Impending lattice results for $B \to D^*\ell\nu$ form factors and their impact in $R(D^*)$ calculations

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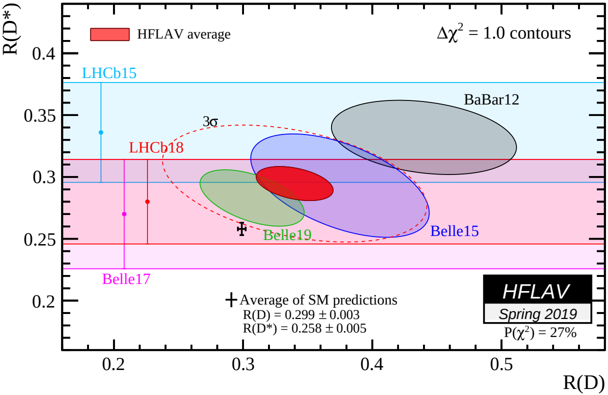
On behalf of the FNAL lattice/MILC collaboration

Status of $R(D)$ - $R(D^*)$

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current $\approx 3\sigma$ tension with the SM

The $B \to D^*\ell\nu$ decay and $R(D^*)$ on the lattice

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B} \to D^*\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \left[\underbrace{K_1(w, m_\ell)}_{\text{Known factors}}\underbrace{\left|\mathcal{F}(w)\right|^2}_{\text{Theory}} + \underbrace{K_2(w, m_\ell)}_{\text{Known factors}}\underbrace{\left|\mathcal{F}_2(w)\right|^2}_{\text{Theory}}\right] \times \left|V_{cb}\right|^2$$

- The amplitudes $\mathcal{F}$,$\mathcal{F}_2$ must be calculated in the theory
- Since $K_2(w, 0) = 0$, $\mathcal{F}_2$ only contributes significantly with the τ
- Knowing these amplitudes, one can extract $|V_{cb}|$ from experiment
  - It is possible to extract $R(D^*)$ without experimental data!

$$R(D^*) = \frac{\int_1^{w_{\mathrm{Max},\tau}} dw \left[K_1(w, m_\tau)\left|\mathcal{F}(w)\right|^2 + K_2(w, m_\tau)\left|\mathcal{F}_2(w)\right|^2\right] \times \cancel{\left|V_{cb}\right|^2}}{\int_1^{w_{\mathrm{Max}}} dw \left[K_1(w, 0)\left|\mathcal{F}(w)\right|^2\right] \times \cancel{\left|V_{cb}\right|^2}}$$

- $|V_{cb}|$ cancels out

The $B \to D^* \ell \nu$ decay and $R(D^*)$ on the lattice

- Form factors

$$\frac{\langle D^*(p_{D^*}, \epsilon^\nu) | \mathcal{V}^\mu \left| \bar{B}(p_B) \right\rangle}{2\sqrt{m_B\, m_{D^*}}} = \frac{1}{2} \epsilon^{\nu *} \varepsilon^{\mu\nu}_{\ \ \rho\sigma} v_B^\rho v_{D^*}^\sigma \boldsymbol{h_V}(w)$$

$$\frac{\langle D^*(p_{D^*}, \epsilon^\nu) | \mathcal{A}^\mu \left| \bar{B}(p_B) \right\rangle}{2\sqrt{m_B\, m_{D^*}}} =$$

$$\frac{i}{2} \epsilon^{\nu *} \left[g^{\mu\nu} (1 + w) \boldsymbol{h_{A_1}}(w) - v_B^\nu \left(v_B^\mu \boldsymbol{h_{A_2}}(w) + v_{D^*}^\mu \boldsymbol{h_{A_3}}(w) \right) \right]$$

- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The h_X enter in the definition of $\mathcal{F}$ and $\mathcal{F}_2$
- We can calculate $h_{A_{1,2,3},V}$ directly from the lattice

