Non-local matrix elements in $B_{(s)} \to \{K^{(*)}, \phi\}\ell^+\ell^-$ decays

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Based on
arXiv:2011.09813
in collaboration with
Danny van Dyk and Javier Virto

Beyond the Flavour Anomalies II
IPPP Durham
ONLINE, 21-Apr-2021

DFG Particle Physics Phenomenology after the Higgs Discovery
Collaborative Research Center TRR 257

UNIVERSITÄT SIEGEN

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Introduction

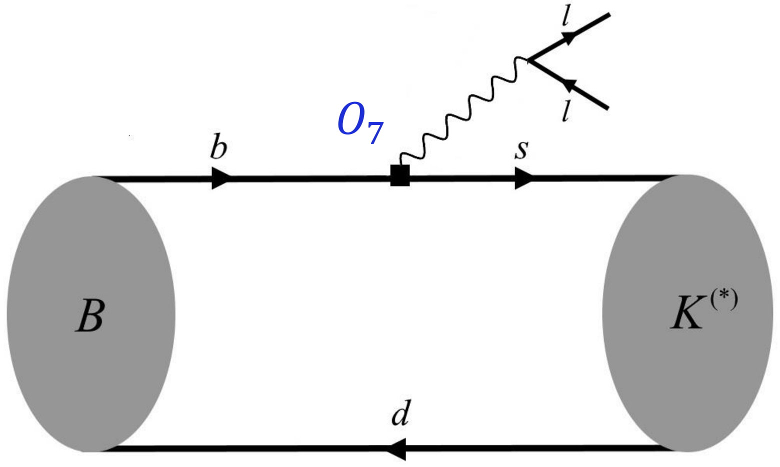
$b \to s\ell\ell$ effective Hamiltonian

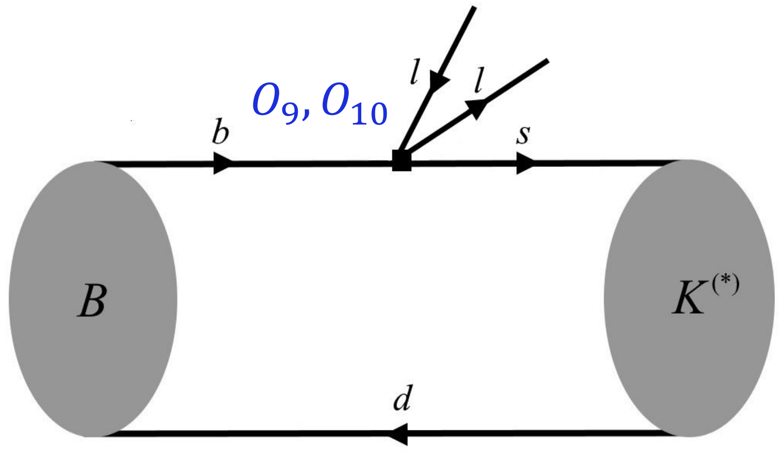
transitions described by the **effective Hamiltonian**

$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \qquad \mu = m_b$$

main contributions to $B_{(s)} \to \{K^{(*)}, \phi\}\ell\ell$ in the SM given by local operators O_7, O_9, O_{10}

$O_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$ $\qquad$ $O_9 = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \sum_l (\bar{\ell}\gamma_\mu \ell)$ $\qquad$ $O_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \sum_l (\bar{\ell}\gamma_\mu \gamma_5 \ell)$

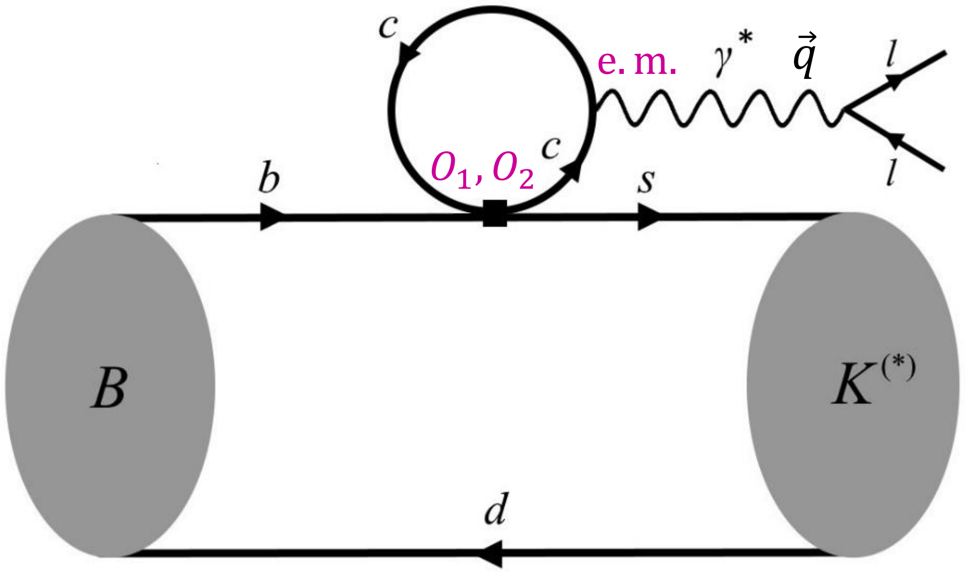




Charm loop in $B \to K^{(*)}\ell\ell$

additional non-local contributions come from O_1^c and O_2^c combined with the e.m. current (charm-loop contribution)

$$O_1^c = (\bar{s}_L \gamma^\mu c_L)(\bar{c}_L \gamma_\mu b_L) \qquad O_2^c = (\bar{s}_L^j \gamma^\mu c_L^i)(\bar{c}_L^i \gamma_\mu b_L^j)$$



Decay amplitude for $B \to K^{(*)}\ell\ell$ decays

calculate decay amplitudes precisely to probe the SM
(B-anomalies: NP or underestimated systematic uncertainties?)

$$\mathcal{A}(B \to K^{(*)}\ell\ell) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right) \boldsymbol{\mathcal{F}_\mu} - \frac{L_V^\mu}{q^2}\left(C_7\, \boldsymbol{\mathcal{F}_{T,\mu}} + \boldsymbol{\mathcal{H}_\mu}\right)\right]$$

local hadronic matrix elements

$$\mathcal{F}_\mu = \langle K^{(*)}(k) | O_{7,9,10} | B(k+q) \rangle$$

non-local hadronic matrix elements

$$\mathcal{H}_\mu = i \int d^4x\, e^{iq\cdot x} \langle K^{(*)}(k) | T\{ j_\mu^{\text{em}}(x), (C_1 O_1^c + C_2 O_2^c)(0)\} | B(k+q) \rangle$$

Form factors definitions

form factors (FFs) parametrize hadronic matrix elements

FFs are functions of the momentum transferred squared q^2

local FFs

$$\mathcal{F}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q)\, \mathcal{F}_\lambda(q^2)$$

computed with Lattice QCD (high q^2) and sum rules (low q^2) with good precision $\sim 10\%$

non-local FFs

$$\mathcal{H}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q) \mathcal{H}_\lambda(q^2)$$

calculated using an Operator Product Expansion (OPE)

