$\Lambda_Q \to \Lambda^{(*)}$ form factors

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b and c baryon semileptonic form factors from unquenched lattice QCD

Transition	m_Q	a [fm]	m_π [MeV]	Reference
$\Lambda_b \to \Lambda$	∞	0.083, 0.111	230–360	WD, DL, SM, MW, arXiv:1212.4827/PRD 2013
$\Lambda_b \to p$	∞	0.083, 0.111	230–360	WD, DL, SM, MW, arXiv:1306.0446/PRD 2013
$\Lambda_b \to p$	phys.	0.083, 0.111	230–360	WD, CL, SM, arXiv:1503.01421/PRD 2015
$\Lambda_b \to \Lambda_c$	phys.	0.083, 0.111	230–360	WD, CL, SM, arXiv:1503.01421/PRD 2015
$\Lambda_b \to \Lambda$	phys.	0.083, 0.111	230–360	WD, SM, arXiv:1602.01399/PRD 2016
$\Lambda_b \to \Lambda^*(1520)$	phys.	0.083, 0.111	300–430	SM, GR, arXiv:2009.09313/PRD 2021
$\Lambda_b \to \Lambda_c^*(2595)$	phys.	0.083, 0.111	300–430	SM, GR, arXiv:2103.08775/PRD 2021
$\Lambda_b \to \Lambda_c^*(2625)$	phys.	0.083, 0.111	300–430	SM, GR, arXiv:2103.08775/PRD 2021
$\Lambda_c \to \Lambda$	phys.	0.083, 0.111, 0.114	**140**–360	SM, arXiv:1611.09696/PRL 2017
$\Lambda_c \to p$	phys.	0.083, 0.111	230–360	SM, arXiv:1712.05783/PRD 2018
$\Xi_c \to \Xi$	phys.	0.080, 0.108	290, 300	Q.-A. Zhang *et al.*, arXiv:2103.07064
$\Lambda_c \to \Lambda^*(1520)$	phys.	0.083, 0.111	300–430	SM, GR, in preparation

WD = W. Detmold, DL = C.-J. D. Lin, SM = S. Meinel, MW = M. Wingate, CL = C. Lehner, GR = G. Rendon

1. $\Lambda_b \to \Lambda$
2. $\Lambda_b \to \Lambda^*(1520)$
3. $\Lambda_c \to \Lambda^*(1520)$

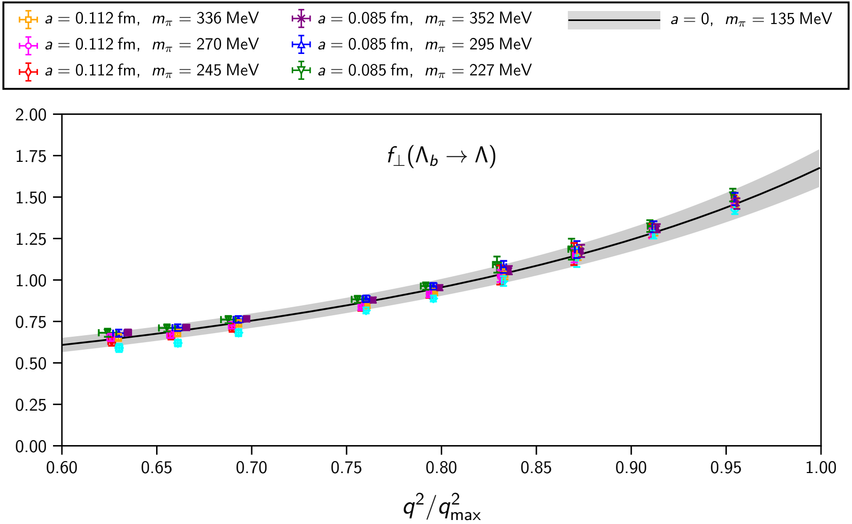
$\Lambda_b \to \Lambda$ form factor definitions

$$
\begin{aligned}
\langle \Lambda | \bar{s}\gamma^\mu b | \Lambda_b \rangle &= \bar{u}_\Lambda \Big[f_0(q^2)\,(m_{\Lambda_b} - m_\Lambda)\frac{q^\mu}{q^2} \\
&\quad + f_+(q^2)\frac{m_{\Lambda_b}+m_\Lambda}{s_+}\left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_\Lambda^2)\frac{q^\mu}{q^2}\right) \\
&\quad + f_\perp(q^2)\left(\gamma^\mu - \frac{2m_\Lambda}{s_+}p^\mu - \frac{2m_{\Lambda_b}}{s_+}p'^\mu\right)\Big] u_{\Lambda_b}, \\
\langle \Lambda | \bar{s}\gamma^\mu \gamma_5 b | \Lambda_b \rangle &= -\bar{u}_\Lambda\,\gamma_5 \Big[g_0(q^2)\,(m_{\Lambda_b} + m_\Lambda)\frac{q^\mu}{q^2} \\
&\quad + g_+(q^2)\frac{m_{\Lambda_b}-m_\Lambda}{s_-}\left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_\Lambda^2)\frac{q^\mu}{q^2}\right) \\
&\quad + g_\perp(q^2)\left(\gamma^\mu + \frac{2m_\Lambda}{s_-}p^\mu - \frac{2m_{\Lambda_b}}{s_-}p'^\mu\right)\Big] u_{\Lambda_b}, \\
iq_\nu \langle \Lambda | \bar{s}\sigma^{\mu\nu} b | \Lambda_b \rangle &= -\bar{u}_\Lambda \Big[h_+(q^2)\frac{q^2}{s_+}\left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_\Lambda^2)\frac{q^\mu}{q^2}\right) \\
&\quad + h_\perp(q^2)\,(m_{\Lambda_b} + m_\Lambda)\left(\gamma^\mu - \frac{2m_\Lambda}{s_+}p^\mu - \frac{2m_{\Lambda_b}}{s_+}p'^\mu\right)\Big] u_{\Lambda_b}, \\
iq_\nu \langle \Lambda | \bar{s}\sigma^{\mu\nu}\gamma_5 b | \Lambda_b \rangle &= -\bar{u}_\Lambda\,\gamma_5 \Big[\widetilde{h}_+(q^2)\frac{q^2}{s_-}\left(p^\mu + p'^\mu - (m_{\Lambda_b}^2 - m_\Lambda^2)\frac{q^\mu}{q^2}\right) \\
&\quad + \widetilde{h}_\perp(q^2)\,(m_{\Lambda_b} - m_\Lambda)\left(\gamma^\mu + \frac{2m_\Lambda}{s_-}p^\mu - \frac{2m_{\Lambda_b}}{s_-}p'^\mu\right)\Big] u_{\Lambda_b},
\end{aligned}
$$

