} n_{\perp})$$

→

$$a_S' = \frac{(n_{\|}^{\dagger} n_S')|n_{\perp}|^2 - (n_{\perp}^{\dagger} n_S')(n_{\|}^{\dagger} n_{\perp})}{|n_{\|}|^2 |n_{\perp}|^2 - |n_{\perp}^{\dagger} n_{\|}|^2}$$

$$b_S' = \frac{(n_{\|}^{\dagger} n_S')(n_{\perp}^{\dagger} n_{\|}) - (n_{\perp}^{\dagger} n_S')|n_{\|}|^2}{|n_{\perp}^{\dagger} n_{\|}|^2 - |n_{\|}|^2 |n_{\perp}|^2}$$

Starting point: $|n_S|^2 = a_S(n_S^{\dagger} n_{\|}) + b_S(n_S^{\dagger} n_{\perp})$

Some 💥#@💢!🤬 later…

$$\begin{aligned} 0 = \; & -3\beta^4 F_S J_{6s}^2 + \Gamma'_{full}[-8(2J_{2s} + J_3)S_{S2}^{r\,2} - 16 J_9 S_{S2}^r S_{S5}^i + 8(-2J_{2s} + J_3)S_{S5}^{i\,2}] \\ & +2\beta^2(6F_S(4J_{2s}^2 - J_3^2 - J_9^2) + \Gamma'_{full}[(-2J_{2s} + J_3)S_{S3}^{r\,2} + 2J_9 S_{S3}^r S_{S4}^i \\ & -(2J_{2s} + J_3)S_{S4}^{i\,2} + 2J_{6s}(S_{S2}^r S_{S3}^r + S_{S4}^i S_{S5}^i)]) \end{aligned}$$

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→

$$a_S' = \frac{(n_{\|}^{\dagger} n_S') |n_{\perp}|^2 - (n_{\perp}^{\dagger} n_S')(n_{\|}^{\dagger} n_{\perp})}{|n_{\|}|^2 |n_{\perp}|^2 - |n_{\perp}^{\dagger} n_{\|}|^2}$$

$$b_S' = \frac{(n_{\|}^{\dagger} n_S')(n_{\perp}^{\dagger} n_{\|}) - (n_{\perp}^{\dagger} n_S') |n_{\|}|^2}{|n_{\perp}^{\dagger} n_{\|}|^2 - |n_{\|}|^2 |n_{\perp}|^2}$$

Starting point: $|n_S|^2 = a_S(n_S^{\dagger} n_{\|}) + b_S(n_S^{\dagger} n_{\perp})$

Massive relations simplify to massless once $\beta \to 1$

Some 💥#@💢!🤬 later…

$$\begin{aligned} 0 = \; & -3\beta^4 F_S J_{6s}^2 + \Gamma'_{full}[-8(2J_{2s} + J_3) S_{S2}^{r\,2} - 16 J_9 S_{S2}^r S_{S5}^i + 8(-2J_{2s} + J_3) S_{S5}^{i\,2}] \\ & + 2\beta^2 (6F_S(4J_{2s}^2 - J_3^2 - J_9^2) + \Gamma'_{full}[(-2J_{2s} + J_3) S_{S3}^{r\,2} + 2J_9 S_{S3}^r S_{S4}^i \\ & - (2J_{2s} + J_3) S_{S4}^{i\,2} + 2J_{6s}(S_{S2}^r S_{S3}^r + S_{S4}^i S_{S5}^i)]) \end{aligned}$$

All relations have been translated in terms of optimised observables

Information from relations

Solving for S^r_{S2} & imposing real solution:

$x = 1 - P_1^2 - 4\beta^2 P_2^2 - 4P_3^2$

$$0 \leq \Delta(S^r_{S2}) = -\beta^2 x (S^r_{S3})^2 - 4x(S^i_{S5})^2 - \beta^2(2P_3 S^r_{S3} + (1+P_1)S^i_{S4} - 4P_2 S^i_{S5})^2 + 3\beta^4 x F_S(1-F_S)F_T(1+P_1)$$

Insensitive to q^2 + safe from details of FFs

Negative terms individually < **Positive term** ⇒ Bounds!

$$|S^r_{S2}| \leq \beta^2 \sqrt{\frac{3}{4} F_S(1-F_S)F_T(1-P_1)} \quad |S^r_{S3}| \leq \beta\sqrt{3F_S(1-F_S)F_T(1+P_1)}$$

$$|S^i_{S4}| \leq \beta\sqrt{3F_S(1-F_S)F_T(1-P_1)} \quad |S^i_{S5}| \leq \beta^2 \sqrt{\frac{3}{4} F_S(1-F_S)F_T(1+P_1)}$$

Same strategy for other relations

$$|S^{r,i}_{S2}| \leq \beta^2 \frac{k_1}{2} \quad |S^{r,i}_{S3}| \leq \beta k_2 \quad |S^{r,i}_{S4}| \leq \beta k_1 \quad |S^{r,i}_{S5}| \leq \beta^2 \frac{k_2}{2}$$

More information from relations

$$S^r_{S4} = \frac{2}{1+P_1}\left(2P_2 S^r_{S5} \pm \frac{1}{\beta}\sqrt{x(\beta^4 \frac{k_2^2}{4} - (S^r_{S5})^2)}\right)$$

Can be tested experimentally

Information from relations

Solving for S^r_{S2} & imposing real solution:

$x = 1 - P_1^2 - 4\beta^2 P_2^2 - 4P_3^2$

$$0 \le \Delta(S^r_{S2}) = -\,\beta^2 x (S^r_{S3})^2 - 4x(S^i_{S5})^2 - \beta^2(2P_3 S^r_{S3} + (1+P_1)S^i_{S4} - 4P_2 S^i_{S5})^2$$
$$+\;\; 3\beta^4 x F_S(1-F_S)F_T(1+P_1)$$

Insensitive to q^2 + safe from details of FFs

Negative terms individually < **Positive term** ⇒ Bounds!

$$|S^r_{S2}| \le \beta^2 \sqrt{\frac{3}{4} F_S(1-F_S)F_T(1-P_1)} \qquad |S^r_{S3}| \le \beta \sqrt{3F_S(1-F_S)F_T(1+P_1)}$$

$$|S^i_{S4}| \le \beta \sqrt{3F_S(1-F_S)F_T(1-P_1)} \qquad |S^i_{S5}| \le \beta^2 \sqrt{\frac{3}{4} F_S(1-F_S)F_T(1+P_1)}$$

Same strategy for other relations

$$|S^{r,i}_{S2}| \le \beta^2 \frac{k_1}{2} \qquad |S^{r,i}_{S3}| \le \beta k_2 \qquad |S^{r,i}_{S4}| \le \beta k_1 \qquad |S^{r,i}_{S5}| \le \beta^2 \frac{k_2}{2}$$

More information from relations

$$S^r_{S4} = \frac{2}{1+P_1}\left(2P_2 S^r_{S5} \pm \frac{1}{\beta}\sqrt{x\left(\beta^4 \frac{k_2^2}{4} - (S^r_{S5})^2\right)}\right)$$

Can be tested experimentally

Information from relations

Solving for S^r_{S2} & imposing real solution:

$x = 1 - P_1^2 - 4\beta^2 P_2^2 - 4P_3^2$

$$0 \leq \Delta(S^r_{S2}) = -\beta^2 x (S^r_{S3})^2 - 4x(S^i_{S5})^2 - \beta^2(2P_3 S^r_{S3} + (1+P_1)S^i_{S4} - 4P_2 S^i_{S5})^2 + 3\beta^4 x F_S(1-F_S)F_T(1+P_1)$$

Negative terms individually < **Positive term** ⇨ Bounds!

Insensitive to q^2 + safe from details of FFs

$$|S^r_{S2}| \leq \beta^2 \sqrt{\frac{3}{4} F_S(1-F_S)F_T(1-P_1)} \qquad |S^r_{S3}| \leq \beta \sqrt{3F_S(1-F_S)F_T(1+P_1)}$$

$$|S^i_{S4}| \leq \beta \sqrt{3F_S(1-F_S)F_T(1-P_1)} \qquad |S^i_{S5}| \leq \beta^2 \sqrt{\frac{3}{4} F_S(1-F_S)F_T(1+P_1)}$$

Same strategy for other relations

$$|S^{r,i}_{S2}| \leq \beta^2 \frac{k_1}{2} \qquad |S^{r,i}_{S3}| \leq \beta k_2 \qquad |S^{r,i}_{S4}| \leq \beta k_1 \qquad |S^{r,i}_{S5}| \leq \beta^2 \frac{k_2}{2}$$

More information from relations

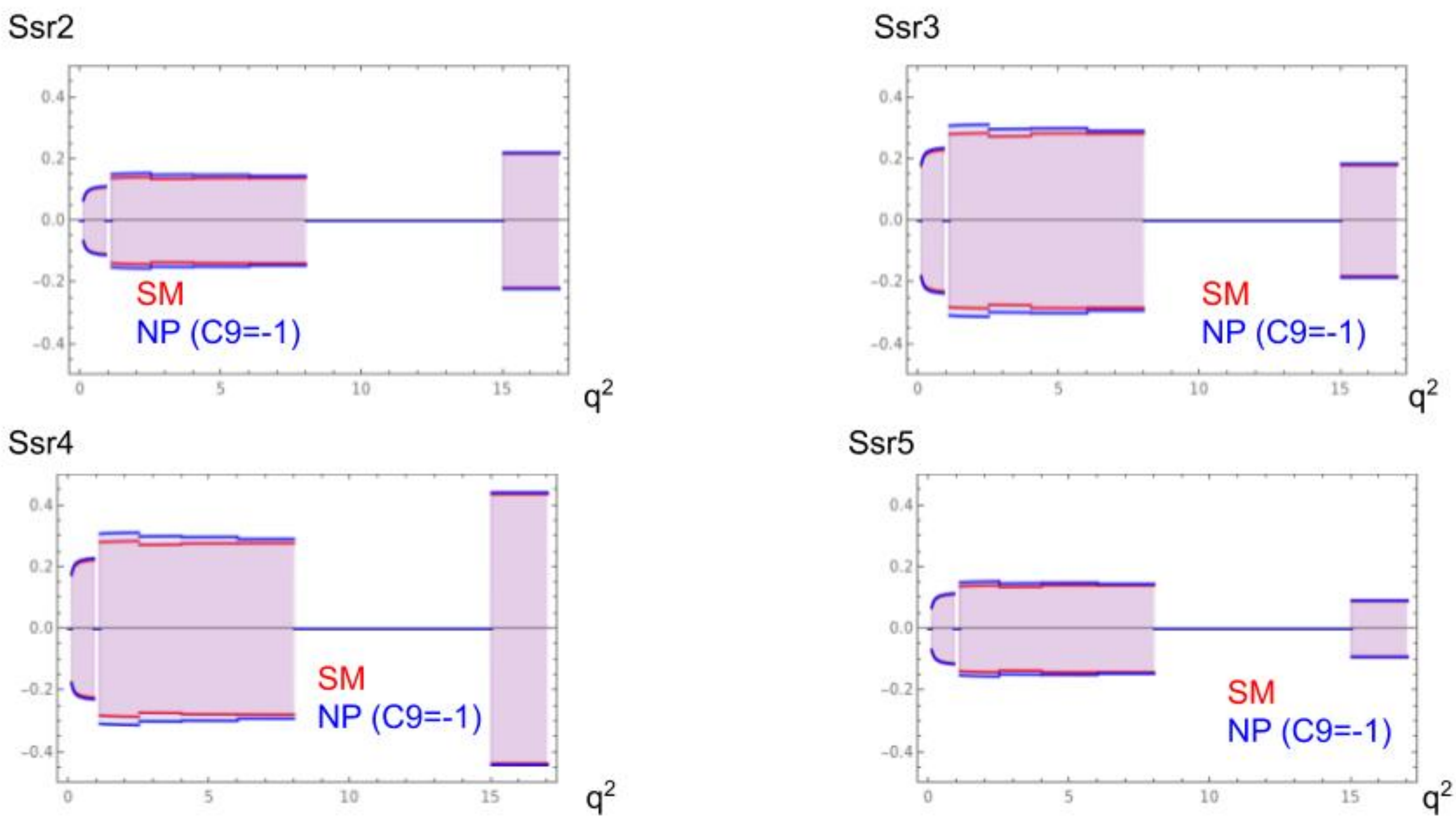
$$S^r_{S4} = \frac{2}{1+P_1}\left(2P_2 S^r_{S5} \pm \frac{1}{\beta}\sqrt{x\left(\beta^4 \frac{k_2^2}{4} - (S^r_{S5})^2\right)}\right)$$