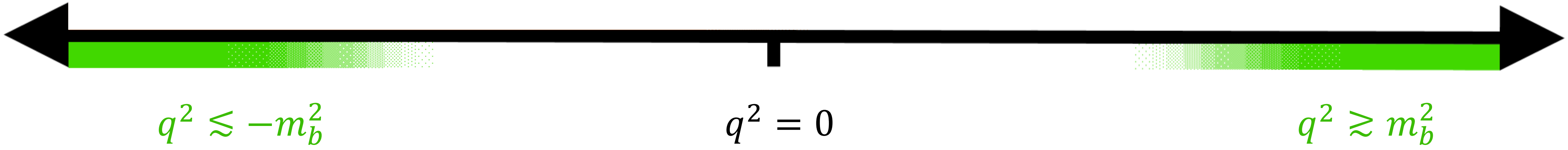
large uncertainties → reduce uncertainties for a better understanding of rare B decays

OPE for non-local matrix elements

two types of OPEs for the non-local FFs $\mathcal{H}_\lambda$

1. local OPE for $|q^2| \gtrsim m_b^2$ [Grinstein/Piryol 2004] [Beylich/Buchalla/Feldmann 2011]

$$\mathcal{H}_\lambda(q^2) = \mathcal{C}_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \mathcal{C}'_\lambda(q^2)\tilde{\mathcal{F}}_\lambda(q^2) + \cdots$$



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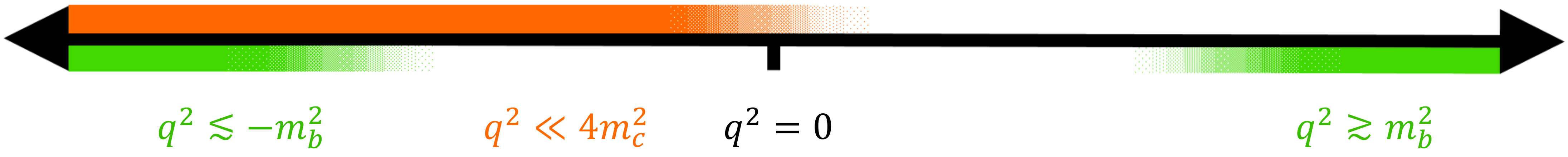
2. light-cone OPE for $q^2 \ll 4m_c^2$ [Khodjamirian/Mannel/Pivovarov/Wang 2010 (KMPW2010)]

$$\mathcal{H}_\lambda(q^2) = C_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{C}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

leading term coincides in the two OPEs



Our main results

1. calculation of the **next-to-leading term in the light-cone OPE** $\boldsymbol{\mathcal{V}_\lambda}$ using for the first time the full set B-meson distribution amplitudes in $B_{(s)} \to \{K^{(*)}, \phi\}\ell^+\ell^-$ and $B_{(s)} \to \{K^*, \phi\}\gamma$ decays

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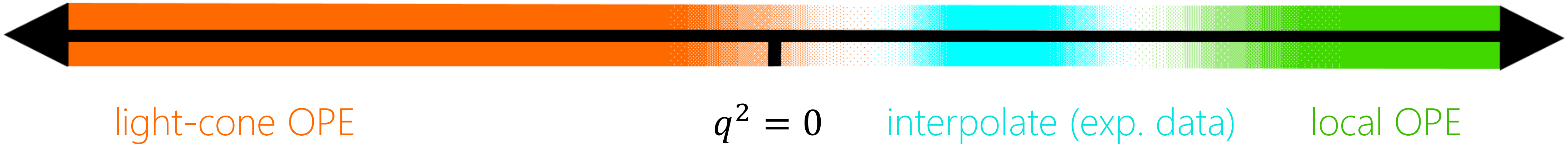
   we obtain results two orders of magnitude smaller than KMPW2010

2. **first model independent constraints on the non-local FFs** $\mathcal{H}_\lambda$ using dispersive bounds

   constrain the coefficients of a new parametrization for $\mathcal{H}_\lambda$ to interpolate in the physical region relevant for B decays and to compare with experimental measurements



Subleading corrections to $\mathcal{H}_\lambda$

Soft-gluon contribution to the charm loop

expand $\mathcal{H}_\lambda$ in a light-cone OPE for $q^2 \ll 4m_c^2$

$$\mathcal{H}_\lambda(q^2) = C_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{C}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

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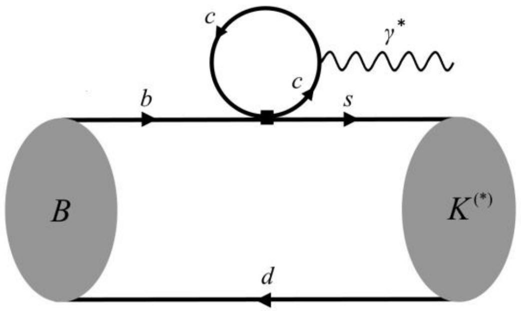
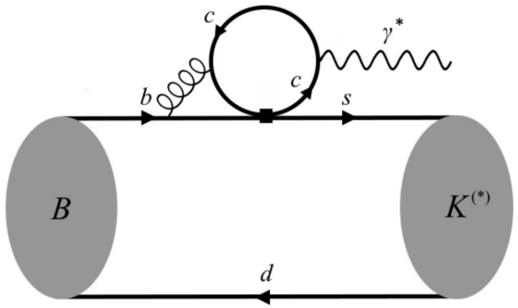
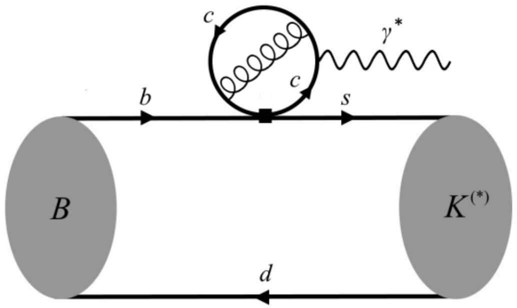
leading power (LO in α_s)



+ hard gluons (α_s) corrections





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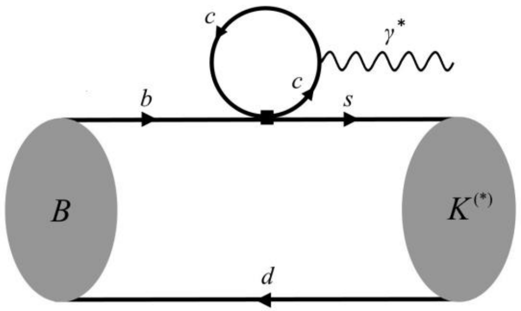