The $B \to D^*\ell\nu$ decay and $R(D^*)$ on the lattice

- Helicity amplitudes

$$H_\pm = \sqrt{m_B\, m_{D^*}}(w+1)\left(\boldsymbol{h_{A_1}}(w) \mp \sqrt{\frac{w-1}{w+1}}\boldsymbol{h_V}(w)\right)$$

$$H_0 = \sqrt{m_B\, m_{D^*}}(w+1)m_B\left[(w-r)\boldsymbol{h_{A_1}}(w) - (w-1)\left(r\,\boldsymbol{h_{A_2}}(w) + \boldsymbol{h_{A_3}}(w)\right)\right]/\sqrt{q^2}$$

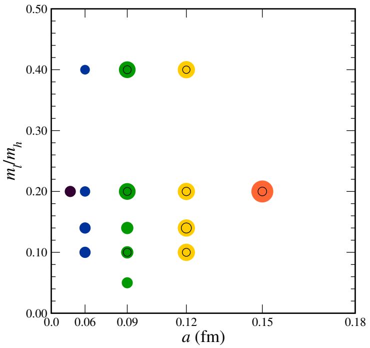
$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}}\left[(1+w)\boldsymbol{h_{A_1}}(w) + (wr-1)\boldsymbol{h_{A_2}}(w) + (r-w)\boldsymbol{h_{A_3}}(w)\right]$$

- Form factor in terms of the helicity amplitudes

$$|\mathcal{F}|^2 = C(w)\left(H_0^2(w) + H_+^2(w) + H_-^2(w)\right) \qquad |\mathcal{F}_2|^2 = C_2(w)H_S^2(w)$$

Available lattice data and simulations

- Using 15 $N_f = 2 + 1$ MILC ensembles of sea asqtad quarks
- The heavy quarks are treated using the Fermilab action



Calculated ratios

$$\frac{\langle D^*(p)|\,\mathbf{V}\,|D^*(0)\rangle}{\langle D^*(p)|\,V_4\,|D^*(0)\rangle} \to x_f, \qquad w = \frac{1+x_f^2}{1-x_f^2}$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle \left\langle \bar{B}(0)\right|\mathbf{A}\left|D^*(p_\perp,\varepsilon_\parallel)\right\rangle}{\langle D^*(0)|\,V_4\,|D^*(0)\rangle \left\langle \bar{B}(0)\right|V_4\left|\bar{B}(0)\right\rangle}^{*} \to R_{A_1}^2, \qquad h_{A_1} = \left(1-x_f^2\right)R_{A_1}$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\perp)\right|\mathbf{V}\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to X_V, \qquad h_V = \frac{2}{\sqrt{w^2-1}}R_{A_1}X_V$$

$$\frac{\left\langle D^*(p_\parallel,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to R_1, \qquad h_{A_3} = \frac{2}{w^2-1}R_{A_1}\left(w-R_1\right)$$

$$\frac{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|A_4\left|\bar{B}(0)\right\rangle}{\left\langle D^*(p_\perp,\varepsilon_\parallel)\right|\mathbf{A}\left|\bar{B}(0)\right\rangle} \to R_0,$$

$$h_{A_2} = \frac{2}{w^2-1}R_{A_1}\left(wR_1-\sqrt{w^2-1}R_0-1\right)$$

* Phys.Rev. **D**66, 01503 (2002)

Lattice challenges: Heavy quarks

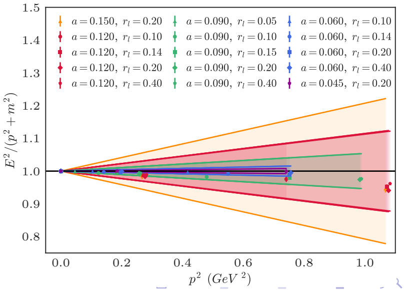
Heavy quark (HQ) treatment in Lattice QCD