Can be tested experimentally

Information from relations

Bounds

$$|S^{r,i}_{S2}| \leq \beta^2 \frac{k_1}{2} \qquad |S^{r,i}_{S3}| \leq \beta k_2 \qquad |S^{r,i}_{S4}| \leq \beta k_1 \qquad |S^{r,i}_{S5}| \leq \beta^2 \frac{k_2}{2}$$



<u>Very</u> preliminary plots 🙂

Take Home 2 🏠

Independent information **must be the same**

⇓

Out of these bases there are only **14 (18)** independent observables, so we find **3 (4) new relations** for $m_\ell = 0$ ($m_\ell \neq 0$)

⇓

Relations provide **bounds** for interference observables $S_{Si}^{r,i}$

New disentangling: correlated zeroes

If no RHC or particular combination $C_{7'} \simeq -\frac{C_7^{\rm eff}}{C_{10} - C_9^{\rm eff}}(C_{10'} - C_{9'})$ ⇒ $P_2^{\rm max}(q^2_{max}) = \frac{1}{2\beta}$ when $n_\perp(q^2_{max}) = n_\parallel(q^2_{max})$

- Several observables are 0 at $q^2_{max} = 2.02\,{\rm GeV}^2$ where $n_\perp(q^2_{max}) = n_\parallel(q^2_{max})$:

$$
\begin{aligned}
X_1 &= P_1 \\
X_2 &= \beta P_5' - P_4' \\
X_3 &= \beta S^r_{S4} - 2S^r_{S5} \\
X_4 &= \beta S^r_{S3} - 2S^r_{S2} \\
Y_1 &= P_3 \\
Y_2 &= \beta P_6' - P_8' \\
Y_3 &= \beta S^i_{S4} - 2S^i_{S5} \\
Y_4 &= \beta S^i_{S3} - 2S^i_{S2}
\end{aligned}
$$

Completely **new** strategy to determine position of q^2_{max} *i.e.* **0 of observables**

⇓

Correlated analysis to all X_i & Y_i observables with more precision

Applies when:

- No or tiny RHC
- RHC fulfill combination above

New disentangling: correlated zeroes

If no RHC or particular combination $C_{7'} \simeq -\frac{C_7^{\text{eff}}}{C_{10} - C_9^{\text{eff}}}(C_{10'} - C_{9'})$

$$P_2^{\max}(q^2_{max}) = \frac{1}{2\beta}$$

when $n_\perp(q^2_{max}) = n_\parallel(q^2_{max})$

- Several observables are 0 at $q^2_{max} = 2.02\,\text{GeV}^2$ where $n_\perp(q^2_{max}) = n_\parallel(q^2_{max})$:

$$\begin{aligned}
X_1 &= P_1 \\
X_2 &= \beta P'_5 - P'_4 \\
X_3 &= \beta S^r_{S4} - 2S^r_{S5} \\
X_4 &= \beta S^r_{S3} - 2S^r_{S2} \\
Y_1 &= P_3 \\
Y_2 &= \beta P'_6 - P'_8 \\
Y_3 &= \beta S^i_{S4} - 2S^i_{S5} \\
Y_4 &= \beta S^i_{S3} - 2S^i_{S2}
\end{aligned}$$

Completely **new** strategy to determine position of q^2_{max} *i.e.* **0 of observables**

- What if **sizeable** RHC?

Correlated analysis to all X_i & Y_i observables with more precision

Applies when:

- No or tiny RHC
- RHC fulfill combination above

New disentangling: correlated zeroes

If sizeable RHC:

$^{\dagger\dagger}x = 1 - P_1^2 - 4\beta^2 P_2^2 - 4P_3^2$

$$x(q_1^2)^{\dagger\dagger} = 0 \quad \Rightarrow \quad P_2(q_1^2) = \frac{\sqrt{1 - P_1^2(q_1^2)}}{2\beta}$$

3 new observables that will be zero at such q_1^2

$$Z_1 = \beta\sqrt{\frac{1-P_1}{1+P_1}}P_5' - P_4'$$

$$Z_2 = \beta\sqrt{\frac{1+P_1}{1-P_1}}S_{S4}^r - 2S_{S5}^r$$

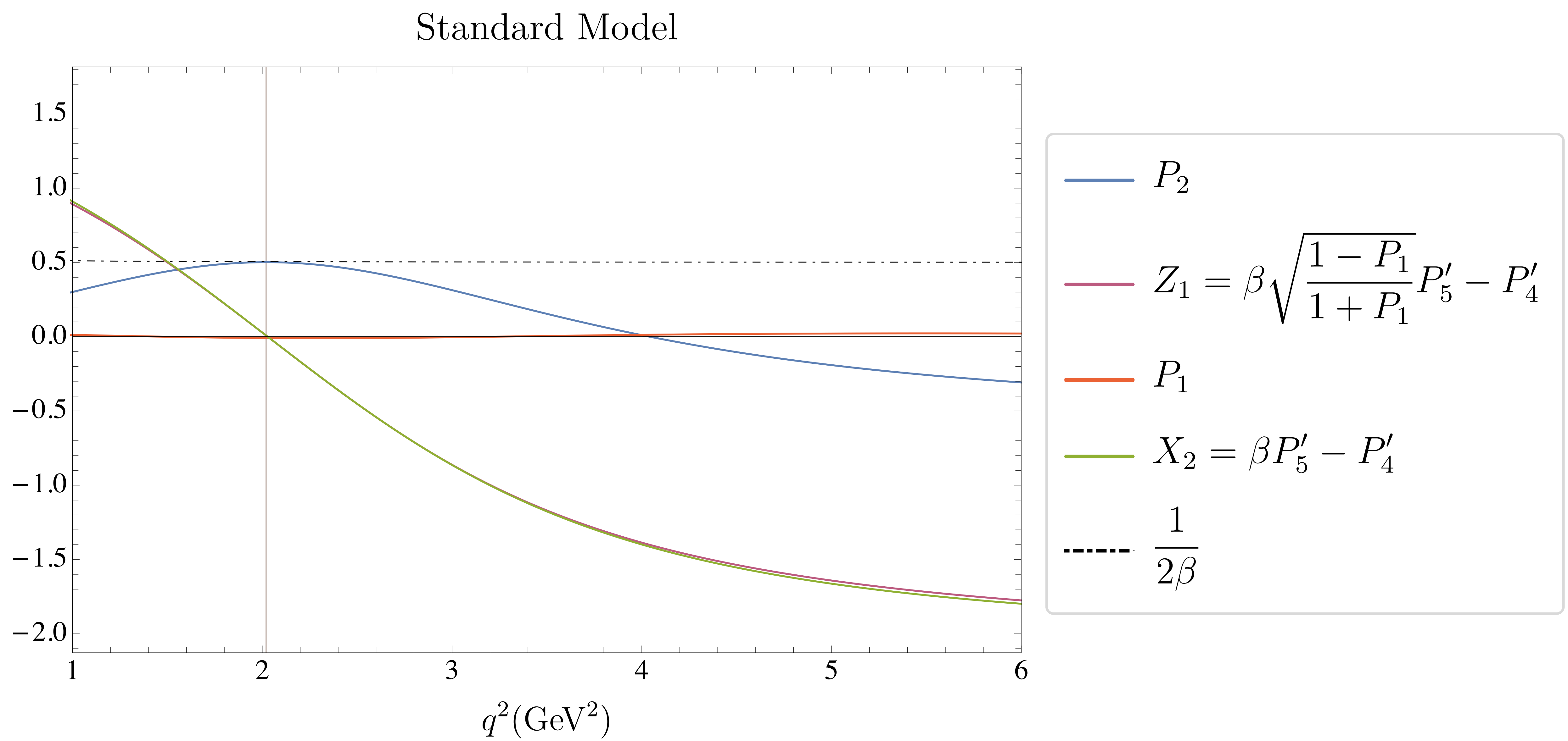
$$Z_3 = \beta\sqrt{\frac{1-P_1}{1+P_1}}S_{S3}^r - 2S_{S2}^r$$

Caveat: quadratically suppressed terms $\mathcal{O}(P_3 S_{Si}^{r,i}, P_3^2)$ are neglected