where $s_\pm = (m_{\Lambda_b} \pm m_\Lambda)^2 - q^2$

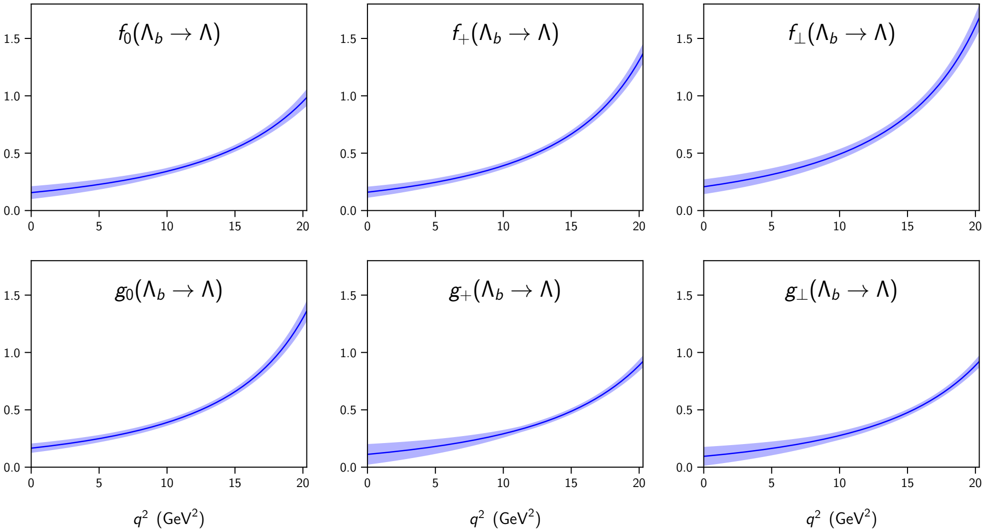
[T. Feldmann and M. W. Y. Yip, arXiv:1111.1844/PRD 2012]

$\Lambda_b \to \Lambda$ form factors from lattice QCD



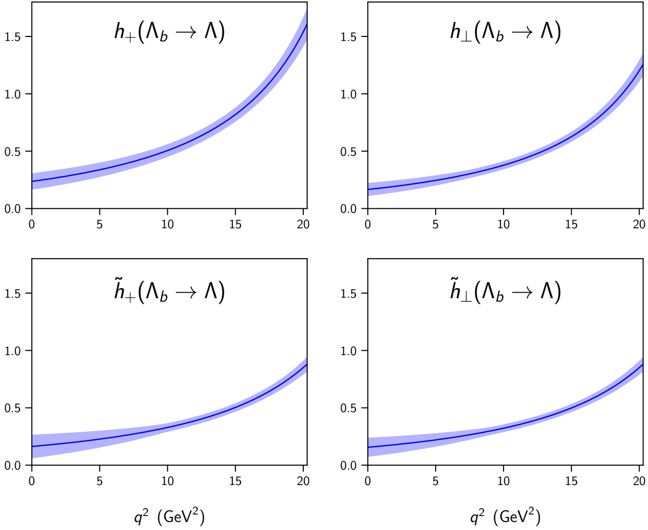
[W. Detmold and S. Meinel, arXiv:1602.01399/PRD 2016]

$\Lambda_b \to \Lambda$ form factors from lattice QCD



[W. Detmold and S. Meinel, arXiv:1602.01399/PRD 2016]

$\Lambda_b \to \Lambda$ form factors from lattice QCD



[W. Detmold and S. Meinel, arXiv:1602.01399/PRD 2016]

Forthcoming improved calculation of $\Lambda_b \to p, \Lambda, \Lambda_c$ form factors

- Remove data sets with $m_{u,d}^{(\mathrm{val})} < m_{u,d}^{(\mathrm{sea})}$, add three new ensembles to better control finite-volume effects, chiral and continuum extrapolations
- For $\Lambda_b \to \Lambda$: physical $m_s^{(\mathrm{val})}$
- More accurate tuning of charm and bottom actions
- All-mode-averaging for higher statistics
- Better source smearing
- Fully nonperturbative renormalization