- For light quarks ($m_l \lesssim \Lambda_{QCD}$), leading discretization errors $\sim \alpha_s^k (a\Lambda_{QCD})^n$
- For heavy quarks ($m_Q > \Lambda_{QCD}$), discretization errors grow as $\sim \alpha_s^k (am_Q)^n$
  - In this work $am_c \sim 0.15 - 0.6$ and $am_b > 1$

Need special actions and ETs to describe the charm/bottom quark

$$a^2 E^2(p_\mu) = (am_1)^2 + \frac{m_1}{m_2}(\mathbf{p}a)^2 + \frac{1}{4}\left[\frac{1}{(am_2)^2} - \frac{am_1}{(am_4)^3}\right](a^2\mathbf{p}^2)^2 - \frac{am_1 w_4}{3}\sum_{i=1}^{3}(ap_i)^4 + O(p_i^6)$$

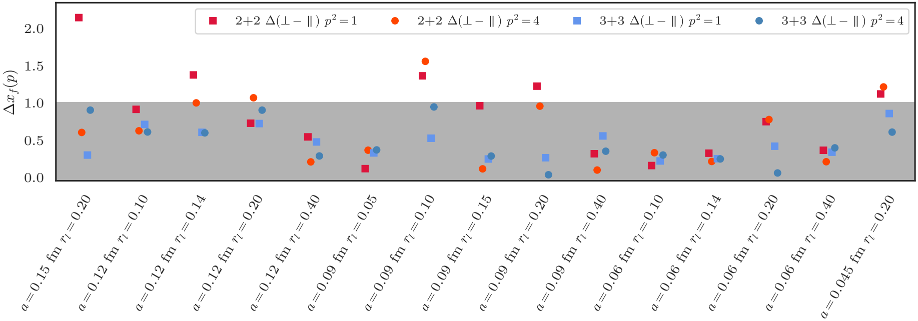
- Relativistic HQ action with tree-level matching, discretization errors $O(\alpha\, m)$
- Breaks the dispersion relation
- Ok to have discretization errors if under control
- Can't check B ($p = 0$), ok in the past



Lattice challenges: Excited states

- Differences in x_f computed using different polarizations inform us about the systematics of the excited states
  - The extra excited states are necessary to handle those systematics
  - Increases the error with respect to our previous analysis at $w = 1$

$$R(t,T) = R_0\left(1 + A_0 e^{-E_0 t} + A_1 e^{-E_1 t} + B_0 e^{-M_0(T-t)} + B_1 e^{-M_1(T-t)} + \cancel{C_0 e^{-(E_0-M_0)t}}\right)$$



Analysis: Chiral-continuum fits

- Our data represents the form factors at non-zero a and unphysical m_π
- Extrapolation to the physical pion mass described by EFTs
  - The EFT describe the a and the m_π dependence
- Functional form explicitly known

$$h_{A_1}(w) = \underbrace{\Bigg[1 + \frac{X_{A_1}(\Lambda_\chi)}{m_c^2} + \frac{g^2_{D^*D\pi}}{48\pi^2 f_\pi^2 r_1^2}\mathrm{logs}_{\mathrm{SU3}}(a, m_l, m_s, \Lambda_{QCD})}_{\mathrm{NLO}\ \chi\mathrm{PT} + \mathrm{HQET}}$$

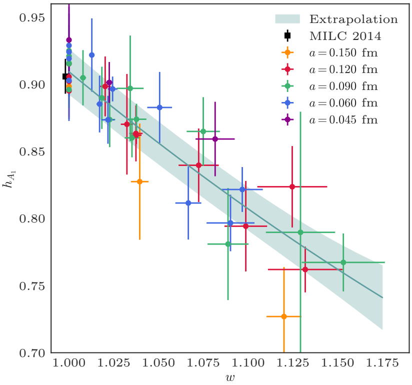
$$\underbrace{+c_1 x_l + c_{a1} x_{a^2}}_{\mathrm{NLO}\ \chi\mathrm{PT}} \underbrace{-\rho^2_{A_1}(w-1) + k_{A_1}(w-1)^2}_{w\ \mathrm{dependence}} \underbrace{+ c_2 x_l^2 + c_{a2} x_{a^2}^2 + c_{a,m} x_l x_{a^2}\Bigg]}_{\mathrm{NNLO}\ \chi\mathrm{PT}} \times$$