Correlated analysis still possible even in presence of arbitrary large RHC

New disentangling: correlated zeroes

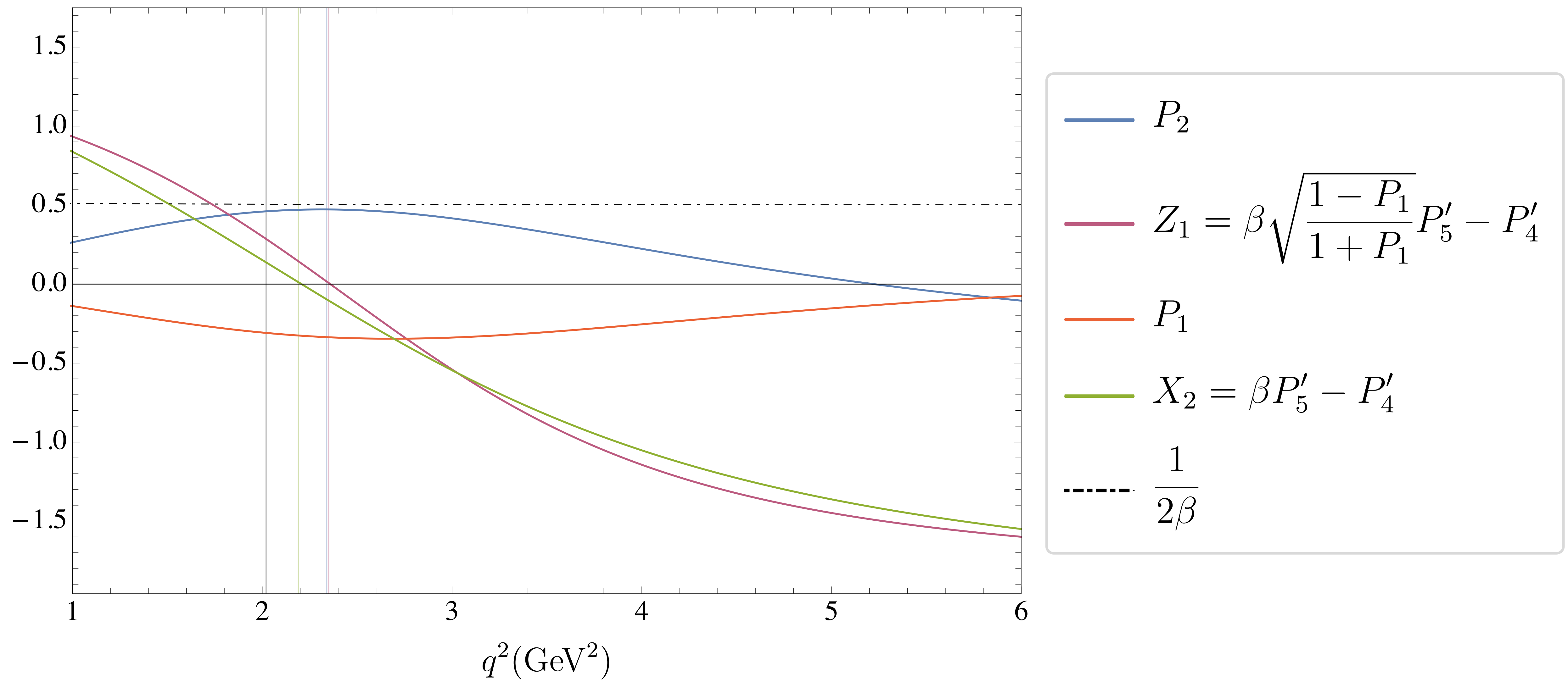
Considering very small q^2−bins



New disentangling: correlated zeroes

Considering very small q^2-bins

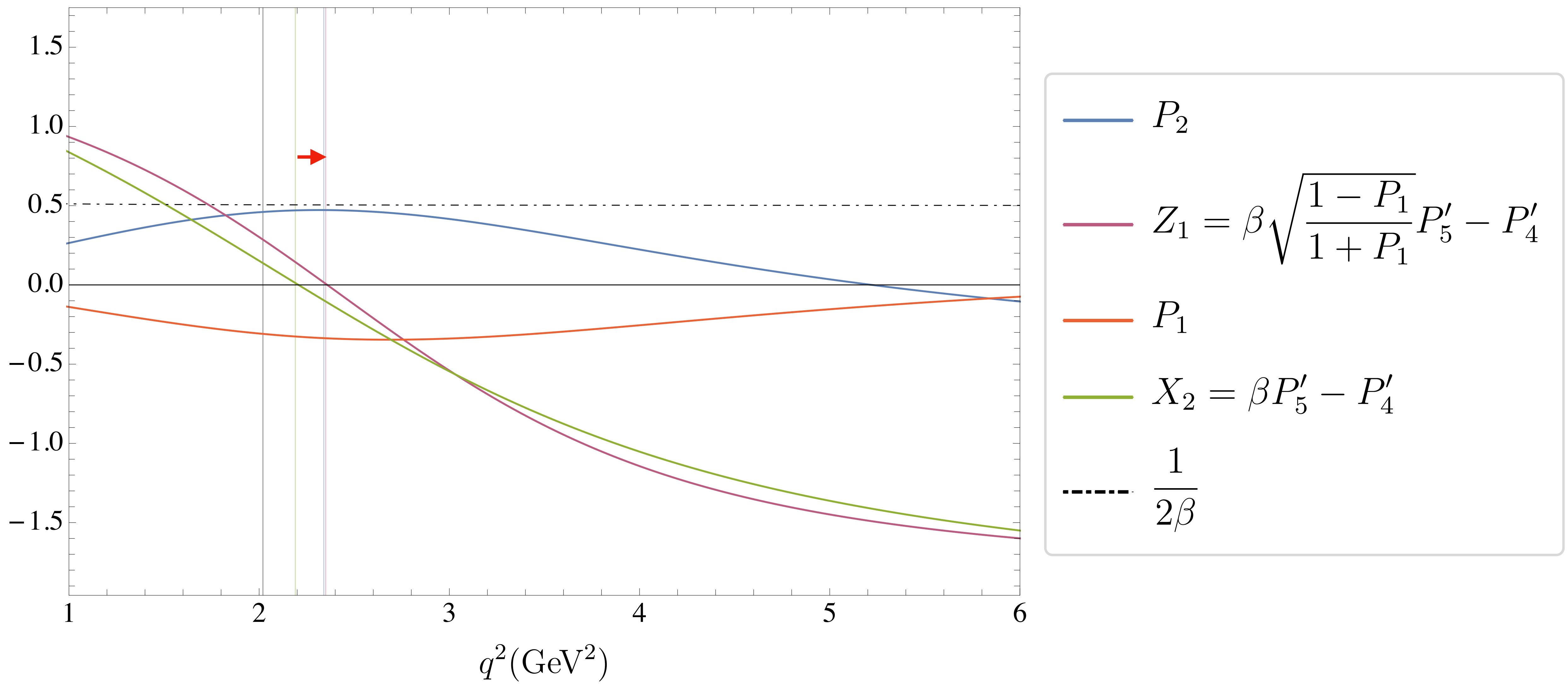
$\mathcal{C}_{9\mu}^{\rm NP} = -\mathcal{C}_{9'\mu} = -1.10,\ \mathcal{C}_{10\mu}^{\rm NP} = \mathcal{C}_{10'\mu} = 0.28$



New disentangling: correlated zeroes

Considering very small q^2-bins

$$\mathcal{C}_{9\mu}^{\rm NP} = -\mathcal{C}_{9'\mu} = -1.10,\ \mathcal{C}_{10\mu}^{\rm NP} = \mathcal{C}_{10'\mu} = 0.28$$



Shift of position of 0 of Z_1 (coincides with maximum of P_2) compared to X_2

Local analysis of zeroes

Local analysis in bin $q^2 \in [1.8,2.5]\ \mathrm{GeV}^2$

We parametrize **new observables** in terms of NP contributions in relevant Wilson Coefficients $\left\{ C_{9\mu}^{\mathrm{NP}}, C_{10\mu}^{\mathrm{NP}}, C_{9'\mu}, C_{10'\mu} \right\}$ and study them **in significant NP scenarios** from Global Fits

(Illustrative exercise, only CV considered)

$\langle X_2^{\mathrm{SM}} \rangle_{[1.8,2.5]}$

$$\langle X_2 \rangle_{[1.8,2.5]} = \langle \beta P_5' - P_4' \rangle_{[1.8,2.5]} \sim -0.14 + 0.22 \left(C_{10\mu}^{\mathrm{NP}} - C_{9\mu}^{\mathrm{NP}} \right) - 0.015 \left[1 + 2 \left(C_{9'\mu} - C_{9\mu}^{\mathrm{NP}} \right) \right]$$

All scenarios from Global Fits [1903.09578] predict $0.03 < \langle X_2 \rangle_{[1.8,2.5]} < 0.14$ except:

- Scenario $C_{10\mu}^{\mathrm{NP}} = 0.57 \Rightarrow \langle X_2 \rangle_{[1.8,2.5]} = -0.01$
- Scenario $\{ C_{9\mu}^{\mathrm{V}} = -C_{10\mu}^{\mathrm{V}} = -0.30\,, C_9^{\mathrm{U}} = -0.92 \}$
  $\Rightarrow \langle X_{2\mu} \rangle_{[1.8,2.5]} = 0.19, \langle X_{2e} \rangle_{[1.8,2.5]} = 0.06$

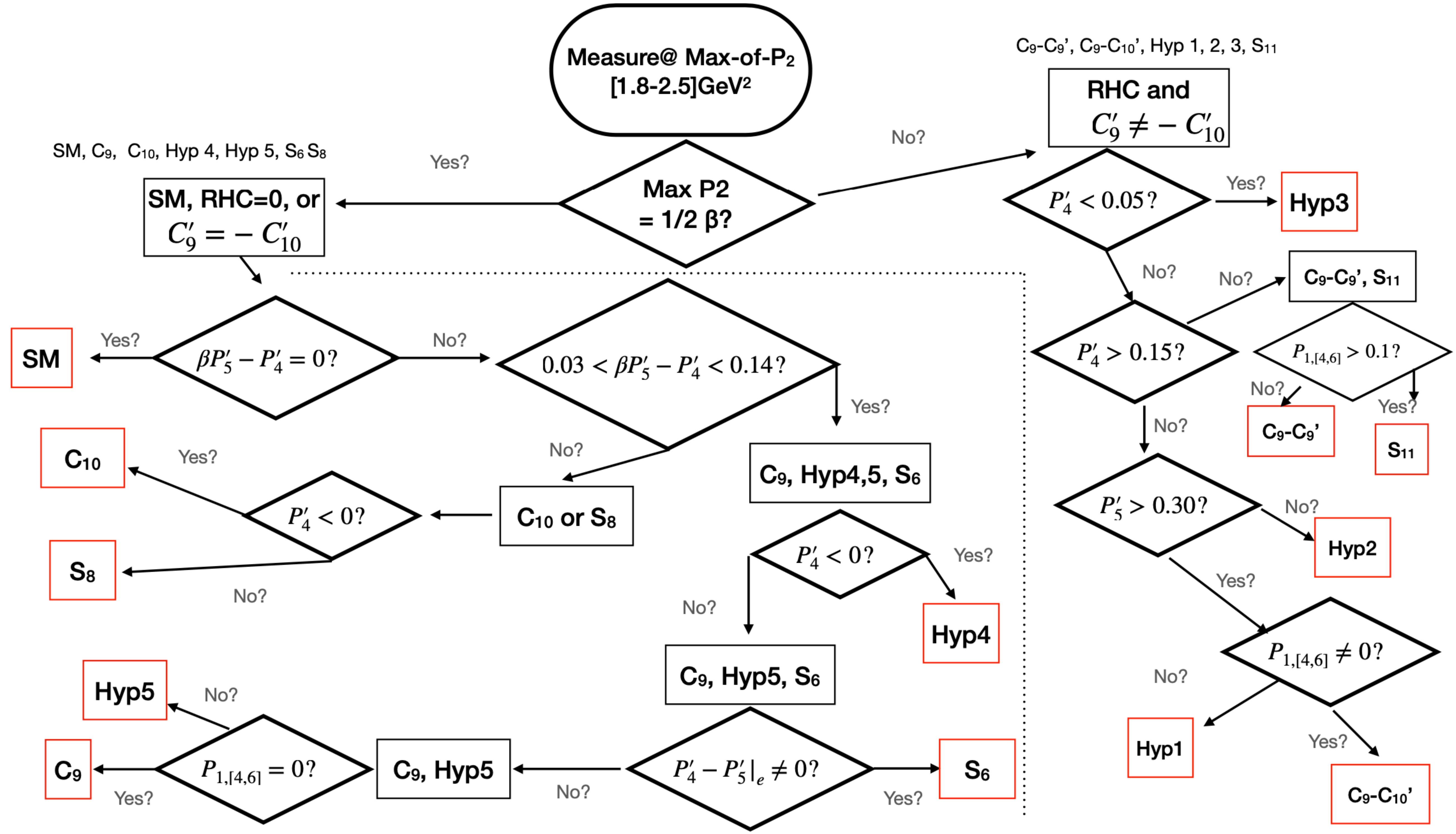
Precision reached by Run 4

X_2 can be a **discriminator** between some of the preferred scenarios (like Q_5)

See talk by P. Stangl & J. Matias

Next step: try to get some info about hadronic contributions (WIP)

Decision Tree from Zeroes



Take Home 3 🏠

Correlated zeroes of different observables give us **new local** information

⇩

Link between **local fits** to zeroes and **global picture** through these observables

⇩

Zeroes can provide a way of **disentangling NP scenarios** from global fits

Experiment

(Pseudo-) Experiment setup I

These plots are aimed at LHCb Run 1 and Run 2 (with some upgrade consideration)

- Minimal and well separated background
- Will equally apply to Belle II with statistics
  - Perhaps with a simpler angular acceptance

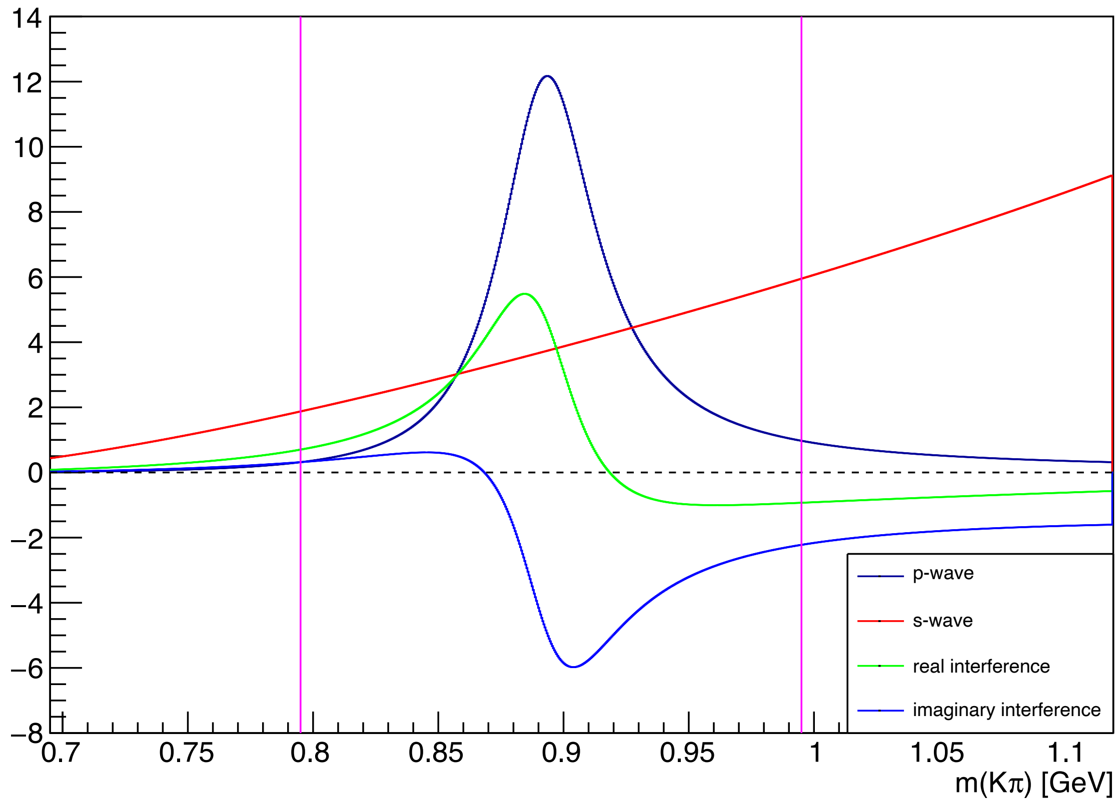
Setup

- Benchmark is expected LHCb Run 1 + Run 2 yield (scaled from published results)
- Representative background - product of Chebychev polynomials + exponential (m_B)
- Representative acceptance function on signal to mode effects of selection
- $0.75 < m_{K\pi} < 1.2\,\mathrm{GeV}$
  - Wide $m_{K\pi}$ window for maximal S-wave and interference control
  - Larger contamination of other P/D-wave
  - Larger contamination of backgrounds