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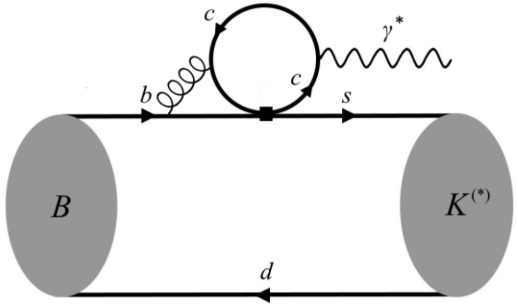
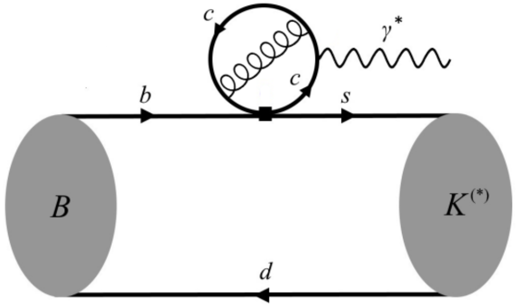
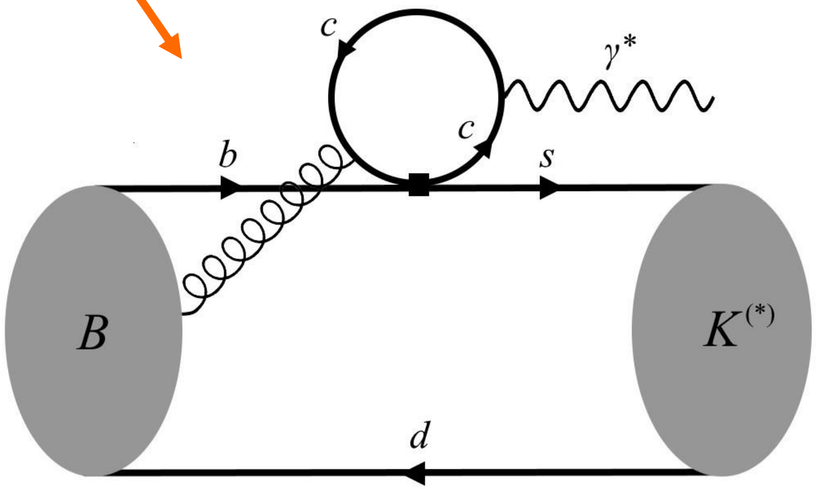




soft gluon correction

non-perturbative

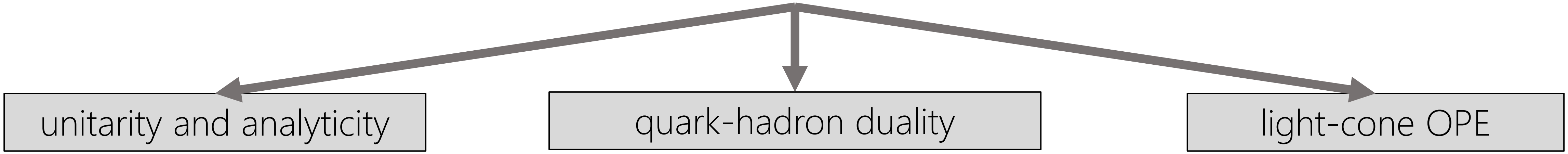
$\Longrightarrow$ not α_s suppressed



Light-cone sum rules in a nutshell

light-cone sum rules (LCSRs) are a method to calculate hadronic matrix elements

method based on:



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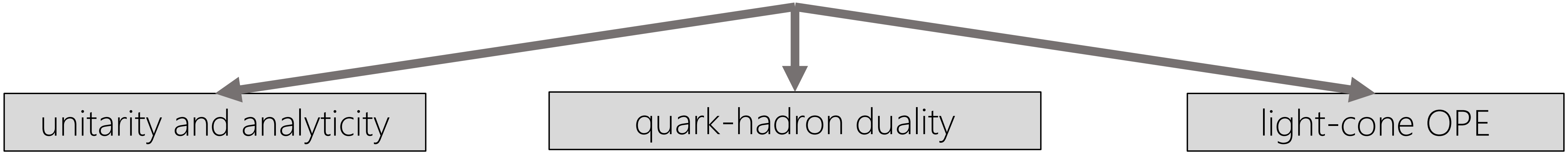

method already applied in KMPW2010 for the matrix elements $\mathcal{V}_\lambda(q^2)$ in $B \to K^{(*)}$
we also apply it to $B_s \to \phi$

we revisit previous calculations to include higher order corrections
in *B*-to-vacuum matrix elements

computation of these non-local matrix elements using Lattice QCD not possible yet

Light-cone distribution amplitudes

the B-meson **light-cone distribution amplitudes** (**LCDAs**) are needed to compute the LCSRs

no two-particle contribution

three-particle contribution — **4 independent LCDAs** in KMPW2010

$$\langle 0|\bar{d}(x)G_{\alpha\beta}(uy)h_v(0)|B(v)\rangle$$
$$= \frac{f_B m_B}{4}\mathrm{Tr}\left\{\gamma_5 P_+ \left[(v_\alpha\gamma_\beta - v_\beta\gamma_\alpha)(\boldsymbol{\Psi_A} - \boldsymbol{\Psi_V}) - i\sigma_{\alpha\beta}\boldsymbol{\Psi_V} - (y_\alpha v_\beta - y_\beta v_\alpha)\frac{\boldsymbol{X_A}}{v\cdot y} + (y_\alpha\gamma_\beta - y_\beta\gamma_\alpha)\frac{\boldsymbol{Y_A}}{v\cdot y}\right]\right\}(\boldsymbol{x}, \boldsymbol{uy})$$

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three-particle contribution — 8 **independent LCDAs** in Braun/Ji/Manashov 2017

$$\langle 0|\bar{d}(x)G_{\alpha\beta}(uy)h_v(0)|B(v)\rangle$$

$$= \frac{f_B m_B}{4}\mathrm{Tr}\Big\{\gamma_5 P_+\Big[(v_\alpha\gamma_\beta - v_\beta\gamma_\alpha)(\boldsymbol{\Psi_A} - \boldsymbol{\Psi_V}) - i\sigma_{\alpha\beta}\boldsymbol{\Psi_V} - (y_\alpha v_\beta - y_\beta v_\alpha)\frac{\boldsymbol{X_A}}{v\cdot y} + (y_\alpha\gamma_\beta - y_\beta\gamma_\alpha)\frac{\boldsymbol{W} + \boldsymbol{Y_A}}{v\cdot y}$$

$$- i\epsilon_{\alpha\beta\sigma\rho}y^\sigma v^\rho\gamma_5\frac{\boldsymbol{\widetilde{X}_A}}{v\cdot y} + i\epsilon_{\alpha\beta\sigma\rho}y^\sigma\gamma^\rho\gamma_5\frac{\boldsymbol{\widetilde{Y}_A}}{v\cdot y} - (y_\alpha v_\beta - y_\beta v_\alpha)y_\sigma\gamma^\sigma\frac{\boldsymbol{W}}{(v\cdot y)^2} + (y_\alpha\gamma_\beta - y_\beta\gamma_\alpha)y_\sigma\gamma^\sigma\frac{\boldsymbol{Z}}{(v\cdot y)^2}\Big]\Big\}(\boldsymbol{x}, \boldsymbol{uy})$$

organize LCDAs in a **twist expansion** (twist = dimension – spin) — $\Psi_A, \Psi_V, X_A, Y_A, \ldots$ have no definite

higher twists are power of $\Lambda_{\mathrm{had}}/m_B$ suppressed

we include contributions up to twist 4

Results and comparison

$\Delta C_9(q^2 = 1\ \text{GeV}^2)$		KMPW2010	GvDV2019
leading power (LO α_s)		0.27	0.27
$B \to K\ell\ell$	$\mathcal{V}_\mathcal{A}$	$-0.09^{+0.06}_{-0.07}$	$(1.9^{+0.6}_{-0.6}) \cdot 10^{-4}$
	$\mathcal{V}_1$	$0.6^{+0.7}_{-0.5}$	$(1.2^{+0.4}_{-0.4}) \cdot 10^{-3}$
$B \to K^*\ell\ell$	$\mathcal{V}_2$	$0.6^{+0.7}_{-0.5}$	$(2.1^{+0.7}_{-0.7}) \cdot 10^{-3}$
	$\mathcal{V}_3$	$1.0^{+1.6}_{-0.8}$	$(3.0^{+1.0}_{-1.0}) \cdot 10^{-3}$
$B_s \to \phi\ell\ell$	$\mathcal{V}_i$	—	see paper