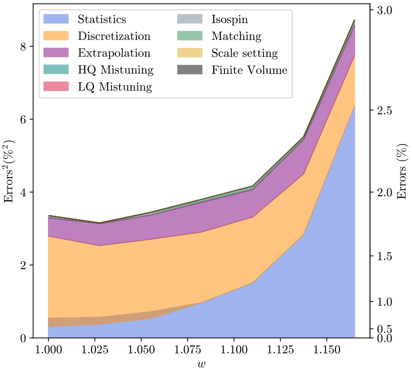
$$\underbrace{\left(1 + \beta_{11}^{A_1}\alpha_s a \Lambda_{\mathrm{QCD}} + \cancel{\beta_{02}^{A_1} a^2 \Lambda_{\mathrm{QCD}}^2} + \beta_{03}^{A_1} a^3 \Lambda_{\mathrm{QCD}}^3\right)}_{\mathrm{HQ\ discretization\ errors}}$$

with

$$x_l = B_0 \frac{m_l}{(2\pi f_\pi)^2}, \qquad x_{a^2} = \left(\frac{a}{4\pi f_\pi r_1^2}\right)^2$$

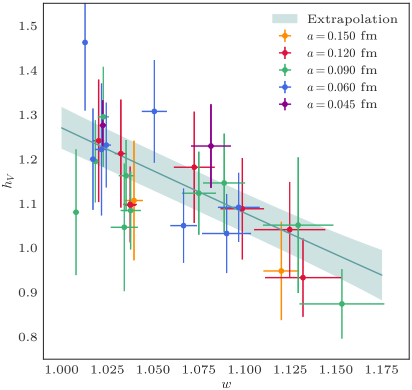
Chiral-continuum fits

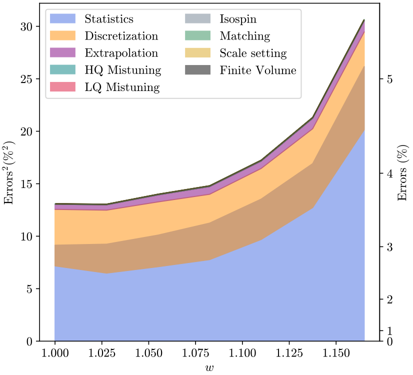




- Preliminary results, combined fit $p - \text{value} = 0.96$
- $h_{A_1}(1) = 0.909(17)$