(Pseudo-) Experiment setup II

The S-wave lineshape you choose matters

- Minimal sensitivity to it in the fit ($\sim\%$ contribution in $\mathcal{O}(10^3)$ events)
- Normalise the $|\mathcal{A}(m_{K\pi})|^2$ P- and S-wave lineshapes over $m_{K\pi}$ window
  - Total rate in the window doesn't depend on $m_{K\pi}$
- $m_{K\pi}$ window changes meaning of S-wave and interference observables
- Choice of $m_{K\pi}$ lineshape changes S-wave normalisation - changes observables
- See K. Petridis talk.

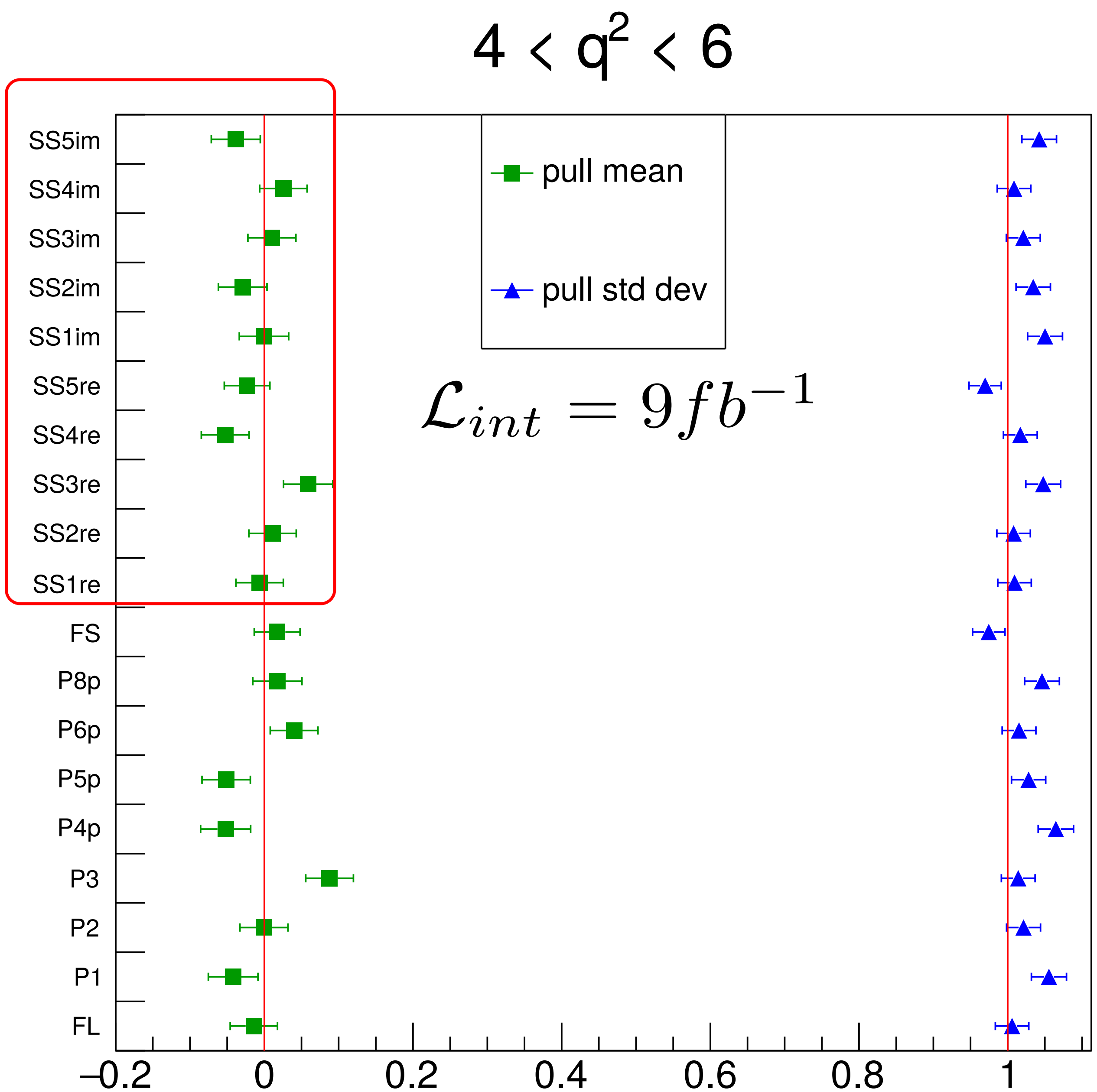


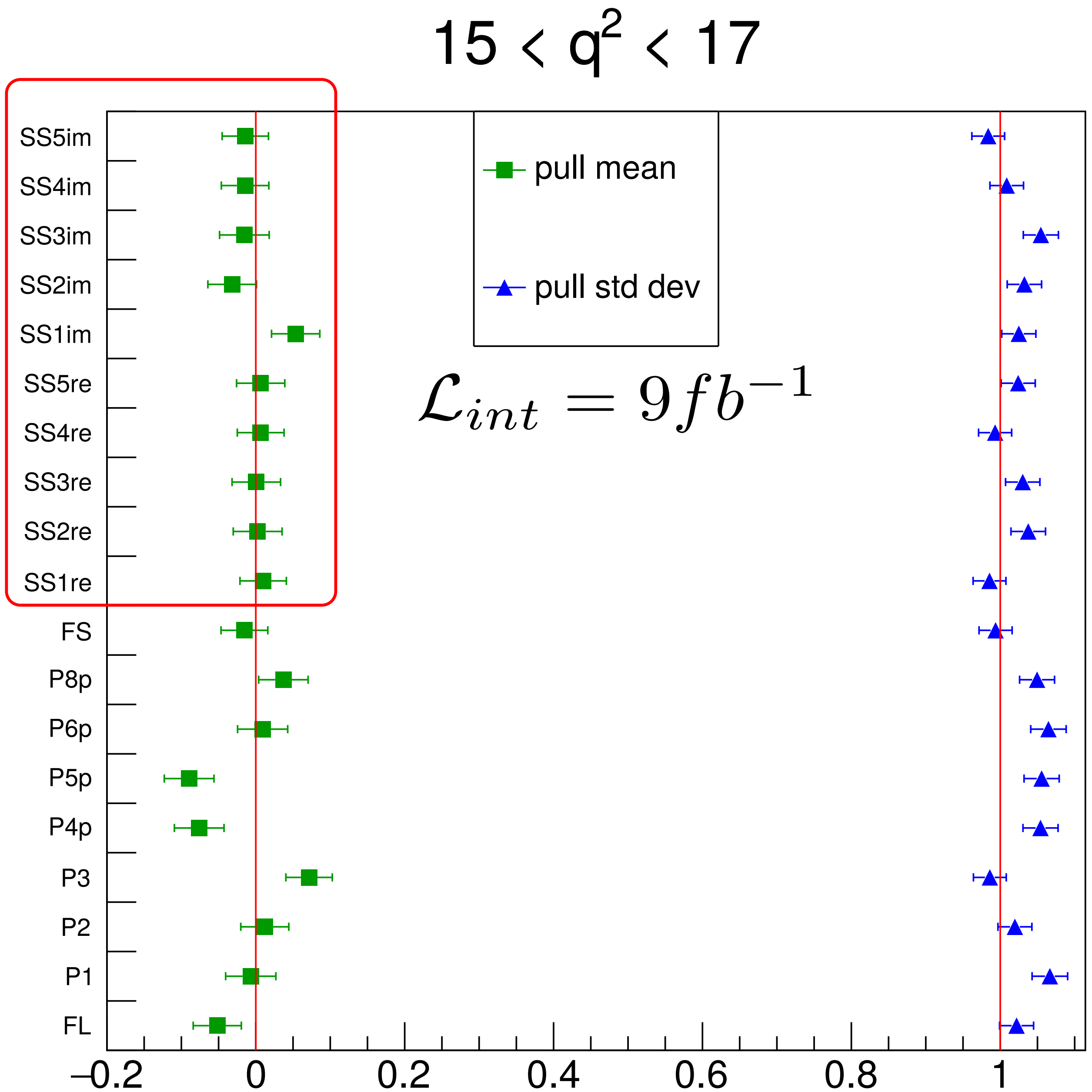
Run 1000 experiments for the various it configurations

- In all cases the fit converges reliably
- We examine biases and error estimation

Massless interference observables

Can we fit the $\mathscr{R}$ and $\mathscr{I}$ part of SS_i ?





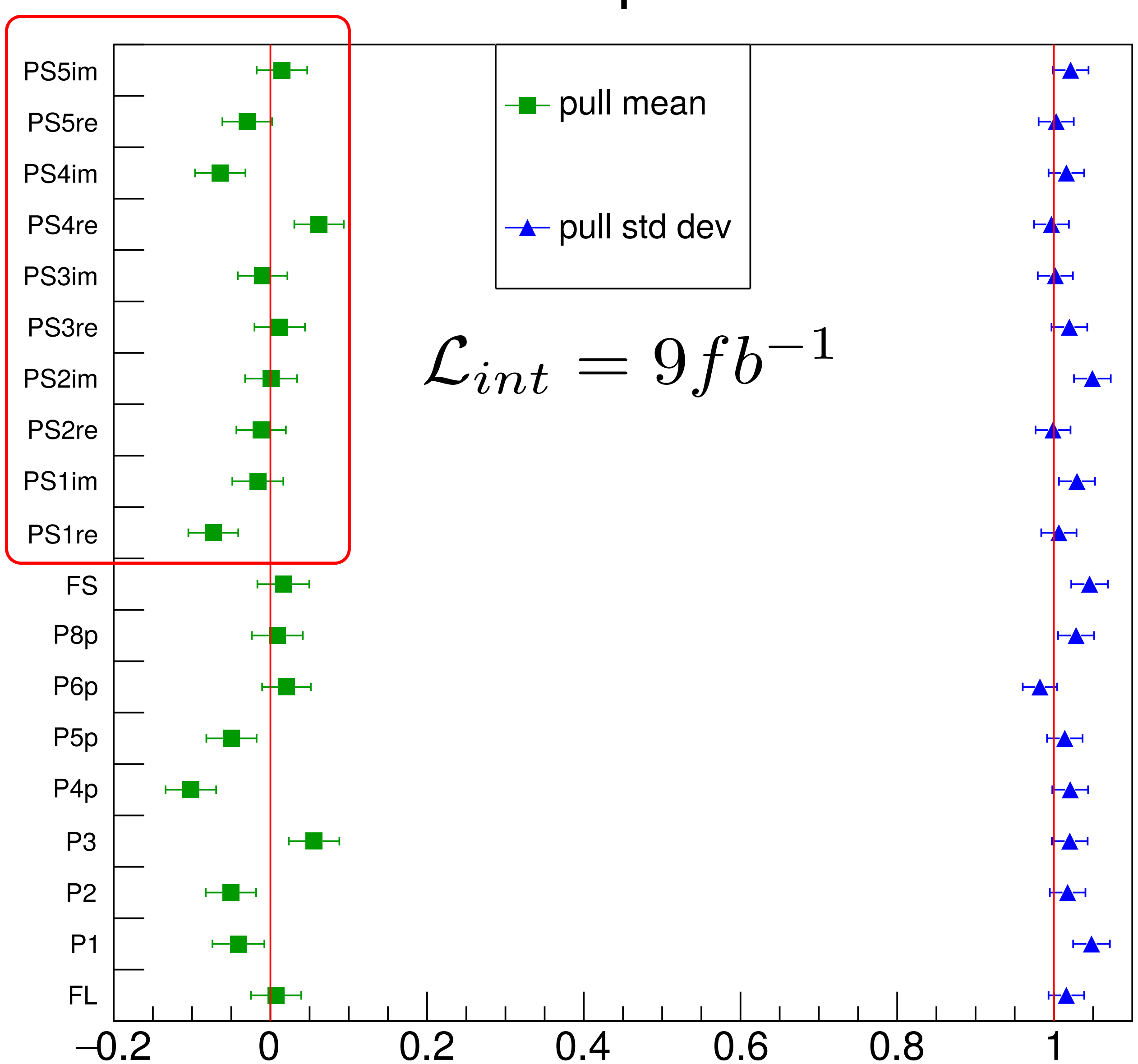
Massless interference observables

Can we fit the optimised observables?