$N_s^3 \times N_t$	β	$am_{u,d}^{(\mathrm{sea})}$	$am_{u,d}^{(\mathrm{val})}$	$am_s^{(\mathrm{sea})}$	a (fm)	$m_\pi^{(\mathrm{sea})}$ (MeV)	$m_\pi^{(\mathrm{val})}$ (MeV)	Status
$32^3 \times 64$	2.13	0.005	0.005	0.04	≈ 0.111	≈ 340	≈ 340	done
$24^3 \times 64$	2.13	0.005	0.005	0.04	≈ 0.111	≈ 340	≈ 340	done
~~$24^3 \times 64$~~	~~2.13~~	~~0.005~~	~~0.002~~	~~0.04~~	~~≈ 0.111~~	~~≈ 340~~	~~≈ 270~~	
~~$24^3 \times 64$~~	~~2.13~~	~~0.005~~	~~0.001~~	~~0.04~~	~~≈ 0.111~~	~~≈ 340~~	~~≈ 250~~	
$48^3 \times 96$	2.13	0.00078	0.00078	0.0362	≈ 0.114	≈ 140	≈ 140	done
$32^3 \times 64$	2.25	0.006	0.006	0.03	≈ 0.083	≈ 360	≈ 360	done
$32^3 \times 64$	2.25	0.004	0.004	0.03	≈ 0.083	≈ 300	≈ 300	done
~~$32^3 \times 64$~~	~~2.25~~	~~0.004~~	~~0.002~~	~~0.03~~	~~≈ 0.083~~	~~≈ 300~~	~~≈ 230~~	
$48^3 \times 96$	2.31	0.002144	0.002144	0.02144	≈ 0.073	≈ 230	≈ 230	$\sim$ 30% done

1. $\Lambda_b \to \Lambda$
2. $\Lambda_b \to \Lambda^*(1520)$
3. $\Lambda_c \to \Lambda^*(1520)$

$\Lambda_b \to \Lambda^*(1520)$

- The $\Lambda^*(1520)$ is the lightest isospin-0, $J^P = \frac{3}{2}^-$ strange baryon resonance. It has a mass of 1519.5 ± 1.0 MeV, a width of 15.6 ± 1.0 MeV, and decays mainly into $N\bar{K}$, $\Sigma\pi$, or $\Lambda\pi\pi$.

- The phenomenology of $\Lambda_b \to \Lambda^*(1520)\mu^+\mu^-$ was recently studied in
  - S. Descotes-Genon, M. Novoa-Brunet, `arXiv:1903.00448`/JHEP 2019
  - D. Das, J. Das, `arXiv:2003.08366`/JHEP 2020
  - Y. Amhis *et al.*, `arXiv:2005.09602` [see Carla's part of the talk]

- The form factors have previously been calculated in a quark model
  - L. Mott, W. Roberts, `arXiv:1108.6129`/IJMPA 2012

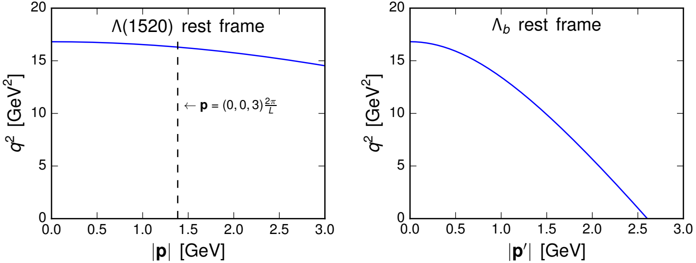
- HQET relations including $1/m_b$ effects have been derived in
  - W. Roberts, NPB **389**, 549–562 (1993)
  - M. Bordone, `arXiv:2101.12028`/Symmetry 2021

$\Lambda_b \to \Lambda^*(1520)$ on the lattice

S. Meinel and G. Rendon, arXiv:2009.09313/PRD 2021

We use the narrow-width approximation: we assume that the lowest finite-volume energy level with the correct quantum numbers corresponds to the $\Lambda^*(1520)$. Even in this approximation, the calculation is substantially more challenging than for $\Lambda_b \to \Lambda$:

- At nonzero momenta, the irreducible representations of the lattice symmetry groups mix positive and negative parities and also mix $J = \frac{1}{2}$ and $J = \frac{3}{2}$. We must therefore work in the $\Lambda^*(1520)$ rest frame and give momentum to the Λ_b instead. This limits us to near $q^2_{\rm max}$.



$\Lambda_b \to \Lambda^*(1520)$ on the lattice

- The simplest choices of three-quark interpolating fields with $I = 0$ and $J^P = \frac{3}{2}^-$ dominantly couple to higher-lying ($S = 3/2$, $L = 0$, flavor-$SU(3)$ octet) states. We found it necessary to use an interpolating field with an ($S = 1/2$, $L = 1$, flavor-$SU(3)$ singlet) structure obtained using covariant spatial derivatives. This requires additional quark propagators with derivative sources.

- Correlation functions for negative-parity "excited" baryons have even more statistical noise than correlation functions for the lightest baryons $\to$ need many samples on many gauge configurations

Data sets and hadron masses

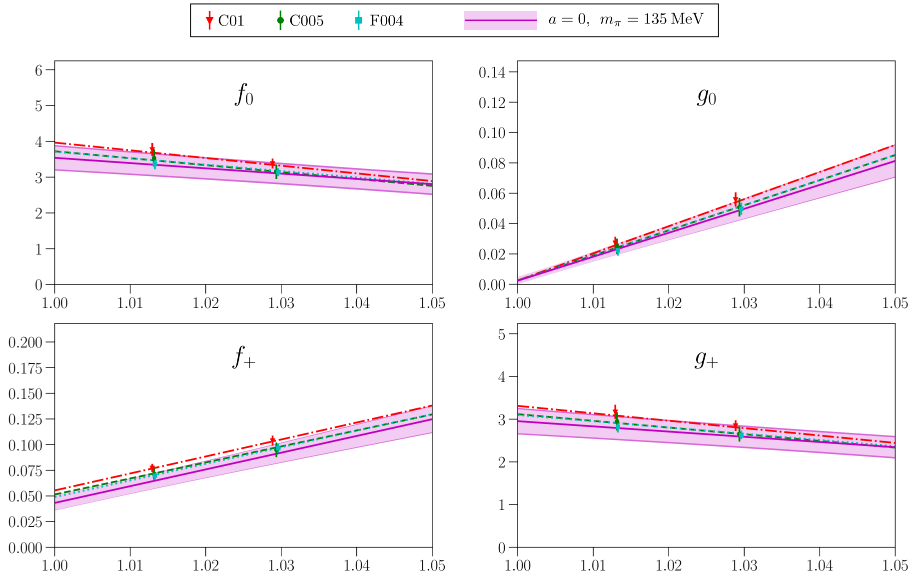
We use gauge-field configurations generated by the RBC and UKQCD Collaborations, with $2+1$ flavors of domain-wall fermions.