- our results are **two orders of magnitude smaller** than in KMWP2010 ($\Longrightarrow$ **smaller unc.**)
- **we can reproduce the analytical results given in KMWP2010 and the differences are well understood**
- quick **convergence of the light-cone OPE**

Why such different results?

different inputs: LCDAs models depend on λ_H^2, λ_E^2 $\longrightarrow$

KMPW10:
$\lambda_H^2 = \lambda_E^2 = 0.31 \pm 0.15~\mathrm{GeV}^2$
$\Longrightarrow$ twist 3 does not contribute

we use $\lambda_E^2 = 0.03 \pm 0.02~\mathrm{GeV}^2$
$\lambda_H^2 = 0.06 \pm 0.03~\mathrm{GeV}^2$
$\Longrightarrow$ ~10 times smaller [Nishikawa/Tanaka 2014]

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three-particle LCDAs twist expansion $\longrightarrow$

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independent 3-particle LCDAs considered → KMPW10: 4 Lorentz structures

all 8 independent Lorentz structures
$\Longrightarrow$partial cancelation (new structures come with an opposite sign)

Dispersive bound for $\mathcal{H}_\lambda$

Dispersive bounds

parametrize $\mathcal{H}_\lambda$
(z or q^2 expansion)

→

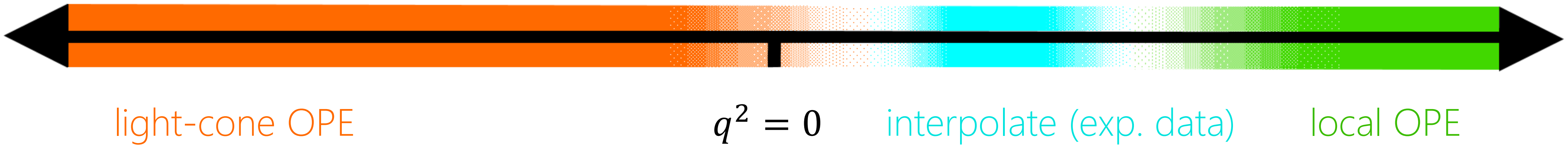
truncate the expansion
estimate truncation error

→

obtain $\mathcal{H}_\lambda$ in the
phenomenologically
relevant region

Dispersive bounds

parametrize $\mathcal{H}_\lambda$ (z or q^2 expansion) → truncate the expansion **estimate truncation error** → obtain $\mathcal{H}_\lambda$ in the phenomenologically relevant region



- **estimate truncation error using dispersive bounds**
- extend method already used for local form factors to non-local form factors $\mathcal{H}_\lambda$

  [BGL 1995] [CLN 1998]
- model independent constraints on $\mathcal{H}_\lambda$ → **control theoretical uncertainties**

Parametrizations for $\mathcal{H}_\lambda$

- q^2 parametrization [Ciuchini et al. 2015]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda(0) + \frac{q^2}{M_B^2}\mathcal{H}_\lambda{}'(0) + \frac{(q^2)^2}{M_B^4}\mathcal{H}_\lambda''(0) + \cdots$$

- dispersion relation [KMPW2010]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda(0) + \sum_{\psi=J/\psi,\psi(2S)} \frac{f_\psi \mathcal{A}_\psi}{M_\psi^2\left(M_\psi^2 - q^2\right)} + \int_{4M_D^2}^{\infty} dt \frac{\rho(t)}{t(t-q^2)}$$

- z expansion [Bobeth/Chrzaszcz/van Dyk/Virto 2017]

$$\mathcal{H}_\lambda(z) = \sum_{n=0}^{\infty} c_n z^n$$

- we propose a new parametrization (z polynomials)

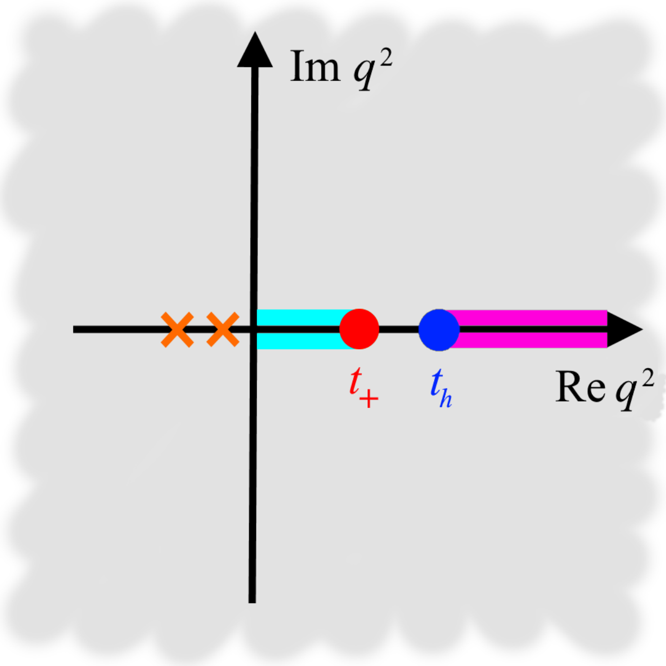
$$\hat{\mathcal{H}}_\lambda(z) = \sum_{n=0}^{\infty} a_n p_n(z)$$

The conformal map

define the z map

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+}}{\sqrt{t_+ - q^2} + \sqrt{t_+}}$$

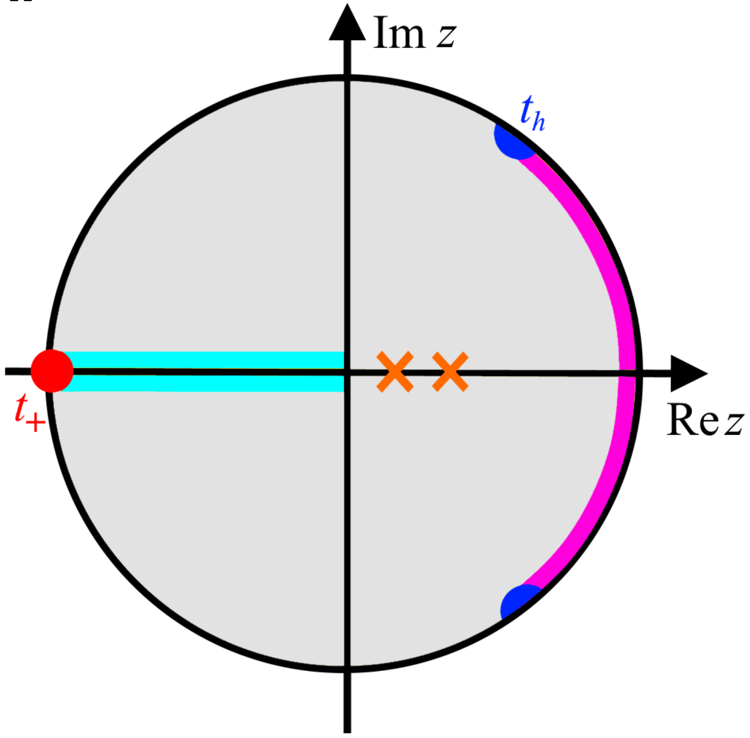
with $t_+ = 4M_D^2$ — note that $t_+ \neq t_h \equiv \left(M_B + M_{K^{(*)}}\right)^2$