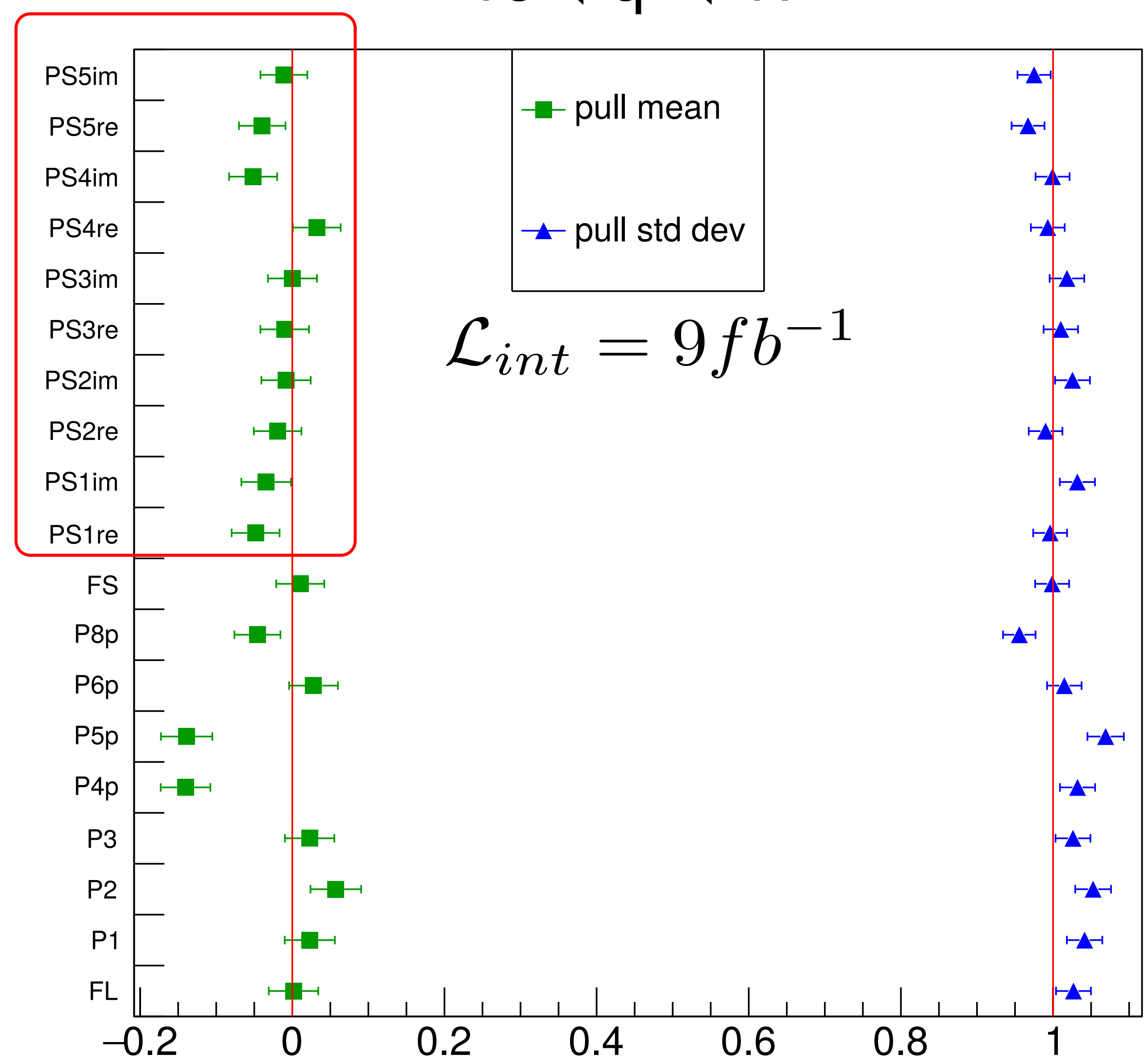
$$PS^{\mathrm{re(im)}}_{S1} = \frac{S^{\mathrm{re(im)}}_{S1}}{\sqrt{\left(1 - 2\tilde{J}^c_{1a} + \frac{2}{3}\tilde{J}^c_{2a}\right) J_{2c}\tilde{J}^c_{2a}}}$$

$$PS^{\mathrm{re(im)}}_{S2-S5} = \frac{S^{\mathrm{re(im)}}_{S2-S5}}{\sqrt{-\left(1 - 2\tilde{J}^c_{1a} + \frac{2}{3}\tilde{J}^c_{2a}\right) J_{2s}\tilde{J}^c_{2a}}}$$