Label	$N_s^3 \times N_t$	a [fm]	m_π [GeV]
C01	$24^3 \times 64$	0.1106(3)	0.4312(13)
C005	$24^3 \times 64$	0.1106(3)	0.3400(11)
F004	$32^3 \times 64$	0.0828(3)	0.3030(12)

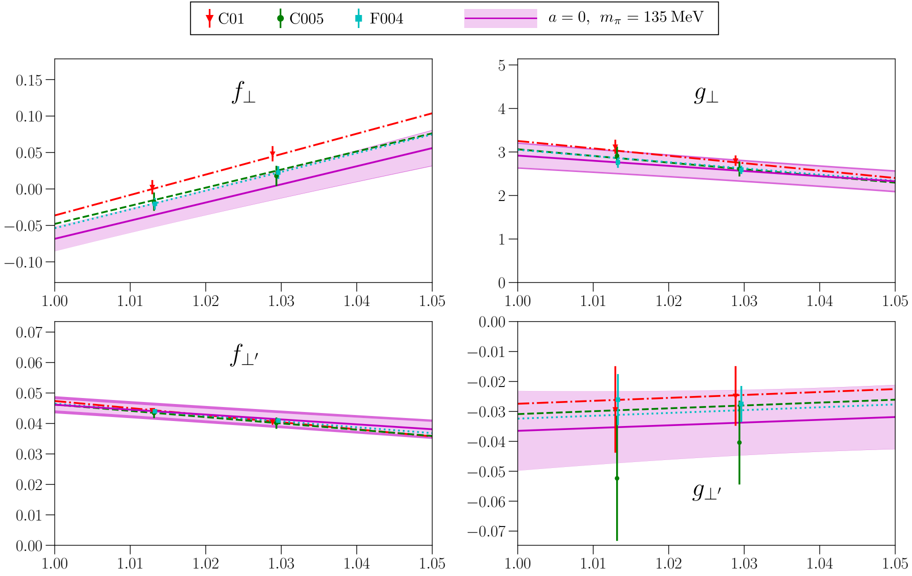
Label	m_K [GeV]	m_N [GeV]	m_Λ [GeV]	m_Σ [GeV]	m_{Λ^*} [GeV]	m_{Λ_b} [GeV]
C01	0.5795(19)	1.2647(51)	1.3494(61)	1.3877(61)	1.825(16)	5.793(17)
C005	0.5501(19)	1.1649(58)	1.2659(66)	1.3173(60)	1.740(17)	5.726(17)
F004	0.5361(24)	1.1197(59)	1.2382(54)	1.303(12)	1.757(15)	5.722(23)

$m_{\Lambda^*} - m_\Sigma - m_\pi$ ranges from approximately $+80$ to $+150$ MeV (physical value: $+192$ MeV),
$m_{\Lambda^*} - m_N - m_K$ ranges from approximately -20 to $+100$ MeV (physical value: $+89$ MeV)

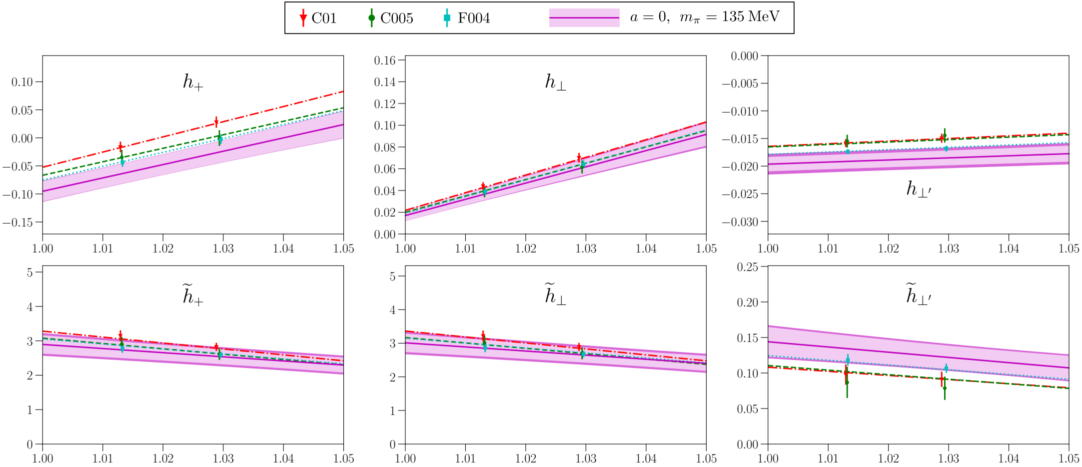
$\Lambda_b \to \Lambda^*(1520)$ form factors as a function of $w = v \cdot v'$