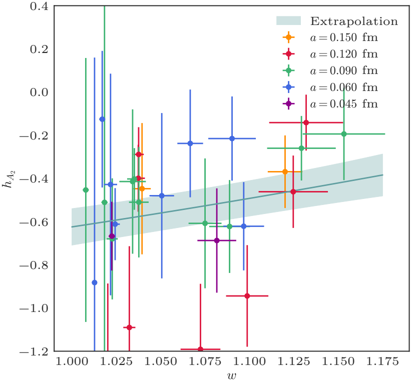
Chiral-continuum fits

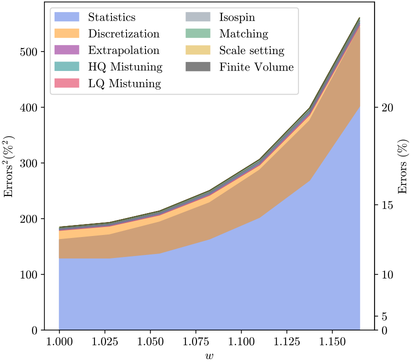




- Preliminary results, combined fit $p -$ value $= 0.96$
- $h_V(1) = 1.270(46)$

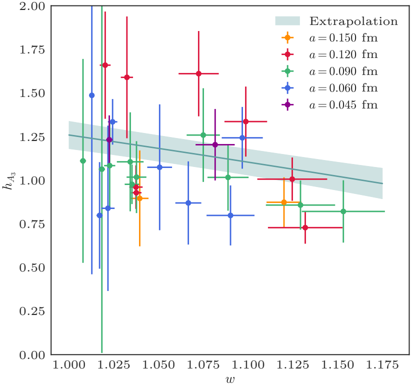
Chiral-continuum fits

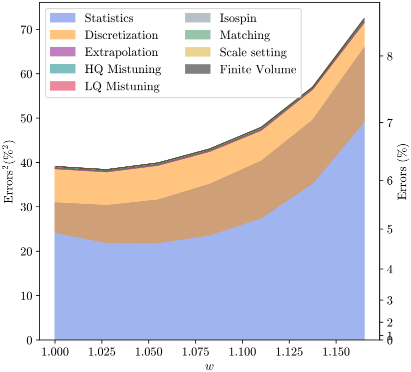




- Preliminary results, combined fit $p - \text{value} = 0.96$
- $h_{A_2}(1) = -0.624(85)$

Chiral-continuum fits





- Preliminary results, combined fit $p - \text{value} = 0.96$
- $h_{A_3}(1) = 1.259(79)$

BGL z-Expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B769**, 441 (2017), Phys.Lett. **B771**, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z) B_g(z)} \sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w) h_{A_1}(w) = \frac{1}{\phi_f(z) B_f(z)} \sum_j b_j z^j$$

$$\mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z) B_{\mathcal{F}_1}(z)} \sum_j c_j z^j$$

$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z) B_{\mathcal{F}_2}(z)} \sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0)$
- Constraint $(1+w) m_B^2 (1-r) \mathcal{F}_1(z = z_{\mathrm{Max}}) = (1+r)\mathcal{F}_2(z = z_{\mathrm{Max}})$
- BGL (weak) unitarity constraints

$$\sum_j a_j^2 \le 1, \qquad \sum_j b_j^2 + c_j^2 \le 1, \qquad \sum_j d_j^2 \le 1$$

Treatment of constraints and number of coefficients

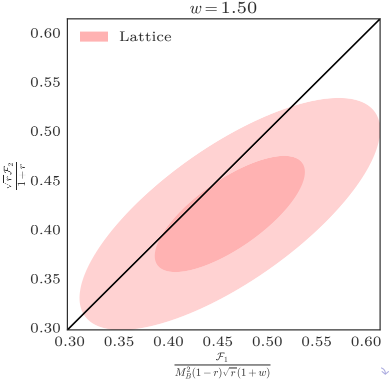
Constraints

- The constraint at zero recoil is used to remove a coefficient of the BGL expansion
- Neither the constraint at maximum recoil nor the unitarity constraints are imposed

How many coefficients in the BGL z-expansion?

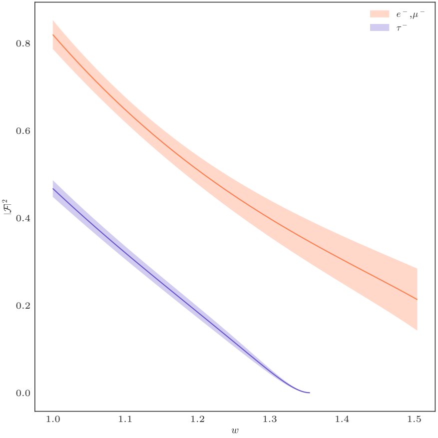
Phys.Rev. **D100 (2019), 013005**

- Add coefficients until
  - We exhaust the degrees of freedom
  - The error is saturated
- Compared linear/quadratic/cubic fits
  - Agreement in the low order coefficients
  - Quadratic saturates error, cubic no new information