q^2 plane
real axis $q^2 > t_h$

$\Longrightarrow$

z plane
arc of unit circle



The dispersive bound

expand $\hat{\mathcal{H}}_\lambda$ in orthogonal polynomials $p_n(z)$

$$\hat{\mathcal{H}}^{B\to K}(z) = \sum_{n=0}^{\infty} a_n^{B\to K}\, p_n^{B\to K}(z)$$

where

$$\hat{\mathcal{H}}^{B\to K}(z) = \mathcal{P}(z)\,\phi^{B\to K}(z)\ \mathcal{H}_\lambda^{B\to K}(z)$$

$$p_0^{B\to K}(z) = \frac{1}{\sqrt{2\alpha_{BK}}}$$

$$p_1^{B\to K}(z) = \left(z - \frac{\sin(\alpha_{BK})}{\alpha_{BK}}\right)\sqrt{\frac{\alpha_{BK}}{2\alpha_{BK}^2+\cos(2\alpha_{BK})-1}}$$

$$p_2^{B\to K}(z) = \left(z^2 + \frac{\sin(\alpha_{BK})(\sin(2\alpha_{BK})-2\alpha_{BK})}{2\alpha_{BK}^2+\cos(2\alpha_{BK})-1} z + \frac{2\,\mathrm{si}\right.$$

$$p_3^{B\to K}(z) = \cdots$$

now the dispersive bound reads

$$1 > 2\sum_{n=0}^{\infty}|a_n^{B\to K}|^2 + \sum_{\lambda}\left(2\sum_{n=0}^{\infty}\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \sum_{n=0}^{\infty}\left|a_{\lambda,n}^{B_s\to\phi}\right|^2\right)$$

the coefficients of the $\hat{\mathcal{H}}_\lambda$ are bounded!

Summary and conclusions

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1. calculation of the next-to-leading term in the light-cone OPE $\mathcal{V}_\lambda$ using for the first time the full set B-meson distribution amplitudes in $B_{(s)} \to \{K^{(*)}, \phi\}\ell^+\ell^-$ and $B_{(s)} \to \{K^*, \phi\}\gamma$ decays

   we obtain results two orders of magnitude smaller than KMPW2010

   results given in a Mathematica file attached to the arXiv version of the paper

   $B_s \to \phi$ local FFs using LCSRs with B-LCDAs at $q^2 = \{-15, -10, -5,\ 0, +5\}\ \mathrm{GeV}^2$

2. first model independent constraints on the non-local FFs $\mathcal{H}_\lambda$ using dispersive bounds

   constrain the coefficients of a new parametrization for $\mathcal{H}_\lambda$ to interpolate in the physical region relevant for B decays and to compare with experimental measurements

   estimate truncation error → control theoretical uncertainties

Thank you!

Dispersion relation

define the correlator

$$\Pi(k,q) = i\int \mathrm{d}^4x\, e^{ikx}\langle 0|T\{\mathcal{O}^\mu(x), \mathcal{O}_\mu(y)\}|0\rangle$$

where

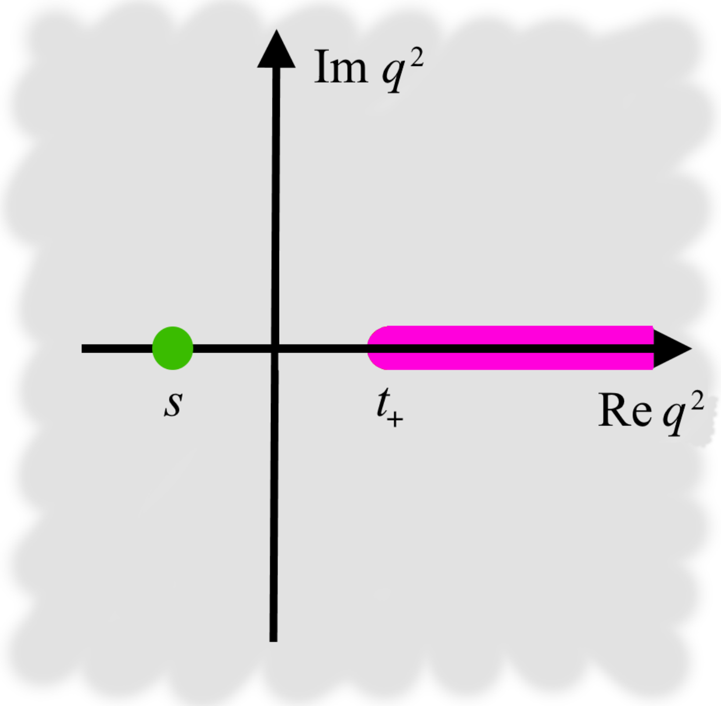
$$\mathcal{O}_\mu \propto \int d^4x\, e^{iq\cdot x}\, T\{j_\mu^{em}(x), (C_1 O_1 + C_2 O_2)(0)\}$$

use a subtracted dispersion relation

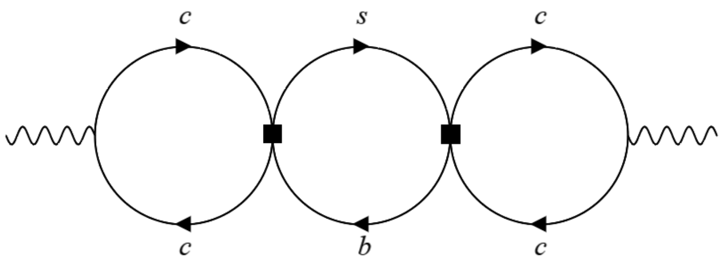
$$\chi(s) \equiv \frac{1}{2}\left(\frac{d}{ds}\right)^2 \Pi(s) \propto \int_{t_+}^{\infty} dq^2 \frac{\mathrm{Disc}_{bs}\Pi(q^2)}{(q^2-s)^3}$$