$4 < q^2 < 6$

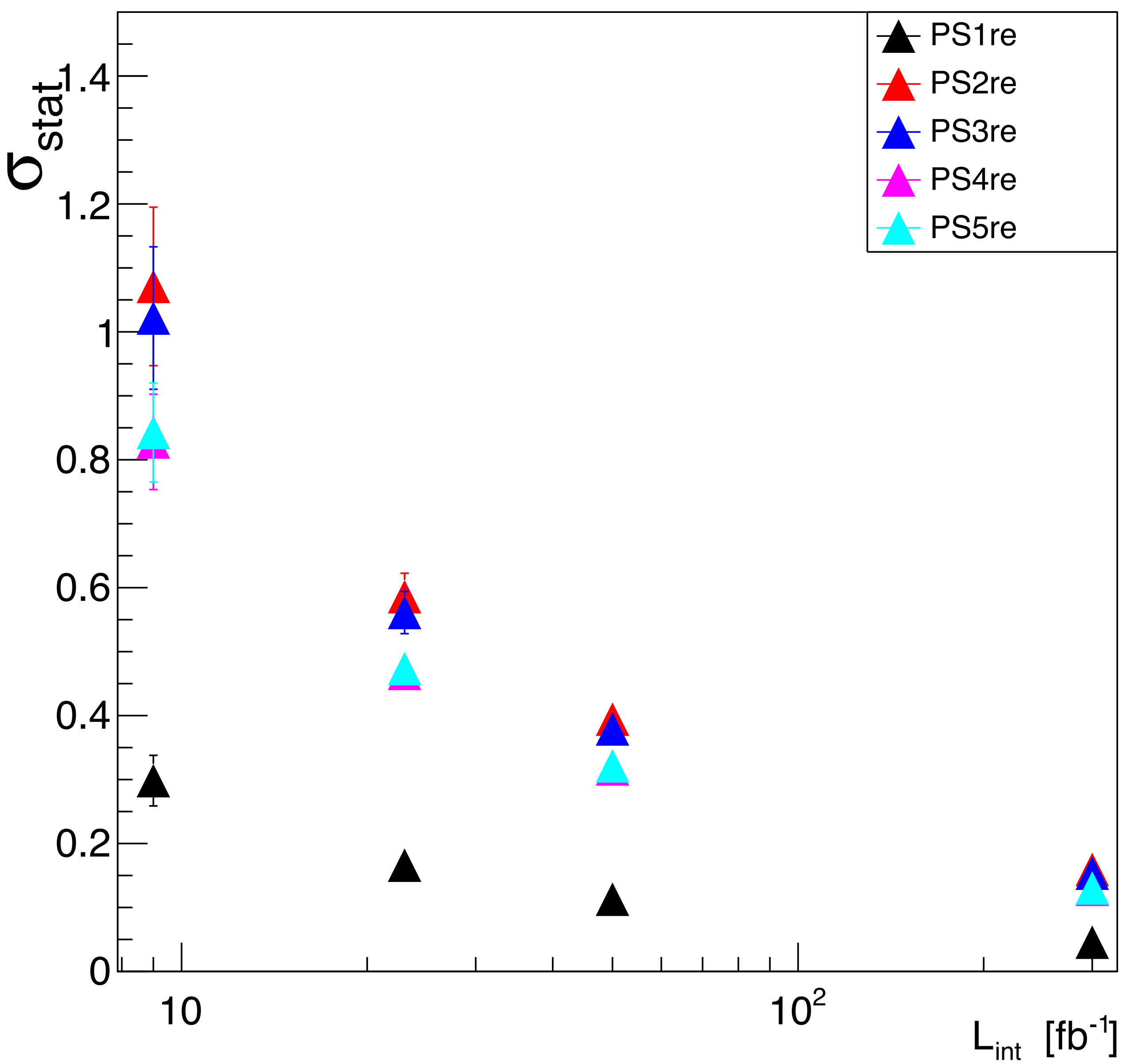


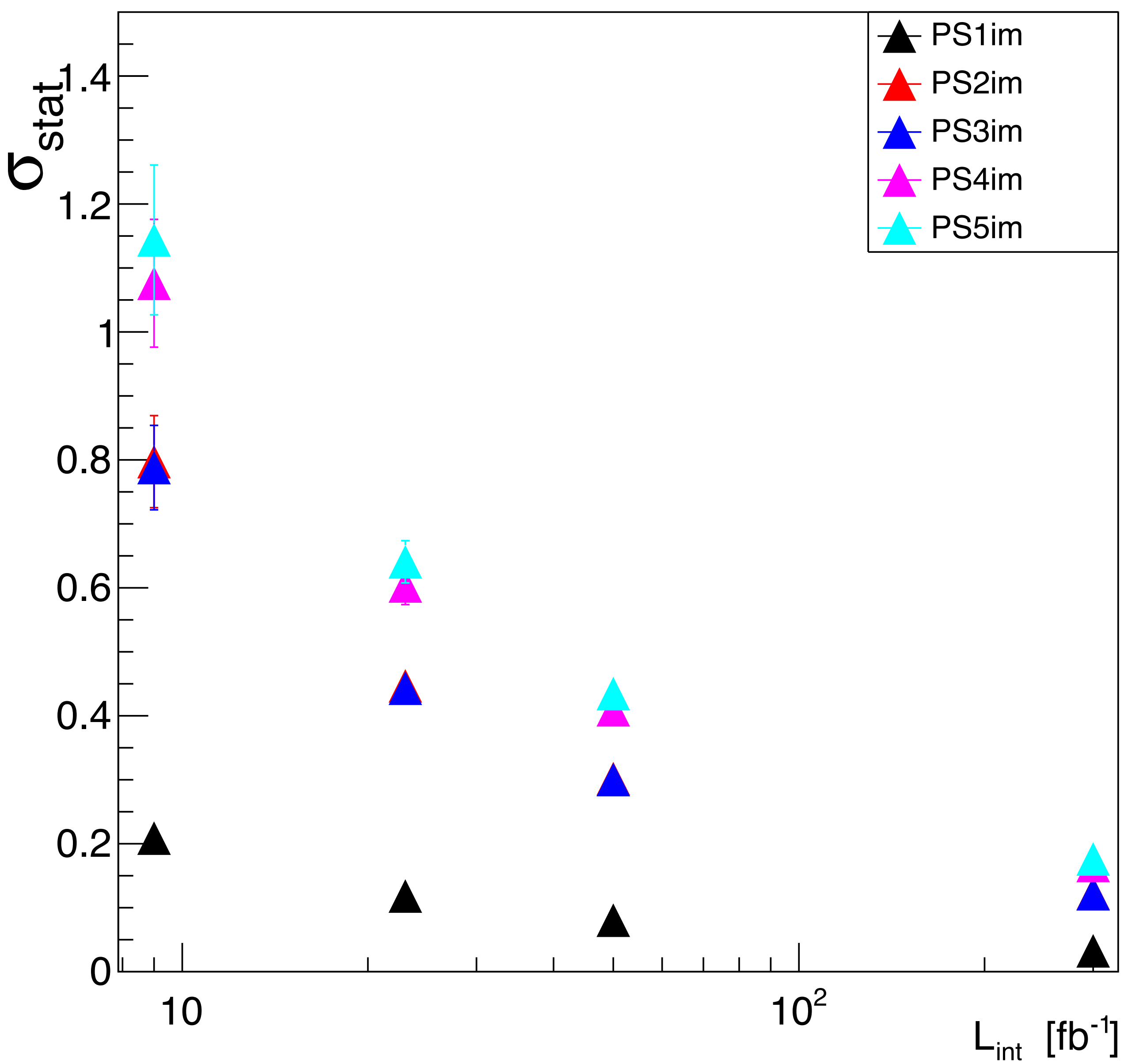
$15 < q^2 < 17$



Massless interference observables

Estimated precisions for $4.0 < q^2 < 6.0\,\mathrm{GeV}^2$

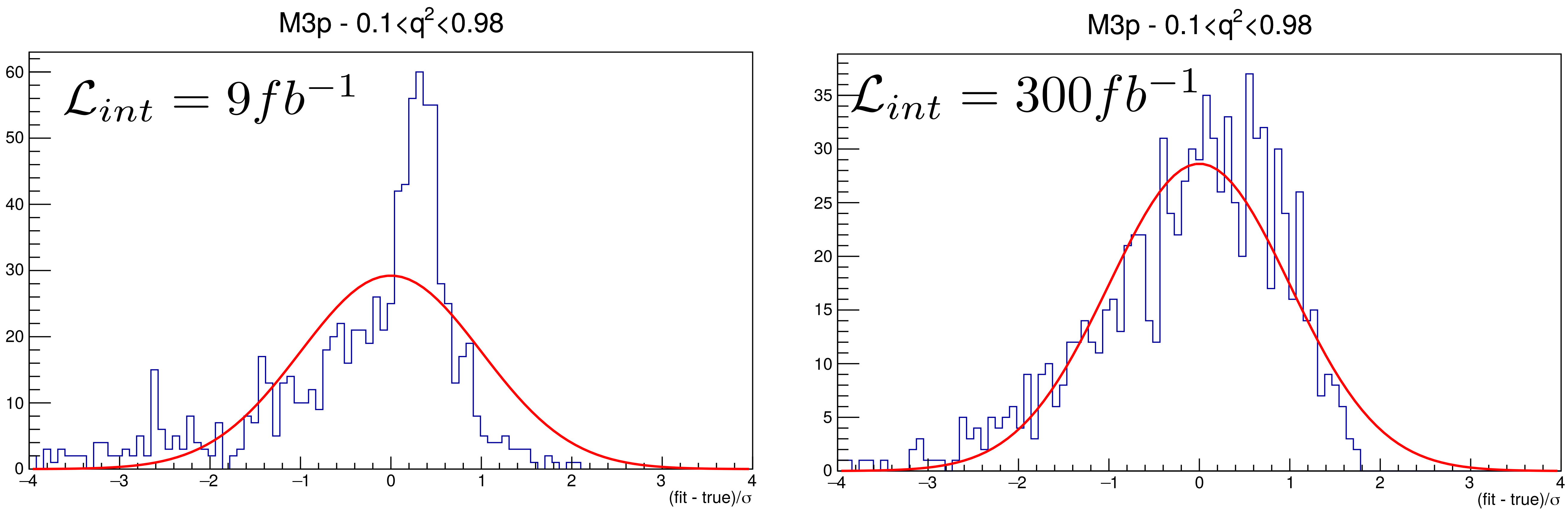




Massive observables: S-wave

$$M_3' = \frac{-\beta^2 \tilde{S}^c_{1a} - \tilde{S}^c_{2a}}{\tilde{S}^c_{2a}}$$

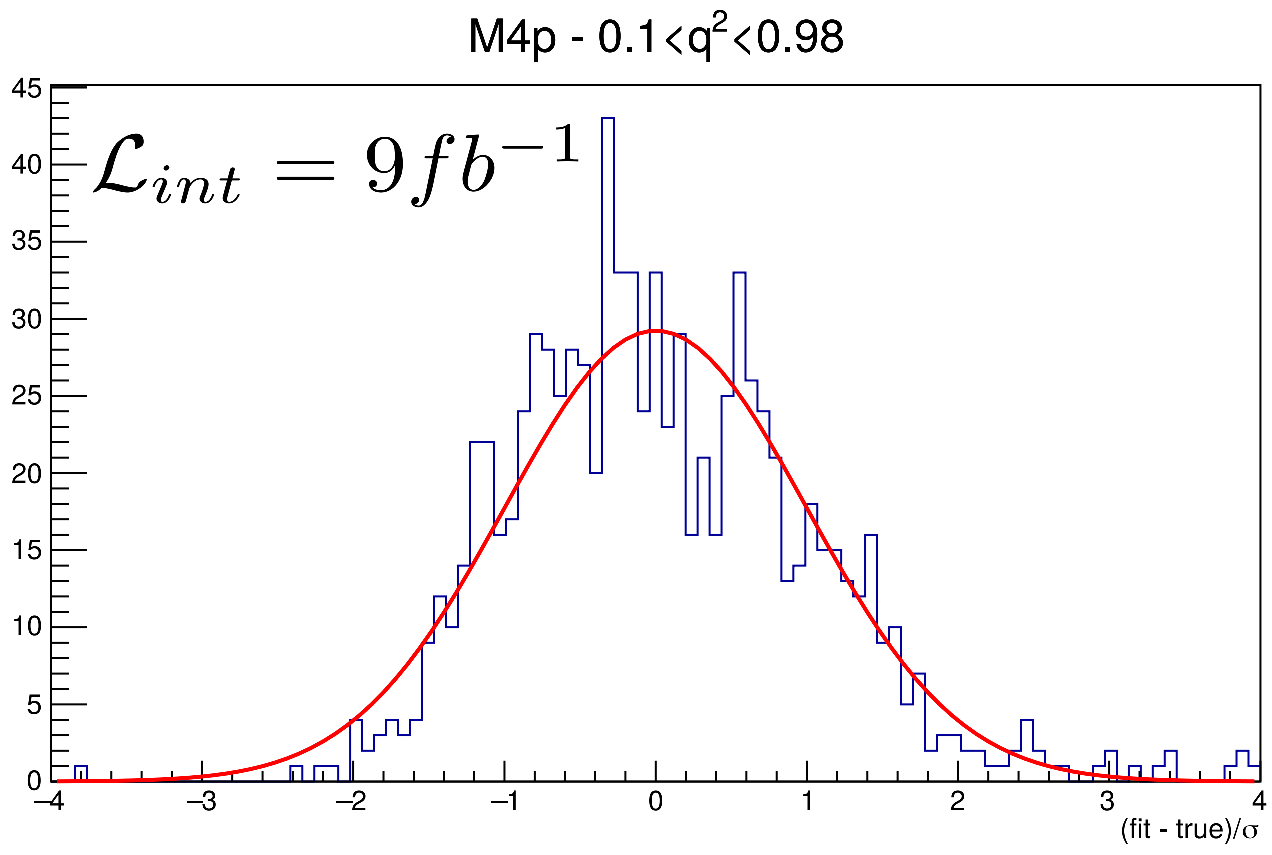
- Essentially a ratio - not ideal experimentally
- Not feasible with Run 2 statistics; still struggle with estimated Run 5, $300\,\mathrm{fb}^{-1}$

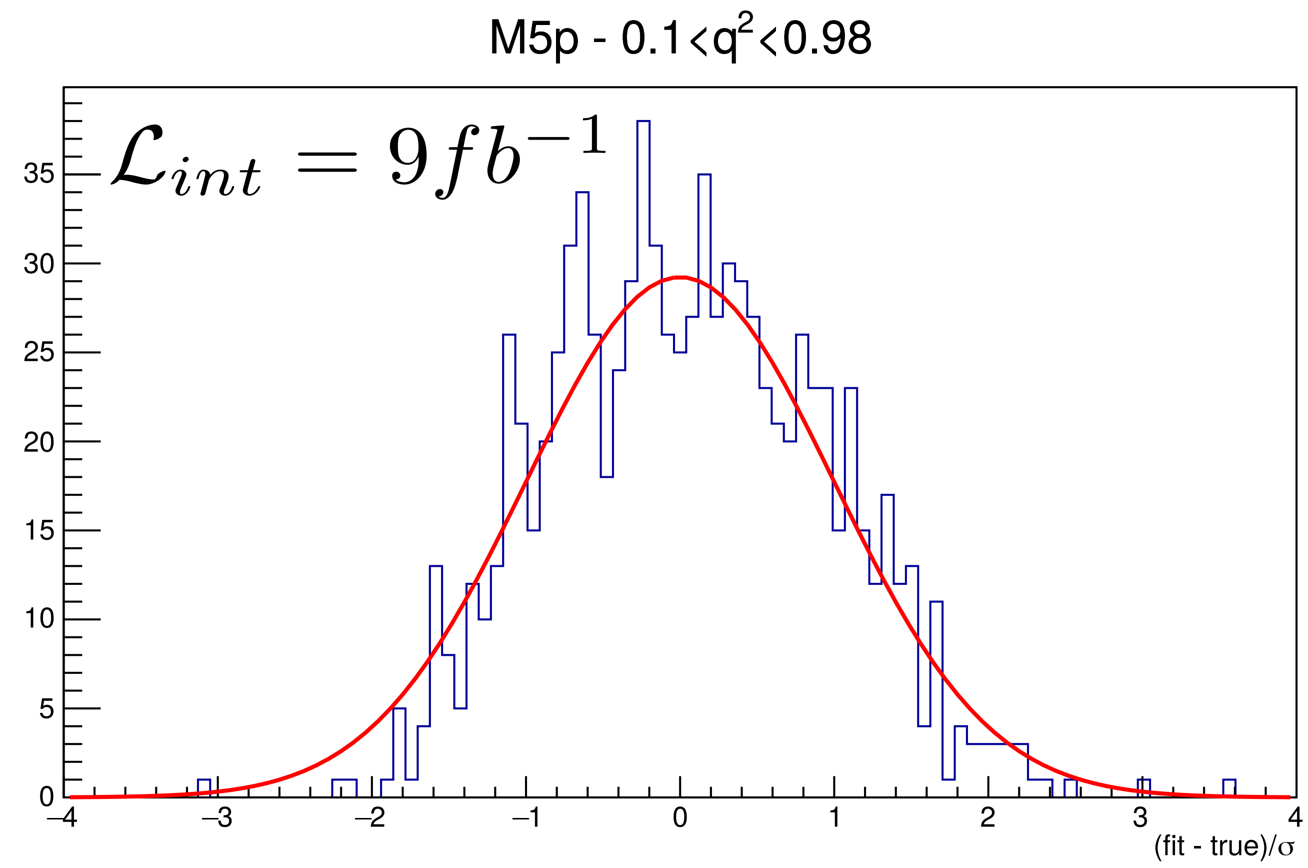


Massive observables: S-wave

$$M_4' = \frac{-\beta^2 \tilde{S}_{1b}^{c,re} - \tilde{S}_{2b}^{c,re}}{\sqrt{(1 - 2\tilde{S}_{1a}^{c} + \frac{2}{3}\tilde{S}_{2a}^{c})S_{2c}\tilde{S}_{2a}^{c}}} \qquad M_5' = \frac{-\beta^2 \tilde{S}_{1b}^{c,im} - \tilde{S}_{2b}^{c,im}}{\sqrt{(1 - 2\tilde{S}_{1a}^{c} + \frac{2}{3}\tilde{S}_{2a}^{c})S_{2c}\tilde{S}_{2a}^{c}}}$$