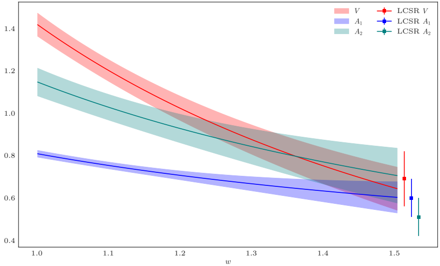


Decay amplitude and form factors

Lattice prediction for the decay amplitude



Comparison with LCSR



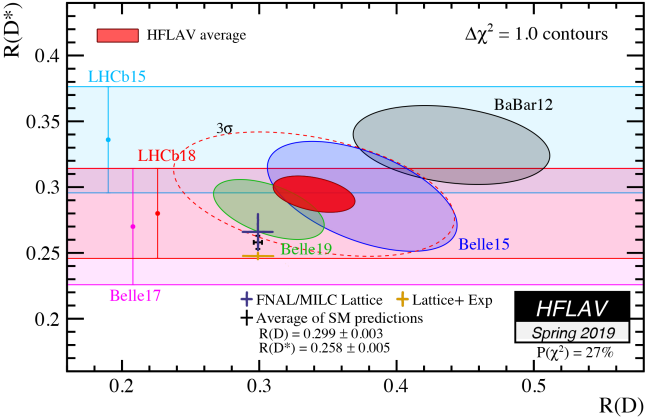
JHEP 01 (2019) 150

- Combined fit $p - \text{value} = 0.89$
- Good agreement for A_1, V
- Reasonable agreement for A_2

$R(D^*)$ results in context

No constraint $w_{\mathbf{Max}}$**:** $R(D^*)_{\mathbf{Lat}} = 0.266(14)$ $R(D^*)_{\mathbf{Lat+Exp}} = 0.2484(13)$
W/ constraint $w_{\mathbf{Max}}$**:** $R(D^*)_{\mathbf{Lat}} = 0.274(10)$ $R(D^*)_{\mathbf{Lat+Exp}} = 0.2492(12)$

Phys.Rev.D 100 (2019), 052007; Phys.Rev.D 103 (2021), 079901; Phys.Rev.Lett. 123 (2019), 091801



Conclusions

- Our $R(D^*)$ value confirms previous theoretical determinations
- The tension with the experimental average is reduced
  - Newest experimental determinations show values closer to the theoretical determination
- Further lattice analysis and refinements of our analysis can potentially settle the $R(D^*)$ issue
  - Pending JLQCD calculation on $B \to D^* \ell\nu$ form factor on the lattice
  - Next FNAL/MILC calculation of $B \to D^{(*)}\ell\nu$ is in the queue
- Our next calculation will allow us to confirm this results and have a better handle on the systematic errors

BACKUP SLIDES

Comparison with an improved CLN

- CLN is much more constraining than BGL, using only 4 fit parameters
- We can relax the constraints by allowing errors in the coefficients
  - We take into account the full correlation between ρ^2, c_{A_1} and d_{A_1}

Nucl.Phys. **B530 (1998), 153**

- Update HQET relations between the form factors

JHEP 11 (2017) 061

$$h_{A_1}(w) = h_{A_1}(1)\left[1 - 8\rho^2 z + \left(64c_{A_1} - 16\rho^2\right)z^2 + \left(512d_{A_1} + 256c_{A_1} - 6\rho^2\right)z^3\right]$$

$$R_0^{\mathrm{CLN}}(w) = 1.25(35) - 0.183(77)\,(w-1) + 0.063(23)\,(w-1)^2$$

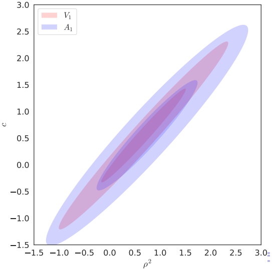
$$R_1^{\mathrm{CLN}}(w) = 1.28(36) - 0.101(51)\,(w-1) + 0.066(24)\,(w-1)^2$$