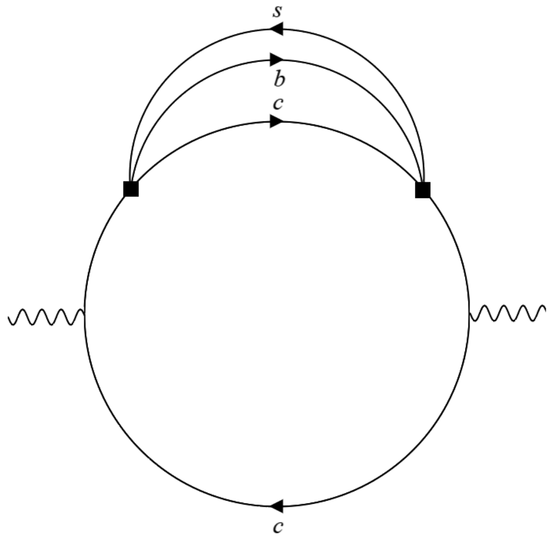
$t_+ = 4M_D^2$ first branch point

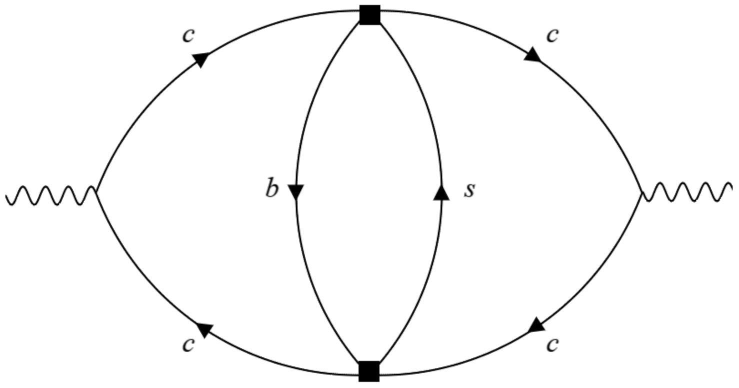
calculate $\chi(s)$ perturbatively and $\mathrm{Disc}_{bs}\Pi(q^2)$ using unitarity



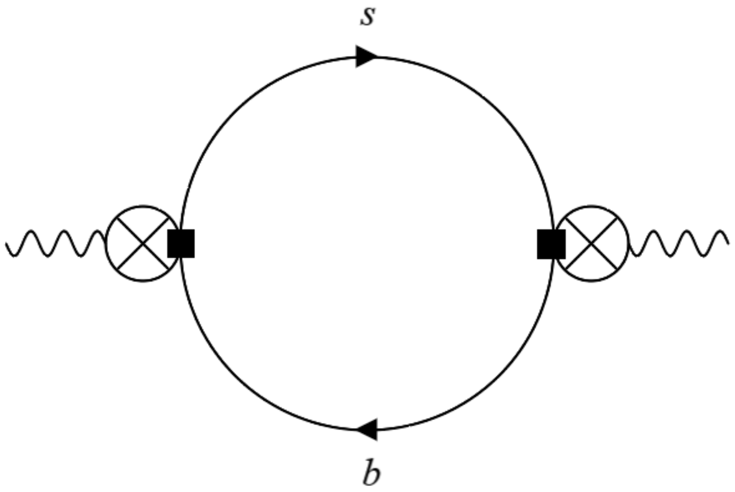
OPE calculation of $\chi^{\mathbf{OPE}}$







calculate χ^{OPE} using the local OPE for $|q^2| \gtrsim m_b^2$ (including α_s corrections)



we obtain at $s = -m_b^2$

$$\chi^{\mathrm{OPE}}(-m_b^2) = (1.81 \pm 0.02) \cdot 10^{-4}\,\mathrm{GeV}^{-2}$$

Hadronic representation of $\mathbf{Disc}_{bs}\Pi$

starting from the correlator

$$\Pi(k,q) = i\int \mathrm{d}^4x\, e^{ikx}\langle 0|T\{\mathcal{O}^\mu(x),\mathcal{O}_\mu(0)\}|0\rangle$$

insert a complete set of states

$$\Pi(k,q) = i\int \mathrm{d}^4x\, e^{ikx}\langle 0|\mathcal{O}^\mu|H_bH_s\rangle\langle H_bH_s|\mathcal{O}_\mu|0\rangle + \cdots$$

using crossing symmetry

$$\langle H_bH_s|\mathcal{O}_\mu|0\rangle \propto \mathcal{H}_\mu^{H_b\to H_s}$$

for $H_bH_s = BK^{(*)} = B_s\phi$

$$\mathrm{Disc}_{bs}\Pi \propto w^{B\to K}\,|\mathcal{H}^{B\to K}|^2 + \sum_\lambda\left(w_\lambda^{B\to K^{(*)}}\left|\mathcal{H}_\lambda^{B\to K^*}\right|^2 + w_\lambda^{B_s\to\phi}\left|\mathcal{H}_\lambda^{B_s\to\phi}\right|^2\right) + \cdots$$

Dispersive bound

matching the OPE result onto the hadronic representations of $\mathbf{Disc}_{bs}\Pi(q^2)$

dispersive bound

$$\chi^{\text{OPE}}(s) > \int_{(M_B+M_K)^2}^{\infty} dq^2 \, \frac{w^{B\to K} \, |\mathcal{H}^{B\to K}|^2}{(q^2-s)^3} + B \to K^* \text{ and } B_s \to \phi \text{ contr.}$$

- **first dispersive bound** for $\mathcal{H}^{B\to K}$, $\mathcal{H}_\lambda^{B\to K^*}$, $\mathcal{H}_\lambda^{B_s\to \phi}$
- model independent constraint
- strengthen the bound adding additional contributions

Exploit the dispersive bound

$$\chi^{\mathrm{OPE}}(s) > \int_{(M_B+M_K)^2}^{\infty} dq^2 \frac{w^{B\to K}\,|\mathcal{H}^{B\to K}|^2}{(q^2-s)^3} + B \to K^* \text{ and } B_s \to \phi \text{ contr.}$$

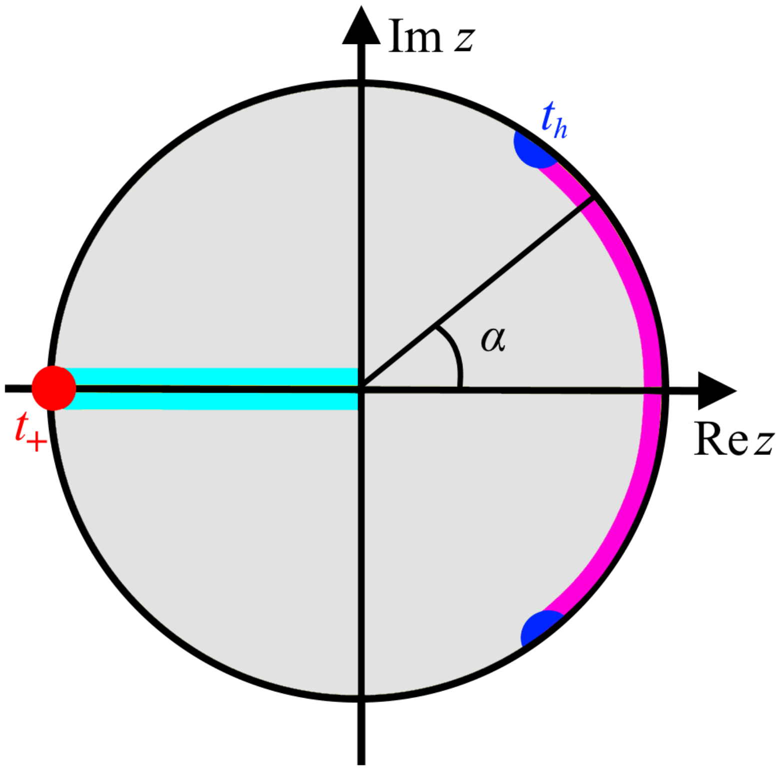
apply the z mapping

$$1 > \int_{-\alpha_{BK}}^{+\alpha_{BK}} d\alpha\, 2 \sum_\lambda \left|\hat{\mathcal{H}}^{B\to K}\right|^2 + B \to K^* \text{ and } B_s \to \phi \text{ contr.}$$

where

$$\hat{\mathcal{H}}^{B\to K}(z) = \mathcal{P}(z)\,\phi^{B\to K}(z)\;\mathcal{H}_\lambda^{B\to K}(z)$$