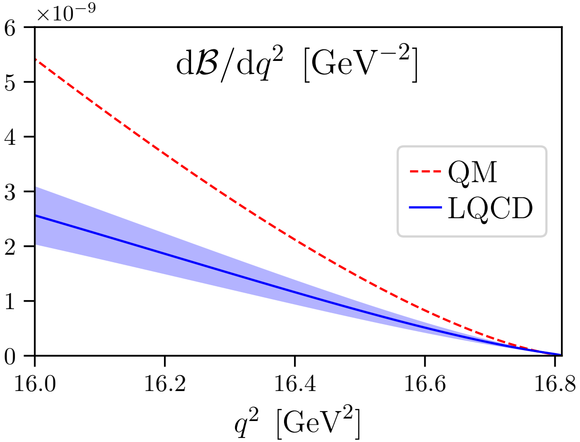
$\Lambda_b \to \Lambda^*(1520)$ form factors as a function of $w = v \cdot v'$



$\Lambda_b \to \Lambda^*(1520)$ form factors as a function of $w = v \cdot v'$

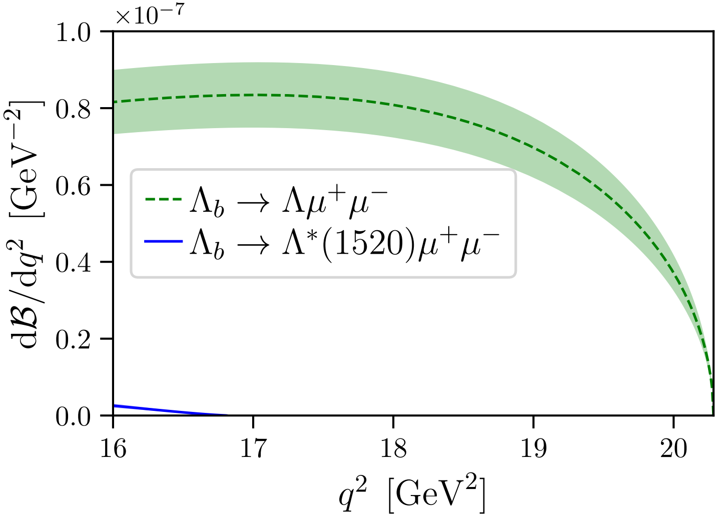


$\Lambda_b \to \Lambda^*(1520)\mu^+\mu^-$ differential branching fraction

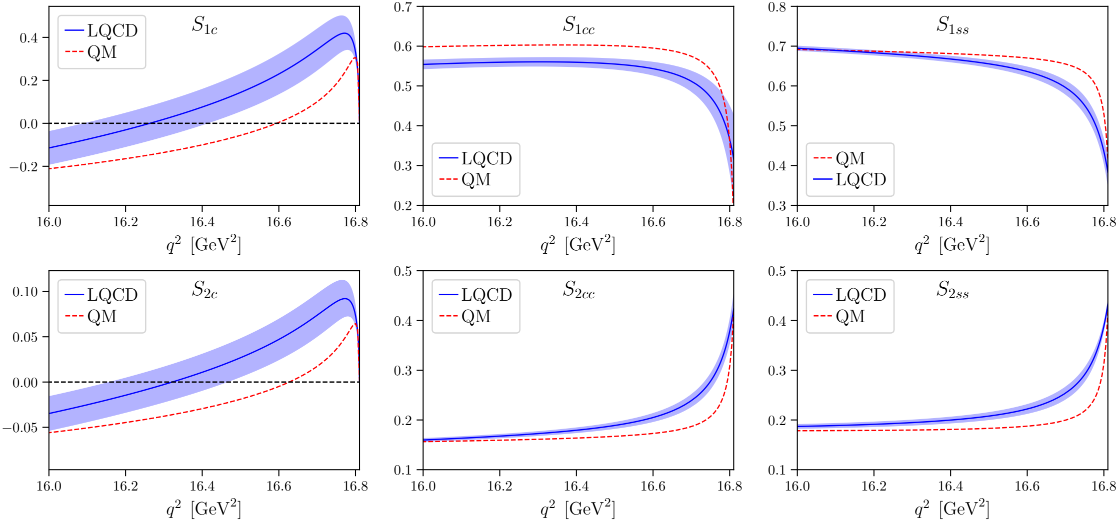


QM = using form factors from [L. Mott, W. Roberts, arXiv:1108.6129/IJMPA 2012]

$\Lambda_b \to \Lambda^*(1520)\mu^+\mu^-$ and $\Lambda_b \to \Lambda\mu^+\mu^-$ differential branching fractions