$$R_2^{\mathrm{CLN}}(w) = 0.744(44) + 0.128(38)\,(w-1) - 0.079(19)\,(w-1)^2$$

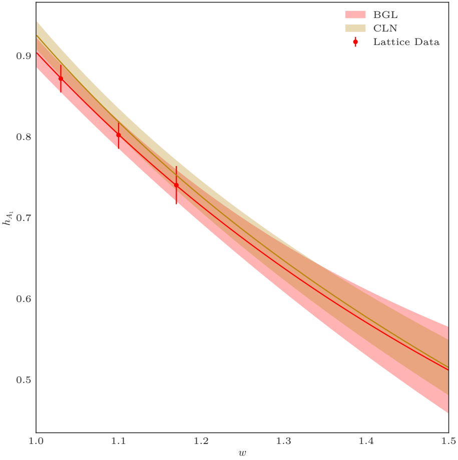
$$R_0^{\mathrm{CLN}}(w) = \frac{\sqrt{r}\mathcal{F}_2(w)}{(1+r)h_{A_1}(w)}$$

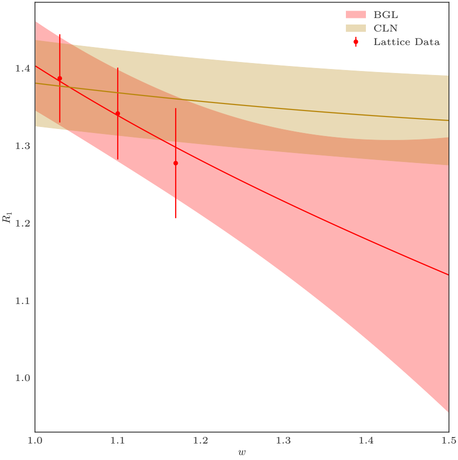
$$R_1^{\mathrm{CLN}}(w) = \frac{h_V(w)}{h_{A_1}(w)}$$

$$R_2^{\mathrm{CLN}}(w) = \frac{\frac{m_{D^*}}{m_B}h_{A_2}(w) + h_{A_3}(w)}{h_{A_1}(w)}$$



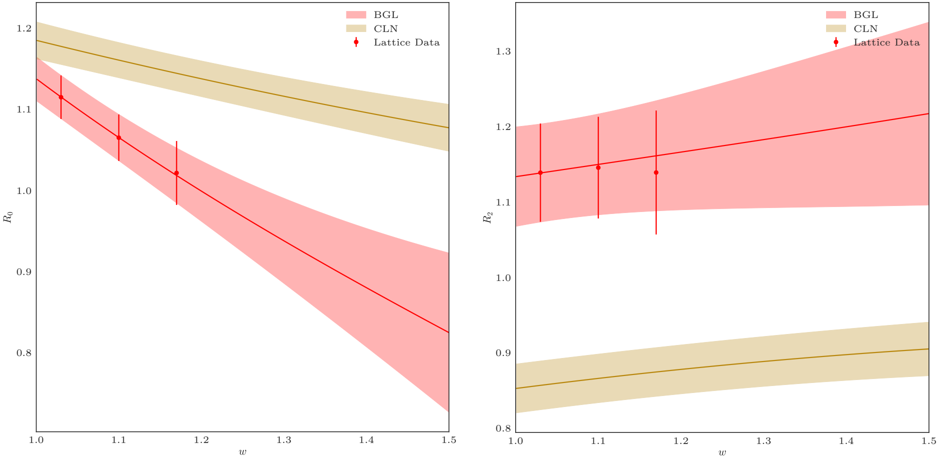
Comparison with an improved CLN





- Lattice only $p-\text{value} =\sim O(10^{-5})$
- Predictions for h_{A_1} and R_1^{CLN} look fine

Comparison with an improved CLN



- Lattice only $p-\text{value} =\sim O(10^{-5})$
- Predictions for R_0^{CLN} and R_2^{CLN} show tensions