Blaschke factor $\mathcal{P}(z)$, outer function $\phi^{B\to K}(z)$



$\widehat{\mathcal{H}}_\lambda$ parametrization

$$1 > \int_{-\alpha_{BK}}^{+\alpha_{BK}} d\alpha\, 2\left|\widehat{\mathcal{H}}^{B\to K}\right|^2 + B \to K^* \text{and } B_s \to \phi \text{ contr.}$$

expand $\widehat{\mathcal{H}}_\lambda$ in orthogonal polynomials $p_n(z)$

$$\widehat{\mathcal{H}}(z) = \sum_{n=0}^{\infty} a_n\, p_n(z)$$

now the dispersive bound reads

$$1 > 2\sum_{n=0}^{\infty} |a_n^{B\to K}|^2 + \sum_\lambda \left(2\sum_{n=0}^{\infty} \left|a_{\lambda,n}^{B\to K^*}\right|^2 + \sum_{n=0}^{\infty} \left|a_{\lambda,n}^{B_s\to \phi}\right|^2 \right)$$

no bound for the z monomials
(coefficient of the Taylor expansion)

$$p_0^{B\to K}(z) = \frac{1}{\sqrt{2\alpha_{BK}}}$$

$$p_1^{B\to K}(z) = \left(z - \frac{\sin(\alpha_{BK})}{\alpha_{BK}}\right)\sqrt{\frac{\alpha_{BK}}{2\alpha_{BK}^2+\cos(2\alpha_{BK})-1}}$$

$$p_2^{B\to K}(z) = \left(z^2 + \frac{\sin(\alpha_{BK})(\sin(2\alpha_{BK})-2\alpha_{BK})}{2\alpha_{BK}^2+\cos(2\alpha_{BK})-1} z + \frac{2\,\mathrm{si}\ldots}{\ldots}\right.$$

$$p_3^{B\to K}(z) = \cdots$$

Application of the bound

estimate the truncation error in the series

$$\widehat{\mathcal{H}}^{B\to K}(z) = \sum_{n=0}^{1} a_n^{B\to K}\, p_n^{B\to K}(z)$$

i.e. the maximal allowed size of $\widehat{\mathcal{H}}^{B\to K}$

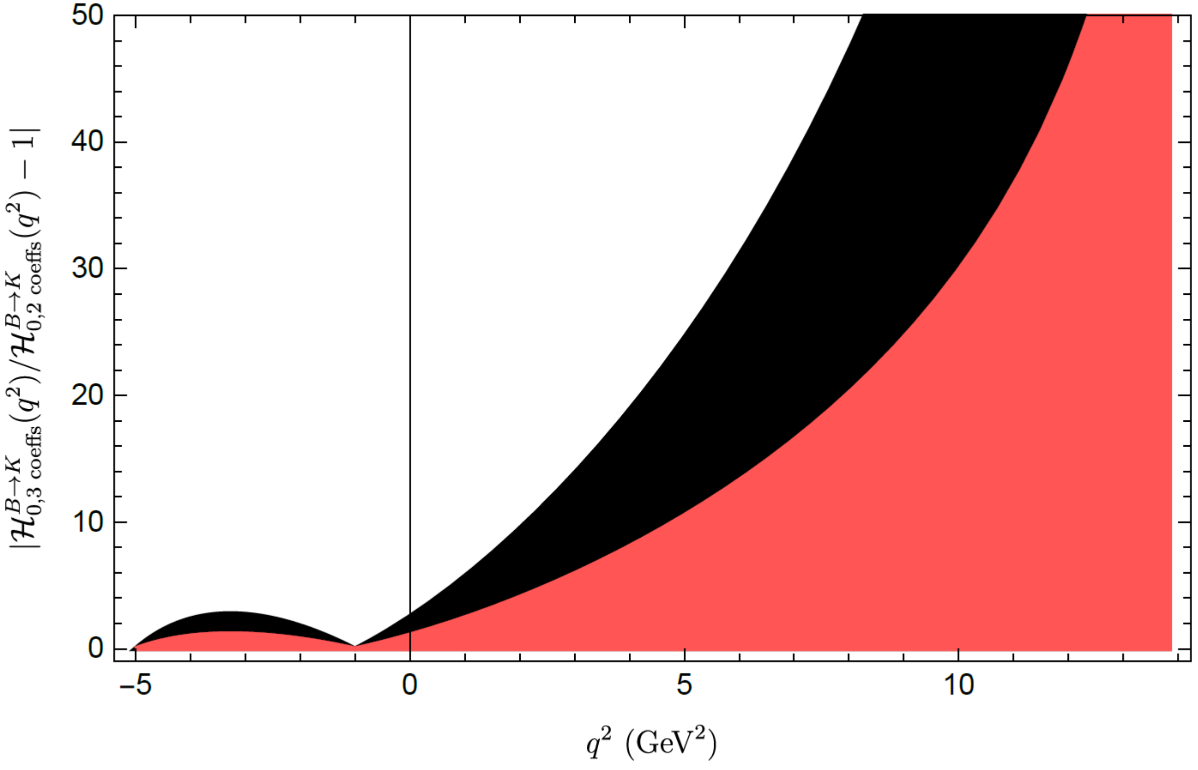
use two theory data points (-5 and $-1\ \mathrm{GeV}^2$)

use the bound

$$\frac{1}{2} > \sum_{n=0}^{\infty} |a_n^{B\to K}|^2$$

assume $B_{(s)} \to \{K^*, \phi\}$ contribute equally to the bound

$$\frac{1}{11} > \sum_{n=0}^{\infty} |a_n^{B\to K}|^2$$



Alignment of the gluon with the $K^{(*)}$ meson

We are interested in the dominant effect of the nonvanishing gluon momenta generated by the exponent in (3.9). Decomposing the covariant derivative in the light-cone vectors

$$\mathcal{D} = (n_+\mathcal{D})\frac{n_-}{2} + (n_-\mathcal{D})\frac{n_+}{2} + \mathcal{D}_\perp, \qquad (3.10)$$

we retain only the n_- component, which corresponds to the gluons emitted antiparallel to q, that is, in the same direction as the s-quark in the B-meson rest frame. We then have

$$\begin{aligned} G^{\alpha\beta}(ux) &\simeq \exp[-iu(n_-x)\frac{(in_+\mathcal{D})}{2}]G^{\alpha\beta} \\ &= \int d\omega \, \exp[-iu(n_-x)\omega] \, \delta[\omega - \frac{(in_+\mathcal{D})}{2}] \, G^{\alpha\beta} \, . \end{aligned} \qquad (3.11)$$

[...]

is represented in a compact unintegrated form, and we use the notation $\tilde{q} = q - u\omega n_-$, so that $\tilde{q}^2 \simeq q^2 - 2u\omega m_b$. Here we take into account that $\omega \ll m_b$, after the hadronic matrix element is taken. Note that the neglected components of $\mathcal{D}$ in (3.10) produce small, $O(\omega/m_b)$ corrections to $\tilde{q}^2$, hence our approximation is well justified.