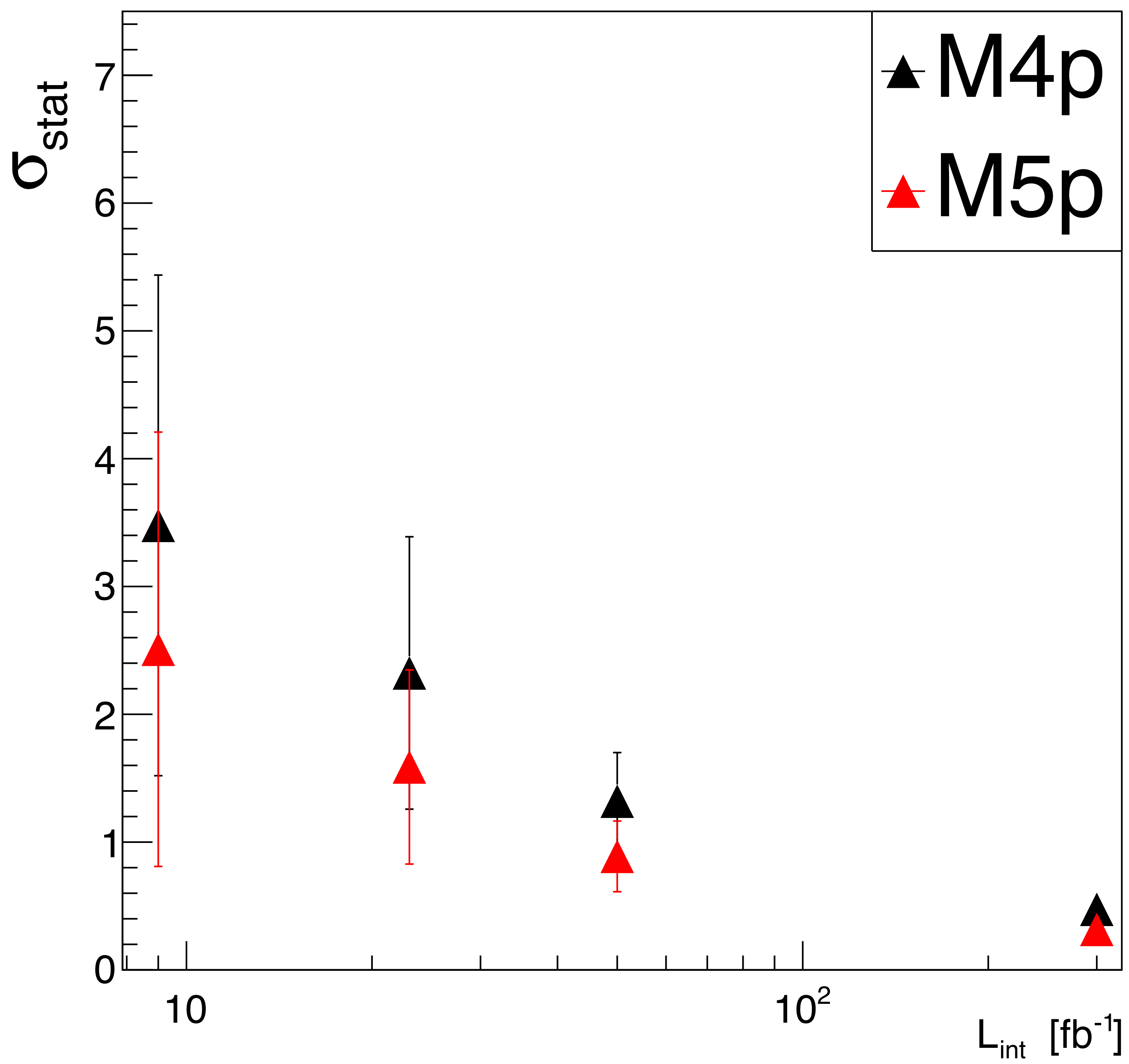
- Constructed more like the P' P-wave optimised observables
- Could be feasibly with Run 2 statistics





Massive observables: S-wave

$$M'_4 = \frac{-\beta^2 \tilde{S}^{c,re}_{1b} - \tilde{S}^{c,re}_{2b}}{\sqrt{(1 - 2\tilde{S}^c_{1a} + \frac{2}{3}\tilde{S}^c_{2a})S_{2c}\tilde{S}^c_{2a}}} \qquad M'_5 = \frac{-\beta^2 \tilde{S}^{c,im}_{1b} - \tilde{S}^{c,im}_{2b}}{\sqrt{(1 - 2\tilde{S}^c_{1a} + \frac{2}{3}\tilde{S}^c_{2a})S_{2c}\tilde{S}^c_{2a}}}$$



Summary

Theory

- Interference observables S_{Si} were not considered and for a fit to decay rate they have to be included
- Splitting P-S wave interference terms in Re & Im allows building new observables
- P-S wave interference obs provide more precise determination of position of zero of observables X_i, Y_i, Z_i from correlated analysis
- Local analysis of bin [1.8,2.5] can help disentangle NP scenarios from Global fits

Experiment

- With the data in hand LHCb can measure all the P/S-wave interference observables
- Can also measure the theory optimised PS_i in all q^2 bins
- For the lowest q^2 bin ($0.1 < q^2 < 0.98\,\mathrm{GeV}^2$) the extra optimised interference observables, M'_4, M'_5 might be measured with the current
  - Certainly feasible with Run 3 and Run 4
- To fit the remaining optimised S-wave observable M'_4 will likely require Run 5 statistics

Thanks!

Thanks!

BACKUP SLIDES

Structure of $B \to K\pi\ell^+\ell^-$ decay

Definitions of $J_i, \tilde{J}_i$:

P-wave

$$J_{1s} = \frac{(2+\beta_\ell^2)}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right] + \frac{4m_\ell^2}{q^2}\mathrm{Re}\left(A_\perp^L A_\perp^{R*} + A_\parallel^L A_\parallel^{R*}\right),$$

$$J_{1c} = |A_0^L|^2 + |A_0^R|^2 + \frac{4m_\ell^2}{q^2}\left[|A_t|^2 + 2\mathrm{Re}(A_0^L A_0^{R*})\right] + \beta_\ell^2\,|A_S|^2\,,$$

$$J_{2s} = \frac{\beta_\ell^2}{4}\left[|A_\perp^L|^2 + |A_\parallel^L|^2 + |A_\perp^R|^2 + |A_\parallel^R|^2\right], \qquad J_{2c} = -\beta_\ell^2\left[|A_0^L|^2 + |A_0^R|^2\right],$$

$$J_3 = \frac{1}{2}\beta_\ell^2\left[|A_\perp^L|^2 - |A_\parallel^L|^2 + |A_\perp^R|^2 - |A_\parallel^R|^2\right], \qquad J_4 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Re}(A_0^L A_\parallel^{L*} + A_0^R A_\parallel^{R*})\right],$$

$$J_5 = \sqrt{2}\beta_\ell\left[\mathrm{Re}(A_0^L A_\perp^{L*} - A_0^R A_\perp^{R*}) - \frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_\parallel^L A_S^* + A_\parallel^{R*} A_S)\right],$$

$$J_{6s} = 2\beta_\ell\left[\mathrm{Re}(A_\parallel^L A_\perp^{L*} - A_\parallel^R A_\perp^{R*})\right], \qquad J_{6c} = 4\beta_\ell\frac{m_\ell}{\sqrt{q^2}}\mathrm{Re}(A_0^L A_S^* + A_0^{R*} A_S)\,,$$

$$J_7 = \sqrt{2}\beta_\ell\left[\mathrm{Im}(A_0^L A_\parallel^{L*} - A_0^R A_\parallel^{R*}) + \frac{m_\ell}{\sqrt{q^2}}\mathrm{Im}(A_\perp^L A_S^* - A_\perp^{R*} A_S))\right],$$

$$J_8 = \frac{1}{\sqrt{2}}\beta_\ell^2\left[\mathrm{Im}(A_0^L A_\perp^{L*} + A_0^R A_\perp^{R*})\right], \qquad J_9 = \beta_\ell^2\left[\mathrm{Im}(A_\parallel^{L*} A_\perp^L + A_\parallel^{R*} A_\perp^R)\right]$$

Structure of $B \to K\pi\ell^+\ell^-$ decay

Definitions of $J_i, \tilde{J}_i$:

S-wave

$$\tilde{J}_{1a}^c = \frac{3}{8}\left[|A_0'^L|^2 + |A_0'^R|^2 + (1-\beta^2)\left(|A_t'|^2 + 2\mathrm{Re}\left[A_0'^L A_0'^{R*}\right]\right)\right]|BW_S|^2\,,$$

$$\tilde{J}_{2a}^c = -\frac{3}{8}\beta^2\left(|A_0'^L|^2 + |A_0'^R|^2\right)|BW_S|^2 = -\frac{3}{8}\beta^2|n_S|^2\,,$$

$$\begin{aligned}\tilde{J}_{1b}^c &= \frac{3}{4}\sqrt{3}\mathrm{Re}\left[\left(A_0'^L A_0^{L*} + A_0'^R A_0^{R*} + (1-\beta^2)\left(A_0'^L A_0^{R*} + A_0^L A_0'^{R*} + A_t' A_t^*\right)\right)BW_S BW_P^*\right]\\ &= \tilde{J}_{1b}^{c\,r}\mathrm{Re}(BW_S BW_P^*) - \tilde{J}_{1b}^{c\,i}\mathrm{Im}(BW_S BW_P^*)\end{aligned}$$

$$\begin{aligned}\tilde{J}_{2b}^c &= -\frac{3}{4}\sqrt{3}\beta^2\mathrm{Re}\left[\left(A_0'^L A_0^{L*} + A_0'^R A_0^{R*}\right)BW_S BW_P^*\right]\\ &= \tilde{J}_{2b}^{c\,r}\mathrm{Re}(BW_S BW_P^*) - \tilde{J}_{2b}^{c\,i}\mathrm{Im}(BW_S BW_P^*)\\ &= -\frac{3}{4}\sqrt{3}\beta^2[\mathrm{Re}(n_0^\dagger n_S)\mathrm{Re}(BW_S BW_P^*) - \mathrm{Im}(n_0^\dagger n_S')\mathrm{Im}(BW_S BW_P^*)]\,,\end{aligned}$$

$$\tilde{J}_4 = \frac{3}{4}\sqrt{\frac{3}{2}}\beta^2\mathrm{Re}\left[(A_0'^L A_\parallel^{L*} + A_0'^R A_\parallel^{R*})BW_S BW_P^*\right] = \frac{3}{4}\sqrt{\frac{3}{2}}\beta^2[\mathrm{Re}(n_\parallel^\dagger n_S)\mathrm{Re}(BW_S BW_P^*) - \mathrm{Im}(n_\parallel^\dagger n_S')\mathrm{Im}(BW_S BW_P^*)\,,$$

$$\tilde{J}_5 = \frac{3}{2}\sqrt{\frac{3}{2}}\beta\mathrm{Re}\left[(A_0'^L A_\perp^{L*} - A_0'^R A_\perp^{R*})BW_S BW_P^*\right] = \frac{3}{2}\sqrt{\frac{3}{2}}\beta[\mathrm{Re}(n_\perp^\dagger n_S)\mathrm{Re}(BW_S BW_P^*) - \mathrm{Im}(n_\perp^\dagger n_S')\mathrm{Im}(BW_S BW_P^*)\,,$$

$$\tilde{J}_7 = \frac{3}{2}\sqrt{\frac{3}{2}}\beta\mathrm{Im}\left[(A_0'^L A_\parallel^{L*} - A_0'^R A_\parallel^{R*})BW_S BW_P^*\right] = \frac{3}{2}\sqrt{\frac{3}{2}}\beta[\mathrm{Im}(n_\parallel^\dagger n_S)\mathrm{Re}(BW_S BW_P^*) + \mathrm{Re}(n_\parallel^\dagger n_S')\mathrm{Im}(BW_S BW_P^*)]\,,$$

$$\tilde{J}_8 = \frac{3}{4}\sqrt{\frac{3}{2}}\beta^2\mathrm{Im}\left[(A_0'^L A_\perp^{L*} + A_0'^R A_\perp^{R*})BW_S BW_P^*\right] = \frac{3}{4}\sqrt{\frac{3}{2}}\beta^2[\mathrm{Im}(n_\perp^\dagger n_S)\mathrm{Re}(BW_S BW_P^*) + \mathrm{Re}(n_\perp^\dagger n_S')\mathrm{Im}(BW_S BW_P^*)$$