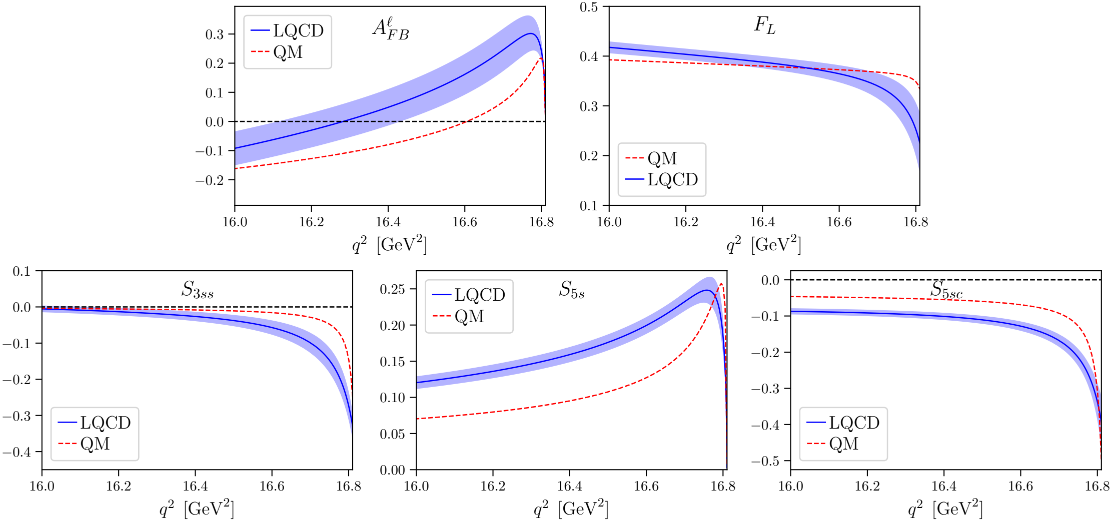


$\Lambda_b \to \Lambda^*(1520)(\to pK^-)\mu^+\mu^-$ angular observables



See [S. Descotes-Genon, M. Novoa-Brunet, arXiv:1903.00448/JHEP 2019] for definitions. The lepton mass is neglected here.

$\Lambda_b \to \Lambda^*(1520)(\to pK^-)\mu^+\mu^-$ angular observables



See [S. Descotes-Genon, M. Novoa-Brunet, arXiv:1903.00448/JHEP 2019] for definitions. The lepton mass is neglected here.

$\Lambda_b \to \Lambda^*(1520)$ form factors in HQET including $1/m_b$ and α_s effects

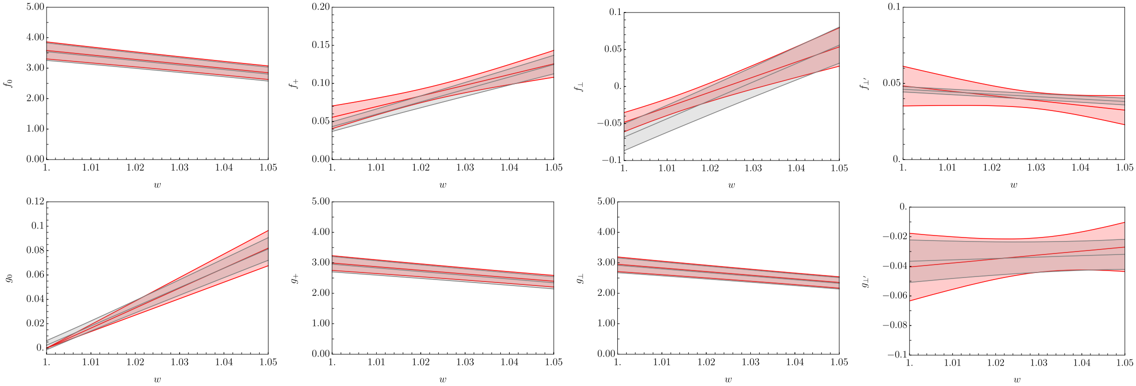
M. Bordone, arXiv:2101.12028/Symmetry 2021

Fit to lattice results for vector and axial-vector form factors

Parameter	Best fit point
$\zeta_1^{(0)}$	0.454 ± 0.070
$\zeta_2^{(0)}$	-0.0303 ± 0.0552
$\zeta_1^{(1)} + \zeta_2^{(1)}$	0.113 ± 0.024
$\zeta_1^{\mathrm{SL},(0)}$	0.125 ± 0.038
$\zeta_1^{\mathrm{SL},(1)}$	0.0487 ± 0.0614
$\zeta_2^{\mathrm{SL},(0)}$	0.0110 ± 0.0363
$\zeta_2^{\mathrm{SL},(1)}$	0.00362 ± 0.06184
$\zeta_3^{\mathrm{SL},(0)}$	0.228 ± 0.190
$\zeta_4^{\mathrm{SL},(0)}$	0.0883 ± 0.185
$\zeta_4^{\mathrm{SL},(1)} - \zeta_3^{\mathrm{SL},(1)}$	-0.0267 ± 0.0773

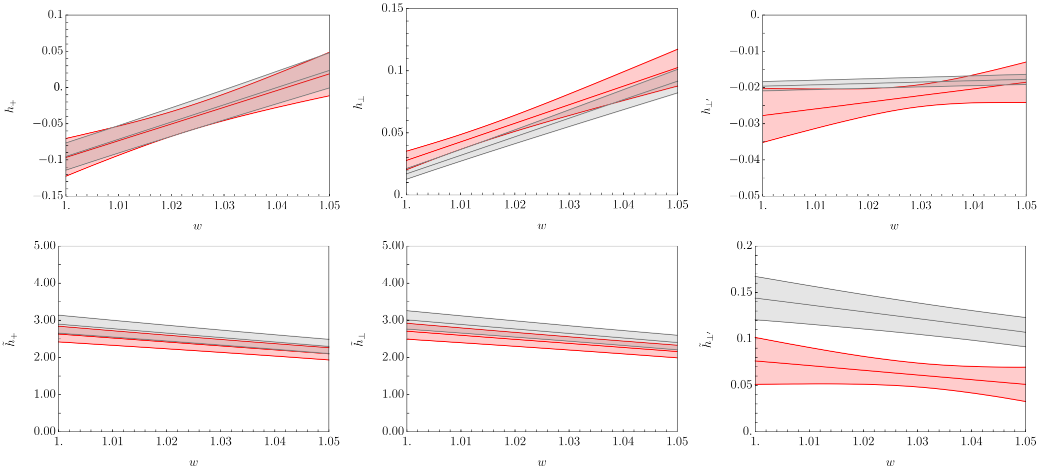
Table 3.1: Best fit points for the HQE parameters.