[Khodjamirian et al. '10]

Where is the mistake?

$$\langle 0|\bar{d}(y)G_{\alpha\beta}(ux)n^{\beta}h_v(0)|B(v)\rangle$$
$$= \frac{f_B m_B}{4}\mathrm{Tr}\left\{\gamma_5 P_+ \left[(v_\alpha\gamma_\beta - v_\beta\gamma_\alpha)(\boldsymbol{\Psi_A} - \boldsymbol{\Psi_V}) - i\sigma_{\alpha\beta}\boldsymbol{\Psi_V} - (y_\alpha v_\beta - y_\beta v_\alpha)\frac{\boldsymbol{X_A}}{v\cdot y} + (y_\alpha\gamma_\beta - y_\beta\gamma_\alpha)\frac{\boldsymbol{Y_A}}{v\cdot y}\right] n^{\beta}\right\}(\boldsymbol{y}, \boldsymbol{ux})$$

[Kawamura et al. '01]

$$\not\Downarrow$$

$$\langle 0|\bar{d}(y)G_{\alpha\beta}(ux)h_v(0)|B(v)\rangle$$
$$= \frac{f_B m_B}{4}\mathrm{Tr}\left\{\gamma_5 P_+ \left[(v_\alpha\gamma_\beta - v_\beta\gamma_\alpha)(\boldsymbol{\Psi_A} - \boldsymbol{\Psi_V}) - i\sigma_{\alpha\beta}\boldsymbol{\Psi_V} - (y_\alpha v_\beta - y_\beta v_\alpha)\frac{\boldsymbol{X_A}}{v\cdot y} + (y_\alpha\gamma_\beta - y_\beta\gamma_\alpha)\frac{\boldsymbol{Y_A}}{v\cdot y}\right]\right\}(\boldsymbol{y}, \boldsymbol{ux})$$

[Khodjamirian et al. '06]

λ_{B_s} extimation

in the exponential model

$$\lambda_{B_q} = \frac{2}{3}\,\bar{\Lambda}_q$$

where

$$\bar{\Lambda}_q = m_{B_q} - m_q + O\left(\frac{1}{m_q}\right)$$

estimate the difference to cancel UV-divergent corrections in fixed-order perturbation theory

$$\lambda_{B_s} = \lambda_{B_d} + \frac{2}{3}(\bar{\Lambda}_s - \bar{\Lambda}_d) = 0.520 \pm 0.110\ \mathrm{GeV}$$

3-particle LCDAs twist basis

models given for LCDAs up to twist 4, twist 5 or higher give corrections of the order $1/m_b^2$

[Braun/Ji/Manashov '17]

$$\Psi_A = \frac{1}{2}(\Phi_3 + \Phi_4)$$

$$\Psi_V = \frac{1}{2}(-\Phi_3 + \Phi_4)$$

$$X_A = \frac{1}{2}(-\Phi_3 - \Phi_4 + 2\Psi_4)$$

$$Y_A = \frac{1}{2}(-\Phi_3 - \Phi_4 + \Psi_4 - \Psi_5)$$

$$\tilde{X}_A = \frac{1}{2}(-\Phi_3 + \Phi_4 - 2\tilde{\Psi}_4)$$

$$\tilde{Y}_A = \frac{1}{2}(-\Phi_3 + \Phi_4 - \tilde{\Psi}_4 + \tilde{\Psi}_5)$$

$$W = \frac{1}{2}(\Phi_4 - \Psi_4 - \tilde{\Psi}_4 + \Psi_5 + \tilde{\Psi}_5 + \tilde{\Phi}_5)$$

$$Z = \frac{1}{4}(-\Phi_3 + \Phi_4 - 2\tilde{\Psi}_4 + 2\tilde{\Psi}_5 + \tilde{\Phi}_5 + \Phi_6)$$

use to compute the sum rule

- all 8 independent Lorentz structures (four of them considered for the first time)
- results using LCDAs up to twist 4
- new models for the LCDAs

3-particle LCDAs models and $\lambda^2_{H,E}$

KMPW2010

$$\Psi_A(\omega_1,\omega_2) = \Psi_V(\omega_1,\omega_2) = \frac{\lambda_E^2}{6\lambda_B^4}\omega_2^2 e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

$$X_A = \frac{\lambda_E^2}{6\lambda_B^4}\omega_2(2\omega_1-\omega_2)e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

$$Y_A = -\frac{\lambda_E^2}{24\lambda_B^4}\omega_2(7\lambda_B-13\omega_1+3\omega_2)e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

Braun/Ji/Manashov

$$\Phi_3(\omega_1,\omega_2) = \frac{\lambda_E^2-\lambda_H^2}{6\lambda_B^4}\omega_1\omega_2^2 e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

$$\Phi_4(\omega_1,\omega_2) = \frac{\lambda_E^2+\lambda_H^2}{6\lambda_B^4}\omega_2^2 e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

$$\Psi_4(\omega_1,\omega_2) = \frac{\lambda_E^2}{3\lambda_B^4}\omega_1\omega_2 e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

$$\widetilde{\Psi}_4(\omega_1,\omega_2) = \frac{\lambda_H^2}{3\lambda_B^4}\omega_1\omega_2 e^{-\frac{\omega_1+\omega_2}{\lambda_B}}$$

$\lambda^2_{H,E}$ definition

$$\langle 0|\bar{d}(0)G_{\alpha\beta}(0)h_v(0)|B(v)\rangle = -\frac{i}{6}f_B\lambda_H^2\mathrm{Tr}\left[\gamma_5\Gamma P_+\sigma_{\alpha\beta}\right] - \frac{i}{6}f_B(\lambda_H^2-\lambda_E^2)\mathrm{Tr}\left[\gamma_5\Gamma P_+\left(v_\alpha\gamma_\beta - v_\beta\gamma_\alpha\right)\right]$$

Threshold s_0 determination

$$f_{K^{(*)}} e^{-\frac{m_{K^{(*)}}}{M^2}} \langle K^{(*)}(k) | \tilde{O}_\mu(0,x) | B(q+k) \rangle = f_B \int_0^{s_0} \mathrm{d}s \, e^{-\frac{s}{M^2}} \sum_{t=3,4} I_t(s,q^2)\, \Psi_t(y,ux) \qquad \mathbf{(1)}$$

derive with respect to $1/M^2$
and divide by $\mathbf{(1)}$

$$m_{K^{(*)}} = \frac{\int_0^{s_0} \mathrm{d}s \, s \, e^{-\frac{s}{M^2}} \sum_{t=3,4} I_t(s,q^2)\, \Psi_t(y,ux)}{\int_0^{s_0} \mathrm{d}s \, e^{-\frac{s}{M^2}} \sum_{t=3,4} I_t(s,q^2)\, \Psi_t(y,ux)}$$

daughter sum rule to extract $\boldsymbol{s_0}$