Structure of $B \to K\pi\ell^+\ell^-$ decay

Definitions of $J_i, \tilde{J}_i$:

S-wave

$$\tilde{J}^c_{1a} = \frac{3}{8}\left[|A_0'^L|^2 + |A_0'^R|^2 + (1-\beta^2)\left(|A_t'|^2 + 2\mathrm{Re}\left[A_0'^L A_0'^{R*}\right]\right)\right]|BW_S|^2\,,$$

$$\tilde{J}^c_{2a} = -\frac{3}{8}\beta^2\left(|A_0'^L|^2 + |A_0'^R|^2\right)|BW_S|^2 = -\frac{3}{8}\beta^2|n_S|^2\,,$$

$$\begin{aligned}\tilde{J}^c_{1b} &= \frac{3}{4}\sqrt{3}\mathrm{Re}\left[\left(A_0'^L A_0^{L*} + A_0'^R A_0^{R*} + (1-\beta^2)\left(A_0'^L A_0^{R*} + A_0^L A_0'^{R*} + A_t' A_t^*\right)\right) BW_S BW_P^*\right] \\ &= \tilde{J}^{c\,r}_{1b}\mathrm{Re}(BW_S BW_P^*) - \tilde{J}^{c\,i}_{1b}\mathrm{Im}(BW_S BW_P^*)\end{aligned}$$

$$\begin{aligned}\tilde{J}^c_{2b} &= -\frac{3}{4}\sqrt{3}\beta^2\mathrm{Re}\left[\left(A_0'^L A_0^{L*} + A_0'^R A_0^{R*}\right) BW_S BW_P^*\right] \\ &= \tilde{J}^{c\,r}_{2b}\mathrm{Re}(BW_S BW_P^*) - \tilde{J}^{c\,i}_{2b}\mathrm{Im}(BW_S BW_P^*) \\ &= -\frac{3}{4}\sqrt{3}\beta^2[\mathrm{Re}(n_0^\dagger n_S)\mathrm{Re}(BW_S BW_P^*) - \mathrm{Im}(n_0^\dagger n_S')\mathrm{Im}(BW_S BW_P^*)]\,,\end{aligned}$$

$$\tilde{J}_4 = \frac{3}{4}\sqrt{\frac{3}{2}}\beta^2\mathrm{Re}\left[\left(A_0'^L A_\parallel^{L*} + A_0'^R A_\parallel^{R*}\right) BW_S BW_P^*\right] = \frac{3}{4}\sqrt{\frac{3}{2}}\beta^2[\mathrm{Re}(n_\parallel^\dagger n_S)\mathrm{Re}(BW_S BW_P^*) - \mathrm{Im}(n_\parallel^\dagger n_S')\mathrm{Im}(BW_S BW_P^*)\,,$$

$$\tilde{J}_5 = \frac{3}{2}\sqrt{\frac{3}{2}}\beta\mathrm{Re}\left[\left(A_0'^L A_\perp^{L*} - A_0'^R A_\perp^{R*}\right) BW_S BW_P^*\right] = \frac{3}{2}\sqrt{\frac{3}{2}}\beta[\mathrm{Re}(n_\perp^\dagger n_S)\mathrm{Re}(BW_S BW_P^*) - \mathrm{Im}(n_\perp^\dagger n_S')\mathrm{Im}(BW_S BW_P^*)\,,$$

$$\tilde{J}_7 = \frac{3}{2}\sqrt{\frac{3}{2}}\beta\mathrm{Im}\left[\left(A_0'^L A_\parallel^{L*} - A_0'^R A_\parallel^{R*}\right) BW_S BW_P^*\right] = \frac{3}{2}\sqrt{\frac{3}{2}}\beta[\mathrm{Im}(n_\parallel^\dagger n_S)\mathrm{Re}(BW_S BW_P^*) + \mathrm{Re}(n_\parallel^\dagger n_S')\mathrm{Im}(BW_S BW_P^*)]\,,$$

$$\tilde{J}_8 = \frac{3}{4}\sqrt{\frac{3}{2}}\beta^2\mathrm{Im}\left[\left(A_0'^L A_\perp^{L*} + A_0'^R A_\perp^{R*}\right) BW_S BW_P^*\right] = \frac{3}{4}\sqrt{\frac{3}{2}}\beta^2[\mathrm{Im}(n_\perp^\dagger n_S)\mathrm{Re}(BW_S BW_P^*) + \mathrm{Re}(n_\perp^\dagger n_S')\mathrm{Im}(BW_S BW_P^*)$$

$BW_S BW_P^*$ introduces mixing S-P wave

d.o.f. and new relations among observables

$$n_c - n_{rel} = 2n_A - n_{sym}$$

Massless case $(m_\ell = 0)$

	P-wave	S-wave
nº coeffs	11	12
trivial rel.	2	1
non-trivial rel.	1	2 + $R^{new}_{m_\ell=0}$
nº amplitudes	8 x 2	
nº symmetries	2	

2 symmetries broken by new $S_i^{r,i}$ coefficients

$$n_c - n_{rel} = 23 - 6 - R^{new}_{m_\ell=0} \stackrel{!}{=} 2n_A - n_{sym} = 14$$

3 new relations!

d.o.f. and new relations among observables

Massive case $(m_\ell \neq 0)$

$$n_c - n_{rel} = 2n_A - n_{sym}$$

	P-wave	S-wave
nº coeffs	11	14
trivial rel.	0	0
non-trivial rel.	1	2 + $R^{new}_{m_\ell=0}$ + $R^{new}_{m_\ell\neq 0}$
nº amplitudes	10 x 2	
nº symmetries	2	

$(R^{new}_{m_\ell=0} = 3)$

1 symmetry from L, R phases
+ 1 extra symmetry $A_t^{(\prime)}$

$n_c - n_{rel}$ = 25 - 6 - $R^{new}_{m_\ell\neq 0}$ $\stackrel{!}{=}$ $2n_A - n_{sym}$ = 18

1 extra relation!

Anatomy of M_1, M_2

$$M_1 = \frac{J_{1s}}{3J_{2s}}$$ → M_1 insensitive to contributions of New Physics

$$M_2 = -\frac{J_{1c}}{J_{2c}}$$ → M_2 potentially sensitive to pseudoscalar (C_P) & scalar (C_S) NP contributions but very insensitive to $C_7^{\rm NP}, C_{9\mu}^{\rm NP}$

$\mathrm{BR}(B_s \to \mu\mu)$ used to constrain possible values of C_P, C_S

[Fleischer et al. arXiv: 1703.10160]

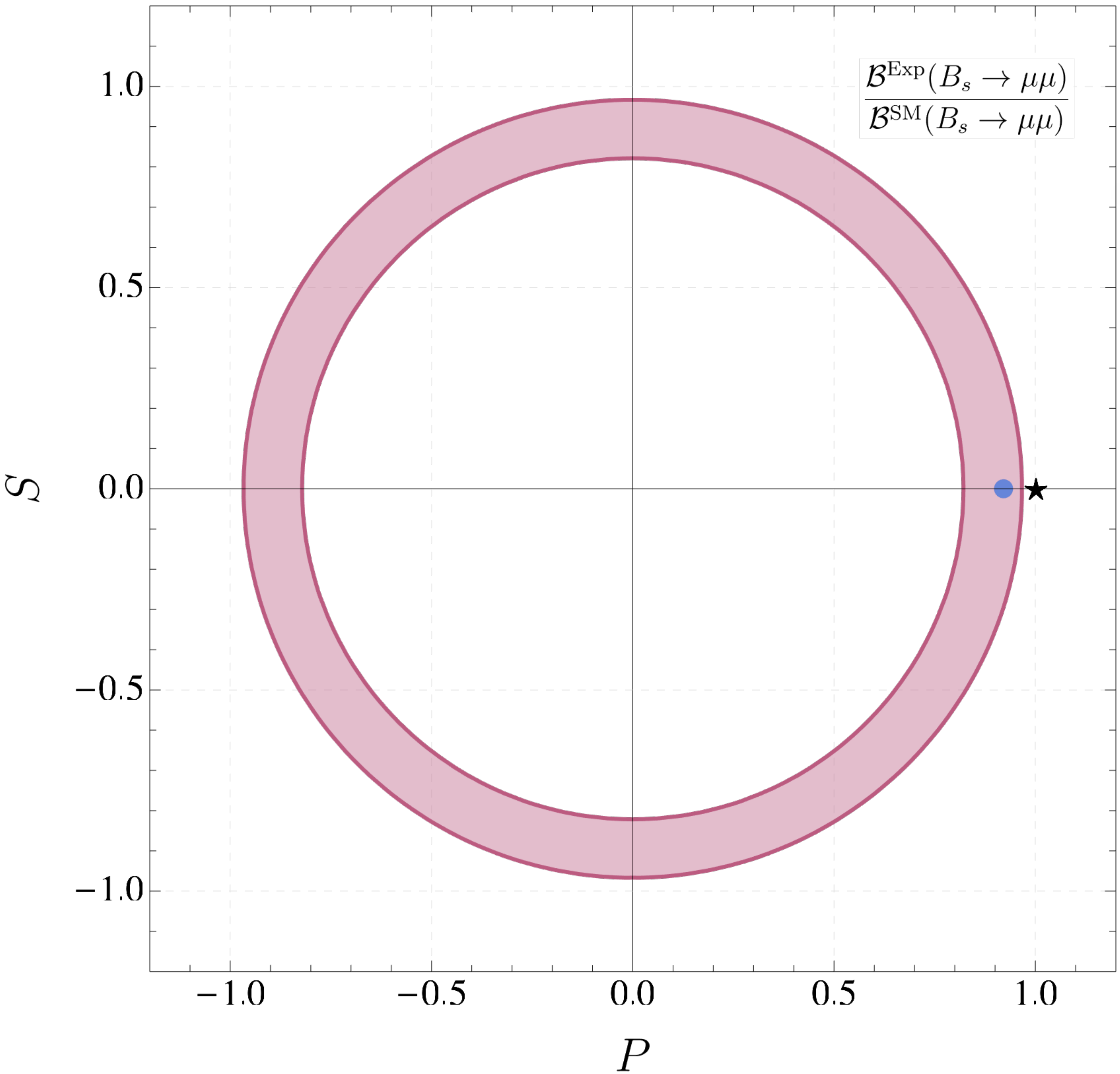
From $R_{B_s\to\mu\mu} = \frac{\mathrm{BR}(B_s\to\mu\mu)}{\mathrm{BR}^{\rm SM}(B_s\to\mu\mu)} = |S|^2 + |P|^2$, where

$$S = \sqrt{1 - 4\frac{m_\mu^2}{m_{B_s}^2}}\,\frac{m_{B_s}^2}{2m_b m_\mu}\left(\frac{C_S - C_{S'}}{C_{10}^{\rm SM}}\right)$$

$$P = \frac{C_{10}^{\rm SM} + C_{10}^{\rm NP} - C_{10'}}{C_{10}^{\rm SM}} + \frac{m_{B_s}^2}{2m_b m_\mu}\left(\frac{C_P - C_{P'}}{C_{10}^{\rm SM}}\right)$$

We take the experimental value of $R_{B_s\to\mu\mu}$ to determine a 1σ region for S, P

Constraints on S, P



$$\frac{\mathrm{BR}^{\rm Exp}(B_s \to \mu\mu)}{\mathrm{BR}^{\rm SM}(B_s \to \mu\mu)} = 0.80 \pm 0.13$$

1σ

★ SM

● $\mathcal{C}^{\rm V}_{10\mu} = 0.34$

$$C_{10'} = C_{P'} = C_{S'} = 0$$

Constraints for C_P, C_S

Same constraint but in terms of WC C_S, C_P