$\Lambda_b \to \Lambda^*(1520)$ form factors in HQET including $1/m_b$ and α_s effects



Gray = lattice QCD vector and axial-vector form factors, Red = HQET fit

$\Lambda_b \to \Lambda^*(1520)$ form factors in HQET including $1/m_b$ and α_s effects



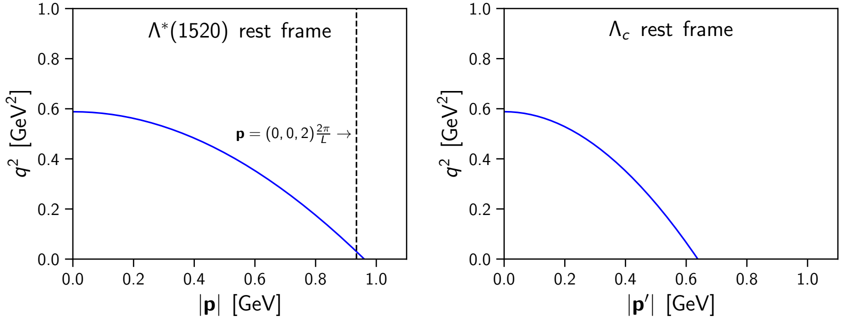
Gray = lattice QCD tensor form factors (not included in the HQET fit), Red = HQET fit

1. $\Lambda_b \to \Lambda$
2. $\Lambda_b \to \Lambda^*(1520)$
3. $\Lambda_c \to \Lambda^*(1520)$

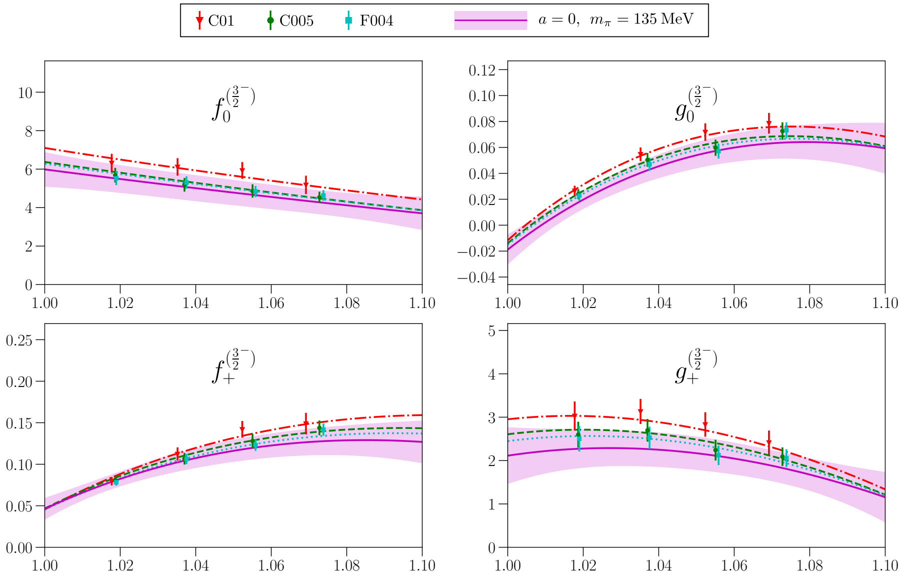
$\Lambda_c \to \Lambda^*(1520)$ form factors from lattice QCD

S. Meinel and G. Rendon, in preparation. The results are preliminary and the analysis of uncertainties is still incomplete.

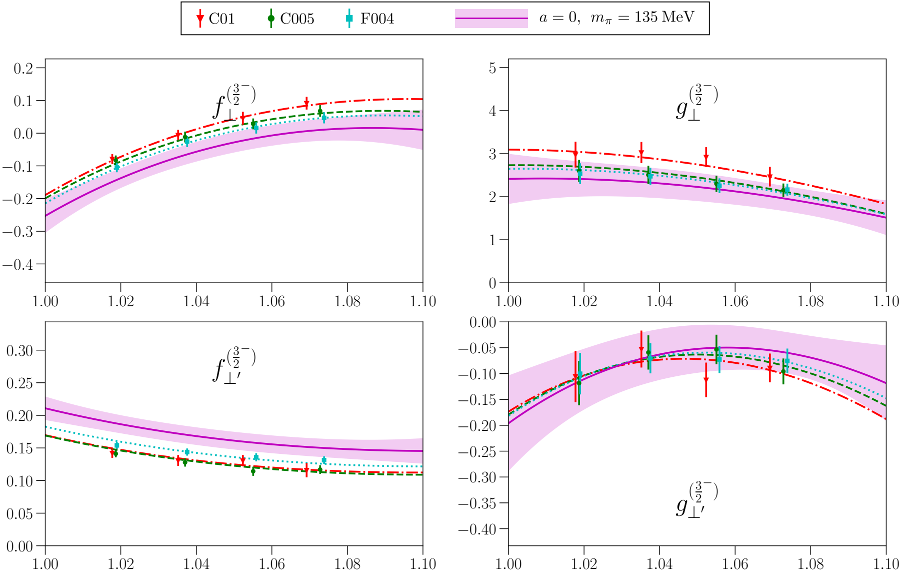
For $\Lambda_c \to \Lambda^*(1520)\ell^+\nu$, we can cover the **full kinematic range** even when working in the $\Lambda^*(1520)$ rest frame! We use four different Λ_c momenta, $\mathbf{p}/(2\pi/L) = (0,0,1),\ (0,1,1),\ (1,1,1),\ (0,0,2)$.



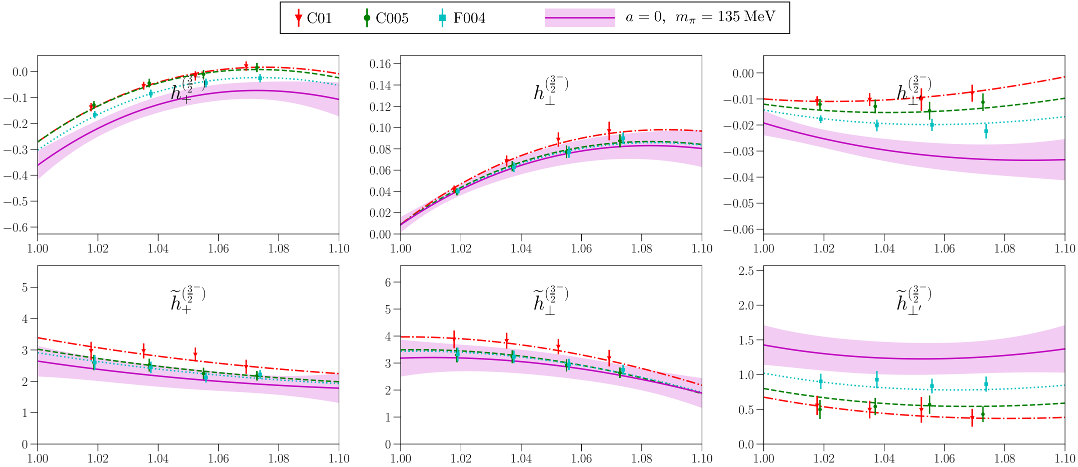
$\Lambda_c \to \Lambda^*(1520)$ form factors as a function of $w = v \cdot v'$



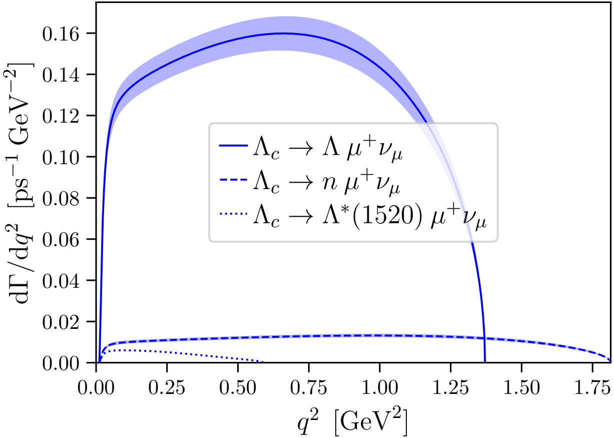
$\Lambda_c \to \Lambda^*(1520)$ form factors as a function of $w = v \cdot v'$



$\Lambda_c \to \Lambda^*(1520)$ form factors as a function of $w = v \cdot v'$



$\Lambda_c \to X\ \mu^+\nu$ differential decay rates predicted by lattice QCD



[$\Lambda_c \to \Lambda^*(1520)$: S. Meinel and G. Rendon, preliminary; uncertainties not yet shown;
$\Lambda_c \to \Lambda$: S. Meinel, arXiv:1611.09696/PRL 2017; $\Lambda_c \to n$: S. Meinel, arXiv:1712.05783/PRD 2018]

Open questions and tasks

$\Lambda_b \to \Lambda \ell^+ \ell^-$ and $\Lambda_b \to \Lambda \gamma$

- An updated measurement of the normalization branching fraction $\mathcal{B}(\Lambda_b \to \Lambda J/\psi)$ is needed. An improved determination of $f(b \to \Lambda_b)$ would also help.
- The $\Lambda_b \to \Lambda \mu^+ \mu^-$ observables at low q^2 should be analyzed using the full LHCb dataset.
- The understanding of nonfactorizable spectator effects at low q^2 needs to be improved.
- $\mathcal{B}(\Lambda_b \to \Lambda \gamma)$ should be studied theoretically using the lattice form factors.
- Higher-precision lattice calculations of the form factors are underway.

Open questions and tasks

$\Lambda_b \to \Lambda^*(1520)\ell^+\ell^-$ and $\Lambda_b \to \Lambda^*(1520)\gamma$

- How well can the $\Lambda^*(1520)$ contribution be isolated from the $\Lambda_b \to pK^-\mu^+\mu^-$ or $\Lambda_b \to pK^-\gamma$ decay distributions, which contain many overlapping Λ^* resonances?
- What is the best way to reach lower q^2 on the lattice? Moving-NRQCD action for the b quark? Or work in Λ_b the rest frame and explicitly deal with the mixing of spin-parity quantum numbers?
- Why does the HQET fit deviate for $\tilde{h}_{\perp'}$?
- Can somewhat lower q^2 for $\Lambda_b \to \Lambda^*(1520)$ be reached using a joint HQET fit to the $\Lambda_b \to \Lambda^*(1520)$ and $\Lambda_c \to \Lambda^*(1520)$ form factors?
- Lattice calculations directly at the physical light-quark masses would be useful (but very expensive) to check the quark-mass extrapolations.
- How useful is the high-q^2 region $15\ \text{GeV}^2 \leq q^2 \leq q^2_{\text{max}} \approx 16.8\ \text{GeV}^2$? Will there be enough events, and is the region wide enough to rely on the OPE treatment of charm-loop effects?
- How accurately can the observables be predicted at low q^2 using SCET/QCDF/LCSR?
- What are the prospects for measuring $\mathcal{B}(\Lambda_c \to \Lambda^*(1520)\mu^+\nu)$ at LHCb, BESIII, Belle II?